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Thermodynamic effects of Bardeen black hole surrounded by perfect fluid dark matter under general uncertainty principle

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The thermodynamics of Bardeen black hole surrounded by perfect fluid dark matter is investigated. We calculate the analytical expresses of corresponding thermodynamic variables, e.g., the Hawking temperature, entropy of the black hole. In addition, we derive the heat capacity to analyze the thermal stability of the black hole. We also compute the rate of emission in terms of photons through tunneling. By numerical method, an obvious phase transition behavior is found. Furthermore, according to the general uncertainty principle, we study the quantum corrections to these thermodynamic quantities and obtain the quantum-corrected entropy containing the logarithmic term. Lastly, we investigate the effects of the magnetic charge g, the dark matter parameter k and the generalized uncertainty principle parameter α on the thermodynamics of Bardeen black hole surrounded by perfect fluid dark matter under general uncertainty principle.

Keywords: general uncertainty principle, quantum-corrected entropy, phase transition

PACS: 04.60.Bc, 04.70.Dy **DOI:** 10.1088/1674-1056/ac3ca8

1. Introduction

Hawking used semiclassical arguments to show that black holes must radiate thermally.[1] It is no doubt that this discovery is another milestone in the development of physics. The thermal nature of this radiation causes the initial pure state to be mapped to the final mixed state in a non-unitary process, which leads to the famous information loss paradox. Hawking temperature can be derived in different ways,[2] the method of quantum tunneling is the most interesting one. In addition, there have been null geodesic approach[3–8] and Hamilton–Jacobi approach,[9–11] etc. Many studies have shown that Hawking temperature has nothing to do with particular coordinate transformation or nature of the particles emitted from black holes.

When the gravitation is combined with quantum theory, the ordinary Heisenberg uncertainty principle could be extended to the generalized uncertainty principle (GUP) accompanying minimum measurable length.[12–19] The tunneling method can be used to compute the quantum-corrected Hawking temperature and the corresponding entropy, in combination with the introduction of generalized uncertainty principle. In Refs. [20–22], Wang *et al.* proposed the approach to understanding the quantum aspects of the black hole entropy by studying the entropy relations of multi-horizons black holes in higher dimensional (A)dS spacetime with maximal symmetries. Majumder[23] and Shen *et al.*[24] studied the quantum-corrected Hawking temperature and entropy of a Schwarzschild black hole under GUP. The quantum correction to the Bekenstein–Hawking entropy is logarithmic, and Romesh *et al.*[25] were interested only in the leading correction to the semiclassical entropy ensued in the limit of arbitrarily large A_H (A_H is the eigenvalue of the area operator). In recent years, the GUP has been applied to several kinds of black holes, such as scale-dependent Hayward black hole,[26] Kehagias–Sfetsos black hole,[27] Schwarzschild–Tangherlini black hole,[28] and sub-Planckian black hole,[29] etc.

One of mysterious properties of black holes is that they have a singularity at the origin of the spacetime.[30] At the spacetime singularity, the curvatures and densities become infinite, and the physical laws are no longer predictive.[30,31] It is generally believed that the spacetime singularities reflect the incompleteness of general relativity and can be resolved with construction of quantum gravity theory. Surprisingly, Bardeen in 1968 obtained a black hole solution without a singularity in a specific form of nonlinear electrodynamics.[32] Later, more regular (non-singular) black holes were proposed, such as Hayward black hole,[33] Ayón–Beato and García black hole,[34] and Berej–Matyjasek–Trynieki–Wornowicz black hole,[35] etc.

The existence of dark energy and dark matter in the universe has been demonstrated in Ref. [36]. According to the standard model of cosmology,[36] our current universe contains mainly of 68.3% dark energy, 26.8% dark matter and 4.9% baryon matter. Therefore, it is important to study the black hole surrounded by dark matter or dark energy. In recent years, the black hole surrounded by quintessence dark energy has attracted great interest. For example, Kiselev[37] studied the Schwarzschild black hole surrounded by the quintessential energy. Then, Toshmatov and Stuchlík[38] extended it to the

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Kerr-like black hole. In Ref. [39], Abdujabbarov *et al.* studied the Hayward black hole surrounded by quintessence. In addition, Kiselev proposed perfect fluid dark matter (PFDM) as a dark matter candidate.[37,40] In this work, we devote to effect of addition of GUP on thermodynamics of the Bardeen black hole surrounded by PFDM.

This paper is organized as follows. In Section 2, we briefly review the basic equations of the black hole thermodynamics. In Section 3, we study the thermodynamic properties of Bardeen black hole in PFDM, we calculate the mass of the black hole, Hawking temperature, surface gravity, the heat capacity at event horizon, and rate of emission of energy through event horizon, etc. We numerically analyze their properties for the Bardeen black hole in PFDM. In Section 4, we reanalyze the thermodynamics of the Bardeen black hole in PFDM under general uncertainty principle. Finally, we discuss and summarize our results in Section 5. Throughout this paper, we set $G=\hbar=c=\kappa_{\rm B}=1$.

2. Basic equations

2.1. Derivation of black hole temperature

At present, there are three main methods in theoretical physics to calculate the Hawking temperature. The first method is quantum tunneling approach. We consider the metric of a BH in four dimensions near the horizon as $\mathrm{d}s^2=-f(r)\mathrm{d}t^2+\frac{\mathrm{d}r^2}{f(r)}+r^2\mathrm{d}\Omega^2$. According to the quantum tunneling effect, particles inside the black hole can pass through the event horizon. In this process, the tunneling probability can be calculated. For example, Hawking radiation of scalar particles is studied by quantum tunneling method.[6,41] In comparison with the Boltzmann factor, the Hawking temperature of the black hole can be calculated by

$$T=\frac{f'(r_{\rm h})}{4\pi}, \tag{1}$$

where $r_{\rm h}$ is the radius of event horizon.

The second method is Hawking's approach.[42] Using the Killing vector, a conserved quantity can be constructed as follows:

$$2\kappa\xi^{\nu}=-\nabla^{\nu}(\xi^{\mu}\xi_{\mu}), \tag{2}$$

where ∇^{ν} is a covariant derivative, κ is surface gravity, and κ is constant along the track ξ, so

$$\ell_{\xi}\kappa=0. \tag{3}$$

The surface gravity κ of a black hole can also be defined as the magnitude of the gradient of the norm of horizon generating Killing field $\chi^a=\zeta^a+\Omega\Psi^a$, evaluated at the horizon. Supposing that the event horizon of a black hole is a Killing horizon, we have

$$\kappa^2:=-(\nabla^a|\Psi|)(\nabla_a|\Psi|). \tag{4}$$

Based on the classical black hole thermodynamics,[1,43–45] the Hawking temperature is given as follows:

$$T=\frac{\kappa}{2\pi}. \tag{5}$$

The third method is thermodynamic approach.[46] The first law of thermodynamics applies to stationary black hole solutions to the Einstein field equation as

$$\mathrm{d}M=\kappa\frac{\mathrm{d}A}{8\pi G}+\Omega\mathrm{d}J+\phi\mathrm{d}Q. \tag{6}$$

The black hole area at event horizon is computed by

$$A=\int_0^{2\pi}\int_0^{\pi}\sqrt{g_{\theta\theta}(r_{\rm h})g_{\phi\phi}(r_{\rm h})}\mathrm{d}\theta\mathrm{d}\phi=4\pi r_{\rm h}^2, \tag{7}$$

and the entropy of a black hole is expressed as

$$S=\frac{A}{4}=\pi r_{\rm h}^2. \tag{8}$$

Combining with

$$\mathrm{d}H=\mathrm{d}M=T\,\mathrm{d}S+V\,\mathrm{d}P, \tag{9}$$

where H is the total gravitational enthalpy, M is the mass of the black hole, and using Eq. (9), we can obtain the black hole temperature as follows:

$$T=\left(\frac{\partial H}{\partial S}\right)_P=\left(\frac{\partial M}{\partial S}\right)_P. \tag{10}$$

From the above three different methods of derivation of temperature, it can be seen that the expressions of temperature are the same.

2.2. Heat capacity

The heat capacity, i.e., the specific heat at event horizon of a black hole is given as

$$\begin{aligned}C_{\rm h}&=\left(\frac{\partial M}{\partial T_{\rm h}}\right)=\left(\frac{\partial M}{\partial r_{\rm h}}\right)\left(\frac{\partial T_{\rm h}}{\partial r_{\rm h}}\right)^{-1}\\&=T_{\rm h}\left(\frac{\partial S_{\rm h}}{\partial T_{\rm h}}\right)=T_{\rm h}\left(\frac{\partial S_{\rm h}}{\partial r_{\rm h}}\right)\left(\frac{\partial T_{\rm h}}{\partial r_{\rm h}}\right)^{-1}.\end{aligned} \tag{11}$$

It is well known that the heat capacity can decide the stability of the thermodynamical system: If the specific heat is positive, the black hole will be thermodynamically stable. If the specific heat is negative, the black hole will be thermodynamically unstable. There is also a critical case in which the specific heat blows up, and it signifies a second-order phase transition for such black hole.

2.3. Generalized uncertainty principle

Heisenberg's uncertainty principle can be generalized in the microphysics regime as[12–19]

$$\Delta x\Delta p \geq \hbar\left(1-\frac{\alpha l_{\rm p}}{\hbar}\Delta p+\frac{\alpha^2 l_{\rm p}^2}{\hbar^2}\Delta p^2\right), \tag{12}$$

where α is a dimensionless positive parameter and $l_{\rm p}=\sqrt{G\hbar/c^3}$ is Planck's length in order of 10^{-35} m, the quadratic form of GUP is given as

$$\Delta x\Delta p \geq \hbar\left(1+\frac{\alpha^2 l_{\rm p}^2}{\hbar^2}\Delta p^2\right), \tag{13}$$

which can also be rewritten as

$$\Delta p \geq \frac{\hbar\Delta x}{2\alpha^2 l_{\rm p}^2}\left(1-\sqrt{1-\frac{4\alpha^2 l_{\rm p}^2}{\Delta x^2}}\right). \tag{14}$$

Taking $\hbar=1$, $l_{\rm p}/\Delta x \ll 1$ and using Taylor series expansion, Eq. (14) can be rewritten as

$$\Delta p \geq \frac{1}{\Delta x}\left(1+\frac{2\alpha^2 l_{\rm p}^2}{\Delta x^2}+\cdots\right). \tag{15}$$

One can call upon a standard textbook argument[47] (also see Ref. [48]) to translate the saturated form of the uncertainty principle ($\Delta x\Delta p \geq 1$) into $E\Delta x \geq 1$, Eq. (15) can be written as

$$E_{\rm G} \geq E\left(1+\frac{2\alpha^2 l_{\rm p}^2}{\Delta x^2}+\cdots\right). \tag{16}$$

Therefore, for a particle with corrected energy $E_{\rm G}$ crossing the event horizon, the tunneling probability will be expressed as[10]

$$\Gamma \simeq \exp[-2\,{\rm Im}\,\tau_{\rm g}] = \exp\left[-\frac{4\pi E_{\rm G}}{f'(r_{\rm h})}\right], \tag{17}$$

where $\tau_{\rm g}$ is the action for the transition from some initial state to a final state, ${\rm Im}\,\tau_{\rm g}$ is an imaginary part of that action. By comparing with the Boltzmann factor $\exp[-\frac{E}{T}]$, the quantum-corrected Hawking temperature becomes

$$T_{\rm G} = T\left(1+\frac{2\alpha^2 l_{\rm p}^2}{\Delta x^2}+\cdots\right)^{-1}. \tag{18}$$

3. Thermodynamic properties of the Bardeen black hole in PFDM

Let us consider the coupling between Einstein gravity and a nonlinear electromagnetic field, it is described by the following equations:

$$G_\mu^\nu = 2\left(\frac{\partial\zeta(F)}{\partial F}F_{\mu\lambda}F^{\nu\lambda}-\delta_\mu^\nu\zeta\right)+8\pi T_\mu^\nu, \tag{19}$$

$$\nabla_\mu\left(\frac{\partial\zeta(F)}{\partial F}F^{\nu\mu}\right)=0, \tag{20}$$

$$\nabla_\mu(*F^{\nu\mu})=0. \tag{21}$$

Here, $F_{\mu\nu}=2\nabla_{[\mu}A_{\nu]}$ and ζ is a function of $F\equiv\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ given by[49]

$$\zeta(F)=\frac{3M}{|\,g\,|^3}\left(\frac{\sqrt{2g^2F}}{1+\sqrt{2g^2F}}\right)^{\frac{5}{2}}, \tag{22}$$

where g and M are magnetic charge and mass, respectively.

In this work, we consider the black holes surrounded by perfect fluid dark matter (PFDM), following Kiselev[37,40] and Li and Yang,[50] the energy density of PFDM is given by

$$T_t^t = T_r^r = \frac{1}{8\pi}\frac{k}{r^3}, \tag{23}$$

where k is the intensity of PFDM.

To obtain a metric satisfying Eqs. (19)–(21), let us commence with a spherically symmetric spacetime

$$\begin{aligned} {\rm d}s^2 &= -f(r){\rm d}t^2+\frac{{\rm d}r^2}{f(r)}+r^2{\rm d}\Omega^2,\\ f(r) &= 1-\frac{2m(r)}{r}, \end{aligned} \tag{24}$$

and use the ansatz for Maxwell field[49]

$$F_{\mu\nu}=(\delta_\mu^\theta\delta_\nu^\varphi-\delta_\nu^\theta\delta_\mu^\varphi)B(r,\theta). \tag{25}$$

Next, using Eqs. (20) and (21), Eq. (25) can be simplified to

$$F_{\mu\nu}=(\delta_\mu^\theta\delta_\nu^\varphi-\delta_\nu^\theta\delta_\mu^\varphi)g\sin\theta, \tag{26}$$

where g is the integration constant. Further, one can get $F=\frac{g^2}{2r^4}$. In order to give a direct physical interpretation to g, for any two-sphere S at infinity, we consider the following integral:

$$\frac{1}{4\pi}\int_S *F=\frac{g}{4\pi}\int_0^\pi\int_0^{2\pi}\sin\theta\,{\rm d}\theta\,{\rm d}\varphi=g. \tag{27}$$

From Eq. (27), one can confirm that g is the magnetic monopole charge.

Using the above equations, the time component of Eq. (19) reduces to

$$-\frac{2}{r^2}\frac{{\rm d}m(r)}{{\rm d}r}=-\frac{6Mg^2}{(r^2+g^2)^{\frac{5}{2}}}+\frac{k}{r^3}. \tag{28}$$

Integrating Eq. (28) from r to ∞ and using $M=\lim_{r\to\infty}(m(r)+\frac{k}{2}\ln\frac{r}{|k|})$, one can finally reach[51]

$$f(r)=1-\frac{2Mr^2}{(r^2+g^2)^{\frac{3}{2}}}+\frac{k}{r}\ln\frac{r}{|k|}. \tag{29}$$

In the case of vanishing g, it reduces to the Schwarzschild black hole surrounded by PFDM.[40,50] In the case of vanishing k, it reduces to the Bardeen black hole.[52] In the case of vanishing g and k, it reduces to the Schwarzschild black hole.

The Bardeen black hole ($k=0$) is regular everywhere as mentioned in the introduction. To test whether this property is

altered by the presence of PFDM, we calculate the following curvature scalars:

$$R = \frac{6Mg^2(4g^2-r^2)}{(g^2+r^2)^{7/2}} - \frac{k}{r^3}, \tag{30}$$

$$R_{\mu\nu}R^{\mu\nu} = \frac{18M^2g^4(8g^4-4g^2r^2+13r^4)}{(g^2+r^2)^7} - \frac{6Mg^2(2g^2+7r^2)k}{r^3(g^2+r^2)^{7/2}} + \frac{5k^2}{2r^6}, \tag{31}$$

$$\begin{aligned}\mathcal{K} = R_{\mu\nu\sigma\tau}R^{\mu\nu\sigma\tau} &= \frac{12k^2\ln^2\frac{r}{|k|}}{r^6} + \frac{13k^2}{r^6} \\ &+ \frac{4k\ln\frac{r}{|k|}\left[\frac{6Mr^5(3g^2-2r^2)}{(g^2+r^2)^{7/2}} - 5k\right]}{r^6} \\ &+ \frac{4kM(-2g^4-37g^2r^2+10r^4)}{r^3(g^2+r^2)^{7/2}} \\ &+ \frac{12M^2(8g^8-4g^6r^2+47g^4r^4-12g^2r^6+4r^8)}{(g^2+r^2)^7}.\end{aligned} \tag{32}$$

The real singularity appears at $r=0$ for the Bardeen black hole surrounded by PFDM. In fact, the future singularity comes from the dark matter background.

The mass of the black hole can be expressed in terms of event horizon r_h as

$$M = \frac{(g^2+r_\mathrm{h}^2)^{\frac{3}{2}}(r_\mathrm{h}+k\ln\frac{r_\mathrm{h}}{|k|})}{2r_\mathrm{h}^3}, \tag{33}$$

which comes from $f(r_\mathrm{h})=0$. The temperature of the black hole at event horizon can be calculated by using Eq. (1) as

$$T_\mathrm{h} = \frac{g^2(k-2r_\mathrm{h})+r_\mathrm{h}^2(k+r_\mathrm{h})-3g^2k\ln\frac{r_\mathrm{h}}{|k|}}{4\pi r_\mathrm{h}^2(g^2+r_\mathrm{h}^2)}. \tag{34}$$

We plot the variation of Hawking temperature T_h with respect to event horizon r_h for different dark matter parameters k with three different values of magnetic charge g in Fig. 1. It can be observed that Hawking temperature T_h increases at a high rate, reaches a maximum, and then decreases at a low rate as event horizon r_h increases. As magnetic charge g increases, the steepness of increase/decrease of Hawking temperature T_h reduces, the maximum appears at a larger radius of event horizon. However, as dark matter parameter k increases, the maximum of Hawking temperature T_h appears at a smaller radius of event horizon. From Eq. (5), we calculate the surface gravity of the black hole at the event horizon as

$$\kappa_\mathrm{h} = \frac{g^2(k-2r_\mathrm{h})+r_\mathrm{h}^2(k+r_\mathrm{h})-3g^2k\ln\frac{r_\mathrm{h}}{|k|}}{2r_\mathrm{h}^2(g^2+r_\mathrm{h}^2)}. \tag{35}$$

According to Eq. (10), the entropy of the black hole at event horizon is expressed as

$$S_\mathrm{h} = \int_0^{r_\mathrm{h}} \frac{\mathrm{d}M}{T_\mathrm{h}} = \int_0^{r_\mathrm{h}} \frac{1}{T_\mathrm{h}}\left(\frac{\mathrm{d}M}{\mathrm{d}r_\mathrm{h}}\right)dr_\mathrm{h}. \tag{36}$$

Here, $\mathrm{d}M/\mathrm{d}r_\mathrm{h}$ is calculated as follows:

$$\frac{\mathrm{d}M}{\mathrm{d}r_\mathrm{h}} = \frac{\sqrt{g^2+r_\mathrm{h}^2}(g^2(k-2r_\mathrm{h})+r_\mathrm{h}^2(k+r_\mathrm{h})-3g^2k\ln\frac{r_\mathrm{h}}{|k|})}{2r_\mathrm{h}^4}. \tag{37}$$

Substituting Eq. (34) into Eq. (36) and combining with Eq. (37), we obtain the entropy of the black hole at event horizon as

$$\begin{aligned}S_\mathrm{h} &= \frac{\pi(-2g^2+r_\mathrm{h}^2)\sqrt{g^2+r_\mathrm{h}^2}}{r_\mathrm{h}} \\ &+ 3g^2\pi\ln(r_\mathrm{h}+\sqrt{g^2+r_\mathrm{h}^2}).\end{aligned} \tag{38}$$

Substituting Eqs. (33) and (34) into Eq. (11), we obtain the specific heat of the black hole as follows:

$$C_\mathrm{h} = -\frac{2\pi(g^2+r_\mathrm{h}^2)^{5/2}(g^2(k-2r_\mathrm{h})+r_\mathrm{h}^2(k+r_\mathrm{h})-3g^2k\ln\frac{r_\mathrm{h}}{|k|})}{r_\mathrm{h}(g^4(5k-2r_\mathrm{h})+7g^2r_\mathrm{h}^2(k-r_\mathrm{h})+r_\mathrm{h}^4(2k+r_\mathrm{h})-6g^2k(g^2+2r_\mathrm{h}^2)\ln\frac{r_\mathrm{h}}{|k|})}. \tag{39}$$

Assuming that the black hole radiates the energy in the form of photon[53,54] and using the Stefan–Boltzmann law in 2D space time, we can express the rate of emission as

$$\frac{\mathrm{d}M}{\mathrm{d}t} \propto T_\mathrm{h}^2, \tag{40}$$

where t is the time interval of the emission occurred. Thus, the rate of emission is given as

$$\frac{\mathrm{d}M}{\mathrm{d}t} \propto \frac{(g^2(k-2r_\mathrm{h})+r_\mathrm{h}^2(k+r_\mathrm{h})-3g^2k\ln\frac{r_\mathrm{h}}{|k|})^2}{16\pi^2r_\mathrm{h}^4(g^2+r_\mathrm{h}^2)^2}. \tag{41}$$

The specific heat C_h with respect to event horizon r_h for different dark matter parameters k with three different values of magnetic charge g is depicted in Fig. 2. As we can see, the specific heat C_h is positive and increases with event horizon r_h firstly, then specific heat C_h diverges and becomes negative. Physically, the black hole is thermodynamically stable and small firstly, then it goes through a second-order phase transition, becomes thermodynamically unstable and large. At the same time, we find that the second-order phase transition occurs at a larger radius of event horizon with the increase of magnetic charge g. However, the increase of dark matter parameter k makes the second-order phase transition shift to a smaller radius of event horizon.

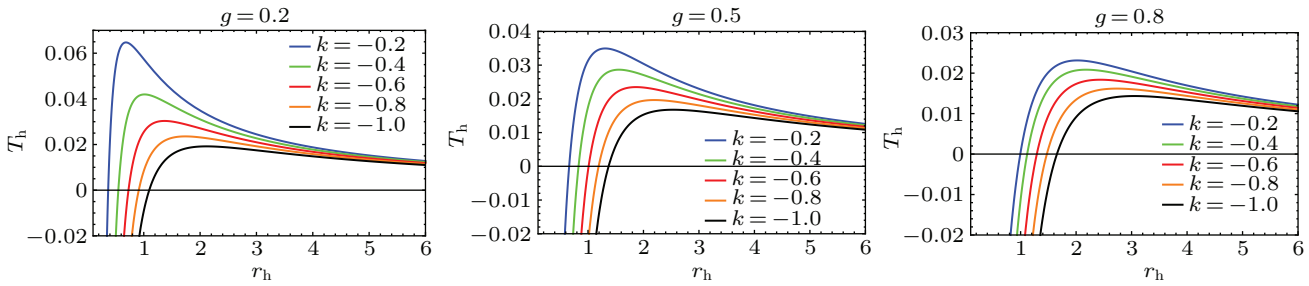


Fig. 1. The variation of Hawking temperature $T_{\rm h}$ with respect to event horizon $r_{\rm h}$ for different dark matter parameters k with different values of magnetic charge g.

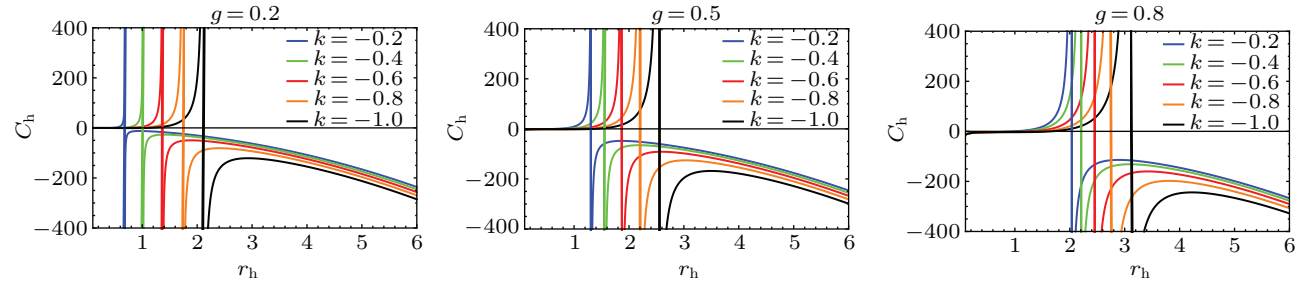


Fig. 2. The variation of specific heat $C_{\rm h}$ with respect to event horizon $r_{\rm h}$ for different dark matter parameters k with different values of magnetic charge g.

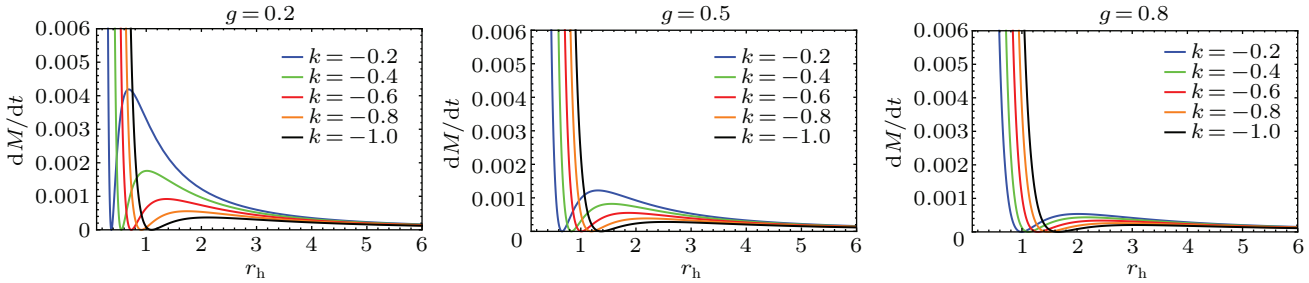


Fig. 3. The variation of rate of emission $\mathrm{d}M/\mathrm{d}t$ with respect to event horizon $r_{\rm h}$ for different dark matter parameters k with different values of magnetic charge g.

The rate of emission $\mathrm{d}M/\mathrm{d}t$ with respect to event horizon $r_{\rm h}$ for different dark matter parameters k with three different values of magnetic charge g is shown in Fig. 3. From the curves, we can see that the rate of emission $\mathrm{d}M/\mathrm{d}t$ decreases rapidly at first, then increases and reaches a maximum, finally decreases slowly with event horizon $r_{\rm h}$. Clearly, one can find that the maximum occurs at a larger radius of event horizon with magnetic charge g. However, the maximum occurs at a smaller radius of event horizon with dark matter parameter k.

4. Thermodynamics of the Bardeen black hole in PFDM under GUP

Supposing that the event uncertainty near the event horizon is given by $\Delta x \simeq 2r_{\rm h}$,[55] the quantum-corrected temperature of the black hole is

$$T_{\rm G} = T_{\rm h}\left(1+\frac{\alpha^2 l_{\rm p}^2}{2r_{\rm h}^2}+\cdots\right)^{-1} = \frac{g^2(k-2r_{\rm h})+r_{\rm h}^2(k+r_{\rm h})-3g^2k\ln\frac{r_{\rm h}}{|k|}}{4\pi r_{\rm h}^2(g^2+r_{\rm h}^2)} \times\left(1+\frac{\alpha^2 l_{\rm p}^2}{2r_{\rm h}^2}+\cdots\right)^{-1}. \tag{42}$$

We have simulated the GUP corrected Hawking temperature $T_{\rm G}$ with respect to event horizon $r_{\rm h}$ for different dark matter parameters k with three different values of magnetic charge g by fixing the generalized uncertainty principle parameter $\alpha = 0.1$, 1 and 10 in Fig. 4. (I) When $\alpha = 0.1$, we observe that $T_{\rm G}$ increases first and after reaching its maximum at a certain $r_{\rm h} = r_{\rm h-crit1}(g,k,\alpha)$, then decreases with event horizon $r_{\rm h}$. If $g_1 > g_2$, we can see $r_{\rm h-crit1}(g_1,k,\alpha) > r_{\rm h-crit1}(g_2,k,\alpha)$. If $k_1 > k_2$, we can find $r_{\rm h-crit1}(g,k_1,\alpha) < r_{\rm h-crit1}(g,k_2,\alpha)$, which means that the maximum value will occur at a larger radius of event horizon with the increase of magnetic charge g, but at a smaller radius of event horizon with the increase of dark matter parameter k. (II) When $\alpha = 1$, the steepness of increase/decrease reduces and the maximum of Hawking temperature $T_{\rm G}$ occurs at a larger radius of event horizon compared with the case I. (III) When $\alpha = 10$, the Hawking temperature $T_{\rm G}$ increases firstly, the maximum occurs very late. Generally, if $\alpha_1 > \alpha_2$, $r_{\rm h-crit1}(g,k,\alpha_1) > r_{\rm h-crit1}(g,k,\alpha_2)$.

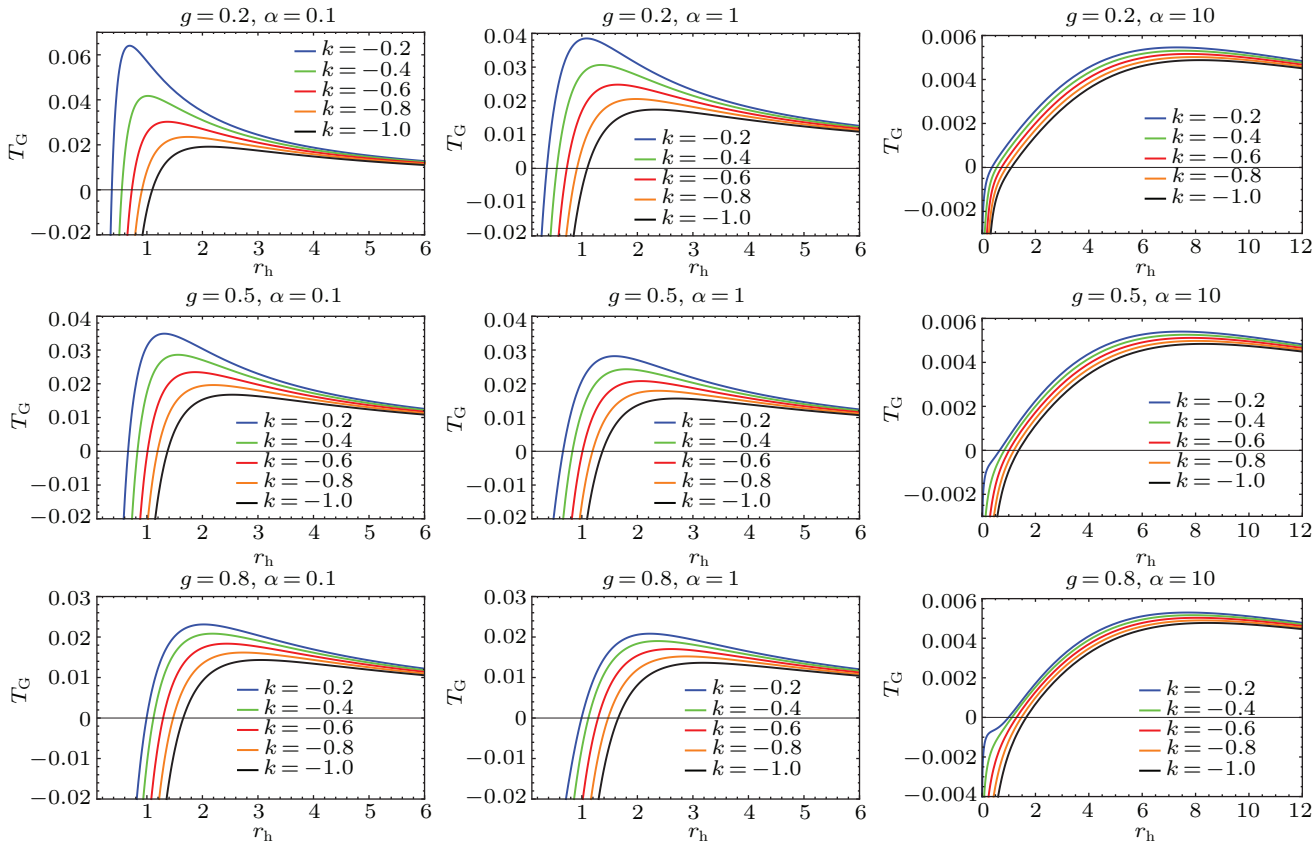


Fig. 4. The variation of GUP corrected Hawking temperature $T_{\rm G}$ with respect to event horizon $r_{\rm h}$ for different dark matter parameters k with different values of magnetic charge g and generalized uncertainty principle parameter α.

The quantum-corrected entropy for the Bardeen black hole in PFDM is calculated by

$$S_{\rm G} = \pi\Bigg(-\frac{\sqrt{g^2+r_{\rm h}^2}(6g^2r_{\rm h}^2-3r_{\rm h}^4+\alpha^2l_{\rm p}^2(g^2+4r_{\rm h}^2))}{3r_{\rm h}^3} + (3g^2+\alpha^2l_{\rm p}^2)\ln(r_{\rm h}+\sqrt{g^2+r_{\rm h}^2})\Bigg). \tag{43}$$

Note that when $\alpha\to 0$, we can obtain the entropy shown in Eq. (38), i.e.,

$$\lim_{\alpha\to 0} S_{\rm G} = S_{\rm h}. \tag{44}$$

We can see that the quantum-corrected entropy term contains the logarithmic term of event horizon $r_{\rm h}$ as mentioned in Ref. [25].

Using the quantum-corrected temperature and quantum-corrected entropy, we can compute the quantum-corrected heat capacity by the following equation:

$$C_{\rm G} = \left(\frac{\partial M}{\partial T_{\rm G}}\right) = \left(\frac{\partial M}{\partial r_{\rm h}}\right)\left(\frac{\partial T_{\rm G}}{\partial r_{\rm h}}\right)^{-1} = T_{\rm G}\left(\frac{\partial S_{\rm G}}{\partial T_{\rm G}}\right) = T_{\rm G}\left(\frac{\partial S_{\rm G}}{\partial r_{\rm h}}\right)\left(\frac{\partial T_{\rm G}}{\partial r_{\rm h}}\right)^{-1}, \tag{45}$$

and the quantum-corrected heat capacity can be expressed as

$$C_{\rm G} = -\Bigg\{\Bigg[\pi(g^2+r_{\rm h}^2)^{5/2}(\alpha^2l_{\rm p}^2+2r_{\rm h}^2)^2\Bigg(g^2(k-2r_{\rm h}) + r_{\rm h}^2(k+r_{\rm h}) - 3g^2k\ln\frac{r_{\rm h}}{|k|}\Bigg)\Bigg]\Bigg/\Bigg[2r_{\rm h}^5(g^4(5k-2r_{\rm h}) + 7g^2r_{\rm h}^2(k-r_{\rm h}) + r_{\rm h}^4(2k+r_{\rm h})) + \alpha^2l_{\rm p}^2r_{\rm h}^3(g^2r_{\rm h}^2(3k-5r_{\rm h}) - r_{\rm h}^5 + g^4(3k+2r_{\rm h})) - 6g^2kr_{\rm h}^5(2g^2+\alpha^2l_{\rm p}^2+4r_{\rm h}^2)\ln\frac{r_{\rm h}}{|k|}\Bigg]\Bigg\}. \tag{46}$$

Figure 5 shows the variation of GUP corrected specific heat $C_{\rm G}$ with event horizon $r_{\rm h}$ for different dark matter parameters k with three different values of magnetic charge g. (I) For $\alpha=0.1$, we observe that $C_{\rm G}$ is positive firstly, then it blows up and becomes negative. Physically, the black hole is thermodynamically stable and small firstly, then it goes through a second-order phase transition, becomes thermodynamically unstable and large. If the phase transition occurs at $r_{\rm h}=r_{\rm h-crit2}(g,k,\alpha)$, then we see $r_{\rm h-crit2}(g_1,k,\alpha)>r_{\rm h-crit2}(g_2,k,\alpha)$ for $g_1>g_2$ and $r_{\rm h-crit2}(g,k_1,\alpha)<r_{\rm h-crit2}(g,k_2,\alpha)$ for $k_1>k_2$, which means that if magnetic charge g increases, the second-order phase transition will occur at a larger value of $r_{\rm h}$, however if dark matter parameter k increases, the second-order phase transition will occur at a smaller value of $r_{\rm h}$. (II) When $\alpha=1$, the second-order phase transition occurs more later than case I. (III) When $\alpha=10$, a stable black hole is presented first, and the phase transition takes place very late. Generally, if $\alpha_1>\alpha_2$, $r_{\rm h-crit2}(g,k,\alpha_1)>r_{\rm h-crit2}(g,k,\alpha_2)$.

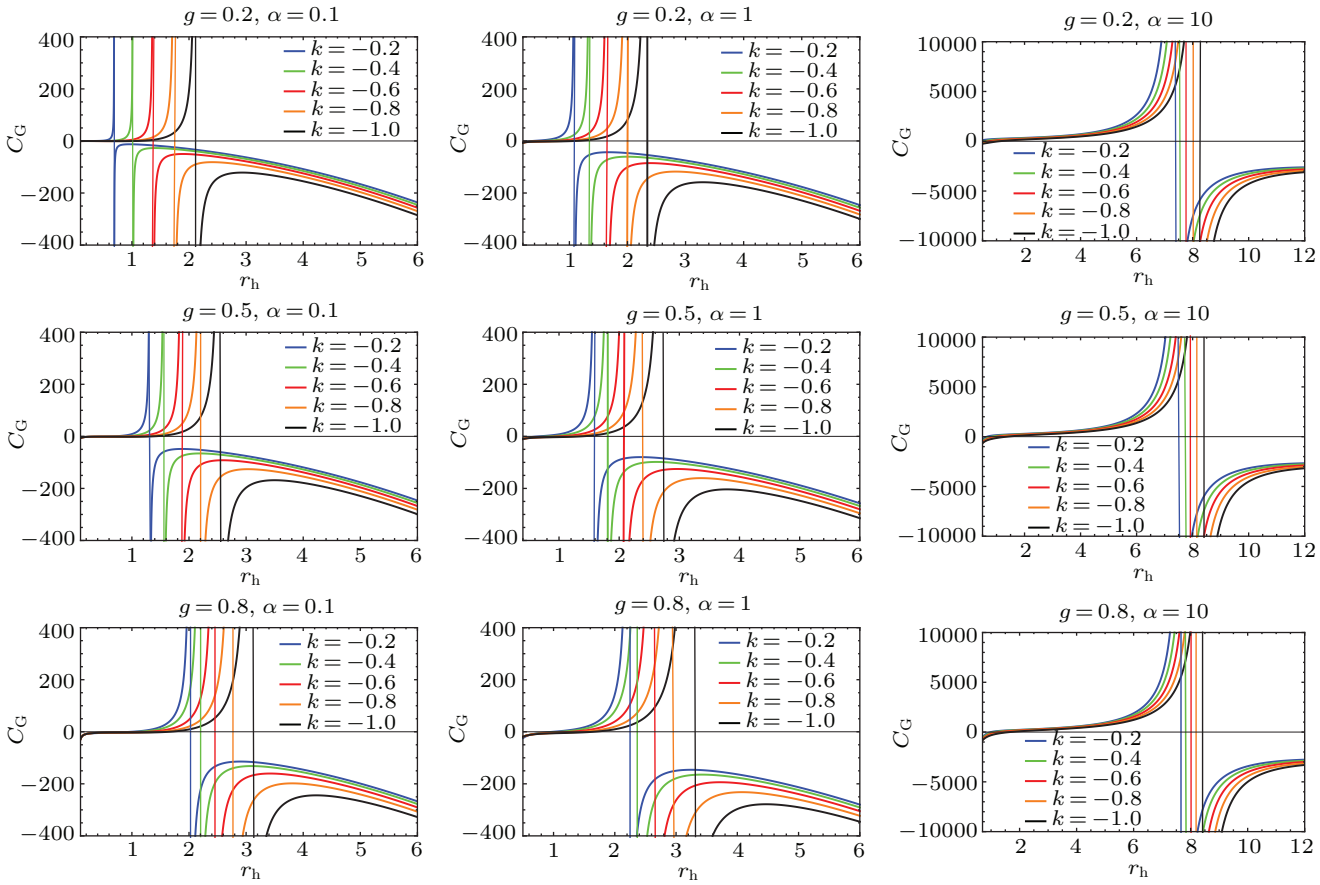


Fig. 5. The variation of GUP corrected specific heat $C_{\rm G}$ with respect to event horizon $r_{\rm h}$ for different dark matter parameters k with different values of magnetic charge g and generalized uncertainty principle parameter α.

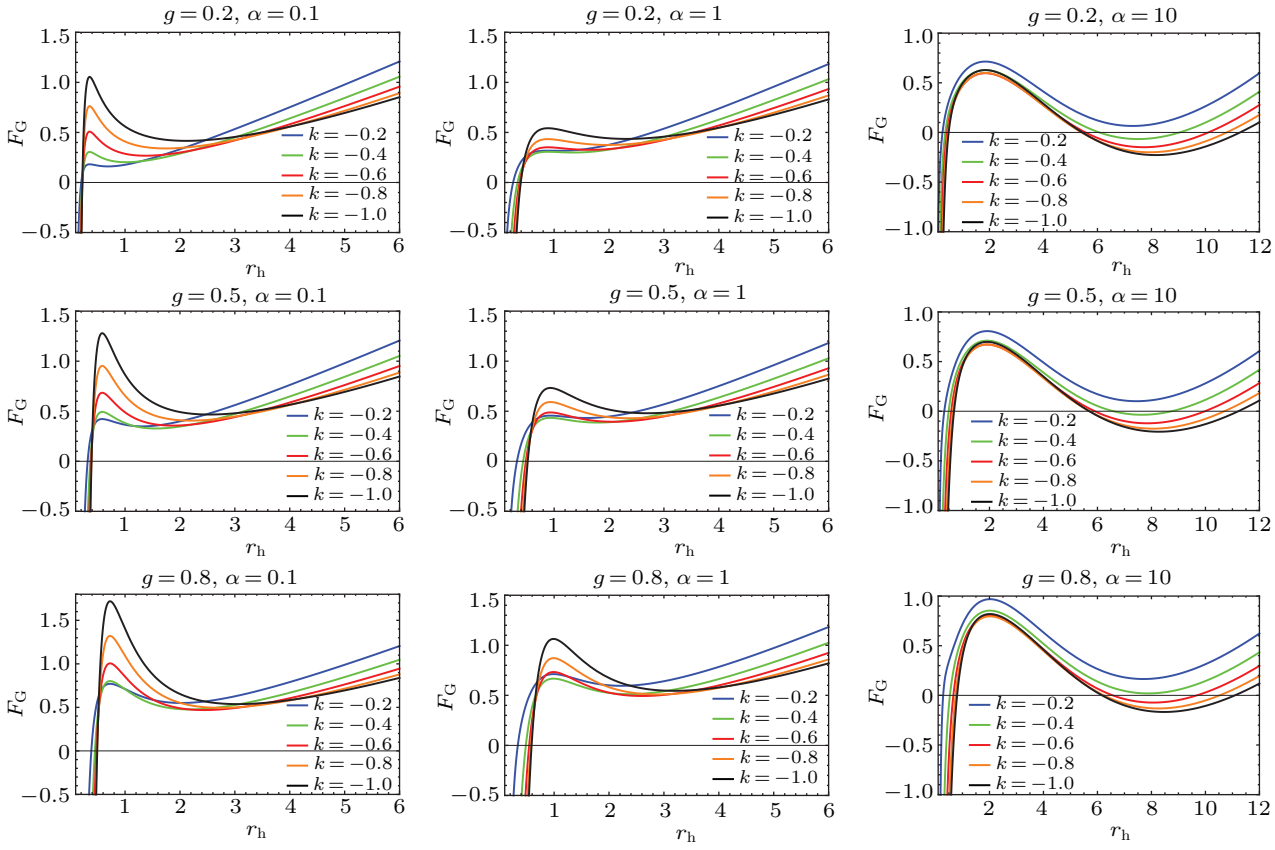


Fig. 6. The variation of GUP corrected Helmholtz free energy $F_{\rm G}$ with respect to event horizon $r_{\rm h}$ for different dark matter parameters k with different values of magnetic charge g and generalized uncertainty principle parameter α.

Obviously, for $\alpha \to 0$, we can obtain the heat capacity shown in Eq. (39), i.e.,

$$\lim_{\alpha\to 0} C_{\rm G} = C_{\rm h}. \tag{47}$$

The quantum-corrected Helmholtz free energy for the Bardeen black hole in PFDM is expressed as

$$F_{\rm G} = M - T_{\rm G} S_{\rm G}. \tag{48}$$

Here, we use the classical black hole thermodynamics to predict $M = U$ (the internal energy of the black hole), so

$$\begin{aligned} F_{\rm G} \;=\; & M - T_{\rm h}\left(1+\frac{\alpha^2 l_{\rm p}^2}{2r_{\rm h}^2}+\cdots\right)^{-1} \\ & \times\left[\pi\left(-\frac{\sqrt{g^2+r_{\rm h}^2}(6g^2r_{\rm h}^2-3r_{\rm h}^4+\alpha^2 l_{\rm p}^2(g^2+4r_{\rm h}^2))}{3r_{\rm h}^3}\right.\right. \\ & \left.\left.+(3g^2+\alpha^2 l_{\rm p}^2)\ln(r_{\rm h}+\sqrt{g^2+r_{\rm h}^2})\right)\right]. \end{aligned} \tag{49}$$

Figure 6 shows the variation of quantum-corrected Helmholtz free energy $F_{\rm G}$ with event horizon $r_{\rm h}$ for different dark matter parameters k with three different values of magnetic charge g. (I) When $\alpha = 0.1$, $F_{\rm G}$ increases at a high rate with event horizon $r_{\rm h}$, reaches a maximum, then decreases, and lastly $F_{\rm G}$ comes back to increase at a slightly lower rate with event horizon $r_{\rm h}$. Clearly, we can see that the maximum occurs at a larger value of $r_{\rm h}$ with the increase of magnetic charge g. (II) For $\alpha = 1$, the rate of this increase and decrease slows down for low value of event horizon $r_{\rm h}$ and the maximum occurs more later compared to the case I. (III) When $\alpha = 10$, the maximum occurs more later compared with the case II.

The quantum-corrected rate of emission is

$$\frac{{\rm d}M}{{\rm d}t} \propto T_{\rm G}^2, \tag{50}$$

and combining with Eq. (42), we can obtain

$$\begin{aligned} \frac{{\rm d}M}{{\rm d}t} \;\propto\; & \frac{(g^2(k-2r_{\rm h})+r_{\rm h}^2(k+r_{\rm h})-3g^2k\ln\frac{r_{\rm h}}{|k|})^2}{16\pi^2 r_{\rm h}^4(g^2+r_{\rm h}^2)^2} \\ & \times\left(1+\frac{\alpha^2 l_{\rm p}^2}{2r_{\rm h}^2}+\cdots\right)^{-2}. \end{aligned} \tag{51}$$

5. Discussion and conclusion

In summary, we have studied the Hawking temperature, surface gravity, and the entropy, etc., of the Bardeen black hole surrounded by perfect fluid dark matter. Moreover, we have re-examined these thermodynamic properties under general uncertainty principle. We have also found that the quantum-corrected entropy term contains the logarithmic term of event horizon $r_{\rm h}$, which was proposed in Ref. [25]. We have observed that the Hawking temperature of the Bardeen black hole in PFDM increases and reaches a local maximum, then decreases. The specific heat diverges at the temperature maximum point at which a second-order phase transition occurs. Considering general uncertainty principle, the maximum occurs at a larger radius of event horizon when the values of magnetic charge g and generalized uncertainty principle parameter α are very high and the value of dark matter parameters k is low. From the specific heat curves, the magnetic charge g makes the second-order phase transition take place at a larger radius of event horizon, but the dark matter parameter k makes the second-order phase transition take place at a smaller radius of event horizon. Furthermore, GUP's effect makes the second-order phase transition occur at a larger radius of event horizon.

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