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Xinyao Yu(于馨瑶), Pingyu Zhu(朱枰谕), Yang Wang(王洋), Miaomiao Yu(余苗苗), Chao Wu(吴超),Shichuan Xue(薛诗川), Qilin Zheng(郑骑林), Yingwen Liu(刘英文), Junjie Wu(吴俊杰), and Ping Xu(徐平)

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Heralded path-entangled NOON states generation from a reconfigurable photonic chip

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Institute for Quantum Information & State Key Laboratory of High Performance Computing, College of Computer Science and Technology, National University of Defense Technology, Changsha 410073, *China*

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Maximal multi-photon entangled states, known as NOON states, play an essential role in quantum metrology. With the number of photons growing, NOON states are becoming increasingly powerful and advantageous for obtaining supersensitive and super-resolved measurements. In this paper, we propose a universal scheme for generating three- and four-photon path-entangled NOON states on a reconfigurable photonic chip via photons subtracted from pairs and detected by heralding counters. Our method is postselection free, enabling phase supersensitive measurements and sensing at the Heisenberg limit. Our NOON-state generator allows for integration of quantum light sources as well as practical and portable precision phase-related measurements.

Keywords: path-entangled NOON state, phase measurement

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1. Introduction

Quantum entanglement,[1] the distinctive feature of quantum systems, plays a crucial role in quantum information processing and lies at the heart of the emerging quantum technologies. In particular, multi-photon maximally entangled states,[2,3] so-called NOON states, have attracted much attention for their great importance in quantum metrology and lithography. The NOON state, denoted as $|N::0\rangle$, takes the form

$$|N::0\rangle_{\mathrm{a,b}} = (|N,0\rangle_{\mathrm{a,b}} + |0,N\rangle_{\mathrm{a,b}})/\sqrt{2}. \tag{1}$$

In other words, a NOON state is an equal superposition of distinct Fock states, i.e., all N photons are either in mode a or b. It has been proven that in optical phase measurements, NOON states have the ability to beat the shot-noise limit (SNL) $\Delta\phi = 1/\sqrt{N}$ and reach the so-called Heisenberg limit (HL) $\Delta\phi = 1/N$ with a $\sqrt{N}$ improvement,[4–6] which shows super-sensitivity, where N is the number of photons used. Moreover, when launched into a Mach–Zehnder interferometer, the NOON state gives an N gain in de Broglie resolution over the Rayleigh diffraction limit using coherent states, showing its super-resolution effect.[7]

Numerous studies have been done on the generation of NOON states, via both bulk optics and integrated optics. A prominent example is the generation of the two-photon NOON state.[8,9] By feeding two single photons into the inputs of a balanced beam splitter, a NOON state with $N = 2$ can be easily and deterministically obtained at the outputs for the bunching effect of photons, which is also known as the Hong–Ou–Mandel effect. However, when $N > 2$, generating NOON states is not straightforward. So far, there have been three main ideas to obtain the NOON state when $N > 2$. The first is harnessing multi-photon interference of classical coherent states and quantum down-converted states;[10,11] by adjusting their ratio over different inherently scalable approaches, high-NOON states with up to $N = 5$ have been generated. The second is a bulk optic-based linear optical scheme: by manipulating polarized-encoded photons,[12–15] realistic experimental methods for generating three- and four-photon NOON states have been proposed. The third are schemes on integrated photonic circuits. Recently, on-chip schemes for generating two- and even multi-mode NOON states have been proposed.[17–20] Among these works, on the one hand, some of them are based on an N-fold photon coincidence detection, that is, via post-selection;[15,16] the required NOON state was separated from the undesired components at the final detection stage, leading to no further applications performed. On the other hand, bulk optical performance is less than satisfactory when it comes to spatial scalability and phase stability. However, the lack of deterministic single-photon sources[17–19] required by some existing on-chip schemes brings makes some experimental demonstrations very difficult. In addition, some of them require off-chip quantum light sources and bulk optical elements, which hinder their portable and robust working fashion.[20]

This paper proposes a fully integrated scheme to generate NOON states encoded in path-mode based on heralding on a reconfigurable photonic chip. In our scheme, we take advantage of the nature of photons in pairs, and utilize the high-order terms of basic two-photon sources; then, on the same chip,

[†]Corresponding author. E-mail: pingxu520@nju.edu.cn

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quantum interference take its role in the cascaded photonic circuits to enable three-photon and four-photon NOON state generation in a heralded fashion. We concretely show the configuration for $N=3$ and $N=4$, which is implementable within current technical conditions. Consisting of on-chip quantum sources and reconfigurable photonic circuits, our proposal is stable and user-friendly. Our scheme involves the explicit generation of the required NOON quantum state, which can be applied to allow for further quantum information processing such as supersensitive and super-resolved single parameter estimation and sensing.

2. Theory and scheme

At the very beginning, we will briefly introduce the key elements supporting the scheme and the main processes occurring on our device. Photon pairs can be produced via a nonlinear process on our photonic chip. By injecting two pump lights with different frequencies into a spiraled waveguide or micro-ring resonator (MRR), the degenerate spontaneous four-wave mixing (SFWM) process occurs, producing pairs of photons with the same frequency; i.e., indistinguishable photon pairs are obtained, which is the key to the NOON states, as shown in Figs. 1(a) and 1(b). Here, we prefer to use micro-rings coupled by a single or dual asymmetric Mach–Zehnder interferometer (AMZI), which ensures spectral purity for generating bright indistinguishable photon pairs; namely, the only the two-photon sources differ or are encoded in path while all the other degrees of freedom are kept identical. Specifically, compared with the spiraled waveguide, via its elaborate design, the AMZI-coupled micro-ring can obtain high spectral purity photons without decreasing the photon rates.[22–25] Next, by combining an unbiased beam splitter (BS), i.e., the symmetric 50:50 BS and two degenerate SFWM-based collinear sources (see Fig. 1(c)), our two-photon state, which also represents the path-entangled NOON state with $N=2$, is obtained through the formula

$$\begin{aligned}|\Psi\rangle_{\mathrm{u,l}} &= g\cdot\left[\frac{1}{2}(a_{\mathrm{l}}^{\dagger 2}-a_{\mathrm{u}}^{\dagger 2})\right]|0\rangle \\ &= g\cdot\frac{1}{\sqrt{2}}\left(|2,0\rangle_{\mathrm{u,l}}-|0,2\rangle_{\mathrm{u,l}}\right),\end{aligned} \tag{2}$$

where $a_{\mathrm{u}}^{\dagger}$ ($a_{\mathrm{l}}^{\dagger}$) is the creation operator for the upper (lower) path mode, $|0\rangle$ denotes the vacuum state and $|n,m\rangle_{\mathrm{u,l}}$ is in the form of the Fock state, meaning n photons in the upper path with m photons in the lower path, and g stands for the two-photon state generation efficiency, which absorbs all the constants relating to the pump power, nonlinear coefficients and the parameters of the MRR.

Another important component of our chip is the Mach–Zehnder interferometer (MZI). As shown in Fig. 1(c), an MZI is composed of two balanced BSs and a phase shifter θ in between, whose evolution matrix is as follows:

$$U=\mathrm{i}e^{\mathrm{i}\theta/2}\begin{bmatrix}\sin\frac{\theta}{2} & \cos\frac{\theta}{2}\\ \cos\frac{\theta}{2} & -\sin\frac{\theta}{2}\end{bmatrix}. \tag{3}$$

It has been shown that the MZI with such structure can realize a BS with variable reflectivity;[21] therefore, by adjusting the phase shifter θ, one can split photon beams in an arbitrary ratio to the two paths.

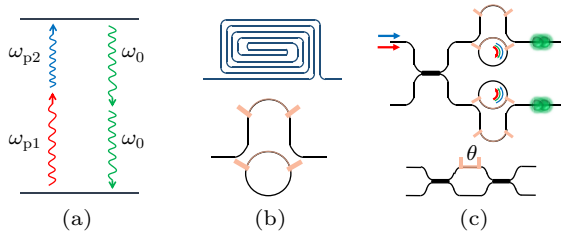


Fig. 1. On-chip key components and processes. (a) Degenerate SFWM process for generating photon pairs with same frequency. (b) Spiraled waveguide (top) and micro-ring resonator (bottom) for on-chip SFWM process. (c) Two-photon NOON state source (top), and the Mach–Zehnder interferometer (bottom).

Now we will turn to our device. Our photonic chip includes three functional parts, as shown in Fig. 2: quantum source, path-entangled NOON-state generation and detection and verification. In the quantum source part, pump lasers are divided equally between the two paths, and degenerate SFWM occurs in two MRRs, producing path-encoded-entangled but frequency uncorrelated photon pairs. Next, the photon pairs transit into the NOON-state generation part to obtain a certain photon-number NOON state. Finally, by observing the corresponding triggered events from the detectors shown in the third part, heralded path-entangled NOON states are generated.

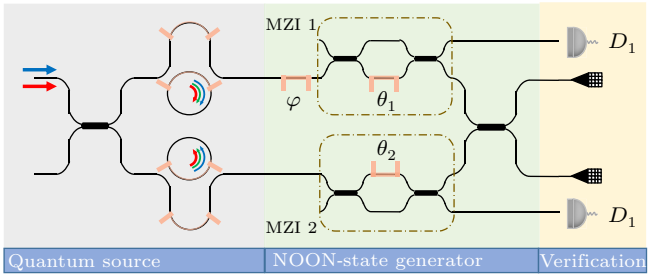


Fig. 2. The universal scheme for generation of on-chip maximal multi-photon path-entangled states.

In our three-photon NOON states generating process, we specify the second-order term to be the quantum source. The superposition state of four-photon terms after two SFWM processes can be expressed by

$$\begin{aligned}|\Psi\rangle_{\mathrm{u,l}} &= g^2\cdot\left[\frac{1}{2}\left(a_{\mathrm{u}}^{\dagger 2}-a_{\mathrm{l}}^{\dagger 2}\right)\right]^2|0\rangle \\ &= g^2\cdot\left[\frac{1}{4}\left(a_{\mathrm{u}}^{\dagger 4}+a_{\mathrm{l}}^{\dagger 4}-2a_{\mathrm{u}}^{\dagger 2}a_{\mathrm{l}}^{\dagger 2}\right)\right]|0\rangle.\end{aligned} \tag{4}$$

Considering that the creation operator will lead to extra coefficients, we append a normalization factor $1/2$; therefore the

initial state for generating three-photon NOON states takes the form of

$$|\Psi\rangle_{\mathrm{u,l}} = g^2 \cdot \left[\frac{1}{8}\left(a_{\mathrm{u}}^{\dagger 4} + a_{\mathrm{l}}^{\dagger 4} - 2a_{\mathrm{u}}^{\dagger 2}a_{\mathrm{l}}^{\dagger 2}\right)\right]|0\rangle. \tag{5}$$

Next, by setting the relative phase φ between the two paths to a proper value as well as letting the photons along the upper path go through a fixed beam splitting ratio BS by setting θ_{l} in MZI1, while the photons along the lower path walk through a waveguide of equal length, that is, setting MZI2 to an all-reflect state, when detecting one and only one photon in the detector D_1, the three-photon state is projected into

$$|\Psi\rangle_{\mathrm{u,l}} = g^2\left[\frac{1}{8}\left(\mathrm{e}^{\mathrm{i}4\varphi}\cdot 4r^3t\cdot a_{\mathrm{u}}^{\dagger 3} - \mathrm{e}^{\mathrm{i}2\varphi}\cdot 4rt\cdot a_{\mathrm{u}}^{\dagger}a_{\mathrm{l}}^{\dagger 2}\right)\right]|0\rangle, \tag{6}$$

where $t(r)$ is the transmittance (reflectivity) of the BS, satisfying $t^2 + r^2 = 1$, determined by the θ used in MZI with $r = \sin(\theta/2)$, and φ is the relative phase.

After that, the two lower-path photons merge with the remaining photons from the upper path by respectively launching into the two inputs of a 50:50 symmetric BS simultaneously, leading to the final state in the form of

$$\begin{aligned}|\Psi\rangle_{\mathrm{u,l}} = g^2\cdot\Big\{\frac{1}{8}\Big[&\sqrt{2}(\mathrm{e}^{\mathrm{i}4\varphi}r^3t + \mathrm{e}^{\mathrm{i}2\varphi}rt)(a_{\mathrm{u}}^{\dagger 3} - \mathrm{i}a_{\mathrm{l}}^{\dagger 3})\\ &+\sqrt{2}(3\mathrm{e}^{\mathrm{i}4\varphi}r^3t - \mathrm{e}^{\mathrm{i}2\varphi}rt)(\mathrm{i}a_{\mathrm{u}}^{\dagger 2}a_{\mathrm{l}}^{\dagger} - a_{\mathrm{u}}^{\dagger}a_{\mathrm{l}}^{\dagger 2})\Big]\Big\}|0\rangle. \end{aligned}\tag{7}$$

Obviously, the final state is a function of θ and φ. Note that here φ includes the extra phase introduced by MZI2. In our current scheme, pure $|1::2\rangle$ states will never be obtained, for there is no feasible solution for equation $\mathrm{e}^{\mathrm{i}4\varphi}r^3t + \mathrm{e}^{\mathrm{i}2\varphi}rt = 0$. Next, by solving the equation $3\mathrm{e}^{\mathrm{i}4\varphi}r^3t - \mathrm{e}^{\mathrm{i}2\varphi}rt = 0$, which guarantees the unwanted terms $|21\rangle$ and $|12\rangle$ be eliminated, we obtain the explicit solution as $\varphi = 0/\pi$ and $r^2 = 1/3$, leading to the three-photon NOON state as

$$\begin{aligned}|\Psi\rangle_{\mathrm{u,l}} &= g^2\cdot\left[\frac{1}{8}\left(\frac{8}{9}a_{\mathrm{u}}^{\dagger 3} - \frac{8}{9}\mathrm{i}a_{\mathrm{l}}^{\dagger 3}\right)\right]|0\rangle\\ &= g^2\cdot\left[\frac{\sqrt{6}}{9}(|3,0\rangle_{\mathrm{u,l}} - \mathrm{i}|0,3\rangle_{\mathrm{u,l}})\right].\end{aligned}\tag{8}$$

The generation efficiency is defined as the square of the modulo probability amplitude. Therefore, the generation efficiency of our scheme is $4/27$. Additionally, the overall generation efficiency is $(4/27)g^2$ by taking the factor g into account, i.e. we obtain a three-photon path-entangled state with the probability of $(4/27)g^2$.

We have given the expression of the output state above, which shows that the final output state is a result of the joint effect of the relative phase φ and the equivalent beam splitting ratio of the AMZI. While, in fact, the key action in our scheme is the interference occurring on the final symmetric BS, the relative phase φ between the two paths needs to be accurately controlled; otherwise pure NOON states cannot be guaranteed. Our reconfigurable photonic chip allows us to achieve this point straightforwardly. Concretely, by fixing the equivalent reflectivity of the MZI to $\sqrt{1/3}$ as mentioned above, the final state is as a function of relative phase φ and takes the following form for generating three-photon path-entangled NOON states:

$$\begin{aligned}|\Psi\rangle_{\mathrm{u,l}} = g^2\cdot\Big\{\frac{1}{36}\Big[&2\mathrm{e}^{\mathrm{i}3\varphi}(2\cos\varphi - \mathrm{i}\sin\varphi)(a_{\mathrm{u}}^{\dagger 3} - \mathrm{i}a_{\mathrm{l}}^{\dagger 3})\\ &+6\mathrm{e}^{\mathrm{i}3\varphi}\sin\varphi(\mathrm{i}a_{\mathrm{u}}^{\dagger 2}a_{\mathrm{l}}^{\dagger} - a_{\mathrm{u}}^{\dagger}a_{\mathrm{l}}^{\dagger 2})\Big]\Big\}|0\rangle. \end{aligned}\tag{9}$$

Figure 3(a) gives the curves intuitively showing that final components change with the relative phase φ, the x-axis represents relative phase φ and the y-axis represents generation probability. By setting different φ, the proportions of different components can be precisely adjusted.

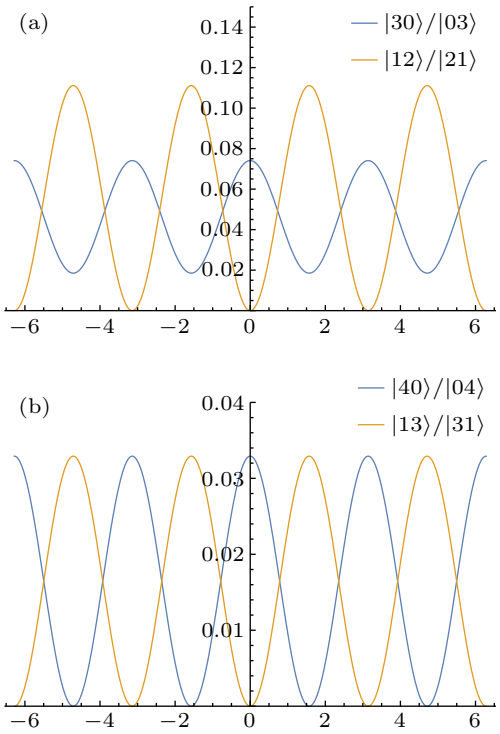


Fig. 3. Curves for final states from schemes for generating (a) three-photon and (b) four-photon path-entangled NOON states. The x-coordinate represents the relative phase φ, and y-coordinate represents the generation probability.

Similarly, after two SFWM processes, the superposition state of six-photon terms can be expressed as follows (also appended a normalization factor as $1/6$):

$$\begin{aligned}|\Psi\rangle_{\mathrm{u,l}} &= \frac{1}{6}g^3\cdot\left[\frac{1}{2}\left(a_{\mathrm{u}}^{\dagger 2} - a_{\mathrm{l}}^{\dagger 2}\right)\right]^3\\ &= g^3\cdot\left[\frac{1}{48}\left(a_{\mathrm{u}}^{\dagger 6} + a_{\mathrm{l}}^{\dagger 6} - 3a_{\mathrm{u}}^{\dagger 4}a_{\mathrm{l}}^{\dagger 2} + 3a_{\mathrm{u}}^{\dagger 2}a_{\mathrm{l}}^{\dagger 4}\right)\right]|0\rangle. \end{aligned}\tag{10}$$

In the NOON-state generation stage, firstly, as before, we introduce a proper relative phase φ between the two paths, and then the photons on both paths separately pass through

an MZI, used as a BS with fixed beam splitting ratio. The processes on the two paths may be performed simultaneously or in steps. Finally, by merging the upper and the lower path mode photons via a balanced BS, under the condition of detecting precisely one and only one single photon at detectors in each of these two heralding modes, we obtain the four-photon path-entangled NOON state from the outputs.

It seems that our proposal for generating four-photon NOON states is similar to that for three-photon NOON states; however, they differ significantly in principle. As mentioned above, in the scenario of the three-photon NOON state generator, the parameter θ of the MZI is to ensure process success, which means that only the $\sqrt{1/3}$-reflectivity BS leads to a pure NOON state. In contrast, as for the four-photon NOON state generator, since our quantum source is symmetric, as long as the symmetry of the structure is guaranteed, i.e., the equivalent reflectivity of both MZIs should be the same, a pure four-photon NOON state will be generated in theory. Specifically, the reflectivity could be arbitrary, other than 0 and 1; as a matter of fact, the probability amplitude of $|4::0\rangle$ is proportional to t^2r^4, where the meaning of $t(r)$ is the same as before. Therefore, $t^2 = 1/3$ is the optimal solution, referred to as the optimum beam splitting ratio, leading to the highest generation efficiency.

Following the procedure above, we set both AMZIs to the optimum beam splitting ratio and explore how the components of our final states from the four-photon NOON state generator vary with the relative phase φ.

After some simple calculation and derivation, the final state evolves into the form of

$$|\Psi\rangle_{\mathrm{u,l}} = g^3 \cdot \left\{ \frac{1}{54} \left[2\mathrm{e}^{\mathrm{i}3\varphi} \sin\varphi \left(a_{\mathrm{u}}^{\dagger 4} - a_{\mathrm{l}}^{\dagger 4} \right) - 4\mathrm{e}^{\mathrm{i}3\varphi} \cos\varphi \left(a_{\mathrm{u}}^{\dagger 3} a_{\mathrm{l}}^{\dagger} + a_{\mathrm{u}}^{\dagger} a_{\mathrm{l}}^{\dagger 3} \right) \right] \right\} |0\rangle. \tag{11}$$

An intuitive curve is shown in Fig. 3(b).

Obviously, by tuning the relative phase φ to zero or π, pure four-photon path-entangled NOON states will be obtained. In the following, we will show the evolution process in detail.

After the relative phase is set to zero, and the photons on both paths go through a BS with $\sqrt{2/3}$-reflectivity separately, when the two detectors are triggered by one and only one single photon, the quantum state of the remaining four photons is projected to

$$|\Psi\rangle_{\mathrm{u,l}} = g^3 \cdot \left[\frac{1}{48} \left(\frac{32}{9} a_{\mathrm{l}}^{\dagger 3} a_{\mathrm{u}}^{\dagger} - \frac{32}{9} a_{\mathrm{l}}^{\dagger} a_{\mathrm{u}}^{\dagger 3} \right) \right] |0\rangle. \tag{12}$$

Then, the photons remix at the symmetric BS, leading to the four-photon path-entangled NOON state described as

$$\begin{aligned} |\Psi\rangle_{\mathrm{u,l}} &= g^3 \cdot \left[\frac{1}{27} (a_{\mathrm{u}}^{\dagger 4} - a_{\mathrm{l}}^{\dagger 4}) \right] |0\rangle \\ &= g^3 \cdot \left[\frac{2\sqrt{6}}{27} (|4,0\rangle_{\mathrm{u,l}} - |0,4\rangle_{\mathrm{u,l}}) \right], \end{aligned} \tag{13}$$

with an overall generation efficiency of $(16/243)g^3$.

From Eq. (11) we also find that by setting the relative phase φ to an appropriate value such as $\pi/2$, pure states in the form of $|1::3\rangle$ will be generated with the same overall generation efficiency as $|4::0\rangle$ states in theory, which is proven to be more robust in the presence of losses.[26]

So far, we have introduced our scheme in detail. Actually, the scheme shown in Fig. 2 is a universal scheme for generating two-, three-, and four-photon NOON states. As listed in Table 1, by reconfiguring the relative phase φ and the MZIs to the all-reflect state or BSs with appropriate reflectivities, as long as the single-photon detected event occurs at the corresponding detector, the success of heralded maximal multi-photon path-entangled state generation is certified.

Table 1. Parameters and efficiency in generating different states via the chip shown in Fig. 2.

Target state	Initial state	φ	$\sin^2(\theta_1/2)$	$\sin^2(\theta_2/2)$	Overall efficiency
$\|2::0\rangle$	$g \cdot \left[(1/2)(a_{\mathrm{u}}^{\dagger 2} - a_{\mathrm{l}}^{\dagger 2}) \right] \|0\rangle$	0	1	1	g
$\|3::0\rangle$	$(1/2)g^2 \cdot \left[(1/2)(a_{\mathrm{u}}^{\dagger 2} - a_{\mathrm{l}}^{\dagger 2}) \right]^2 \|0\rangle$	$0/\pi$	1/3	1	$(4/27)g^2$
$\|4::0\rangle$	$(1/6)g^3 \cdot \left[(1/2)(a_{\mathrm{u}}^{\dagger 2} - a_{\mathrm{l}}^{\dagger 2}) \right]^3 \|0\rangle$	$0/\pi$	2/3	2/3	$(16/243)g^3$
$\|3::1\rangle$	$(1/6)g^3 \cdot \left[(1/2)(a_{\mathrm{u}}^{\dagger 2} - a_{\mathrm{l}}^{\dagger 2}) \right]^3 \|0\rangle$	$\pi/2$	2/3	2/3	$(16/243)g^3$

3. Discussion and conclusion

We have theoretically demonstrated a compact scheme for generating two-, three-, and four-photon NOON states on our reconfigurable photonic chip. Compared with the previous work,[20] our structure is fully integrated, including SFWM photon sources and path-entangled NOON state generators composed of a series of linear optical elements such as MZIs. Our proposal is of higher portability, integration and practicability with no extra requirements of off-chip bulk nonlinear crystals and optical elements. Furthermore, the two-photon sources on our chip have a higher spectral purity which ensures higher visibility of quantum interference for the phase sensing measurements. This scheme is feasible since all of the above-mentioned elements are demonstrated

with high performance. The AMZI-coupled micro-rings have been wildly studied in previous works by our group and other groups,[22–25] and high spectral purity is experimentally achieved.[22] For the liner optical circuits composed by MZIs, a large number of studies have proved them to be of high precision and reconfigurability.[27,28] Considering the generation rate of this heralded three-photon or four-photon NOON state, which actually provides four-photon or six-photon probability, a high generation rate is expected since the on-chip multi-photon quantum states with up to eight photons have been demonstrated.[29–31]

As for $N \geq 5$, the circumstances and outcomes of interference are more diverse while such a two-mode interference beam splitter has limited capacity for eliminating unwanted terms. In spite of this, adding quantum sources and regarding the generator as a universal linear optical network may be a potential method to achieve high-NOON states. As photon numbers increase, although the generation efficiency decrease, our generation efficiency is still productive. Furthermore, by harnessing our previous work,[22–24] the factor g can be optimized for higher overall generation efficiency. In summary, our heralded proposal is a new method for generating maximal multi-photon path-entangled states and has a wide range of applications in the field of quantum metrology and lithography, especially highly integrated, miniaturized and precision measurements as well as quantum sensing.

Acknowledgments

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