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Kinetic Alfvén waves in a deuterium-tritium fusion plasma with slowing-down distributed α-particles

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The dispersion relation and damping rate of kinetic Alfvén waves (KAWs) in a deuterium-tritium fusion plasma with slowing-down distributed α-particles are investigated using the kinetic theory. The variations of wave frequency and damping rate with respect to the α concentration (n_α/n_e) and perpendicular wave number ($k_\perp$) are studied from a numerical way. The results show that the fluctuation of α concentration slightly affects the frequency and damping rate of KAWs at low n_α/n_e. In addition, the frequency and the damping rate increase as the $k_\perp$ and the background temperature T_e increase. For comparison, the calculations are performed also in the case of α-particles following an equivalent Maxwellian distribution. For a given $k_\perp$, the value of the frequency obtained in the slowing-down distribution case is smaller than that obtained in the Maxwellian distribution case. Conversely, the value of the damping rate obtained in the slowing-down distribution case is slightly larger than that obtained in the Maxwellian distribution case.

Keywords: kinetic Alfvén wave, dispersion relation, deuterium-tritium fusion plasma

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1. Introduction

Kinetic Alfvén waves (KAWs) are low-frequency electromagnetic waves invoked by Alfvén waves through mode conversion, playing an extremely important role in many dynamical processes occurring in space,[1] astrophysics[2] and laboratory[3,4] plasmas. Theoretical studies of KAWs in burning plasmas are aimed at understanding heating plasmas[5–7] and driving current[8,9] in direction of ambient magnetic field B_0. Due to the presence of local spatial charge, ions can no longer move along the magnetic lines. Electrons, however, remain attached to the magnetic field lines because of their small Larmor radius. Therefore, KAWs have a nonzero electric field parallel to B_0. Furthermore, the nonzero electric field enables the Landau resonance, where charged particles can gain or lose energy by interacting with the waves. Undoubtedly, KAWs are essential for the energy transfer processes in burning plasmas. In fact, KAWs were originally proposed by Hasegawa and Chen[10] for the purpose of fusion plasma heating. Subsequently, heating of tokamak plasmas by KAWs was confirmed in the experiments of Wesien *et al.*[11] Understanding the generation, propagation and interaction of KAWs is critical to unraveling and predicting the physics associated with these fundamental processes.

In general, KAWs are small-scale dispersive Alfvén waves with perpendicular wavelength comparable to the thermal ion gyroradius ρ_i, but with a parallel wavelength much longer than the ion inertial length λ_i. On the one hand, dispersive properties are the fundamental subject of Alfvén waves in plasma physics. The propagation, instability, refraction and absorption of plasma waves can be studied based on the dispersion. A lot of work related to the dispersion of KAWs has been carried out in various plasma environments.[12–16] Recently, Belova *et al.*[17] proposed an energy-channeling mechanism to explain the flattening of electron temperature profiles in NSTX tokamak by investigating the dispersion relation of KAWs modified by beam-ion effects. On the other hand, since the perpendicular wavelength plays an important role for KAWs, there should be a bound on $k_\perp$. Perkins *et al.*[18] estimated that the perpendicular wave number of the Tokamak Fusion Test Reactor (TFTR) should vary within the certain range of $0 < k_\perp < 1.5$ cm^{-1}. Soon thereafter, the experimental measurements of the perpendicular wave number in the TFTR plasmas were reported.[19–22]

In D-T fusion plasmas, the energy of fusion-born α-particles is typically two orders of magnitude higher than that of the D-T fuel ions. Compared to thermal particles, the α-particles usually deviate from Maxwellian distribution, and obey a slowing-down distribution when correctly described.[23–27] In addition, α-particles are the dominant minority particles due to their low density, large kinetic energy and high pressure in the fusion reactors. The α-particles are expected to play a key role in current driving, plasma heating, energy transfer and plasma stability. Therefore, it is particularly important to study α-particles physics in burning plas-

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mas. Li *et al.*,[28] using kinetic theory, calculated the frequencies ($\omega_r \sim 10^6$ s^{-1}) and damping rate ($\gamma_e \sim 10^4$ s^{-1}) of KAWs in the absence of α-particles and concluded that electron Landau damping is important for KAWs. Later, Ding *et al.*[29] further investigated the effects of α-particles on KAWs and found that α-particles made the propagation of KAWs to the central region of the tokamak plasma more difficult and reduced the KAWs amplitude.

In the present paper, the dispersion and damping rate of KAWs in a D-T fusion plasma with the slowing-down distributed α-particles, the Maxwellian distributed ions (D and T) and electrons are investigated under the framework of low frequency and short vertical wavelength assumptions, and the kinetic dispersion relation for a low-β plasma is taken into account. The paper is organized as follows. The detailed calculations of the dispersion of the KAWs are presented in Section 2. In Section 3, the analytical results of the damping rate of KAWs are given. The numerical calculations and discussions based on the D-T fusion parameters are described in Section 4. Finally, the main results are summarized in Section 5.

2. Dispersion of the KAWs

Assume that all bulk-plasma populations are expressed by the Maxwellian distribution functions

$$f_j = \frac{1}{v_{\mathrm{T}j}^3 \pi^{3/2}} \exp\left(-\frac{v^2}{v_{\mathrm{T}j}^2}\right), \tag{1}$$

where $v_{\mathrm{T}j} = \sqrt{2k_{\mathrm{B}}T_j/m_j}$ is the thermal velocity for each species j (D ions, T ions and e electrons) with k_{B} being the Boltzmann constant, m_j and T_j are the mass and temperature of the jth species.

In contrast, the α-particles, born at 3.52 MeV, are slowed down by collisions between the α-particles and the background plasmas. Thus, the α-particles are appropriately described by the slowing-down distribution function[24]

$$f_\alpha = \frac{1}{4\pi I}\frac{1}{v_{\mathrm{c}}^3 + v^3}\left(|v| < v_\alpha\right), \tag{2}$$

where

$$v_{\mathrm{c}} = \left(\frac{3\sqrt{\pi}}{4}\frac{m_{\mathrm{e}}}{m_\alpha}Z_{\mathrm{I}}\right)^{1/3} v_{\mathrm{Te}} \tag{3}$$

and

$$I = \frac{1}{3}\ln\left(\frac{v_\alpha^3}{v_{\mathrm{c}}^3} + 1\right) \tag{4}$$

are the slowing-down critical velocity and the constant of normalization, respectively. $Z_{\mathrm{I}} = m_\alpha \sum_{j=\mathrm{D,T}} n_j\left(Z_j^2/m_j\right)/n_{\mathrm{e}}$ is an effective charge, with Z_j and n_j representing the charge number and density of species j, and $Z_{\mathrm{I}} = 5/3$ for a mixture of 50% D and 50% T; v_α denotes the birth velocity of α-particles corresponding to an energy $E_\alpha = m_\alpha v_\alpha^2/2 = 3.5$ MeV for fusion α-particles.

Before proceeding further, we must give an effective slowing-down temperature T_{eff}, defined as $3k_{\mathrm{B}}T_{\mathrm{eff}} = \int m_\alpha v^2 f_\alpha \mathrm{d}^3 v$. In what follows, we will proceed to investigate the effect of the α slowing-down distribution on the physical properties of KAWs compared to that of the α Maxwellian distribution on it at the same slowing-down temperature.

For a low frequency wave mode with a long parallel wavelength in a low-β plasma, the nondiagonal components of the dielectric tensor can be neglected. Heretofore, the kinetic dispersion equation is given by[12,14]

$$D = \varepsilon_{zz}\left(\varepsilon_{xx} - \frac{c^2 k_\parallel^2}{\omega^2}\right) - \varepsilon_{xx}\frac{c^2 k_\perp^2}{\omega^2} = 0. \tag{5}$$

In Eq. (5), $k_{\parallel(\perp)}$ refers to the parallel (perpendicular) wave number along (across) the background magnetic field, and c is the speed of light in vacuum. Moreover, the dielectric tensor elements along and across the unperturbed ambient magnetic field are expressed as[30]

$$\varepsilon_{xx} = 1 + \sum_j \frac{\omega_{pj}^2}{\omega}\int v_\perp \mathrm{d}v^3 \sum_{n=-\infty}^{\infty}\frac{n^2}{\mu^2}\frac{J_n^2(\mu)}{\omega - k_\parallel v_\parallel - n\Omega_j}\frac{\partial f_j}{\partial v_\perp}, \tag{6}$$

$$\varepsilon_{zz} = 1 + \sum_j \frac{\omega_{pj}^2}{\omega}\int v_\perp \mathrm{d}v^3 \sum_{n=-\infty}^{\infty}\frac{v_\parallel}{v_\perp}\frac{J_n^2(\mu)}{\omega - k_\parallel v_\parallel - n\Omega_j}\frac{\partial f_j}{\partial v_\parallel}, \tag{7}$$

where $\mu = k_\perp v_\perp/\Omega_j$, and $\Omega_j = \left|Z_j\right| eB_0/m_j$ are, respectively, the gyrofrequency and plasma frequency of j species. $J_n(\mu) = \sum_{k=0}^{\infty}(-1)^k(\mu/2)^{2k+n}/k!\Gamma(k+n+1)$ is the j-th order Bessel function of the first kind. The sum on j extends over all plasma species.

Substituting $J_{-n}^2(\mu) = J_n^2(\mu)$ and $\mathrm{d}v^3 = 2\pi v_\perp \mathrm{d}v_\perp \mathrm{d}v_\parallel$ into Eq. (6), we have

$$\varepsilon_{xx} = 1 + \sum_j \chi_j, \tag{8}$$

with

$$\chi_j = \frac{2\pi\omega_{pj}^2}{\omega}\int\int v_\perp^2 \mathrm{d}v_\perp \mathrm{d}v_\parallel \sum_{n=1}^{\infty}\frac{n^2 J_n^2(\mu)}{\mu^2} \times\left(\frac{1}{\omega - k_\parallel v_\parallel - n\Omega_j} + \frac{1}{\omega - k_\parallel v_\parallel + n\Omega_j}\right)\frac{\partial f_j}{\partial v_\perp}. \tag{9}$$

For Maxwellian distributed deuterium ions, tritium ions and electrons, i.e., $j = $ D, T and e, Eq. (9) can be rewritten as

$$\chi_j = \frac{\omega_{pj}^2}{\omega^2}\frac{\omega}{k_\parallel v_{\mathrm{T}j}}\sum_{n=1}^{\infty}\frac{n^2 A_n\left(k_\perp^2\rho_j^2\right)}{k_\perp^2\rho_j^2} \times\left[Z\left(\frac{\omega - n\Omega_j}{k_\parallel v_{\mathrm{T}j}}\right) + Z\left(\frac{\omega + n\Omega_j}{k_\parallel v_{\mathrm{T}j}}\right)\right]. \tag{10}$$

In Eq. (10), $A_n\left(k_\perp^2\rho_j^2\right) = \exp\left(-k_\perp^2\rho_j^2\right) I_n\left(k_\perp^2\rho_j^2\right)$, and $I_n\left(k_\perp^2\rho_j^2\right)$ is the modified Bessel function with $\rho_j = v_{\mathrm{T}j}/\Omega_j$. $Z(x)$ denotes the usual plasma dispersion function with the asymptotic forms as $Z(x) \approx -1/x - 1/2x^3$ for $x \gg 1$. For the low frequency shear Alfvén wave, $\Omega_j \gg \omega, k_\parallel v_{\mathrm{T}j}$, thus $\left|\omega \pm n\Omega_j\right| / k_\parallel v_{\mathrm{T}j} \gg 1$ (for $n \neq 0$). Therefore, we have

$$Z\left(\frac{\omega \pm n\Omega_j}{k_\parallel v_{\mathrm{T}j}}\right) \approx -\frac{k_\parallel v_{\mathrm{T}j}}{\omega \pm n\Omega_j}. \tag{11}$$

Now Eq. (10) can be further simplified to

$$\chi_j \approx \frac{\omega_{pj}^2}{\Omega_j^2}\left(1 - \frac{3}{4}k_\perp^2\rho_j^2\right). \tag{12}$$

For α-particles with a slowing-down distribution, after integrating the term on $v_\perp$ in Eq. (9) by parts, we have

$$\begin{aligned}\chi_\alpha =& \frac{2\pi\omega_{p\alpha}^2}{\omega}\frac{\Omega_\alpha^2}{k_\perp^2}\sum_{n=1}^{\infty} n^2 f_\alpha \int_{-v_\alpha}^{v_\alpha} \mathrm{d}v_\parallel \\ &\times \left(\frac{1}{\omega - k_\parallel v_\parallel - n\Omega_\alpha} + \frac{1}{\omega - k_\parallel v_\parallel + n\Omega_\alpha}\right) \\ &\times J_n^2\left(\frac{k_\perp}{\Omega_\alpha}\sqrt{v_\alpha^2 - v_\parallel^2}\right) \\ &- \frac{2\pi\omega_{p\alpha}^2}{\omega}\frac{\Omega_\alpha}{k_\perp}\sum_{n=1}^{\infty} n^2 \int_{-v_\alpha}^{v_\alpha} \mathrm{d}v_\parallel \\ &\times \left(\frac{1}{\omega - k_\parallel v_\parallel - n\Omega_\alpha} + \frac{1}{\omega - k_\parallel v_\parallel + n\Omega_\alpha}\right) \\ &\times \int_0^{\sqrt{v_\alpha^2 - v_\parallel^2}} \mathrm{d}v_\perp \frac{\partial J_n^2(\mu)}{\partial \mu} f_\alpha. \end{aligned} \tag{13}$$

For small gyroradius, i.e., $k_\perp\rho_\alpha \ll 1$ ($\rho_\alpha = v_\alpha/\Omega_\alpha$), we expand

$$J_n(\mu) \approx \frac{1}{\Gamma(n+1)}\left(\frac{\mu}{2}\right)^n - \frac{1}{\Gamma(n+2)}\left(\frac{\mu}{2}\right)^{n+2} + \cdots.$$

Furthermore, in the low frequency limit of $\Omega_\alpha \gg \omega, k_\parallel v_{\mathrm{T}\alpha}$, the first term on the right-hand side of Eq. (13) becomes

$$\begin{aligned}&\frac{2\pi\omega_{p\alpha}^2}{\omega}\frac{\Omega_\alpha^2}{k_\perp^2}\sum_{n=1}^{\infty} n^2 f_\alpha \int_{-v_\alpha}^{v_\alpha} \mathrm{d}v_\parallel \\ &\times \left(\frac{1}{\omega - k_\parallel v_\parallel - n\Omega_\alpha} + \frac{1}{\omega - k_\parallel v_\parallel + n\Omega_\alpha}\right) \\ &\times J_n^2\left(\frac{k_\perp}{\Omega_\alpha}\sqrt{v_\alpha^2 - v_\parallel^2}\right) \approx -\frac{1}{3}\frac{\omega_{p\alpha}^2}{\Omega_\alpha^2}\frac{1}{I}\frac{1}{1 + v_{\mathrm{c}}^3/v_\alpha^3}. \end{aligned} \tag{14}$$

Similarly, by using the relation

$$\frac{\partial J_n^2(\mu)}{\partial \mu} = J_n(\mu)\left[J_{n-1}(\mu) - J_{n+1}(\mu)\right],$$

the second term on the right-hand side of Eq. (13) is approximated by

$$\begin{aligned}&\frac{2\pi\omega_{p\alpha}^2}{\omega}\frac{\Omega_\alpha}{k_\perp}\sum_{n=1}^{\infty} n^2 \int_{-v_\alpha}^{v_\alpha} \mathrm{d}v_\parallel \\ &\times \left(\frac{1}{\omega - k_\parallel v_\parallel - n\Omega_\alpha} + \frac{1}{\omega - k_\parallel v_\parallel + n\Omega_\alpha}\right) \\ &\times \int_0^{\sqrt{v_\alpha^2 - v_\parallel^2}} \mathrm{d}v_\perp \frac{\partial J_n^2(\mu)}{\partial \mu} f_\alpha \\ &\approx -\frac{\omega_{p\alpha}^2}{\Omega_\alpha^2}\frac{1}{I}\left[\varsigma_1 - \frac{1}{8}(3\varsigma_2 - \varsigma_1)k_\perp^2\rho_\alpha^2\right], \end{aligned} \tag{15}$$

where

$$\varsigma_1 = v_\alpha \int_0^{v_\alpha} \mathrm{d}v \frac{v}{v_{\mathrm{c}}^3 + v^3}, \quad \varsigma_2 = \frac{1}{v_\alpha}\int_0^{v_\alpha} \mathrm{d}v \frac{v^3}{v_{\mathrm{c}}^3 + v^3}.$$

Substituting Eqs. (14) and (15) into Eq. (13) yields

$$\chi_\alpha = \frac{\omega_{p\alpha}^2}{\Omega_\alpha^2}\frac{1}{I}\left[\varsigma_1 - \frac{1}{3}\frac{1}{1 + v_{\mathrm{c}}^3/v_\alpha^3} - \frac{1}{8}(3\varsigma_2 - \varsigma_1)k_\perp^2\rho_\alpha^2\right]. \tag{16}$$

Then inserting Eqs. (12) and (16) into Eq. (8) gives the result

$$\begin{aligned}\varepsilon_{xx} =& 1 + \sum_{j=\mathrm{D,T,e}} \frac{\omega_{pj}^2}{\Omega_j^2}\left(1 - \frac{3}{4}k_\perp^2\rho_j^2\right) + \frac{\omega_{p\alpha}^2}{\Omega_\alpha^2}\frac{1}{I} \\ &\times \left[\left(\varsigma_1 - \frac{1}{3}\frac{1}{1 + v_{\mathrm{c}}^3/v_\alpha^3}\right) - \frac{1}{8}(3\varsigma_2 - \varsigma_1)k_\perp^2\rho_\alpha^2\right]. \end{aligned} \tag{17}$$

Considering the fact that

$$\sum_{j=\mathrm{D,T}} \frac{\omega_{pj}^2}{\Omega_j^2}\left(1 - \frac{3}{4}k_\perp^2\rho_j^2\right) + \frac{\omega_{p\alpha}^2}{\Omega_\alpha^2}\frac{\varsigma_1}{I} \gg \left[1, \frac{\omega_{pe}^2}{\Omega_{\mathrm{e}}^2}\right]_{\max},$$

we can further simplify Eq. (17) to

$$\varepsilon_{xx} \approx \sum_{j=\mathrm{D,T,e}} \frac{\omega_{pj}^2}{\Omega_j^2}\left(1 - \frac{3}{4}k_\perp^2\rho_j^2\right) + \frac{\omega_{p\alpha}^2}{\Omega_\alpha^2}\left(\zeta_1 - \zeta_2 k_\perp^2\rho_\alpha^2\right), \tag{18}$$

where

$$\zeta_1 = \frac{1}{I}\left(\varsigma_1 - \frac{1}{3}\frac{1}{1 + v_{\mathrm{c}}^3/v_\alpha^3}\right), \quad \zeta_2 = \frac{1}{8I}(3\varsigma_2 - \varsigma_1).$$

In addition, Eq. (7) can be rewritten as

$$\varepsilon_{zz} = 1 + \sum_j \chi'_j, \tag{19}$$

with

$$\chi'_j = \frac{\omega_{pj}^2}{\omega}\int v_\perp \mathrm{d}v^3 \sum_{n=-\infty}^{\infty} \frac{v_\parallel}{v_\perp}\frac{J_n^2(\mu)}{\omega - k_\parallel v_\parallel - n\Omega_j}\frac{\partial f_j}{\partial v_\parallel}. \tag{20}$$

For Maxwellian distributed deuterium ions, tritium ions and electrons, i.e., $j = \mathrm{D}$, T and e, Eq. (20) can be expressed as

$$\begin{aligned}\chi'_j \approx& \frac{2\omega_{pj}^2}{\omega^2} Z\left(\frac{\omega}{k_\parallel v_{\mathrm{T}j}}\right)\left(\frac{\omega}{k_\parallel v_{\mathrm{T}j}}\right)^3 A_0\left(k_\perp^2\rho_j^2\right) \\ &- \frac{4\omega_{pj}^2}{\omega^2}\left(\frac{\omega}{k_\parallel v_{\mathrm{T}j}}\right)^2 \sum_{n=1}^{\infty} \Gamma_n\left(k_\perp^2\rho_j^2\right) \\ &+ \frac{2\omega_{pj}^2}{\omega^2}\left(\frac{\omega}{k_\parallel v_{\mathrm{T}j}}\right)^2. \end{aligned} \tag{21}$$

Using $2\sum_{n=1}^{+\infty} A_n\left(k_\perp^2\rho_j^2\right) = 1 - A_0\left(k_\perp^2\rho_j^2\right)$, Eq. (21) can be further simplified to

$$\chi_j' \approx \frac{2\omega_{pj}^2}{\omega^2} Z\left(\frac{\omega}{k_\parallel v_{\mathrm{T}j}}\right)\left(\frac{\omega}{k_\parallel v_{\mathrm{T}j}}\right)^3 A_0\left(k_\perp^2\rho_j^2\right) + \frac{2\omega_{pj}^2}{\omega^2}\left(\frac{\omega}{k_\parallel v_{\mathrm{T}j}}\right)^2 A_0\left(k_\perp^2\rho_j^2\right). \tag{22}$$

For D and T ions, after following the condition $\omega/k_\parallel v_{\mathrm{TD,TT}} \gg 1$ and neglecting the ion Landau damping, we have

$$\chi_{\mathrm{D,T}}' \approx -\frac{\omega_{pD,pT}^2}{\omega^2}\left(1 - k_\perp^2\rho_{\mathrm{D,T}}^2\right). \tag{23}$$

For electrons, recognizing that $\omega/k_\parallel v_{\mathrm{Te}} \ll 1$ and utilizing the relation

$$Z\left(\frac{\omega}{k_\parallel v_{\mathrm{Te}}}\right) \approx -\frac{2\omega}{k_\parallel v_{\mathrm{Te}}} + \mathrm{i}\sqrt{\pi}\exp\left(-\frac{\omega^2}{k_\parallel^2 v_{\mathrm{Te}}^2}\right),$$

we have

$$\chi_{\mathrm{e}}' \approx \frac{1}{k_\parallel^2\lambda_{\mathrm{de}}^2} A_0\left(k_\perp^2\rho_{\mathrm{e}}^2\right) + \mathrm{i}\sqrt{\pi}\frac{1}{k_\parallel^2\lambda_{\mathrm{de}}^2}\frac{\omega}{k_\parallel v_{\mathrm{Te}}}\exp\left(-\frac{\omega^2}{k_\parallel^2 v_{\mathrm{Te}}^2}\right) A_0\left(k_\perp^2\rho_{\mathrm{e}}^2\right), \tag{24}$$

where $\lambda_{\mathrm{de}} = v_{\mathrm{Te}}/\omega_{pe}$ is the Debye length of electrons.

For α-particles with a slowing-down distribution, after taking account of $\Omega_\alpha \gg \omega, k_\parallel v_\alpha$, we neglect term of $n \neq 0$ and integrate the term on $v_\perp$ in Eq. (20) by parts. Therefore,

$$\chi_\alpha' = -\frac{2\pi\omega_{p\alpha}^2}{\omega}\int_0^{v_\alpha} J_0^2(\mu)\, v_\perp \mathrm{d}v_\perp \times\left[\int_{-\sqrt{v_\alpha^2 - v_\perp^2}}^{\sqrt{v_\alpha^2 - v_\perp^2}} \mathrm{d}v_\parallel f_\alpha \frac{d}{\mathrm{d}v_\parallel}\left(\frac{v_\parallel}{\omega - k_\parallel v_\parallel}\right) - \frac{2\omega\sqrt{v_\alpha^2 - v_\perp^2}}{\omega^2 - k_\parallel^2\left(v_\alpha^2 - v_\perp^2\right)} f_\alpha\right]. \tag{25}$$

In the case of $k_\perp^2\rho_\alpha^2 \ll 1$, after recognizing the fact that $\omega/k_\parallel v_\alpha \ll 1$ and integrating Eq. (25) with respect to $v_\perp$ and then $v_\parallel$, Eq. (25) can be simplified to

$$\chi_\alpha' \approx \zeta_3\frac{\omega_{p\alpha}^2}{k_\parallel^2 v_\alpha^2} + \mathrm{i}\zeta_4\frac{\pi}{2}\frac{\omega_{p\alpha}^2}{k_\parallel^2 v_\alpha^2}\frac{\omega}{k_\parallel v_\alpha}. \tag{26}$$

where

$$\zeta_3 = \frac{1}{I}\left[\varsigma_1 - \frac{v_\alpha^3}{v_{\mathrm{c}}^3 + v_\alpha^3} - \left(\varsigma_1 + \varsigma_2 - \frac{2}{3}\frac{v_\alpha^3}{v_{\mathrm{c}}^3 + v_\alpha^3}\right)\frac{k_\perp^2\rho_\alpha^2}{2}\right],$$

$$\zeta_4 = \frac{1}{I}\left(\frac{v_\alpha^3}{v_{\mathrm{c}}^3 + v_\alpha^3} + \varsigma_1 k_\perp^2\rho_\alpha^2\right).$$

Now substituting Eqs. (23), (24) and (26) into Eq. (19), and considering the case of $k_\parallel\lambda_{\mathrm{de}} \ll 1$, we obtain

$$\varepsilon_{zz} = \frac{A_0\left(k_\perp^2\rho_{\mathrm{e}}^2\right)}{k_\parallel^2\lambda_{\mathrm{de}}^2} + \frac{\omega_{p\alpha}^2\zeta_3}{k_\parallel^2 v_\alpha^2} + \mathrm{i}\frac{\pi}{2}\frac{\omega_{p\alpha}^2}{k_\parallel^2 v_\alpha^2}\frac{\omega\zeta_4}{k_\parallel v_\alpha} + \mathrm{i}\sqrt{\pi}\frac{A_0\left(k_\perp^2\rho_{\mathrm{e}}^2\right)}{k_\parallel^2\lambda_{\mathrm{de}}^2}\frac{\omega}{k_\parallel v_{\mathrm{Te}}} \times \exp\left(-\frac{\omega^2}{k_\parallel^2 v_{\mathrm{Te}}^2}\right). \tag{27}$$

In deriving Eq. (27), under the condition of $\frac{\omega^2}{k_\parallel^2\lambda_{\mathrm{de}}^2} \gg \omega_{pD,pT}^2$, we conventionally ignore the effect of ions on the dielectric tensor ε_{zz}.

Combining Eqs. (5), (18) and (27), we finally obtain the dispersion of KAWs as follows:

$$\frac{\omega_{\mathrm{s}}^2}{k_\parallel^2 c^2} = \frac{1}{\sum_{j=\mathrm{D,T,e}}\frac{\omega_{pj}^2}{\Omega_j^2}\left(1 - \frac{3}{4}k_\perp^2\rho_j^2\right) + \frac{\omega_{p\alpha}^2}{\Omega_\alpha^2}\left(\zeta_1 - \zeta_2 k_\perp^2\rho_\alpha^2\right) + \frac{k_\perp^2}{k_\parallel^2}\frac{1}{\frac{A_0\left(k_\perp^2\rho_{\mathrm{e}}^2\right)}{k_\parallel^2\lambda_{\mathrm{de}}^2} + \frac{\zeta_3\omega_{p\alpha}^2}{k_\parallel^2 v_\alpha^2}}}. \tag{28}$$

For the same reason, if the α-particles take the equivalent Maxwellian distribution at the same slowing-down temperature, the dispersion relation of KAWs takes the form

$$\frac{\omega_{\mathrm{m}}^2}{k_\parallel^2 c^2} = \frac{1}{\sum_{j=\mathrm{D,T,e},\alpha}\frac{\omega_{pj}^2}{\Omega_j^2}\left(1 - \frac{3}{4}k_\perp^2\rho_j^2\right) + \frac{k_\perp^2}{k_\parallel^2}\frac{1}{\frac{A_0\left(k_\perp^2\rho_{\mathrm{e}}^2\right)}{k_\parallel^2\lambda_{\mathrm{de}}^2} + \frac{A_0\left(k_\perp^2\rho_\alpha^2\right)}{k_\parallel^2\lambda_{\mathrm{d}\alpha}^2}}}. \tag{29}$$

Different from the MHD Alfvén waves, the dispersion relation of the KAWs depends on $k_\perp$, which means that the waves can propagate obliquely to the ambient magnetic field.

3. Dampling of the KAWs

Assuming that $\omega = \omega_{\mathrm{r}} + \mathrm{i}\gamma$ with $|\gamma| \ll \omega_{\mathrm{r}}$, we find the expression for the collisionless damping rate of the KAWs to be given by[31,32]

$$\gamma = -\left.\frac{\mathrm{Im}D}{\partial\mathrm{Re}D/\partial\omega}\right|_{\omega=\omega_{\mathrm{r}}}. \tag{30}$$

Thus, we obtain the expressions for the damping rates of KAWs as follows:

$$\gamma_{\mathrm{s}} = -\frac{k_\perp^2 c^2}{2}\frac{\left[\frac{\sqrt{\pi}}{k_\parallel v_{\mathrm{Te}}}\frac{A_0\left(k_\perp^2\rho_{\mathrm{e}}^2\right)}{k_\parallel^2\lambda_{\mathrm{de}}^2}\exp\left(-\frac{\omega_{\mathrm{s}}^2}{k_\parallel^2 v_{\mathrm{Te}}^2}\right) + \frac{\pi}{k_\parallel v_\alpha}\frac{\zeta_4\omega_{p\alpha}^2}{2k_\parallel^2 v_\alpha^2}\right]}{\left[\frac{A_0\left(k_\perp^2\rho_{\mathrm{e}}^2\right)}{k_\parallel^2\lambda_{\mathrm{de}}^2} + \frac{\varsigma_3\omega_{p\alpha}^2}{k_\parallel^2 v_\alpha^2}\right]^2} \tag{31}$$

in the case of slowing-down distribution, and

$$\gamma_{\rm m}=-\frac{k_\perp^2c^2}{2}\frac{\left[\frac{\sqrt{\pi}}{k_\parallel v_{\rm Te}}\frac{A_0\left(k_\perp^2\rho_{\rm e}^2\right)}{k_\parallel^2\lambda_{\rm de}^2}\exp\left(-\frac{\omega_{\rm rn}^2}{k_\parallel^2v_{\rm Te}^2}\right)+\frac{\sqrt{\pi}}{k_\parallel v_{{\rm T}\alpha}}\frac{A_0\left(k_\perp^2\rho_\alpha^2\right)}{k_\parallel^2\lambda_{{\rm d}\alpha}^2}\exp\left(-\frac{\omega_{\rm rn}^2}{k_\parallel^2v_{{\rm T}\alpha}^2}\right)\right]}{\left[\frac{A_0\left(k_\perp^2\rho_{\rm e}^2\right)}{k_\parallel^2\lambda_{\rm de}^2}+\frac{A_0\left(k_\perp^2\rho_\alpha^2\right)}{k_\parallel^2\lambda_{{\rm d}\alpha}^2}\right]^2} \tag{32}$$

in the case of Maxwellian distribution. Note that the first term in the numerator of Eq. (32) denotes electron damping and the second term denotes α-particles damping. Strictly speaking, the parallel phase velocity of KAWs is much lower than the thermal velocity of electrons (i.e., $\omega/k_\parallel \ll v_{\rm Te}$), so we take $\exp\left(-\omega_{\rm r}^2/k_\parallel^2v_{\rm Te}^2\right)\sim 1$. Furthermore, after neglecting the α-particle damping, Eqs. (31) and (32) will be reduced to $\gamma=-\frac{\sqrt{\pi}}{2}\frac{k_\perp^2c^2}{k_\parallel v_{\rm Te}}\frac{k_\parallel^2\lambda_{\rm de}^2}{A_0\left(k_\perp^2\rho_{\rm e}^2\right)}$, which is consistent with Eq. (2.265) in Ref. [33].

4. Results and discussion

To further elucidate the nature of KAWs in a D-T plasma, we will describe the dispersion relations in Eqs. (28) and (29) and damping rates in Eqs. (31) and (32). We take typical TFTR tokamak parameters with[28,29] $B_0=5.2T$, $n_{\rm e}=3.2\times10^{20}$ m^{-3}, $T_{\rm e}=T_{\rm D}=T_{\rm T}=10$ keV. For the TFTR device, $k_\parallel\sim 1/R$ with $R=2.48$ m being the major radius. Thus $k_\parallel\approx 0.4$ m^{-1}. Additionally, the plasma quasi-neutrality goes as $2n_\alpha+n_{\rm D}+n_{\rm T}=n_{\rm e}$. Here, we choose $n_{\rm D}=n_{\rm T}$, thus $2n_\alpha+2n_{\rm D}=n_{\rm e}$.

Figure 1 shows the effect of α concentration on the KAWs. From Figs. 1(a) and 1(b), it can be found that the frequency of KAWs increases with the increase of normalized α density ($n_\alpha/n_{\rm e}$), and likewise the damping rate increases. Moreover, the values of both the frequency and damping obtained with slowing-down distributed α-particles are slightly lower than those of the equivalent Maxwellian case for a certain α concentration. However, since the α density in D-T plasmas is generally less than one percent of the bulk plasma density,[24] the magnitudes of the frequency and the damping rate do not change significantly with increasing α concentration.

From Fig. 2(a), it can be seen that the frequency increases with the increase of the perpendicular wave number ($k_\perp\rho_{\rm D}$). Meanwhile, it can be found from Fig. 2(b) that the Landau damping is enhanced with the increasing $k_\perp\rho_{\rm D}$. For a given $k_\perp\rho_{\rm D}$, the value of frequency in the slowing-down case is significantly smaller compared to that in the Maxwellian case. However, the damping rate in the large wave number region is slightly larger than that in the Maxwellian case, which indicates that the Landau damping in the Maxwellian distribution plasma system is slightly suppressed compared to that in the case of slowing-down distribution. In addition, for a given $k_\perp\rho_{\rm D}$, as the parallel wave number increases, the frequency and damping rate increase in both the distribution cases.

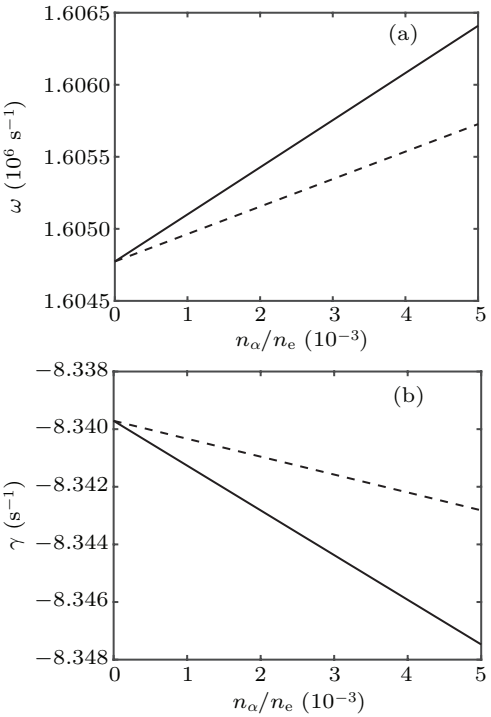


Fig. 1. (a) Frequency (ω) and (b) damping rate (γ) vs normalized α density ($n_\alpha/n_{\rm e}$) for $k_\perp=10$ m^{-1}, $k_\parallel=0.4$ m^{-1} and $T_{\rm e}=T_{\rm D}=T_{\rm T}=10$ keV. The solid and dashed lines denote the Maxwellian and slowing-down distributions, respectively.

As mentioned above, the perpendicular wave number ($k_\perp$), with $0.01<k_\perp<1$ cm^{-1}, is appropriate in tokamak plasma confinement experiments. Figures 3(a) and 3(b) provide the information on the variation of frequency and damping rate versus the perpendicular wave number ($k_\perp$) under different background temperatures. From Figs. 3(a) and 3(b), it can be concluded that the higher background temperature will lead to a larger wave frequency and damping rate in both the distribution cases. Physically, when the temperature increases the finite Larmor radius of the ions increases, which leads to an increase in the real frequency. On the other hand, the dispersion of the KAWs is induced by the thermal motion of the particles, and the Landau damping is determined by the number of charge particles in the vicinity of the resonant phase velocity of plasma waves. Apparently, it can be found from Eq. (31) that the damping rate is proportional to $T_{\rm e}^{1/2}$. The elevated background temperature intensifies the thermal motion of charged particles. Then, the number of the electrons that satisfy the resonant condition is increased, and the Landau damping is enhanced.

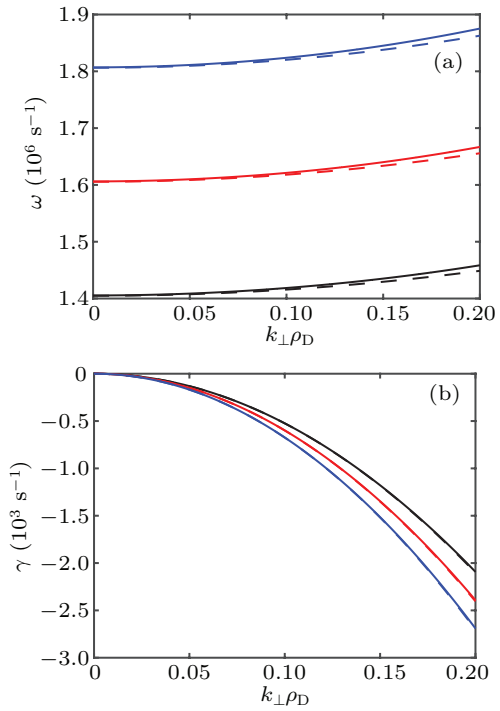


Fig. 2. (a) Frequency (ω) and (b) damping rate (γ) vs normalized perpendicular wave number ($k_\perp\rho_D$) for $n_\alpha/n_e = 0.005$ and $T_e = T_D = T_T =$ 10 keV. The black, red and blue lines correspond to $k_\parallel = 0.35$, 0.40 and 0.45 m^{-1}, respectively, while the solid and dashed lines denote the Maxwellian and slowing-down distributions.

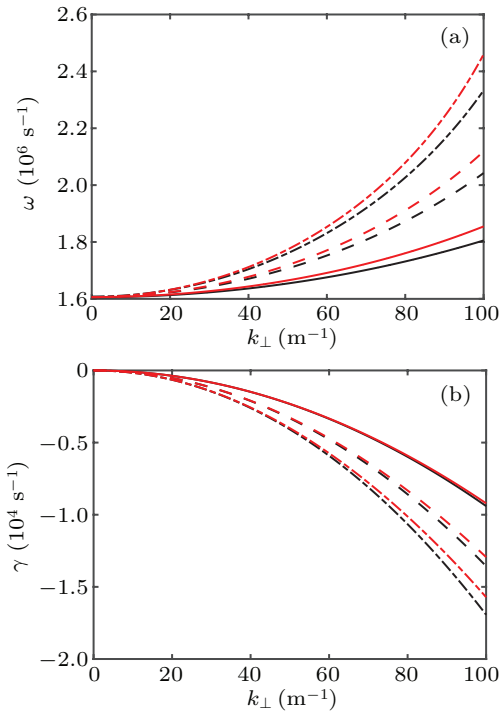


Fig. 3. (a) Frequency (ω) and (b) damping rate (γ) vs perpendicular wave number ($k_\perp$) for $k_\parallel = 0.4$ m^{-1} and $n_\alpha/n_e = 0.005$. The solid, dotted and dash-dotted lines correspond to $T_e = T_D = T_T = 10$, 20 and 30 keV, respectively, while the red and black lines denote the Maxwellian and slowing-down distributions.

5. Conclusions

In this work, using the kinetic theory we have studied the KAWs in D-T fusion plasmas with the α-particles described by a slowing-down distribution function, as well as a Maxwellian distribution function. For low frequency $\omega < \Omega_j$, short perpendicular and long parallel wavelength $k_\parallel \ll k_\perp$, and in the case of $k_\perp\rho_j \ll 1$, the dispersion and damping rate of the KAWs are derived and numerically evaluated.

The present results indicate that both the frequency and the damping rate increase with the increase of α concentration. However, at the low α concentration, i.e., $n_\alpha/n_e \leq 10^{-2}$, the fluctuation of the α concentration has less effect on the frequency and damping rate of KAWs. In addition, the frequencies of KAWs obtained from both the slowing-down distribution and the equivalent Maxwellian distribution gradually increase as the $k_\perp$ increases, and the value of the frequency in the case of slowing-down distribution is smaller than that in the Maxwellian distribution. Inversely, the value of the damping rate in the slowing-down distribution is slightly larger compared to that in the Maxwellian distribution. Furthermore, it can be found that the higher the background temperature T_e, the larger the wave frequency and damping rate. The present work is conductive to the understanding of KAWs in D-T fusion plasmas, which may also be related to the nonlinear process associated with the low-frequency Alfvén wave modes in burning plasmas, and will be investigated in our future work.

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