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Deterministic remote state preparation of arbitrary three-qubit state through noisy cluster-GHZ channel

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We propose a novel scheme for remote state preparation of an arbitrary three-qubit state with unit success probability, utilizing a nine-qubit cluster-GHZ state without introducing auxiliary qubits. Furthermore, we proceed to investigate the effects of different quantum noises (e.g., amplitude-damping, phase-damping, bit-flip and phase-flip noises) on the systems. The fidelity results of three-qubit target state are presented, which are usually used to illustrate how close the output state is to the target state. To compare the different effects between the four common types of quantum noises, the fidelities under one specific identical target state are also calculated and discussed. It is found that the fidelity of the phase-flip noisy channel drops the fastest through the four types of noisy channels, while the fidelity is found to always maintain at 1 in bit-flip noisy channel.

Keywords: deterministic remote state preparation, arbitrary three-qubit state, cluster state, quantum noises

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1. Introduction

As a fundamental property of quantum mechanics, quantum entanglement has led to many important discoveries, such as quantum teleportation (QT),[1–3] quantum key distribution (QKD),[4,5] quantum secret sharing,[6] quantum dialogue,[7] quantum dense coding,[8] and remote state preparation (RSP).[9,10] Compared with their classic methods, in principle, quantum protocols can guarantee a higher security. Research in the fields of quantum system control[11–16] and quantum information processing[17–20] have been extensively studied in recent years. The idea of QT was first proposed by Bennet *et al.*[1] in 1993, in which the sender wished to transmit an arbitrary single-qubit state to a distant receiver via the maximally entangled Bell state. Neither the sender nor the receiver knows any information about the state to be transmitted. Later, the concept of remote state preparation was proposed by Lo *et al.*[9] in 2000, where the sender could remotely prepare an arbitrary single-qubit state for the receiver by classical communication and a pre-shared entangled state. Different from QT, the sender prepares a known state for the receiver with a pre-shared quantum channel and appropriate measurements. Until now, there are already many different kinds of RSP protocols reported, such as deterministic RSP (DRSP),[21,22] joint RSP (JRSP)[23,24] and controlled RSP (CRSP).[25,26] The main advantage of DRSP over RSP protocol is that DRSP protocols can guarantee unit success probability. Various DRSP protocols have been proposed over the past few years and many entangled states (e.g., Einstein–Podolsky–Rosen (EPR) pairs,[27–30] GHZ state,[31,32] cluster state,[33,34] χ state,[22] Brown state[35] and Bell state[36]) have been used to implement DRSP. In 2013, Zhan *et al.*[37] proposed a DRSP scheme by using six EPR pairs, where a four-qubit cluster state was remotely prepared. In 2015, Hou *et al.*[38] presented several DRSP schemes of four-qubit W state via EPR pairs and GHZ states. In 2017, Zhang *et al.*[22] and Ma *et al.*[23] put forward two DRSP protocols, in which arbitrary two- and three-qubit states were prepared by taking χ state and Brown state as the entangled channels. In 2018, Wei *et al.*[39] gave an efficient DRSP scheme for preparing an arbitrary n-qubit equatorial state via multiple two-qubit maximally entangled states. In 2020, Zha *et al.*[40] proposed two DRSP protocols, where an arbitrary four-qubit cluster state could be prepared using two Bell states. In these schemes, the ideal channels were used. However, noises inevitably influence the process of the quantum information transmission.

Recently, researchers have examined the effects of quantum noise on DRSP of one-qubit,[41] two-qubit[42,43] and three-qubit[44,45] states. In 2019, Zhang *et al.*[42] proposed two DRSP schemes of arbitrary two-qubit state via the χ state channel in noisy environment. In 2020, Qian *et al.*[41] presented a DRSP scheme, in which arbitrary one-qubit state working in noisy environment was prepared via one intermediate node. Dash *et al.*[43] investigated the effects of different quantum noises on DRSP of arbitrary two-qubit system. The effects of quantum noise have been taken into consideration in these DRSP schemes. There have been many studies previously published on the RSP of arbitrary three-qubit state.[44–47] In 2016, Falaye *et al.*[44] put forward a JRSP protocol of three-qubit states via three tripartite GHZ entangled states in noisy quantum channel, by putting some special cases

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into consideration, in which the success probability could be ameliorated to 1. In 2018, Sun *et al.*[45] studied the effects of noise on two DRSP protocols of an arbitrary three-qubit state, where χ state and Brown state were used as the quantum channel, respectively.

Cluster states have played a crucial role in quantum information processing tasks in recent years. It is worth noting that there has been no study on the DRSP of three-qubit states when the cluster-GHZ channel are used so far. Here, we present a deterministic remote state preparation protocol for remotely preparing arbitrary three-qubit state in ideal environment, where the sender Alice and the receiver Bob share a composite cluster-GHZ state as the quantum channel. Alice performs two three-qubit measurements based on her knowledge of the original states. Finally, according to Alice's measurement results, the receiver Bob can reconstruct the respective target state by preforming appropriate unitary operations. In this paper, we also analyze the impact of four kinds of quantum noises, such as amplitude-damping, phase-damping, bit-flip and phase-flip noises, on DRSP of an arbitrary three-qubit state via a nine-qubit cluster-GHZ state channel. Further, we provide the fidelity results of arbitrary three-qubit target state. The results of this study indicate that the fidelities depend on the noise parameters, the amplitude parameters and the phase information of the initial state. The detailed analysis will be given in Section 3. The structure of the paper is arranged as follows. In Section 2, we introduce a novel DRSP protocol of arbitrary three-qubit state via a nine-qubit cluster-GHZ state channel. In Section 3, we proceed to investigate the effects of noises on the quantum channel and analyze the fidelities obtained. Finally, we discuss the performance of our scheme and close the paper with a brief summary in Section 4.

2. Remote state preparation of an arbitrary three qubit state

In this section, we discuss the remote state preparation of an arbitrary three-qubit state under ideal channels. Suppose that there are two participants: Alice and Bob. Alice intends to prepare an arbitrary three-qubit state for Bob. Here the state is known by Alice. The receiver does not have any knowledge about the original state at all. The desired state is described as follows:

$$\begin{aligned}|\chi\rangle_{b_1b_2b_3} =\ & x_0\,\mathrm{e}^{\mathrm{i}\theta_0}|000\rangle + x_1\,\mathrm{e}^{\mathrm{i}\theta_1}|001\rangle + x_2\,\mathrm{e}^{\mathrm{i}\theta_2}|010\rangle \\ & + x_3\,\mathrm{e}^{\mathrm{i}\theta_3}|011\rangle + x_4\,\mathrm{e}^{\mathrm{i}\theta_4}|100\rangle + x_5\,\mathrm{e}^{\mathrm{i}\theta_5}|101\rangle \\ & + x_6\,\mathrm{e}^{\mathrm{i}\theta_6}|110\rangle + x_7\,\mathrm{e}^{\mathrm{i}\theta_7}|111\rangle, \end{aligned} \tag{1}$$

where x_i and $\theta_i \in [0, 2\pi)$, $(i = 1, 2, 3, \ldots, 7)$ are real numbers, and satisfy the normalization condition $\sum_{i=0}^{7} x_i^2 = 1$. In addition, in order to realize the DRSP, we assume that the quantum channel $|\psi\rangle$ shared initially by the two participants consists of one six-qubit cluster state and one GHZ state. The entire quantum system can be rewritten as

$$\begin{aligned}|\psi\rangle =\ & \frac{1}{2}(|000000\rangle + |000111\rangle + |111000\rangle \\ & - |111111\rangle)_{a_1a_2b_1a_3a_4b_2} \\ & \otimes \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{a_5a_6b_3} \\ =\ & \frac{1}{2\sqrt{2}}(|000000000\rangle + |000111000\rangle + |111000000\rangle \\ & - |111111000\rangle + |000000111\rangle + |000111111\rangle \\ & + |111000111\rangle - |111111111\rangle)_{a_1a_2b_1a_3a_4b_2a_5a_6b_3} \\ =\ & \frac{1}{2\sqrt{2}}(|000\rangle \otimes |000000\rangle + |001\rangle \otimes |001001\rangle \\ & + |010\rangle \otimes |010010\rangle + |011\rangle \otimes |011011\rangle \\ & + |100\rangle \otimes |100100\rangle + |101\rangle \otimes |101101\rangle - |110\rangle \\ & \otimes |110110\rangle - |111\rangle \otimes |111111\rangle)_{a_1a_3a_5a_2a_4a_6b_1b_2b_3}. \end{aligned} \tag{2}$$

Here, qubits a_1, a_2, a_3, a_4, a_5 and a_6 belong to Alice and b_1, b_2 and b_3 belong to Bob, respectively.

In order to realize the DRSP of arbitrary three-qubit state for Bob, Alice firstly performs a projective measurement on her particles with the mutually orthogonal base $\{|\varphi_i\rangle\}$ $(i = 0, 1, 2, \ldots, 7)$:

$$\begin{bmatrix}|\varphi_0\rangle\\|\varphi_1\rangle\\|\varphi_2\rangle\\|\varphi_3\rangle\\|\varphi_4\rangle\\|\varphi_5\rangle\\|\varphi_6\rangle\\|\varphi_7\rangle\end{bmatrix} = \begin{bmatrix} x_0 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ x_1 & -x_0 & x_3 & -x_2 & x_5 & -x_4 & x_7 & -x_6 \\ x_2 & -x_3 & -x_0 & x_1 & -x_6 & x_7 & x_4 & -x_5 \\ x_3 & x_2 & -x_1 & -x_0 & x_7 & x_6 & -x_5 & -x_4 \\ x_4 & -x_5 & x_6 & -x_7 & -x_0 & x_1 & -x_2 & x_3 \\ x_5 & x_4 & -x_7 & -x_6 & -x_1 & -x_0 & x_3 & x_2 \\ x_6 & -x_7 & -x_4 & x_5 & x_2 & -x_3 & -x_0 & x_1 \\ x_7 & x_6 & x_5 & x_4 & -x_3 & -x_2 & -x_1 & -x_0 \end{bmatrix} \begin{bmatrix}|000\rangle\\|001\rangle\\|010\rangle\\|011\rangle\\|100\rangle\\|101\rangle\\|110\rangle\\|111\rangle\end{bmatrix}. \tag{3}$$

Next, according to her first measurement result, Alice chooses the projective measurement bases to measure her qubits, where a set of mutually orthogonal basis is given by

$$\begin{bmatrix}|\eta_0^{(i)}\rangle\\|\eta_1^{(i)}\rangle\\|\eta_2^{(i)}\rangle\\|\eta_3^{(i)}\rangle\\|\eta_4^{(i)}\rangle\\|\eta_5^{(i)}\rangle\\|\eta_6^{(i)}\rangle\\|\eta_7^{(i)}\rangle\end{bmatrix} = G^{(i)} \begin{bmatrix}|000\rangle\\|001\rangle\\|010\rangle\\|011\rangle\\|100\rangle\\|101\rangle\\|110\rangle\\|111\rangle\end{bmatrix}, \tag{4}$$

where $G^{(i)}$ is an 8×8 matrix,

$$\begin{aligned} G^{(0)} &= G(r_0, r_1, r_2, r_3, r_4, r_5, r_6, r_7), \\ G^{(1)} &= G(r_1, r_0, r_3, r_2, r_5, r_4, r_7, r_6), \\ G^{(2)} &= G(r_2, r_3, r_0, r_1, r_6, r_7, r_4, r_5), \\ G^{(3)} &= G(r_3, r_2, r_1, r_0, r_7, r_6, r_5, r_4), \\ G^{(4)} &= G(r_4, r_5, r_6, r_7, r_0, r_1, r_2, r_3), \\ G^{(5)} &= G(r_5, r_4, r_7, r_6, r_1, r_0, r_3, r_2), \end{aligned}$$

$$G^{(6)} = G(r_6, r_7, r_4, r_5, r_2, r_3, r_0, r_1),$$
$$G^{(7)} = G(r_7, r_6, r_5, r_4, r_3, r_2, r_1, r_0), \tag{5}$$

with

$$r_k = \mathrm{e}^{-\mathrm{i}\theta_k}, \quad (k = 0, 1, 2, \ldots, 7).$$

The matrix $G(r_0, r_1, r_2, r_3, r_4, r_5, r_6, r_7)$ is given by

$$G^{(0)} = \frac{1}{2\sqrt{2}} \begin{bmatrix} r_0 & r_1 & r_2 & r_3 & r_4 & r_5 & x_6 & r_7 \\ r_0 & -r_1 & r_2 & -r_3 & r_4 & -r_5 & r_6 & -r_7 \\ r_0 & -r_1 & -r_2 & r_3 & -r_4 & r_5 & r_6 & -r_7 \\ r_0 & r_1 & -r_2 & -r_3 & r_4 & r_5 & -r_6 & -r_7 \\ r_0 & -r_1 & r_2 & -r_3 & -r_4 & r_5 & -r_6 & r_7 \\ r_0 & r_1 & -r_2 & -r_3 & -r_4 & -r_5 & r_6 & r_7 \\ r_0 & -r_1 & -r_2 & r_3 & r_4 & -r_5 & -r_6 & r_7 \\ r_0 & r_1 & r_2 & r_3 & -r_4 & -r_5 & -r_6 & -r_7 \end{bmatrix}. \tag{6}$$

Obviously, the remaining matrices will be easily written by the law. The corresponding relationship between Alice's measurement results $\{|\varphi_i\rangle\}$ and the measuring basis $\{|\eta_j^{(i)}\rangle\}$ can be described as $\{|\varphi_i\rangle\} \to \{|\eta_j^{(i)}\rangle\}$, $(i, j = 0, 1, 2, \ldots, 7)$. After Alice's measurements, she informs Bob of her two measurement results by the classical channel. Bob first applies the following unitary transformation on his particles b_1, b_2 and b_3:

$$U_{b_1 b_2 b_3} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}. \tag{7}$$

In this case, it is easy to see that Bob can always reestablish the target state with the same probability, provided that Alice's projective measurement result is anyone of the sixty-four results. Then, according to the result, he can recover the original state $|\chi\rangle_{b_1 b_2 b_3}$ by suitable unitary operations. For instance, without loss of generality, when Alice's measurement outcomes are $|\varphi_1\rangle \otimes |\eta_2^{(1)}\rangle$, Bob needs to perform the unitary operation $Z_{b_1} Z_{b_2} X_{b_3}$ to restore the target state shown in Eq. (1). The relationship between the measurement results (MR) obtained by Alice and the necessary unitary operations performed by Bob (U_b) is shown in Table 1.

Table 1. Unitary operations (U_b) performed by Bob according to the measurement result (MR) of Alice. Here, ξ_i and $\zeta_j^{(i)}$ represent Alice's measurement results $|\varphi_i\rangle_{a_1 a_3 a_5}$ and $|\eta_j^{(i)}\rangle_{a_2 a_4 a_6}$, respectively.

MR	U_b	MR	U_b	MR	U_b	MR	U_b
$\xi_0\zeta_0^{(0)}$	$I_{b_1} I_{b_2} I_{b_3}$	$\xi_2\zeta_0^{(2)}$	$Z_{b_1}(ZX)_{b_2} Z_{b_3}$	$\xi_4\zeta_0^{(4)}$	$(ZX)_{b_1} I_{b_2} Z_{b_3}$	$\xi_6\zeta_0^{(6)}$	$X_{b_1}(ZX)_{b_2} Z_{b_3}$
$\xi_0\zeta_1^{(0)}$	$I_{b_1} I_{b_2} Z_{b_3}$	$\xi_2\zeta_1^{(2)}$	$Z_{b_1} Z_{b_2} X_{b_3}$	$\xi_4\zeta_1^{(4)}$	$(ZX)_{b_1} I_{b_2} I_{b_3}$	$\xi_6\zeta_1^{(6)}$	$X_{b_1}(ZX)_{b_2} Z_{b_3}$
$\xi_0\zeta_2^{(0)}$	$I_{Z_1} Z_{b_2} Z_{b_3}$	$\xi_2\zeta_2^{(2)}$	$I_{b_1} X_{b_2} I_{b_3}$	$\xi_4\zeta_2^{(4)}$	$X_{b_1} Z_{b_2} Z_{b_3}$	$\xi_6\zeta_2^{(6)}$	$(ZX)_{b_1} X_{b_2} I_{b_3}$
$\xi_0\zeta_3^{(0)}$	$I_{b_1} Z_{b_2} I_{b_3}$	$\xi_2\zeta_3^{(2)}$	$Z_{b_1} X_{b_2} Z_{b_3}$	$\xi_4\zeta_3^{(4)}$	$(ZX)_{b_1} Z_{b_2} Z_{b_3}$	$\xi_6\zeta_3^{(6)}$	$X_{b_1} X_{b_2} Z_{b_3}$
$\xi_0\zeta_4^{(0)}$	$Z_{b_1} I_{b_2} Z_{b_3}$	$\xi_2\zeta_4^{(2)}$	$I_{b_1} Z_{b_2}(XZ)_{b_3}$	$\xi_4\zeta_4^{(4)}$	$X_{b_1} I_{b_2} I_{b_3}$	$\xi_6\zeta_4^{(6)}$	$(ZX)_{b_1}(ZX)_{b_2} I_{b_3}$
$\xi_0\zeta_5^{(0)}$	$Z_{b_1} Z_{b_2} I_{b_3}$	$\xi_2\zeta_5^{(2)}$	$I_{b_1} X_{b_2} Z_{b_3}$	$\xi_4\zeta_5^{(4)}$	$X_{b_1} Z_{b_2} Z_{b_3}$	$\xi_6\zeta_5^{(6)}$	$(ZX)_{b_1} X_{b_2} Z_{b_3}$
$\xi_0\zeta_6^{(0)}$	$I_{b_1} Z_{b_2} Z_{b_3}$	$\xi_2\zeta_6^{(2)}$	$Z_{b_1} X_{b_2} I_{b_3}$	$\xi_4\zeta_6^{(4)}$	$Z_{b_1} X_{b_2} Z_{b_3}$	$\xi_6\zeta_6^{(6)}$	$X_{b_1} X_{b_2} I_{b_3}$
$\xi_0\zeta_7^{(0)}$	$Z_{b_1} I_{b_2} I_{b_3}$	$\xi_2\zeta_7^{(2)}$	$I_{b_1}(ZX)_{b_2} Z_{b_3}$	$\xi_4\zeta_7^{(4)}$	$X_{b_1} I_{b_2} Z_{b_3}$	$\xi_6\zeta_7^{(6)}$	$(ZX)_{b_1}(ZX)_{b_2} Z_{b_3}$
$\xi_1\zeta_0^{(1)}$	$I_{b_1} I_{b_2}(ZX)_{b_3}$	$\xi_3\zeta_0^{(3)}$	$I_{b_1}(ZX)_{b_2}(ZX)_{b_3}$	$\xi_5\zeta_0^{(5)}$	$(ZX)_{b_1} Z_{b_2} X_{b_3}$	$\xi_7\zeta_0^{(7)}$	$(ZX)_{b_1} X_{b_2} X_{b_3}$
$\xi_1\zeta_1^{(1)}$	$I_{b_1} I_{b_2} X_{b_3}$	$\xi_3\zeta_1^{(3)}$	$I_{b_1}(ZX)_{b_2}(ZX)_{b_3}$	$\xi_5\zeta_1^{(5)}$	$(ZX)_{b_1} Z_{b_2}(ZX)_{b_3}$	$\xi_7\zeta_1^{(7)}$	$(ZX)_{b_1} X_{b_2}(ZX)_{b_3}$
$\xi_1\zeta_2^{(1)}$	$Z_{b_1} Z_{b_2} X_{b_3}$	$\xi_3\zeta_2^{(3)}$	$Z_{b_1} X_{b_2}(ZX)_{b_3}$	$\xi_5\zeta_2^{(5)}$	$X_{b_1} I_{b_2}(ZX)_{b_3}$	$\xi_7\zeta_2^{(7)}$	$(ZX)_{b_1} X_{b_2}(ZX)_{b_3}$
$\xi_1\zeta_3^{(1)}$	$I_{b_1} Z_{b_2}(ZX)_{b_3}$	$\xi_3\zeta_0^{(3)}$	$I_{b_1} X_{b_2} X_{b_3}$	$\xi_5\zeta_3^{(5)}$	$(ZX)_{b_1} I_{b_2} X_{b_3}$	$\xi_7\zeta_3^{(7)}$	$(XZ)_{b_1} X_{b_2} X_{b_3}$
$\xi_1\zeta_4^{(1)}$	$Z_{b_1} I_{b_2} X_{b_3}$	$\xi_3\zeta_4^{(3)}$	$Z_{b_1}(ZX)_{b_2}(ZX)_{b_3}$	$\xi_5\zeta_4^{(5)}$	$X_{b_1} Z_{b_2}(ZX)_{b_3}$	$\xi_7\zeta_4^{(7)}$	$(ZX)_{b_1}(ZX)_{b_2}(ZX)_{b_3}$
$\xi_1\zeta_5^{(1)}$	$Z_{b_1} Z_{b_2}(ZX)_{b_3}$	$\xi_3\zeta_5^{(3)}$	$Z_{b_1} X_{b_2} X_{b_3}$	$\xi_5\zeta_5^{(5)}$	$X_{b_1} I_{b_2} X_{b_3}$	$\xi_7\zeta_5^{(7)}$	$(XZ)_{b_1} X_{b_2}(ZX)_{b_3}$
$\xi_1\zeta_6^{(1)}$	$I_{b_1} Z_{b_2} X_{b_3}$	$\xi_3\zeta_6^{(3)}$	$X_{b_1} Z_{b_2} X_{b_3}$	$\xi_5\zeta_6^{(5)}$	$(ZX)_{b_1} I_{b_2}(ZX)_{b_3}$	$\xi_7\zeta_6^{(7)}$	$(ZX)_{b_1}(ZX)_{b_2}(ZX)_{b_3}$
$\xi_1\zeta_7^{(1)}$	$Z_{b_1} I_{b_2}(ZX)_{b_3}$	$\xi_3\zeta_7^{(3)}$	$Z_{b_1}(ZX)_{b_2} I_{b_3}$	$\xi_5\zeta_7^{(5)}$	$X_{b_1} Z_{b_2}(ZX)_{b_3}$	$\xi_7\zeta_7^{(7)}$	$X_{b_1} X_{b_2} X_{b_3}$

3. RSP of an arbitrary three-qubit state in noisy environment

Quantum noise should be taken into account in actual communication. This section describes four types of quantum noises that are common in quantum channels: amplitude-damping, phase-damping, bit-flip and phase-flip noises. Quantum noise can be considered as a specific quantum operation that can be expressed in the form of operator summation. In order to determine the information loss, the fidelity in each situation is calculated. The DRSP of arbitrary three-qubit state is considered as follows. Alice is the source generator. She keeps qubits $(a_1, a_2, a_3, a_4, a_5, a_6)$, and sends (b_1, b_2, b_3) to Bob via the noisy channels. The noise on each channel is identical. Since the qubit pair $(a_1, a_2, a_3, a_4, a_5, a_6)$ is not transmitted, it only takes into account the effect of the channel noise on the qubits (b_1, b_2, b_3).

3.1. Density operators representation

We analyze the DRSP of an arbitrary three-qubit state in the form of density operators.[41] Alice distributes qubits to Bob through four kinds of noisy channels (bit-flip, phase-flip, amplitude-damping and phase-damping noisy channels,

respectively). The prepared state can be written as

$$\rho_{\text{target}} = |\chi\rangle\langle\chi|. \tag{8}$$

The density matrix of a quantum channel has the form

$$\rho_{\text{source}} = |\psi\rangle\langle\psi|. \tag{9}$$

The first measurement base of Alice is

$$MA_i = |\varphi_i\rangle\langle\varphi_i|, \quad i \in \{0,1,2,\ldots,7\}. \tag{10}$$

The second measurement base of Alice is

$$MB_j = \left|\eta_j^{(i)}\right\rangle\left\langle\eta_j^{(i)}\right|, \quad j \in \{0,1,2,\ldots,7\}. \tag{11}$$

The steps of the remote preparation of arbitrary three-qubit state can be written as follows:

Step 1 Alice measures qubits (a_1, a_3, a_5) using measurement operators MA_i, and the system becomes

$$\rho_{Q_i} = \text{tr}_{a_1 a_3 a_5}\left(\frac{MA_i \cdot \rho_{\text{source}} \cdot MA_i^{\dagger}}{\text{tr}\left(MA_i^{\dagger} \cdot MA_i \cdot \rho_{\text{source}}\right)}\right). \tag{12}$$

Step 2 Alice measures qubits (a_2, a_4, a_6) using the measurement operators MB_j, and the system becomes

$$\rho'_{Q_i} = \text{tr}_{a_2 a_4 a_6}\left(\frac{MB_j \cdot \rho_{Q_i} \cdot MB_j^{\dagger}}{\text{tr}\left(MB_j^{\dagger} \cdot MB_j \cdot \rho_{\text{source}}\right)}\right). \tag{13}$$

Step 3 Bob recovers the prepared state by performing another corresponding operation U and $R_j^{(i)}$,

$$\rho_{\text{out}} = R_j^{(i)} \cdot U \rho'_{Q_i} \cdot U^{\dagger} \cdot R_j^{(i)\dagger}. \tag{14}$$

Step 4 The fidelity of the system is

$$F = \langle\chi|\rho_{\text{out}}|\chi\rangle. \tag{15}$$

The fidelity ranges from 0 to 1, indicating that the smaller fidelity, the more information lost in noisy environment.

In the following discussions, we assume that Alice's measurement results are $MA_0 = |\varphi_0\rangle\langle\varphi_0|$ and $MB_0 = |\eta_0^{(0)}\rangle\langle\eta_0^{(0)}|$, respectively.

3.2. The amplitude-damping noise

The amplitude damping noise describes the effect of energy dissipation due to energy loss in the quantum system.[45] The Kraus operators of the amplitude-damping noise are expressed as follows:

$$E_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\lambda_{\text{AD}}} \end{pmatrix}, \quad E_0 = \begin{pmatrix} 0 & \sqrt{\lambda_{\text{AD}}} \\ 0 & 0 \end{pmatrix}, \tag{16}$$

where $0 \le \lambda_{\text{AD}} \le 1$ indicates the noise factor which characterizes the error probability. The influence of the amplitude-damping channel can be written as

$$\xi(\rho) = \sum_i \sum_j \sum_k E_i^{b_1} E_j^{b_2} E_k^{b_3} \rho E_i^{b_1\dagger} E_j^{b_2\dagger} E_k^{b_3\dagger}, \tag{17}$$

where ρ denotes the density matrix of the state described in Eq. (2), and E_i, E_j, E_k denote the noise operators acting on the transmitted qubits, and the superscripts represent the transmitted qubits. Following the similar steps (step 1 to 4 in this section), in the amplitude-damping noisy environment, the output state is

$$\begin{aligned} \rho_{\text{out}}^{\text{AD}} = {} & A_0\left|P_0^{\text{AD}}\right\rangle A_0'\left\langle P_0^{\text{AD}}\right| + A_0\left|P_1^{\text{AD}}\right\rangle A_0'\left\langle P_1^{\text{AD}}\right| \\ & + A_0\left|P_2^{\text{AD}}\right\rangle A_0'\left\langle P_2^{\text{AD}}\right| + A_0\left|P_3^{\text{AD}}\right\rangle A_0'\left\langle P_3^{\text{AD}}\right| \\ & + A_0\left|P_4^{\text{AD}}\right\rangle A_0'\left\langle P_4^{\text{AD}}\right| + A_0\left|P_5^{\text{AD}}\right\rangle A_0'\left\langle P_5^{\text{AD}}\right| \\ & + A_0\left|P_6^{\text{AD}}\right\rangle A_0'\left\langle P_6^{\text{AD}}\right| + A_0\left|P_7^{\text{AD}}\right\rangle A_0'\left\langle P_7^{\text{AD}}\right|, \end{aligned} \tag{18}$$

$$\begin{aligned} A_0 &= \left[x_0 e^{i\theta_0}, x_1 e^{i\theta_1}, x_2 e^{i\theta_2}, x_3 e^{i\theta_3}, x_4 e^{i\theta_4}, x_5 e^{i\theta_5}, x_6 e^{i\theta_6}, x_7 e^{i\theta_7}\right], \\ A_0' &= \left[x_0 e^{-i\theta_0}, x_1 e^{-i\theta_1}, x_2 e^{-i\theta_2}, x_3 e^{-i\theta_3}, x_4 e^{-i\theta_4}, x_5 e^{-i\theta_5}, x_6 e^{-i\theta_6}, x_7 e^{-i\theta_7}\right], \\ \left|P_0^{\text{AD}}\right\rangle &= [|000\rangle, \sqrt{1-\lambda_{\text{AD}}}|001\rangle, \sqrt{1-\lambda_{\text{AD}}}|010\rangle, (1-\lambda_{\text{AD}})|011\rangle, \\ &\quad \sqrt{1-\lambda_{\text{AD}}}|100\rangle, (1-\lambda_{\text{AD}})|101\rangle, (1-\lambda_{\text{AD}})|110\rangle, \sqrt{1-\lambda_{\text{AD}}}^3|111\rangle]^{\text{T}}, \\ \left|P_1^{\text{AD}}\right\rangle &= \left[0, \sqrt{\lambda_{\text{AD}}}|000\rangle, 0, \sqrt{\lambda_{\text{AD}}(1-\lambda_{\text{AD}})}|010\rangle, 0, \sqrt{\lambda_{\text{AD}}(1-\lambda_{\text{AD}})}|100\rangle, 0, \sqrt{\lambda_{\text{AD}}}(1-\lambda_{\text{AD}})|110\rangle\right]^{\text{T}}, \\ \left|P_2^{\text{AD}}\right\rangle &= \left[0, 0, \sqrt{\lambda_{\text{AD}}}|000\rangle, \sqrt{\lambda_{\text{AD}}(1-\lambda_{\text{AD}})}|001\rangle, 0, 0, \sqrt{\lambda_{\text{AD}}(1-\lambda_{\text{AD}})}|100\rangle, \sqrt{\lambda_{\text{AD}}}(1-\lambda_{\text{AD}})|101\rangle\right]^{\text{T}}, \\ \left|P_3^{\text{AD}}\right\rangle &= \left[0, 0, 0, \lambda_{\text{AD}}|000\rangle, 0, 0, 0, \sqrt{\lambda_{\text{AD}}}(1-\lambda_{\text{AD}})|100\rangle\right]^{\text{T}}, \\ \left|P_4^{\text{AD}}\right\rangle &= \left[0, 0, 0, 0, \sqrt{\lambda_{\text{AD}}}|000\rangle, \sqrt{\lambda_{\text{AD}}(1-\lambda_{\text{AD}})}|001\rangle, \sqrt{\lambda_{\text{AD}}(1-\lambda_{\text{AD}})}|010\rangle, \sqrt{\lambda_{\text{AD}}}(1-\lambda_{\text{AD}})|011\rangle\right]^{\text{T}}, \\ \left|P_5^{\text{AD}}\right\rangle &= \left[0, 0, 0, 0, 0, \lambda_{\text{AD}}|000\rangle, 0, \sqrt{\lambda_{\text{AD}}}(1-\lambda_{\text{AD}})|010\rangle\right]^{\text{T}}, \\ \left|P_6^{\text{AD}}\right\rangle &= \left[0, 0, 0, 0, 0, 0, \lambda_{\text{AD}}|000\rangle, \sqrt{\lambda_{\text{AD}}}(1-\lambda_{\text{AD}})|001\rangle\right]^{\text{T}}, \end{aligned}$$

$$\left|P_7^{\mathrm{AD}}\right\rangle = \left[0,0,0,0,0,0,0,\sqrt{\lambda_{\mathrm{AD}}}^{3}\left|000\right\rangle\right]^{\mathrm{T}}. \tag{19}$$

The fidelity is calculated as follows:

$$\begin{aligned}
F^{\mathrm{AD}} = {} & x_0^4+2\sqrt{1-\lambda_{\mathrm{AD}}}x_0^2\left(x_1^2+x_2^2+x_4^2\right)+2\sqrt{1-\lambda_{\mathrm{AD}}}^{5}x_7^2\left(x_3^2+x_5^2+x_6^2\right)\\
& +(1-\lambda_{\mathrm{AD}})\left[\left(x_1^4+x_2^4+x_4^4\right)+2x_0^2\left(x_3^2+x_5^2+x_6^2\right)+2\left(x_1^2x_2^2+x_1^2x_4^2+x_2^2x_4^2\right)\right]\\
& +2\sqrt{1-\lambda_{\mathrm{AD}}}^{3}\left[\left(x_1^2+x_2^2+x_4^2\right)\left(x_3^2+x_5^2+x_6^2\right)+x_7^2x_0^2\right]+(1-\lambda_{\mathrm{AD}})^3x_7^4\\
& +(1-\lambda_{\mathrm{AD}})^2\left[x_3^2\left(x_3^2+2x_5^2+2x_6^2\right)+\left(x_5^2+x_6^2\right)^2+2x_7^2\left(x_1^2+x_2^2+x_4^2\right)\right]\\
& +\lambda_{\mathrm{AD}}x_0^2\left(x_1^2+x_2^2+x_3^2\right)+\lambda\left(1-\lambda\right)\left[\left(x_2^2x_3^2+x_4^2x_5^2+x_1^2x_3^2+x_4^2x_6^2+x_1^2x_5^2+x_2^2x_6^2\right)\right]\\
& +2\lambda\left(1-\lambda_{\mathrm{AD}}\right)\cdot\left(x_0x_1x_6x_7\cos\left(\theta_1+\theta_6-\theta_0-\theta_7\right)+x_2x_3x_4x_5\cos\left(\theta_3+\theta_4-\theta_2-\theta_5\right)\right.\\
& +x_0x_2x_5x_7\cos\left(\theta_2+\theta_5-\theta_0-\theta_7\right)+x_1x_3x_4x_6\cos\left(\theta_3+\theta_4-\theta_1-\theta_6\right)\\
& \left.+x_0x_3x_4x_7\cos\left(\theta_3+\theta_4-\theta_0-\theta_7\right)+x_1x_2x_5x_6\cos\left(\theta_2+\theta_5-\theta_1-\theta_6\right)\right)\\
& +4\lambda\sqrt{1-\lambda_{\mathrm{AD}}}\cdot\left(x_0x_1x_2x_3\cos\left(\theta_1+\theta_2-\theta_0-\theta_3\right)+x_0x_1x_4x_5\cos\left(\theta_1+\theta_4-\theta_0-\theta_5\right)\right.\\
& \left.+x_0x_2x_4x_6\cos\left(\theta_2+\theta_4-\theta_0-\theta_6\right)\right)+\lambda_{\mathrm{AD}}(1-\lambda_{\mathrm{AD}})^2x_7^2\left(x_6^2+x_5^2+x_3^2\right)\\
& +4\lambda\sqrt{1-\lambda_{\mathrm{AD}}}^{3}\cdot\left(x_2x_3x_6x_7\cos\left(\theta_3+\theta_6-\theta_2-\theta_7\right)+x_4x_5x_6x_7\cos\left(\theta_5+\theta_6-\theta_4-\theta_7\right)\right.\\
& \left.+x_1x_3x_5x_7\cos\left(\theta_3+\theta_5-\theta_1-\theta_7\right)\right)+\lambda_{\mathrm{AD}}^2\left(1-\lambda_{\mathrm{AD}}\right)x_7^2\left(x_1^2+x_2^2+x_4^2\right)\\
& +2\lambda_{\mathrm{AD}}^2\sqrt{1-\lambda_{\mathrm{AD}}}\left(x_0x_3x_4x_7\cos\left(\theta_3+\theta_4-\theta_0-\theta_7\right)+x_0x_2x_5x_7\cos\left(\theta_0+\theta_7-\theta_2-\theta_7\right)\right.\\
& \left.+x_0x_1x_6x_7\cos\left(\theta_1+\theta_6-\theta_0-\theta_7\right)\right)+\lambda_{\mathrm{AD}}^2x_0^2\left(x_3^2+x_5^2+x_6^2\right)+\lambda_{\mathrm{AD}}^3x_0^2x_7^2.
\end{aligned} \tag{20}$$

3.3. The phase-damping noise

In contrast to the amplitude-damping noise, the phase-damping noise simulates an interaction with the surroundings when energy loss is not involved.[44] The behavior of this noise is characterized by the following set of Kraus operators:

$$E_0=\sqrt{1-\lambda_{\mathrm{PD}}}\begin{pmatrix}1&0\\0&1\end{pmatrix},\quad E_1=\sqrt{\lambda_{\mathrm{PD}}}\begin{pmatrix}1&0\\0&0\end{pmatrix},\quad E_2=\sqrt{\lambda_{\mathrm{PD}}}\begin{pmatrix}0&0\\0&1\end{pmatrix}, \tag{21}$$

where $0\le\lambda_{\mathrm{PD}}\le 1$ indicates the noise factor of phase damping noise. The influence of the phase-damping channel is described in Eq. (17). Based on Eqs. (8)–(14), we can obtain the output state

$$\begin{aligned}
\rho_{\mathrm{out}}^{\mathrm{PD}} = {} & (1-\lambda)^3A_0\left|P_0^{\mathrm{PD}}\right\rangle A_0'\left\langle P_0^{\mathrm{PD}}\right|+\lambda^3\left[A_0\left|P_{13}^{\mathrm{PD}}\right\rangle A_0'\left\langle P_{13}^{\mathrm{PD}}\right|+A_0\left|P_{14}^{\mathrm{PD}}\right\rangle A_0'\left\langle P_{14}^{\mathrm{PD}}\right|+A_0\left|P_{16}^{\mathrm{PD}}\right\rangle A_0'\left\langle P_{16}^{\mathrm{PD}}\right|\right.\\
& \left.+A_0\left|P_{17}^{\mathrm{PD}}\right\rangle A_0'\left\langle P_{17}^{\mathrm{PD}}\right|+A_0\left|P_{22}^{\mathrm{PD}}\right\rangle A_0'\left\langle P_{22}^{\mathrm{PD}}\right|+A_0\left|P_{23}^{\mathrm{PD}}\right\rangle A_0'\left\langle P_{23}^{\mathrm{PD}}\right|+A_0\left|P_{25}^{\mathrm{PD}}\right\rangle A_0'\left\langle P_{25}^{\mathrm{PD}}\right|+A_0\left|P_{26}^{\mathrm{PD}}\right\rangle A_0'\left\langle P_{26}^{\mathrm{PD}}\right|\right]\\
& +\lambda(1-\lambda)^2\left[A_0\left|P_1^{\mathrm{PD}}\right\rangle A_0'\left\langle P_1^{\mathrm{PD}}\right|+A_0\left|P_2^{\mathrm{PD}}\right\rangle A_0'\left\langle P_2^{\mathrm{PD}}\right|+A_0\left|P_3^{\mathrm{PD}}\right\rangle A_0'\left\langle P_3^{\mathrm{PD}}\right|\right.\\
& \left.+A_0\left|P_6^{\mathrm{PD}}\right\rangle A_0'\left\langle P_6^{\mathrm{PD}}\right|+A_0\left|P_9^{\mathrm{PD}}\right\rangle A_0'\left\langle P_9^{\mathrm{PD}}\right|+A_0\left|P_{18}^{\mathrm{PD}}\right\rangle A_0'\left\langle P_{18}^{\mathrm{PD}}\right|\right]\\
& +\lambda^2(1-\lambda)\left[A_0\left|P_4^{\mathrm{PD}}\right\rangle A_0'\left\langle P_4^{\mathrm{PD}}\right|+A_0\left|P_5^{\mathrm{PD}}\right\rangle A_0'\left\langle P_5^{\mathrm{PD}}\right|+A_0\left|P_7^{\mathrm{PD}}\right\rangle A_0'\left\langle P_7^{\mathrm{PD}}\right|+A_0\left|P_8^{\mathrm{PD}}\right\rangle A_0'\left\langle P_8^{\mathrm{PD}}\right|\right.\\
& +A_0\left|P_{10}^{\mathrm{PD}}\right\rangle A_0'\left\langle P_{10}^{\mathrm{PD}}\right|+A_0\left|P_{11}^{\mathrm{PD}}\right\rangle A_0'\left\langle P_{11}^{\mathrm{PD}}\right|+A_0\left|P_{12}^{\mathrm{PD}}\right\rangle A_0'\left\langle P_{12}^{\mathrm{PD}}\right|+A_0\left|P_{15}^{\mathrm{PD}}\right\rangle A_0'\left\langle P_{15}^{\mathrm{PD}}\right|\\
& \left.+A_0\left|P_{19}^{\mathrm{PD}}\right\rangle A_0'\left\langle P_{19}^{\mathrm{PD}}\right|+A_0\left|P_{20}^{\mathrm{PD}}\right\rangle A_0'\left\langle P_{20}^{\mathrm{PD}}\right|+A_0\left|P_{21}^{\mathrm{PD}}\right\rangle A_0'\left\langle P_{21}^{\mathrm{PD}}\right|+A_0\left|P_{24}^{\mathrm{PD}}\right\rangle A_0'\left\langle P_{24}^{\mathrm{PD}}\right|\right],
\end{aligned} \tag{22}$$

where

$$\begin{aligned}
\left|P_0^{\mathrm{PD}}\right\rangle &= [\left|000\right\rangle,\left|001\right\rangle,\left|010\right\rangle,\left|011\right\rangle,\left|100\right\rangle,\\
&\quad \left|101\right\rangle,\left|110\right\rangle,\left|111\right\rangle]^{\mathrm{T}},\\
\left|P_1^{\mathrm{PD}}\right\rangle &= [\left|000\right\rangle,0,\left|010\right\rangle,0,\left|100\right\rangle,0,\left|110\right\rangle,0]^{\mathrm{T}},\\
\left|P_2^{\mathrm{PD}}\right\rangle &= [0,\left|001\right\rangle,0,\left|011\right\rangle,0,\left|101\right\rangle,0,\left|111\right\rangle]^{\mathrm{T}},\\
\left|P_3^{\mathrm{PD}}\right\rangle &= [\left|000\right\rangle,\left|001\right\rangle,0,0,\left|100\right\rangle,\left|101\right\rangle,0,0]^{\mathrm{T}},\\
\left|P_4^{\mathrm{PD}}\right\rangle &= [\left|000\right\rangle,0,0,0,\left|100\right\rangle,0,0,0]^{\mathrm{T}},\\
\left|P_5^{\mathrm{PD}}\right\rangle &= [0,\left|001\right\rangle,0,0,0,\left|101\right\rangle,0,0]^{\mathrm{T}},\\
\left|P_6^{\mathrm{PD}}\right\rangle &= [0,0,\left|010\right\rangle,\left|011\right\rangle,0,0,\left|110\right\rangle,\left|111\right\rangle]^{\mathrm{T}},\\
\left|P_7^{\mathrm{PD}}\right\rangle &= [0,0,\left|010\right\rangle,0,0,0,\left|110\right\rangle,0]^{\mathrm{T}},\\
\left|P_8^{\mathrm{PD}}\right\rangle &= [0,0,0,\left|011\right\rangle,0,0,0,\left|111\right\rangle]^{\mathrm{T}},\\
\left|P_9^{\mathrm{PD}}\right\rangle &= [\left|000\right\rangle,\left|001\right\rangle,\left|010\right\rangle,\left|011\right\rangle,0,0,0,0]^{\mathrm{T}},\\
\left|P_{10}^{\mathrm{PD}}\right\rangle &= [\left|000\right\rangle,0,\left|010\right\rangle,0,0,0,0,0]^{\mathrm{T}},\\
\left|P_{11}^{\mathrm{PD}}\right\rangle &= [0,\left|001\right\rangle,0,\left|011\right\rangle,0,0,0,0]^{\mathrm{T}},
\end{aligned}$$

$$
\begin{aligned}
&\left|P_{12}^{\mathrm{PD}}\right\rangle=[|000\rangle,|001\rangle,0,0,0,0,0,0]^{\mathrm{T}},\\
&\left|P_{13}^{\mathrm{PD}}\right\rangle=[|000\rangle,0,0,0,0,0,0,0]^{\mathrm{T}},\\
&\left|P_{14}^{\mathrm{PD}}\right\rangle=[0,|001\rangle,0,0,0,0,0,0]^{\mathrm{T}},\\
&\left|P_{15}^{\mathrm{PD}}\right\rangle=[0,0,|010\rangle,|011\rangle,0,0,0,0]^{\mathrm{T}},\\
&\left|P_{16}^{\mathrm{PD}}\right\rangle=[0,0,|010\rangle,0,0,0,0,0]^{\mathrm{T}},\\
&\left|P_{17}^{\mathrm{PD}}\right\rangle=[0,0,0,|011\rangle,0,0,0,0]^{\mathrm{T}},\\
&\left|P_{18}^{\mathrm{PD}}\right\rangle=[0,0,0,0,|100\rangle,|101\rangle,|110\rangle,|111\rangle]^{\mathrm{T}},\\
&\left|P_{19}^{\mathrm{PD}}\right\rangle=[0,0,0,0,|100\rangle,0,|110\rangle,0]^{\mathrm{T}},\\
&\left|P_{20}^{\mathrm{PD}}\right\rangle=[0,0,0,0,0,|101\rangle,0,|111\rangle]^{\mathrm{T}},\\
&\left|P_{21}^{\mathrm{PD}}\right\rangle=[0,0,0,0,|100\rangle,|101\rangle,0,0]^{\mathrm{T}},\\
&\left|P_{22}^{\mathrm{PD}}\right\rangle=[0,0,0,0,|100\rangle,0,0,0]^{\mathrm{T}},\\
&\left|P_{23}^{\mathrm{PD}}\right\rangle=[0,0,0,0,0,|101\rangle,0,0]^{\mathrm{T}},\\
&\left|P_{24}^{\mathrm{PD}}\right\rangle=[0,0,0,0,0,0,|110\rangle,|111\rangle]^{\mathrm{T}},\\
&\left|P_{25}^{\mathrm{PD}}\right\rangle=[0,0,0,0,0,0,0,|111\rangle]^{\mathrm{T}},\\
&\left|P_{26}^{\mathrm{PD}}\right\rangle=[0,0,0,0,0,0,0,|111\rangle]^{\mathrm{T}}.
\end{aligned}
\tag{23}
$$

Following the similar steps, the fidelity is calculated as follows:

$$
\begin{aligned}
F^{\mathrm{PD}} =\ & (1-\lambda_{\mathrm{PD}})^3+\lambda_{\mathrm{PD}}^3\left(x_0^4+x_1^4+x_2^4+x_3^4+x_4^4+x_5^4+x_6^4+x_7^4\right)\\
&+\lambda_{\mathrm{PD}}(1-\lambda_{\mathrm{PD}})^2\left(\left(x_0^2+x_2^2+x_4^2+x_6^2\right)^2+\left(x_1^2+x_3^2+x_5^2+x_7^2\right)^2+\left(x_0^2+x_1^2+x_4^2+x_5^2\right)^2\right.\\
&\left.+\left(x_2^2+x_4^2+x_6^2+x_7^2\right)^2+\left(x_0^2+x_1^2+x_2^2+x_3^2\right)^2+\left(x_4^2+x_5^2+x_6^2+x_7^2\right)^2\right)\\
&+\lambda_{\mathrm{PD}}^2(1-\lambda_{\mathrm{PD}})\left(\left(x_0^2+x_4^2\right)^2+\left(x_1^2+x_5^2\right)^2+\left(x_2^2+x_6^2\right)^2+\left(x_3^2+x_7^2\right)^2+\left(x_0^2+x_2^2\right)^2+\left(x_1^2+x_3^2\right)^2\right.\\
&\left.+\left(x_0^2+x_1^2\right)^2+\left(x_4^2+x_6^2\right)^2+\left(x_5^2+x_7^2\right)^2+\left(x_4^2+x_5^2\right)^2+\left(x_6^2+x_7^2\right)^2+\left(x_2^2+x_3^2\right)^2\right).
\end{aligned}
\tag{24}
$$

3.4. The bit-flip noise

The bit-flip noise changes a qubit from $|0\rangle\rightarrow|1\rangle$ or from $|1\rangle\rightarrow|0\rangle$ with the probability λ_{BF},[45] and it can be described by the Kraus operators as

$$
E_0=\sqrt{1-\lambda_{\mathrm{BF}}}\begin{pmatrix}1&0\\0&1\end{pmatrix},\quad E_1=\sqrt{\lambda_{\mathrm{BF}}}\begin{pmatrix}0&1\\1&0\end{pmatrix},
\tag{25}
$$

where λ_{BF} is the decoherence rate satisfying $0\leq\lambda_{\mathrm{BF}}\leq 1$. The influence of the bit-flip channel is described in Eq. (17). In the bit-flip noisy channel, the output state can be calculated as follows:

$$
\begin{aligned}
\rho_{\mathrm{out}}^{\mathrm{BF}} =\ & (1-\lambda)^3A_0\left|P_0^{\mathrm{BF}}\right\rangle A_0'\left\langle P_0^{\mathrm{BF}}\right|+\lambda(1-\lambda)^2\left[A_0\left|P_1^{\mathrm{BF}}\right\rangle A_0'\left\langle P_1^{\mathrm{BF}}\right|+A_0\left|P_2^{\mathrm{BF}}\right\rangle A_0'\left\langle P_2^{\mathrm{BF}}\right|+A_0\left|P_4^{\mathrm{BF}}\right\rangle A_0'\left\langle P_4^{\mathrm{BF}}\right|\right]\\
&+\lambda^2(1-\lambda)\left[A_0\left|P_3^{\mathrm{BF}}\right\rangle A_0'\left\langle P_3^{\mathrm{BF}}\right|+A_0\left|P_5^{\mathrm{BF}}\right\rangle A_0'\left\langle P_5^{\mathrm{BF}}\right|+A_0\left|P_6^{\mathrm{BF}}\right\rangle A_0'\left\langle P_6^{\mathrm{BF}}\right|\right]+\lambda^3A_0\left|P_7^{\mathrm{BF}}\right\rangle A_0'\left\langle P_7^{\mathrm{BF}}\right|,
\end{aligned}
\tag{26}
$$

where

$$
\begin{aligned}
&\left|P_0^{\mathrm{BF}}\right\rangle=[|000\rangle,|001\rangle,|010\rangle,|011\rangle,|100\rangle,|101\rangle,|110\rangle,|111\rangle]^{\mathrm{T}},\\
&\left|P_1^{\mathrm{BF}}\right\rangle=[|001\rangle,|000\rangle,|011\rangle,|010\rangle,|101\rangle,|100\rangle,|111\rangle,|110\rangle]^{\mathrm{T}},\\
&\left|P_2^{\mathrm{BF}}\right\rangle=[|010\rangle,|011\rangle,|000\rangle,|001\rangle,|110\rangle,|111\rangle,|100\rangle,|101\rangle]^{\mathrm{T}},\\
&\left|P_3^{\mathrm{BF}}\right\rangle=[|011\rangle,|010\rangle,|001\rangle,|000\rangle,|111\rangle,|110\rangle,|101\rangle,|100\rangle]^{\mathrm{T}},\\
&\left|P_4^{\mathrm{BF}}\right\rangle=[|100\rangle,|101\rangle,|110\rangle,|111\rangle,|000\rangle,|001\rangle,|010\rangle,|011\rangle]^{\mathrm{T}},\\
&\left|P_5^{\mathrm{BF}}\right\rangle=[|101\rangle,|100\rangle,|111\rangle,|110\rangle,|001\rangle,|000\rangle,|011\rangle,|010\rangle]^{\mathrm{T}},\\
&\left|P_6^{\mathrm{BF}}\right\rangle=[|110\rangle,|111\rangle,|100\rangle,|101\rangle,|010\rangle,|011\rangle,|000\rangle,|001\rangle]^{\mathrm{T}},\\
&\left|P_7^{\mathrm{BF}}\right\rangle=[|111\rangle,|110\rangle,|101\rangle,|100\rangle,|011\rangle,|010\rangle,|001\rangle,|000\rangle]^{\mathrm{T}}.
\end{aligned}
\tag{27}
$$

In the bit-flip noisy environment, the fidelity is too dedious to present here. We put it in Appendix A.

3.5. The phase-flip noise

The phase damping noise changes the qubit $|1\rangle\rightarrow-|1\rangle$ with the probability λ_{PF}, which describes the loss of quantum information that does not contain information loss caused by energy loss,[44] and its Kraus operators are

$$
E_0=\sqrt{1-\lambda_{\mathrm{PF}}}\begin{pmatrix}1&0\\0&1\end{pmatrix},\quad E_1=\sqrt{\lambda_{\mathrm{PF}}}\begin{pmatrix}1&0\\0&-1\end{pmatrix},
\tag{28}
$$

where $0 \leq \lambda_{\rm PF} \leq 1$ indicates the decoherence rate. The influence of the phase-flip channel is described in Eq. (17). In the phase-flip noisy environment, we can also calculate the output state

$$\begin{aligned}\rho_{\rm out}^{\rm PF} =\ & (1-\lambda)^3 A_0 \left|P_0^{\rm PF}\right\rangle A_0' \left\langle P_0^{\rm PF}\right| + \lambda^3 [A_0 \left|P_7^{\rm PF}\right\rangle A_0' \left\langle P_7^{\rm PF}\right|] + \lambda(1-\lambda)^2 [A_0 \left|P_1^{\rm PF}\right\rangle A_0' \left\langle P_1^{\rm PF}\right| + A_0 \left|P_2^{\rm PF}\right\rangle A_0' \left\langle P_2^{\rm PF}\right| \\ & + A_0 \left|P_4^{\rm PF}\right\rangle A_0' \left\langle P_4^{\rm PF}\right|] + \lambda^2(1-\lambda)[A_0 \left|P_3^{\rm PF}\right\rangle A_0' \left\langle P_3^{\rm PF}\right| + A_0 \left|P_5^{\rm PF}\right\rangle A_0' \left\langle P_5^{\rm PF}\right| + A_0 \left|P_6^{\rm PF}\right\rangle A_0' \left\langle P_6^{\rm PF}\right|]. \end{aligned} \tag{29}$$

where

$$\begin{aligned}
\left|P_0^{\rm PF}\right\rangle &= [|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle]^{\rm T}, \\
\left|P_1^{\rm PF}\right\rangle &= [|001\rangle, -|000\rangle, |011\rangle, -|010\rangle, |101\rangle, -|100\rangle, |111\rangle, -|110\rangle]^{\rm T}, \\
\left|P_2^{\rm PF}\right\rangle &= [|010\rangle, |011\rangle, -|000\rangle, -|001\rangle, |110\rangle, |111\rangle, -|100\rangle, -|101\rangle]^{\rm T}, \\
\left|P_3^{\rm PF}\right\rangle &= [|011\rangle, -|010\rangle, -|001\rangle, |000\rangle, |111\rangle, -|110\rangle, -|101\rangle, |100\rangle]^{\rm T}, \\
\left|P_4^{\rm PF}\right\rangle &= [|100\rangle, |101\rangle, |110\rangle, |111\rangle, -|000\rangle, -|001\rangle, -|010\rangle, -|011\rangle]^{\rm T}, \\
\left|P_5^{\rm PF}\right\rangle &= [|101\rangle, -|100\rangle, |111\rangle, -|110\rangle, -|001\rangle, |000\rangle, -|011\rangle, |010\rangle]^{\rm T}, \\
\left|P_6^{\rm PF}\right\rangle &= [|110\rangle, |111\rangle, -|100\rangle, -|101\rangle, -|010\rangle, -|011\rangle, |000\rangle, |001\rangle]^{\rm T}, \\
\left|P_7^{\rm PF}\right\rangle &= [|111\rangle, -|110\rangle, -|101\rangle, |100\rangle, -|011\rangle, |010\rangle, |001\rangle, -|000\rangle]^{\rm T},
\end{aligned} \tag{30}$$

and the fidelity is calculated as follows:

$$\begin{aligned}F^{\rm PF} =\ & (1-\lambda_{\rm PF})^3 + \lambda_{\rm PF}(1-\lambda_{\rm PF})^2 \big((x_0^2 - x_1^2 + x_2^2 - x_3^2 + x_4^2 - x_5^2 + x_6^2 - x_7^2)^2 \\ & + (x_0^2 + x_1^2 - x_2^2 - x_3^2 + x_4^2 + x_5^2 - x_6^2 - x_7^2)^2 + (x_0^2 + x_1^2 + x_2^2 + x_3^2 - x_4^2 - x_5^2 - x_6^2 - x_7^2)^2\big) \\ & + \lambda_{\rm PF}^2(1-\lambda_{\rm PF})\big((x_0^2 - x_1^2 - x_2^2 + x_3^2 + x_4^2 - x_5^2 - x_6^2 + x_7^2)^2 + (x_0^2 - x_1^2 + x_2^2 - x_3^2 - x_4^2 + x_5^2 - x_6^2 + x_7^2)^2 \\ & + (x_0^2 + x_1^2 - x_2^2 - x_3^2 - x_4^2 - x_5^2 + x_6^2 + x_7^2)^2\big) + \lambda_{\rm PF}^3 (x_0^2 - x_1^2 - x_2^2 + x_3^2 - x_4^2 + x_5^2 + x_6^2 - x_7^2)^2. \end{aligned} \tag{31}$$

3.6. Analysis of noise impact

It can be seen that the fidelities depend on the coefficients of the amplitude information of the target state and the noise parameter under the phase-flip noise and phase-damping noisy channels. Meanwhile, the decoherence rate, the amplitude parameter and the phase information of the initial state can affect the fidelities in bit-flip and amplitude-damping noisy channels. For simplicity, when the target state of DRSP is $x_0|000\rangle + \sqrt{\frac{1}{2} - x_0^2}|001\rangle + \frac{1}{2}|010\rangle + \frac{1}{2}|011\rangle$ ($x_0 \in (0, \frac{\sqrt{2}}{2}]$, $x_1 = \sqrt{\frac{1}{2} - x_0^2}$, $x_3 = x_4 = \frac{1}{2}$, $x_4 = x_5 = x_6 = x_7 = 0$), the variations of the fidelities with the decoherence rate and the amplitude parameter of the initial state are illustrated in Figs. 1(a)–1(d). It is obvious that there is a clear decreasing trend of the fidelities when the decoherence rates increase, respectively.

We can get a clear picture of how four types of quantum noises affect the system when preparing the same target state. In Fig. 2, the four types of quantum noises are compared when the target state is $\frac{1}{2\sqrt{2}}(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle)$. As shown in the figure, in the early stage, the fidelity of the phase-damping noise decreases faster than that of amplitude-damping noise. However, when $\lambda = 1$, both the fidelities of the phase-damping noise and the amplitude-damping noise drop to 0.1. For bit-flip noisy environment, it is interesting that the fidelity always maintains at 1, indicating that there will be no loss of information in this case. Among the four types of quantum noises, the fidelity drops the fastest in the phase-flip noise and approaches 0 after λ reaches 0.85.

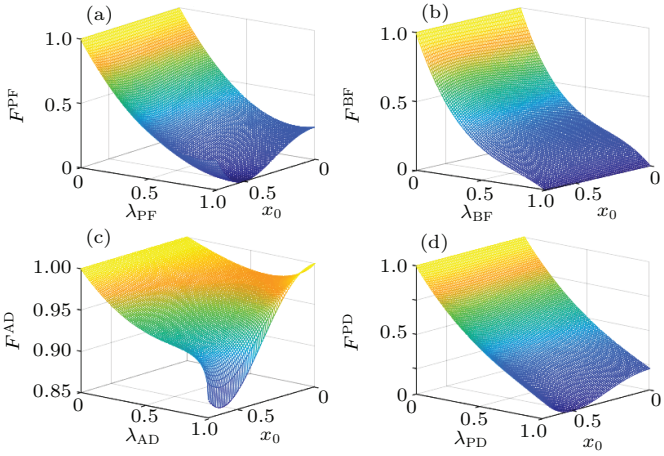


Fig. 1. The fidelities of four-type noise channels with x_0 and λ change for $x_0 \in (0, \frac{\sqrt{2}}{2}]$, $x_1 = \sqrt{\frac{1}{2} - x_0^2}$, $x_3 = x_4 = \frac{1}{2}$ and $x_4 = x_5 = x_6 = x_7 = 0$.

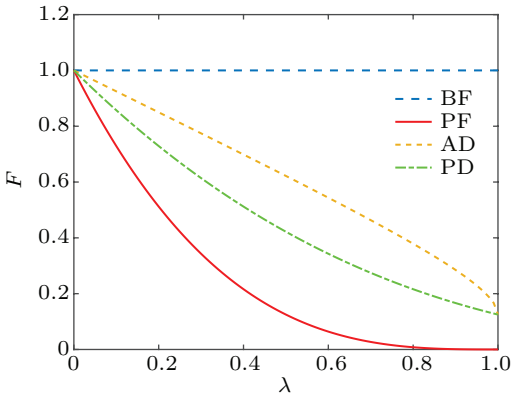


Fig. 2. Plots of the fidelities as a function of decoherence rate of amplitude-damping (yellow dotted line), phase-damping (greeen dot-dash line), bit-flip (bule dashed line), and phase-flip (red solid line) noise channels with $x_0 = x_1 = \cdots = x_7 = \frac{1}{2\sqrt{2}}$.

4. Conclusion

In summary, we have put forward a novel protocol for deterministic remote state preparation of arbitrary three-qubit state via one nine-qubit cluster-GHZ entangled state in ideal environment. We can realize the transmission of arbitrary three-qubit state which guarantees unit success probability in the meantime. In this study, two three-qubit projective measurements and appropriate unitary operations are required and only two classic bits are cost in classical communication. In addition, some attempts have been made to construct multi-particle channels, including cluster states and GHZ states,[31,33] which have made significant progress in both theoretical and experimental terms. In one sense, it shows the feasibility in the experiment in future.

The process of information transmission is inevitably affected by the surrounding environment in the real environment. One aim of the present research is to examine the influence of common four-type noises on the DRSP of arbitrary three-qubit state and obtains some interesting results. The results of this investigation show that the fidelities depend on the coefficients of the amplitude information and the noise parameter under the phase-flip and the phase-damping noises, while the decoherence rate, the amplitude parameter and the phase information of the initial state can affect the fidelities in the bit-flip and the amplitude-damping noise channels. The most obvious finding emerging from the analysis is that the fidelities decrease with the decoherence rates rising. The second finding is that the loss of information is minimal for the bit-flip channel and is maximal for the phase-flip channel when preparing the target state $\frac{1}{2\sqrt{2}}(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle)$. It may have significant implications for real systems to avoid influence of noise.

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Appendix A

$$
\begin{aligned}
F^{\mathrm{BF}} = {} & (1-\lambda_{\mathrm{BF}})^3 + 2\lambda_{\mathrm{BF}}(1-\lambda_{\mathrm{BF}})^2 \left\{x_0^2x_1^2\left[1+\cos 2(\theta_0-\theta_1)\right] + x_2^2x_3^2\left[1+\cos 2(\theta_2-\theta_3)\right] + x_4^2x_5^2\left[1+\cos 2(\theta_4-\theta_5)\right]\right. \\
& + x_6^2x_7^2\left[1+\cos 2(\theta_6-\theta_7)\right] + x_0^2x_2^2\left[1+\cos 2(\theta_0-\theta_2)\right] + x_1^2x_3^2\left[1+\cos 2(\theta_1-\theta_3)\right] \\
& + x_4^2x_6^2\left[1+\cos 2(\theta_4-\theta_6)\right] + x_5^2x_7^2\left[1+\cos 2(\theta_5-\theta_7)\right] + x_0^2x_4^2\left[1+\cos 2(\theta_0-\theta_4)\right] \\
& + x_1^2x_5^2\left[1+\cos 2(\theta_1-\theta_5)\right] + x_2^2x_6^2\left[1+\cos 2(\theta_2-\theta_6)\right] + x_3^2x_7^2\left[1+\cos 2(\theta_3-\theta_7)\right] \\
& + 2x_0x_1x_2x_3\left[\cos(\theta_0+\theta_3-\theta_1-\theta_2)+\cos(\theta_0+\theta_2-\theta_1-\theta_3)\right] \\
& + 2x_0x_1x_4x_5\left[\cos(\theta_0+\theta_5-\theta_1-\theta_4)+\cos(\theta_0+\theta_4-\theta_1-\theta_5)\right] \\
& + 2x_0x_1x_6x_7\left[\cos(\theta_0+\theta_7-\theta_1-\theta_6)+\cos(\theta_0+\theta_6-\theta_1-\theta_7)\right] \\
& + 2x_2x_3x_4x_5\left[\cos(\theta_3+\theta_5-\theta_2-\theta_4)+\cos(\theta_3+\theta_4-\theta_2-\theta_5)\right] \\
& + 2x_2x_3x_6x_7\left[\cos(\theta_3+\theta_7-\theta_2-\theta_6)+\cos(\theta_3+\theta_6-\theta_2-\theta_7)\right] \\
& + 2x_4x_5x_6x_7\left[\cos(\theta_4+\theta_7-\theta_5-\theta_6)+\cos(\theta_4+\theta_6-\theta_5-\theta_7)\right] \\
& + 2x_0x_1x_2x_3\left[\cos(\theta_0+\theta_3-\theta_1-\theta_2)+\cos(\theta_0+\theta_2-\theta_1-\theta_3)\right] \\
& + 2x_0x_2x_4x_6\left[\cos(\theta_0+\theta_6-\theta_2-\theta_4)+\cos(\theta_0+\theta_4-\theta_2-\theta_6)\right] \\
& + 2x_0x_2x_5x_7\left[\cos(\theta_0+\theta_7-\theta_2-\theta_5)+\cos(\theta_0+\theta_5-\theta_2-\theta_7)\right] \\
& + 2x_1x_3x_4x_6\left[\cos(\theta_1+\theta_6-\theta_3-\theta_4)+\cos(\theta_1+\theta_4-\theta_3-\theta_6)\right]
\end{aligned}
$$

$$
\begin{aligned}
&+2x_1x_3x_5x_7\left[\cos\left(\theta_1+\theta_7-\theta_3-\theta_5\right)+\cos\left(\theta_1+\theta_5-\theta_3-\theta_7\right)\right]\\
&+2x_4x_5x_6x_7\left[\cos\left(\theta_4+\theta_7-\theta_5-\theta_6\right)+\cos\left(\theta_4+\theta_5-\theta_6-\theta_7\right)\right]\\
&+2x_0x_1x_4x_5\left[\cos\left(\theta_0+\theta_5-\theta_1-\theta_4\right)+\cos\left(\theta_0+\theta_1-\theta_4-\theta_5\right)\right]\\
&+2x_0x_2x_4x_6\left[\cos\left(\theta_0+\theta_6-\theta_2-\theta_4\right)+\cos\left(\theta_0+\theta_2-\theta_4-\theta_6\right)\right]\\
&+2x_0x_3x_4x_7\left[\cos\left(\theta_0+\theta_7-\theta_4-\theta_3\right)+\cos\left(\theta_0+\theta_3-\theta_4-\theta_7\right)\right]\\
&+2x_1x_2x_5x_6\left[\cos\left(\theta_1+\theta_6-\theta_2-\theta_5\right)+\cos\left(\theta_1+\theta_2-\theta_5-\theta_6\right)\right]\\
&+2x_1x_3x_5x_7\left[\cos\left(\theta_1+\theta_7-\theta_3-\theta_5\right)+\cos\left(\theta_1+\theta_3-\theta_5-\theta_7\right)\right]\\
&+2x_2x_3x_6x_7\left[\cos\left(\theta_3+\theta_6-\theta_2-\theta_7\right)+\cos\left(\theta_2+\theta_3-\theta_6-\theta_7\right)\right]\}\\
&+2\lambda_{\mathrm{BF}}^2\left(1-\lambda_{\mathrm{BF}}\right)\left\{x_0^2x_3^2\left[1+\cos2\left(\theta_0-\theta_3\right)\right]+x_1^2x_2^2\left[1+\cos2\left(\theta_1-\theta_2\right)\right]+x_4^2x_7^2\left[1+\cos2\left(\theta_4-\theta_7\right)\right]\right.\\
&+x_5^2x_6^2\left[1+\cos2\left(\theta_5-\theta_6\right)\right]+x_0^2x_5^2\left[1+\cos2\left(\theta_0-\theta_5\right)\right]+x_1^2x_4^2\left[1+\cos2\left(\theta_1-\theta_4\right)\right]\\
&+x_2^2x_7^2\left[1+\cos2\left(\theta_2-\theta_7\right)\right]+x_3^2x_6^2\left[1+\cos2\left(\theta_3-\theta_6\right)\right]+x_0^2x_6^2\left[1+\cos2\left(\theta_0-\theta_6\right)\right]\\
&+x_1^2x_7^2\left[1+\cos2\left(\theta_1-\theta_7\right)\right]+x_2^2x_4^2\left[1+\cos2\left(\theta_2-\theta_4\right)\right]+x_3^2x_5^2\left[1+\cos2\left(\theta_3-\theta_5\right)\right]\\
&+2x_0x_1x_2x_3\left[\cos\left(\theta_0+\theta_2-\theta_1-\theta_3\right)+\cos\left(\theta_0+\theta_1-\theta_2-\theta_3\right)\right]\\
&+2x_0x_3x_4x_7\left[\cos\left(\theta_0+\theta_7-\theta_3-\theta_4\right)+\cos\left(\theta_0+\theta_4-\theta_3-\theta_7\right)\right]\\
&+2x_0x_3x_5x_6\left[\cos\left(\theta_0+\theta_6-\theta_3-\theta_5\right)+\cos\left(\theta_0+\theta_5-\theta_3-\theta_6\right)\right]\\
&+2x_1x_2x_4x_7\left[\cos\left(\theta_1+\theta_7-\theta_2-\theta_4\right)+\cos\left(\theta_1+\theta_4-\theta_2-\theta_7\right)\right]\\
&+2x_1x_2x_5x_6\left(\left[\cos\left(\theta_1+\theta_6-\theta_5-\theta_2\right)+\cos\left(\theta_1+\theta_5-\theta_2-\theta_6\right)\right]\right.\\
&+2x_4x_5x_6x_7\left[\cos\left(\theta_4+\theta_6-\theta_5-\theta_7\right)+\cos\left(\theta_4+\theta_5-\theta_6-\theta_7\right)\right]\\
&+2x_0x_1x_4x_5\left[\cos\left(\theta_0+\theta_4-\theta_1-\theta_5\right)+\cos\left(\theta_0+\theta_1-\theta_4-\theta_5\right)\right]\\
&+2x_0x_2x_5x_7\left[\cos\left(\theta_0+\theta_7-\theta_2-\theta_5\right)+\cos\left(\theta_0+\theta_2-\theta_5-\theta_7\right)\right]\\
&+2x_0x_3x_5x_6\left[\cos\left(\theta_0+\theta_6-\theta_3-\theta_5\right)+\cos\left(\theta_0+\theta_3-\theta_5-\theta_6\right)\right]\\
&+2x_1x_2x_4x_7\left[\cos\left(\theta_1+\theta_7-\theta_2-\theta_4\right)+\cos\left(\theta_1+\theta_2-\theta_4-\theta_7\right)\right]\\
&+2x_1x_3x_4x_6\left[\cos\left(\theta_3+\theta_4-\theta_1-\theta_6\right)+\cos\left(\theta_1+\theta_3-\theta_4-\theta_6\right)\right]\\
&+2x_2x_3x_6x_7\left[\cos\left(\theta_3+\theta_7-\theta_2-\theta_6\right)+\cos\left(\theta_2+\theta_3-\theta_6-\theta_7\right)\right]\\
&+2x_0x_1x_6x_7\left[\cos\left(\theta_0+\theta_7-\theta_1-\theta_6\right)+\cos\left(\theta_0+\theta_1-\theta_6-\theta_7\right)\right]\\
&+2x_0x_2x_4x_6\left[\cos\left(\theta_0+\theta_4-\theta_2-\theta_6\right)+\cos\left(\theta_0+\theta_2-\theta_4-\theta_6\right)\right]\\
&+2x_0x_3x_5x_6\left[\cos\left(\theta_0+\theta_5-\theta_3-\theta_6\right)+\cos\left(\theta_0+\theta_3-\theta_5-\theta_6\right)\right]\\
&+2x_1x_2x_4x_7\left[\cos\left(\theta_1+\theta_4-\theta_2-\theta_7\right)+\cos\left(\theta_1+\theta_2-\theta_4-\theta_7\right)\right]\\
&+2x_1x_3x_5x_7\left[\cos\left(\theta_1+\theta_5-\theta_3-\theta_7\right)+\cos\left(\theta_1+\theta_3-\theta_5-\theta_7\right)\right]\\
&+2x_2x_3x_4x_5\left[\cos\left(\theta_3+\theta_4-\theta_2-\theta_5\right)+\cos\left(\theta_2+\theta_3-\theta_4-\theta_5\right)\right]\}\\
&+2\lambda_{\mathrm{BF}}^3\left\{x_0^2x_7^2\left[1+\cos2\left(\theta_0-\theta_7\right)\right]+x_1^2x_6^2\left[1+\cos2\left(\theta_1-\theta_6\right)\right]\right.\\
&+x_3^2x_4^2\left[1+\cos2\left(\theta_3-\theta_4\right)\right]+x_2^2x_5^2\left[1+\cos2\left(\theta_2-\theta_5\right)\right]\\
&+2x_0x_1x_6x_7\left[\cos\left(\theta_0+\theta_6-\theta_1-\theta_7\right)+\cos\left(\theta_0+\theta_1-\theta_6-\theta_7\right)\right]\\
&+2x_0x_2x_5x_7\left[\cos\left(\theta_0+\theta_5-\theta_2-\theta_7\right)+\cos\left(\theta_0+\theta_2-\theta_5-\theta_7\right)\right]\\
&+2x_0x_3x_4x_7\left[\cos\left(\theta_0+\theta_4-\theta_3-\theta_7\right)+\cos\left(\theta_0+\theta_3-\theta_4-\theta_7\right)\right]\\
&+2x_1x_2x_5x_6\left[\cos\left(\theta_1+\theta_5-\theta_2-\theta_6\right)+\cos\left(\theta_1+\theta_2-\theta_5-\theta_6\right)\right]\\
&+2x_1x_3x_4x_6\left[\cos\left(\theta_1+\theta_4-\theta_3-\theta_6\right)+\cos\left(\theta_1+\theta_3-\theta_4-\theta_6\right)\right]\\
&\left.+2x_2x_3x_4x_5\left[\cos\left(\theta_3+\theta_5-\theta_2-\theta_4\right)+\cos\left(\theta_2+\theta_3-\theta_4-\theta_5\right)\right]\right\}.
\end{aligned}
$$

References

[1] Bennett C H, Brassard G, C Crepeau *et al.* 1993 *Phys. Rev. Lett.* **70** 1895

[2] Karlsson A and Bourennane M 2002 *Phys. Rev. A* **58** 4394

[3] Zhang S L, Jin C H, Shi J H *et al.* 2017 *Chin. Phys. Lett.* **34** 040302

[4] Wang J, Zhang Q, Tang C J 2006 *International Conference on Computational Intelligence and Security*, November 3–6, 2006, Guangzhou, China, pp. 69–80

[5] Xu P, Bao W S, Li H W *et al.* 2017 *Chin. Phys. Lett.* **34** 020302

[6] Zhang Z J and Man Z X 2004 *Phys. Rev. A* **72** 022303

[7] Xiao M, Cao Y R and Song X L 2017 *Chin. Phys. Lett.* **34** 030302
[8] Mattle K, Weinfurter H, Kwiat P G *et al.* 1996 *Phys. Rev. Lett.* **76** 4656
[9] Lo H K 2000 *Phys. Rev. A* **62** 012313
[10] Bennett C H, DiVincenzo D P, Shor P W *et al.* 2001 *Phys. Rev. Lett.* **87** 077902
[11] Qi B, Pan H and Guo L 2012 *IEEE Trans. Autom. Control* **58** 1349
[12] Dong D, Chen C, Tarn T J *et al.* 2008 *IEEE Trans. Systems, Man, and Cybernetics, Part B (Cybernetics)* **38** 957
[13] Wang Y, Dong D, Qi B *et al.* 2017 *IEEE Trans. Autom. Control* **63** 1388
[14] Wu R B and Rabitz H 2012 *J. Phys. A: Math. Theor.* **45** 485303
[15] Xue S B, Tan L Y, Wu R B *et al.* 2012 *Phys. Rev. A* **102** 042227
[16] Cong S and Yang F 2013 *J. Phys. A: Math. Theor.* **46** 075305
[17] Long G ad Feng G 2019 *Quantum Engin.* **1** e27
[18] Yang L and Liu Y C 2020 *Chin. Phys. B* **29** 060301
[19] Sheng Y B and Zhou L 2018 *Phys. Rev. A* **98** 052343
[20] Zhang X P, Shen L T, Zhang Y *et al.* 2021 *Sci. China Phys. Mech. Astron.* **64** 1
[21] Wei J H, Dai H Y and Zhang M 2014 *Quantum Inform. Process.* **13** 2115
[22] Zhang P, Li X, Ma S Y *et al.* 2017 *Commun. Theor. Phys.* **67** 498
[23] Ma S Y, Gao C, Zhang P *et al.* 2017 *Quantum Inform. Process.* **16** 1
[24] Nguyen B A and Kim J 2008 *J. Phys. B: At. Mol. Opt. Phys.* **41** 095501
[25] Chen X B, Ma S Y, Su Y *et al.* 2012 *Quantum Inform. Process.* **11** 1653
[26] Zhang Y W, Yi M L, Xue Q Z *et al.* 2009 *Phys. Rev. Lett.* **52** 235
[27] Kiess T E, Shih Y H, Sergienko A V *et al.* 1993 *Phys. Rev. Lett.* **71** 3893
[28] An N B, Cao T B and Don N V 2011 *Phys. Lett. A* **375** 3570
[29] Chen Q Q, Xia Y and Song J 2012 *J. Phys. A: Math. Theor.* **45** 055303
[30] Bich C T, Don N V and An N B 2012 *Int. J. Theor. Phys.* **51** 2272
[31] Resch K J, Walther P and Zeilinger A 2005 *Phys. Rev. Lett.* **94** 070402
[32] Peng J Y, Luo M X and Mo Z W 2013 *Quantum Inform. Process.* **12** 2325
[33] Jia H Y, Wen Q Y, Wu X *et al.* 2012 *Int. J. Theor. Phys.* **51** 2086
[34] Wang M M, Chen X B and Yang Y X 2013 *Commun. Theor. Phys.* **59** 568
[35] Ding M, Jiang M, Huang X and Liu Y 2016 *35th Chinese Control Conference (CCC)*, July 27–29, 2016, Chengdu, China, pp. 9105–9108
[36] Zha X W, Wang M R and Jiang R X 2020 *Chin. Phys. B* **29** 040304
[37] Zhan Y B, Hao F, Li X W *et al.* 2013 *Int. J. Theor. Phys.* **52** 2615
[38] Hou K, Yu J Y and Yan F 1993 *Int. J. Theor. Phys.* **54** 3092
[39] Wei J, Shi L, Zhu Y *et al.* 2018 *Quantum Inform. Process.* **17** 70
[40] Zha X W, Wang M R and Jiang R X 2020 *Int. J. Theor. Phys.* **59** 960
[41] Qian Y, Xue S and Jiang M 2020 *Phys. Lett. A* **384** 126204
[42] Zhang P, Ma S and Gong L 2019 *Int. J. Theor. Phys.* **58** 2795
[43] Dash T, Sk R and Panigrahi P K 2020 *Opt. Commun.* **464** 125518
[44] Falaye B J, Sun G H, Camacho-Nieto O *et al.* 2016 *Int. J. Quantum Inform.* **14** 1650034
[45] Le S, Wu S, Qu Z *et al.* 2018 *Quantum Inform. Process.* **17** 283
[46] Wang D, Zha X W, Qi J X and He Y 2012 *Acta Photon. Sin.* **41** 335 (in Chinese)
[47] Zhan Y B and Ma P C 2013 *Quantum Inform. Process.* **12** 997