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Citation: Chin. Phys. B, 2022, 31 (**5**): 050306. DOI: 10.1088/1674-1056/ac2f36
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Chin. Phys. B, 2022, 31 (**3**): 030307. DOI: 10.1088/1674-1056/ac40f8

Effects of colored noise on the dynamics of quantum entanglement of a one-parameter qubit–qutrit system

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(Received 18 June 2021; revised manuscript received 12 September 2021; accepted manuscript online 13 October 2021)

We analyzed the effect of colored noise on the negativity dynamics of a hybrid system consisting of a qubit–qutrit and not interacting, prepared from the start in an entangled one-parameter state and acting with noise in local and non-local environments. In this pink and brown noise we investigated two different situations: in the first situation, the noise is produced by a bistable oscillator with an unknown exchange rate; however, in the second situation, the noise is generated by a set of bistable oscillators. We found that entanglement decreases with time to zero, and undergoes the phenomenon of sudden death and rebirth. The pink noise is more prone to entanglement than the brown noise and the non-local environment is more prone to entanglement than the local one. When the number of fluctuators is increased, entanglement decays faster and finally, for certain parameters of the initial state, the subsystems are not affected by the noise.

Keywords: entanglement, hybrid system, qutrit, colored noise

PACS: 03.67.Mn **DOI:** 10.1088/1674-1056/ac2f36

1. Introduction

Over many years, several studies have been done on the study of quantum correlations with the aim to control and reduce the phenomenon of decoherence, or either to maximize the coherence time or even to completely eliminate the decoherence in the system. Among these correlations we have quantum entanglement, which was defined for the first time by Schrödinger, as one of the most considered non-classical effects of quantum mechanics, which makes it possible to differentiate between the quantum world and the classical world. However, the inevitable interaction of subsystems with noise induces decoherence, which leads to the ineffectiveness of quantum memory in the superpositions of state.[1] When an entangled quantum system is in contact with its environment, it loses its entanglement and we consider it decoherence. This entanglement can be repaired using a probabilistic quantum operation called entanglement distillation.[2] In the same vein, Jin *et al.* demonstrated in 2016 that quantum resonance leads to the generation of entanglement, the region of the chaotic parameter leads to an increase in the generation speed, and the symmetries of the initial probability of distribution determine the final degree of entanglement.[3] Among these researchers we can cite Benedetti *et al.*, who in 2013 studied the dynamics of quantum correlations in colored noise and showed that the spectrum and the microscopic structure of noise are valid for the engineering of non-Gaussian colored environments. In addition, for a set of fluctuators, the colored noise is produced by a linear combination of random telegraph noise each with a specific switching rate and, for a single fluctuator, this rate is random.[4] In 2017, Sheng *et al.* worked on continuous variable quantum teleportation in the beam-wandering modeled atmosphere channel by taking the beam-wandering model as an example and studying the possibility of improving its distribution of entanglement from the atmospheric channel, and there they found that an entanglement pretreatment trial improves more practical entanglement distribution among free space channels.[5] This same group also worked on the decoy state quantum key distribution using the beam-wandering modeled atmosphere channel in 2016 and showed that atmospheric turbulence is not the only channel with a fluctuating loss rate, and that even in communication fibers of hundreds of kilometers, keeping the loss rate at a fixed level is a rather difficult problem.[6]

In other words, noise can either destroy or conserve quantum entanglement and quantum discord, so it is very significant to study the effect of diverse types of environment on the dynamics of quantum correlations in the subsystems, which can lead to sudden death and rebirth.[7] In other words, the interaction of the subsystems with noise causes decoherence and loss of quantum properties of the

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system.[8,9] Therefore, special attention was paid to the testing, features, and regulation of the dynamics of quantum entanglement and quantum discord in physical subsystems,[10] with quantum optics,[11–15] nuclear magnetic resonance,[16,17] physics of nanoparticles,[18,19] and biology.[20,21] When quantum noise is acting on two-part open quantum subsystems, quantum correlations can lead to sudden death and rebirth phenomena, imprisonment and transition phenomena.[7,22–26] For non-interacting qubits, these phenomena come from the non-Markovian regime of noise.[27] Several researchers have also worked on coherence, which is also fundamental for quantum information. Among these we cited Jianwei, who showed in 2020 that we cannot obtain new measures of coherence by a function acting on a given coherence measure, except in the case of qubit states.[28] Quantum entanglement has several properties including the monogamy relation, which is characterized by the distribution of entanglement in multipartite systems.[29] The relationships of monogamy and polygamy help to distribute entanglement in multipartite systems. These relationships of monogamy and polygamy are closer than those related to the i-th power of the entanglement-based measure in Renyi entropy.[30] Entanglement can also be generated between two distant qubits using entanglement exchange, and the entanglement power is therefore used to measure the average amount of entanglement exchanged over all possible pure initial states.[31] There is also a reference diagram and data making it possible to have a good multipartite EPR piloting in experimentation and which can advance the applications of quantum piloting in the field of quantum information processing.[32] In 2020 Si *et al.* showed how relativistic motion affects the quantum fluctuation of entanglement for two entangled Unruh–Dewitt detectors when one of them is accelerated and which is in interaction with the external scalar field neighbor. Hence, they found that the quantum fluctuation of entanglement increases first by Unruh thermal noise, then decreases directly when the acceleration reaches a very high value.[33] The hybrid system is more effective in the sense that unlike other bipartite systems (two qubits and two qutrits) and for specific regions, its initial entanglement is perfectly isolated from noise detrimental effects, without the need for any new strategy. In addition, the quantum properties of a hybrid quantum system can perfectly avoid environmental noise degrading effects. Entanglement is a fundamental resource which has several applications, namely, quantum information processing, quantum computing,[34] quantum cryptography,[34,35] and quantum teleportation.[36–38]

In 1925, $1/f^{\alpha}$ noise was discovered for the first time by Johnson when he was conducting analyses on the current oscillations in a thermionic tunnel.[39] Since then, $1/f^{\alpha}$ noise has been the subject of many studies. To date, $1/f^{\alpha}$ noise has been measured in semiconductors, semimetals, normal metals, superconductors, tunnel junctions, etc.[40,41] This noise, which comes from a set of fluctuations on the dynamics of entanglement and discord, is a function of parameter α. When α is small, these correlations are destroyed in a constant way, and for higher values of the parameter, these quantum correlations decrease with the oscillations. The more the number of fluctuations increases, the stronger the decoherence becomes. When the noise is described by bistable probabilistic fluctuators, a strong dependence on the number of probabilistic fluctuators for the dynamics of quantum correlations is found. In particular, for a single fluctuator, revivals appear for all considered values of α. However, the increase in the number of fluctuators leads to a behavior in agreement with the case of a set of bistable oscillators with fixed exchange rates.[42]

Random telegraph noise (RTN) represents the basis for construction of colored noise, which causes loss of coherence in quantum semiconductor circuits.[41,43,44] Colored noise is a combination of several sources of RTN with various exchange rates. The value of the parameter α determines the color of the noise. Thus, when $\alpha = 1$, the noise has a pink color of type $1/f$ which comes from a collection of RTN. When $\alpha = 2$, the spectrum is of type $1/f^2$ and the noise has a brown color in relation to Brownian motion.[45] When the system is associated with several bistable oscillators, with a determined probability of exchange rates, colored noise is obtained.[4] In the outline with several bistable oscillators, the presence of pink and brown noise leads to the phenomena of sudden death and rebirth.[46] The variations are due to the various numbers of decoherence gutters in the two outlines. By making an appropriate choice of probabilistic terms as a function of time in the Hamiltonian, we will describe the effects of both local and non-local environments on the dynamics of the system. In the first case, the subsystems are in an independent environment, while in the second case both subsystems are in a common environment. Environmental effects due to colored noise have been observed practically in electronic circuits at the nanometric scale, where only one electron tunneling arises to be infected by charge oscillations.[47,48]

The choice of our subject is motivated by the fact that several studies have been intensively conducted in the field, most of them using the same energy and the same coupling constant in the subsystems. However, they have used composite systems and did not take into account hybridization. Hybrid quantum systems seek to combine the strength of their constituents to respond to the fundamental contradictory requirements of quantum technology: fast and accurate systems controlled together with perfect protection from the environment, including the measurement apparatus, to achieve long-term quantum

coherence.[49] These systems are also very robust against the effects of the environment.

What will be the effect of colored noise on the dynamics of quantum entanglement in our system? Answering this question will be the subject of this work.

This work is organized as follows. In Section 2, we present the physical configuration of a qubit–qutrit acting with colored noise in independent and common environments. In Section 3, we report results and discussions, and we end the work in Section 4 with a conclusion.

2. Physical model

In this part of our work, we describe the configuration of one qubit and one qutrit, initially entangled, subjected to colored noise. Specifically, the local interaction and the non-local interaction between the two subsystems and environments are considered.

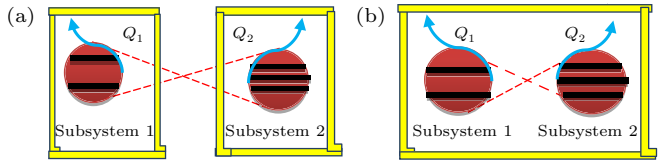


Fig. 1. (a) Qubit–qutrit in different environments (de), where every subsystem interacts independently with its own local environment. (b) Qubits–qutrit in common environment (ce), where the two subsystems interact with the same source of noise.

The red dashed lines represent the entanglement between the subsystems and the blue wavy arrows represent the interaction between the subsystems and the colored noise.

The dichotomy of our system is given by the Hamiltonian

$$H(t) = H_1(t) \otimes I_2 + I_1 \otimes H_2(t). \tag{1}$$

Here, $H_1(t)$ is the Hamiltonian of the qubit and $H_2(t)$ is the Hamiltonian of the single qutrit defined as

$$H_1(t) = [\varepsilon_1 + J_1 c_1(t)]\,\sigma_z^{(1)}, \quad H_2(t) = [\varepsilon_2 + J_2 c_2(t)]\,S_z^{(2)}, \tag{2}$$

$\sigma_z^{(1)}$, $I_{1(2)}$, and $S_z^{(2)}$ are the Pauli operator of the qubit, the identity matrix, and the spin operator of the qutrit, respectively. ε_i is the energy of each subsystem. J_i is the coupling constant of each subsystem with it environment. $c_i(t)$ is the probabilistic parameter of each subsystem, connected to the specification of the noise.

The corresponding evolution matrix of the subsystems is given by $U(c_1(t), c_2(t), t) = \exp(-\frac{\mathrm{i}}{\hbar}\int H(t')\,\mathrm{d}t')$ (hereafter $\hbar = 1$). The system is initially prepared in the one-parameter family state[50–52] given by

$$\begin{aligned}\rho_0 = \frac{p}{2}\big[&|0_1 0_2\rangle\langle 0_1 0_2| + |0_1 1_2\rangle\langle 0_1 1_2| + |1_1 2_2\rangle\langle 1_1 2_2| \\ &+ |1_1 1_2\rangle\langle 1_1 1_2| + |0_1 1_2\rangle\langle 1_1 1_2| + |1_1 1_2\rangle\langle 0_1 1_2| \\ &+ |0_1 0_2\rangle\langle 1_1 2_2| + |1_1 2_2\rangle\langle 0_1 0_2|\big] \\ &+ \frac{1-2p}{2}\big[|0_1 2_2\rangle\langle 0_1 2_2| + |0_1 2_2\rangle\langle 1_1 0_2| \\ &+ |1_1 0_2\rangle\langle 0_1 2_2| + |1_1 0_2\rangle\langle 1_1 0_2|\big].\end{aligned} \tag{3}$$

We will now introduce the effects of environmental noise in the representation of the dynamics of our system.

2.1. The $1/f^\alpha$ noise from only one fluctuator

The probabilistic parameter $c_i(t)$ varies between $+1$ and -1 in the case of a single oscillator.[4,53] Here our system represents the bodily model in which each subsystem acts with only one oscillator for an independent environment, or with the alike oscillator in a common environment. In the case where the environments are common, the subsystems are achieved by one spring of noise, and for this reason there is a single decoherence fairway. A number of sequence $c(t)$ is created for a value of the exchange rate taken from the probability of distribution $p_\alpha(\gamma)$. Abel *et al.* proved that the probabilistic attitude of $c(t)$ produces in the advancement of the state of the system a phase of a random noise[54] $\phi_1^*(t) = -J_1\phi_1(t) = -J_1\int_0^t c_1(t')\,\mathrm{d}t'$ and $\phi_2^*(t) = -J_2\phi_2(t) = -J_2\int_0^t c_2(t')\,\mathrm{d}t'$. Therefore $\phi_i^*(t)$ is a random variable which can take values in the interval $[-J_i t, J_i t]$. The autocorrelation function of the probabilistic parameter $c(t)$ is a function which decreases exponentially with time, $\langle \delta c(t)\delta c(0)\rangle = \exp(-2\gamma|t|)$, and its frequencies present the Lorentzian form $4\gamma/(\omega^2 + 4\gamma^2)$, where γ is the coefficient of autocorrelation. Hence, here the stochastic variable $c(t)$ characterizes a linear association of bistable oscillators $c(t) = \sum_{i=1}^{N_\mathrm{f}} c_i(t)$. N_f is the number of fluctuators, and each $c_i(t)$ has a spectrum $S(f)$, where

$$S(f) = \sum_{i=1}^{N_\mathrm{f}} S_i(f;\gamma_i) = \sum_{i=1}^{N_\mathrm{f}} \frac{\gamma_i}{\gamma_i^2 + 4\pi^2 f} \propto \frac{1}{f^\alpha}. \tag{4}$$

If γ is a constant the noise is referred to as RTN, and if γ is different from a constant the noise is referred to as colored noise.

2.2. The $1/f^\alpha$ noise from a set of fluctuators

This noise comes from several sources of RTN with exchange rates given by $\{\gamma_i\}_{i=1}^N$. The noise spectrum comes from the association of a large number of fluctuators with the system. Each of these fluctuators is characterized by a specific exchange rate, chosen from the probability $p_\alpha(\gamma)$ in the range $[\gamma_\mathrm{a}, \gamma_\mathrm{b}]$.[55–57] $p_\alpha(\gamma)$ is the probability of the exchange rate and its value depends on the type of noise,

$$P_\alpha(\gamma) = \begin{cases} \dfrac{1}{\gamma \ln(\gamma_\mathrm{b}/\gamma_\mathrm{a})}, & \alpha = 1, \\ \dfrac{(\alpha-1)}{\gamma^\alpha}\left[\dfrac{(\gamma_\mathrm{a}\gamma_\mathrm{b})^{\alpha-1}}{\gamma_\mathrm{b}^{\alpha-1} - \gamma_\mathrm{a}^{\alpha-1}}\right], & 1 < \alpha \le 2. \end{cases} \tag{5}$$

For this reason, to have a $1/f^{\alpha}$ frequency, it is important to take a great number of oscillators, and each γ_i must be a probability sample $p_\alpha(\gamma_i)$ in the interval $[\gamma_a, \gamma_b]$. The dynamic is obtained by averaging the density operator of the qubit–qutrit for the specific parameters of the exchange rate on the γ in the interval $[\gamma_a, \gamma_b]$.

(I) Different environments Depending on the approach used in Ref. [58], the density matrix $\rho_{de}(\gamma_1,\gamma_2,t)$ of the system at time t in local environments of colored noise is given by

$$\overline{\rho_{de}(\gamma_1,\gamma_2,t)} = \int_{\gamma_a}^{\gamma_b}\int_{\gamma_a}^{\gamma_b} \rho_{de}(\gamma_1,\gamma_2,t)P_\alpha(\gamma_1)P_\alpha(\gamma_2)\,d\gamma_1\,d\gamma_2 = \begin{pmatrix} \frac{p}{2} & 0 & 0 & 0 & 0 & \Lambda_{16} \\ 0 & \frac{p}{2} & 0 & 0 & \Lambda_{25} & 0 \\ 0 & 0 & \frac{1-2p}{2} & \Lambda_{34} & 0 & 0 \\ 0 & 0 & \Lambda_{34}^{+} & \frac{1-2p}{2} & 0 & 0 \\ 0 & \Lambda_{25}^{+} & 0 & 0 & \frac{p}{2} & 0 \\ \Lambda_{16}^{+} & 0 & 0 & 0 & 0 & \frac{p}{2} \end{pmatrix}, \tag{6}$$

where

$$\Lambda_{16} = \int_{\gamma_a}^{\gamma_b}\int_{\gamma_a}^{\gamma_b} \frac{p}{2}\exp\{-2i(\varepsilon_1+\varepsilon_2)t\}D_{2J_1}(\gamma_1,t)D_{2J_2}(\gamma_2,t)P_\alpha(\gamma_1)P_\alpha(\gamma_2)\,d\gamma_1\,d\gamma_2,$$

$$\Lambda_{16}^{+} = \int_{\gamma_a}^{\gamma_b}\int_{\gamma_a}^{\gamma_b} \frac{p}{2}\exp\{2i(\varepsilon_1+\varepsilon_2)t\}D_{2J_1}(\gamma_1,t)D_{2J_2}(\gamma_2,t)P_\alpha(\gamma_1)P_\alpha(\gamma_2)\,d\gamma_1\,d\gamma_2,$$

$$\Lambda_{25} = \int_{\gamma_a}^{\gamma_b}\int_{\gamma_a}^{\gamma_b} \frac{p}{2}\exp\{-2i\varepsilon_1 t\}D_{1J_1}(\gamma_1,t)P_\alpha(\gamma_1)P_\alpha(\gamma_2)\,d\gamma_1\,d\gamma_2,$$

$$\Lambda_{25}^{+} = \int_{\gamma_a}^{\gamma_b}\int_{\gamma_a}^{\gamma_b} \frac{p}{2}\exp\{2i\varepsilon_1 t\}D_{1J_1}(\gamma_1,t)P_\alpha(\gamma_1)P_\alpha(\gamma_2)\,d\gamma_1\,d\gamma_2,$$

$$\Lambda_{34} = \int_{\gamma_a}^{\gamma_b}\int_{\gamma_a}^{\gamma_b} \frac{(1-2p)}{2}\exp\{2i(\varepsilon_2-\varepsilon_1)t\}D_{2J_1}(\gamma_1,t)D_{2J_2}(\gamma_2,t)P_\alpha(\gamma_1)P_\alpha(\gamma_2)\,d\gamma_1\,d\gamma_2,$$

$$\Lambda_{43}^{+} = \int_{\gamma_a}^{\gamma_b}\int_{\gamma_a}^{\gamma_b} \frac{(1-2p)}{2}\exp\{-2i(\varepsilon_2-\varepsilon_1)t\}D_{2J_1}(\gamma_1,t)D_{2J_2}(\gamma_2,t)P_\alpha(\gamma_1)P_\alpha(\gamma_2)\,d\gamma_1\,d\gamma_2.$$

(II) Common environment In this case the density matrix $\rho_{ce}(\gamma,t)$ of the system at time t is given by

$$\overline{\rho_{ce}(\gamma,t)} = \int_{\gamma_a}^{\gamma_b} \rho_{ce}(\gamma,t)P_\alpha(\gamma)\,d\gamma = \begin{pmatrix} \frac{p}{2} & 0 & 0 & 0 & 0 & \Lambda_{16} \\ 0 & \frac{p}{2} & 0 & 0 & \Lambda_{25} & 0 \\ 0 & 0 & \frac{(1-2p)}{2} & \frac{(1-2p)}{2}\exp\{2i(\varepsilon_2-\varepsilon_1)t\} & 0 & 0 \\ 0 & 0 & \frac{(1-2p)}{2}\exp\{-2i(\varepsilon_2-\varepsilon_1)t\} & \frac{(1-2p)}{2} & 0 & 0 \\ 0 & \Lambda_{25}^{+} & 0 & 0 & \frac{p}{2} & 0 \\ \Lambda_{16}^{+} & 0 & 0 & 0 & 0 & \frac{p}{2} \end{pmatrix}, \tag{7}$$

where

$$\Lambda_{16} = \int_{\gamma}^{\gamma'} \frac{p}{2}\exp\{-2i(\varepsilon_1+\varepsilon_2)t\}D_{4J}(\gamma,t)P_\alpha(\gamma)\,d\gamma, \quad \Lambda_{16}^{+} = \int_{\gamma_a}^{\gamma_b} \frac{p}{2}\exp\{2i(\varepsilon_1+\varepsilon_2)t\}D_{4J}(\gamma,t)P_\alpha(\gamma)\,d\gamma,$$

$$\Lambda_{25} = \int_{\gamma_a}^{\gamma_b} \frac{p}{2}\exp\{-2i\varepsilon_1 t\}D_{2J}(\gamma,t)P_\alpha(\gamma)\,d\gamma, \quad \Lambda_{25}^{+} = \int_{\gamma_a}^{\gamma_b} \frac{p}{2}\exp\{2i\varepsilon_1 t\}D_{2J}(\gamma,t)P_\alpha(\gamma)\,d\gamma.$$

2.3. Estimators of quantum entanglement

Quantum entanglement is estimated by negativity[59] defined as

$$N = \sum_i |\lambda_i| - 1, \tag{8}$$

where λ_i are the eigenvalues of the partial transpose of the density operator of the system. Negativity is equal to zero for separable states, and to one for maximally entangled states. This negativity has recently been extended to tripartite identical systems.[60]

3. Results and discussion

Here we give the analytical and simulation outcomes of negativity when our two subsystems are affected by colored noise in local and non-local environments. The explicit derivation of density matrices is given in Appendix A.

When $\alpha = 1$, i.e., the $1/f$ spectrum, the noise is referred to as pink noise.

(I) For independent environments Because the system is in contact with colored noise, the negativity of the system is corrupted by the effects of this noise, and this is explained by the decrease in this negativity. Here the entanglement presents an oscillatory movement. We observed that negativity decreases exponentially over time to zero and then undergoes rebirth and sudden death phenomena with a decrease in amplitude. These oscillations clearly demonstrate the non-Markovian character of colored noise. For an initial parameter p $(0 < p < 1/3)$ we observed that the initial amount of entanglement decreases with increasing p, and for $(1/3 < p \leq 1/2)$ it increases with increasing p. Therefore, we can conclude that the entanglement here depends on the parameter of the initial state p.

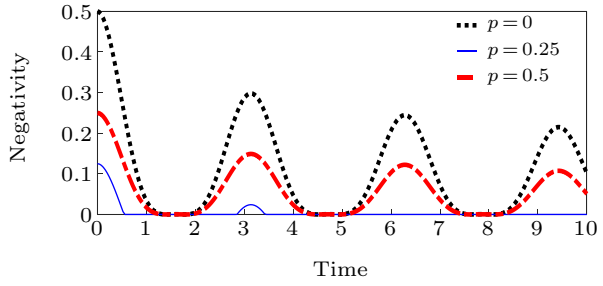


Fig. 2. Dynamics of entanglement for qubit–qutrit acting with pink noise in independent environments.

(II) For common environment In Fig. 3, we find that entanglement exists and decays in an oscillating manner over time and the amount of entanglement initially present in the system also decreases over time. We also observe that with only one decoherence gutters, a rebirth of entanglement appears. Here there are two remarkable revivals of entanglement: one with a slow amplitude and the other one with a height amplitude. If the back action is strong enough, the revival amplitudes of entanglement will be high. We also found that in the opposition with the case of local environments where the negativity is affected by the noise when $0 < p < 1/3$, here the system does not perceive the effects of the noise. For $p = 0$ and for $p = 0.25$, entanglement does not vary; it remains constant for all values of time. This clearly demonstrates once more that different environments affecting the subsystems contribute more to their entanglement than a common environment acting on them.

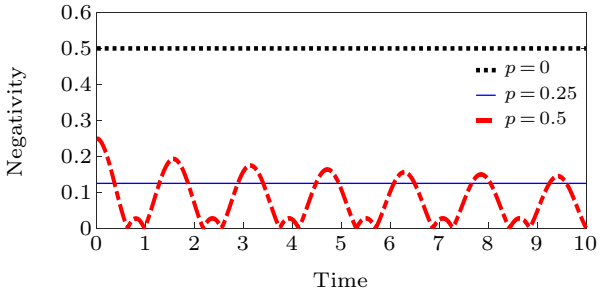


Fig. 3. Dynamics of entanglement for qubit–qutrit acting with pink noise in common environments.

Another interesting case in colored noise is brown noise, when $\alpha = 2$.

(I) For different environments In Fig. 4, we observe the same phenomena as in Fig. 2, but here, quantum entanglement undergoes the rebirth and sudden death phenomena with constant amplitudes. We also find that entanglement is an oscillating function of time. This oscillation occurs periodically with a constant amplitude. Given that there are several types of initial states according to the configuration of the systems, the dynamics of the entanglement of the system therefore depend not only on the environment but also on the parameter of the initial state of this system. This is in agreement with the results of Arjmandi, who worked on the investigation of quantum correlations in the Dirac field and found that the controllable factor of the initial state allows changes to be determined in quantum correlations.[61]

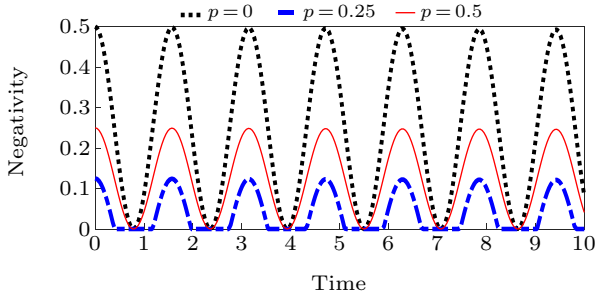


Fig. 4. Dynamics of entanglement for qubit–qutrit acting with brown noise in independent environments.

(II) For common environment Figure 5 presents the same phenomena as observed in Fig. 3, but here we see that

the amplitudes of the oscillations are constant. We also find that the entanglement of the system does not perceive the presence of brown noise when $0 < p < 1/3$, which is not the case in different environments, so entanglement is more prominent in a common environment than in independent ones.

We can see that in pink noise, negativity decreases in a damped oscillation with a slow decrease in amplitude, while in brown noise, the amplitudes of the oscillations are constant; we can therefore conclude that brown noise preserves entanglement more than pink noise.

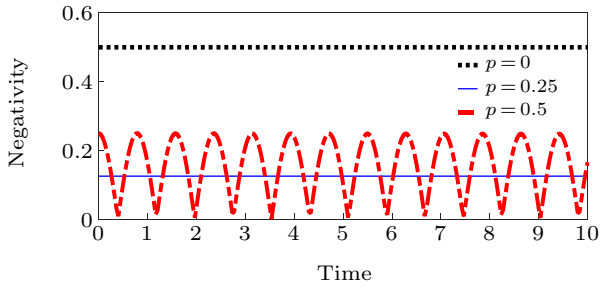


Fig. 5. Dynamics of entanglement for qubit–qutrit acting with brown noise in common environments.

For the case of a collection fluctuator we have the following results.

(I) For independent environments In Fig. 6, we study the evolution of quantum entanglement for 20 fluctuators, and we find that there is suppression of quantum entanglement at long times. We also see the phenomenon of sudden death and revival. Entanglement also decreases in an oscillating and periodic manner with decreasing picks. For all the values of p chosen, one observes the same behavior of oscillations except that these oscillations do not have the same amplitude.

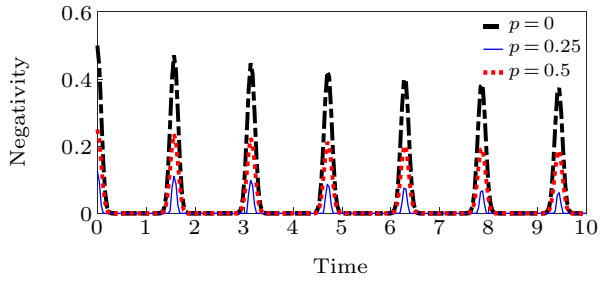


Fig. 6. Dynamics of entanglement for qubit–qutrit acting with a set of N bistable oscillators in independent environments ($N = 20$).

In Fig. 7 we plot the evolution of quantum entanglement for 100 fluctuators and we find that, when the number of oscillators increases, entanglement decreases faster. We thus see that the knowledge of the frequency is not adequate to describe the behavior of entanglement.

In Fig. 8 we plot the evolution of quantum entanglement for 20 and 100 fluctuators and we find that, when our system acts with a set of oscillators, the dynamics depend also on the frequency of the noise: as the number of fluctuators is increased, the negativity decreases faster. The phenomena of sudden death and rebirth arise for quantum entanglement and, as the number of oscillators increases, the amplitudes of the peaks decrease.

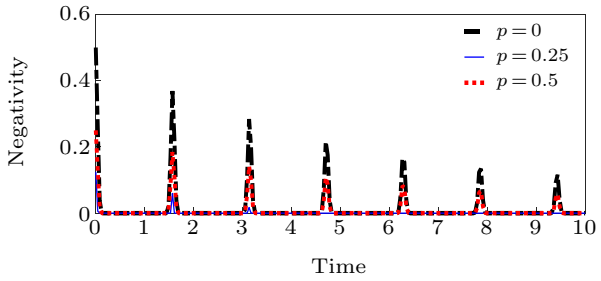


Fig. 7. Dynamics of entanglement for qubit–qutrit acting with a set of N bistable oscillators in independent environments ($N = 100$).

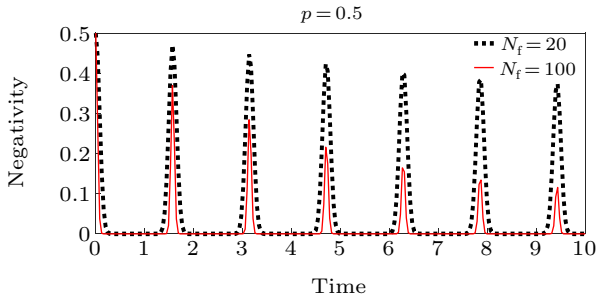


Fig. 8. Dynamics of entanglement for qubit–qutrit acting with a set of N bistable oscillators in independent environments ($N = 20$ and $N = 100$).

(II) For common environment In Fig. 9, entanglement shows the phenomena of sudden death and rebirth. The negativity decay to zero at a finite time, and remains constant up to a certain time and then goes to zero monotonically. We also note that for certain values of p, the system does not perceive the effects of the environment, i.e., that the negativity of the system remains constant. By comparing Figs. 9 and 6, we can conclude that a common environment is more robust to the effects of the environment than different environments.

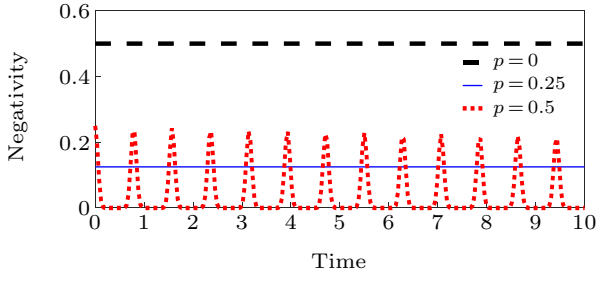


Fig. 9. Dynamics of entanglement for qubit–qutrit acting with a set of N bistable oscillators in common environments ($N = 20$).

In Fig. 10, we observe the same phenomena as in Fig. 9, but here the amplitude of the pick decreases a little faster. For certain values of the parameter p, we see that quantum entanglement is not perceived the presence of colored noise. Compared to different environments, common environments better preserve the entanglement of the system.

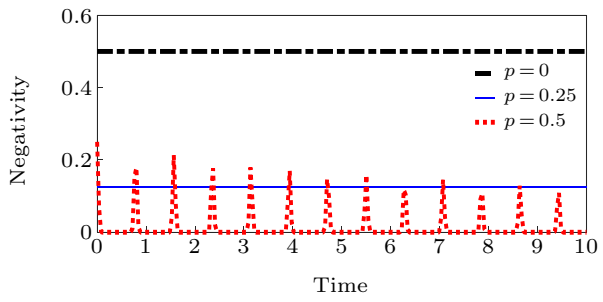


Fig. 10. Dynamics of entanglement for qubit–qutrit acting with a set of N bistable oscillators in common environments ($N=100$).

4. Conclusion

In our work we have studied the effect of colored noise on the entanglement dynamics of a non-interacting qubit–qutrit system coupled in different and in common environments. We assessed negativity by using analytical and numerical methods, and we found that when our system interacts with a colored noise environment, entanglement decreases with time. This decrease in entanglement is done in several ways: for an initial parameter p $(0<p<1/3)$ we observed that the initial amount of entanglement decreases with increasing p, and for $(1/3<p\leq 1/2)$ it increases with increasing p. In some cases, there are two notable revivals of entanglement, one with a slow amplitude and the other one with a height amplitude due to the fact that the back action is strong enough, while in other cases, quantum entanglement exhibits revivals and sudden death phenomena with constant amplitudes.

In addition, we found that entanglement may be degraded with time, reaching a zero value asymptotically, or it can decay to zero in a finite time; however, in certain cases it is possible to see a rebirth of entanglement, i.e., entanglement can decrease to zero and then revive.

On the other hand, in certain cases the results showed the attenuation of entanglement over a prolonged time. We also found that when the system interacts with one bistable oscillator, entanglement exhibits oscillations for pink and brown noise and that the effect of a non-local environment is to better protect entanglement and to increase the amount of rebirth comparatively to the case of different environments. The amplitude of the oscillations decreases faster in the case of pink noise than in the case of brown noise.

When our system interacts with a set of bistable oscillators, entanglement shows phenomena of sudden death and rebirth. The action of local or non-local environments also has several consequences on the robustness of entanglement, and it is in agreement with the outcome collected by Refs. [62,63]. The outcome of this work also shows that the behavior of entanglement is forced by the frequency of the noise and by the number of oscillators used to shape it. When the number of decoherence channels is increased, the loss of information becomes very great and we do not observe rebirth. Finally, we found that there are some parameters of the initial state for which the system does not perceive the presence of the external environment, and this agrees with previous results obtained in Ref. [64].

Appendix A: Explicit derivation of the system density matrix

Given the Hamiltonian in Eqs. (1) and (2), the corresponding evolution operator of the system reads

$$
\begin{aligned}
U &= U_1(c_1(t),t)\otimes U_2(c_2(t),t)\\
&= \begin{bmatrix}
\exp[-\mathrm{i}(M+N)] & 0 & 0 & 0 & 0 & 0\\
0 & \exp[-\mathrm{i}(N)] & 0 & 0 & 0 & 0\\
0 & 0 & \exp[\mathrm{i}(M-N)] & 0 & 0 & 0\\
0 & 0 & 0 & \exp[-\mathrm{i}(M-N)] & 0 & 0\\
0 & 0 & 0 & 0 & \exp[\mathrm{i}(N)] & 0\\
0 & 0 & 0 & 0 & 0 & \exp[\mathrm{i}(M+N)]
\end{bmatrix}
\end{aligned}
\tag{A1}
$$

with $M=\phi_2+\theta_2$ and $N=\phi_1+\theta_1$.

From Eq. (6), $\rho_{\mathrm{de}}(\gamma_1,\gamma_2,t)$ is given by

$$
\rho_{\mathrm{de}}(\gamma_1,\gamma_2,t)=\begin{pmatrix}
\frac{p}{2} & 0 & 0 & 0 & 0 & \rho_{16}(\gamma_1,\gamma_2,t)\\
0 & \frac{p}{2} & 0 & 0 & \rho_{25}(\gamma_1,\gamma_2,t) & 0\\
0 & 0 & \frac{1-2p}{2} & \rho_{34}(\gamma_1,\gamma_2,t) & 0 & 0\\
0 & 0 & \rho_{34}^{+}(\gamma_1,\gamma_2,t) & \frac{1-2p}{2} & 0 & 0\\
0 & \rho_{25}^{+}(\gamma_1,\gamma_2,t) & 0 & 0 & \frac{p}{2} & 0\\
\rho_{16}^{+}(\gamma_1,\gamma_2,t) & 0 & 0 & 0 & 0 & \frac{p}{2}
\end{pmatrix},
\tag{A2}
$$

where

$$\rho_{16}(\gamma_1,\gamma_2,t)=\frac{p}{2}\exp\{-2\mathrm{i}(\varepsilon_1+\varepsilon_2)t\}D_{2J_1}(\gamma_1,t)D_{2J_2}(\gamma_2,t),$$
$$\rho_{16}^{+}(\gamma_1,\gamma_2,t)=\frac{p}{2}\exp\{2\mathrm{i}(\varepsilon_1+\varepsilon_2)t\}D_{2J_1}(\gamma_1,t)D_{2J_2}(\gamma_2,t),$$
$$\rho_{34}(\gamma_1,\gamma_2,t)=\frac{1-2p}{2}\exp\{-2\mathrm{i}(\varepsilon_2-\varepsilon_1)t\}D_{2J_1}(\gamma_1,t)D_{2J_2}(\gamma_2,t),$$
$$\rho_{34}^{+}(\gamma_1,\gamma_2,t)=\frac{1-2p}{2}\exp\{2\mathrm{i}(\varepsilon_2-\varepsilon_1)t\}D_{2J_1}(\gamma_1,t)D_{2J_2}(\gamma_2,t),$$
$$\rho_{25}^{+}(\gamma_1,\gamma_2,t)=\frac{p}{2}\exp\{2\mathrm{i}\varepsilon_1 t\}D_{1J_1}(\gamma_1,t),$$
$$\rho_{25}(\gamma_1,\gamma_2,t)=\frac{p}{2}\exp\{-2\mathrm{i}\varepsilon_1 t\}D_{1J_1}(\gamma_1,t).$$

From Eq. (5), $\rho_{\mathrm{ce}}(\gamma,t)$ is given by

$$\rho_{\mathrm{ce}}(\gamma,t)=\begin{pmatrix}\frac{p}{2}&0&0&0&0&\rho_{16}(\gamma,t)\\0&\frac{p}{2}&0&0&\rho_{25}(\gamma,t)&0\\0&0&\frac{1-2p}{2}&\rho_{34}(\gamma,t)&0&0\\0&0&\rho_{34}^{+}(\gamma,t)&\frac{1-2p}{2}&0&0\\0&\rho_{25}^{+}(\gamma,t)&0&0&\frac{p}{2}&0\\\rho_{16}^{+}(\gamma,t)&0&0&0&0&\frac{p}{2}\end{pmatrix},\tag{A3}$$

where

$$\rho_{16}(\gamma,t)=\frac{p}{2}\exp\{-2\mathrm{i}(\varepsilon_1+\varepsilon_2)t\}D_{4J}(\gamma,t),$$
$$\rho_{16}^{+}(\gamma,t)=\frac{p}{2}\exp\{2\mathrm{i}(\varepsilon_1+\varepsilon_2)t\}D_{4J}(\gamma,t),$$
$$\rho_{34}(\gamma,t)=\frac{1-2p}{2}\exp\{2\mathrm{i}(\varepsilon_2-\varepsilon_1)t\},$$
$$\rho_{34}^{+}(\gamma,t)=\frac{1-2p}{2}\exp\{-2\mathrm{i}(\varepsilon_2-\varepsilon_1)t\},$$
$$\rho_{25}^{+}(\gamma,t)=\frac{p}{2}\exp\{2\mathrm{i}\varepsilon_1 t\}D_{2J}(\gamma,t),$$
$$\rho_{25}(\gamma,t)=\frac{p}{2}\exp\{-2\mathrm{i}\varepsilon_1 t\}D_{2J}(\gamma,t).$$

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