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Quantum multicast communication over the butterfly network

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We propose a scheme where one can exploit auxiliary resources to achieve quantum multicast communication with network coding over the butterfly network. In this paper, we propose the quantum 2-pair multicast communication scheme, and extend it to k-pair multicast communication over the extended butterfly network. Firstly, an EPR pair is shared between each adjacent node on the butterfly network, and make use of local operation and classical communication to generate entangled relationship between non-adjacent nodes. Secondly, each sender adds auxiliary particles according to the multicast number k, in which the CNOT operations are applied to form the multi-particle entangled state. Finally, combined with network coding and free classical communication, quantum multicast communication based on quantum measurements is completed over the extended butterfly network. Not only the bottleneck problem is solved, but also quantum multicast communication can be completed in our scheme. At the same time, regardless of multicast number k, the maximum capacity of classical channel is 2 bits, and quantum channel is used only once.

Keywords: quantum nondemolition measurement, special single particle basis, quantum network coding, quantum multicast communication

PACS: 03.67.Hk, 03.65.Ud, 03.65.Aa, 03.67.–a **DOI:** 10.1088/1674-1056/ac20c6

1. Introduction

In 2000, Ahlswede *et al.*[1] proposed the idea of classical network coding for the first time, which is of epoch-making significance and brings new opportunities for the development of information communication.

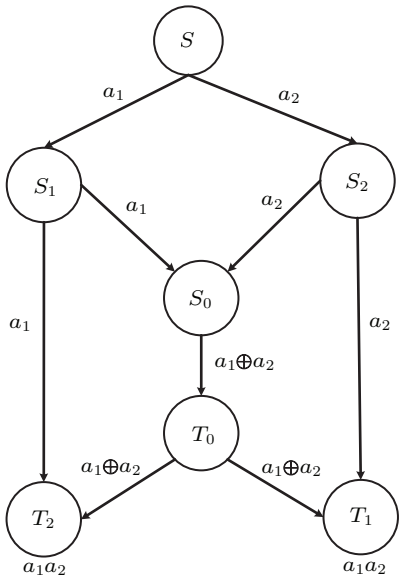


Fig. 1. Classic network coding.

As shown in Fig. 1, by performing an XOR operation at the bottleneck node, the receivers T_1 and T_2 can get bits a_1 and a_2 simultaneously. The protocol completes two pairs of multicast communication over the butterfly network. The network coding solves the congestion problem at the bottleneck node, and improves the transmission efficiency and the total network throughput.[2–6]

In 2007, Hayashi *et al.*[7] initiated proposed a quantum network coding protocol considering the advantages of network coding. As shown in Fig. 2, senders S_1 and S_2 transmit pure states, while receivers receive the mixed states. Although this protocol can achieve quantum 2-pair multi-unicast communication, and the fidelity of the output states cannot reach 1. It is impossible in principle to replicate exactly an unknown quantum state due to the non-cloning theorem.[8] Subsequently, Kobayashi *et al.*[9–11] showed that as long as classical communication is free over the butterfly network, the perfect transmission of quantum states[12–15] can be completed. This result provides a theoretical basis for perfect quantum network coding.[12,16–19] So far, a large number of quantum network coding schemes[20–24] have been proposed, some of which have been implemented in experiments.[25,26] In Ref. [25], Lu *et al.* have demonstrated Hayashi'protocol[12] by employing eight photons generated via spontaneous parametric down-conversion. The average fidelities of cross state transmission and cross entanglement distribution achieved exceed the theoretical upper bounds permitted without prior

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entanglement. In Ref. [26], Pathumsoot *et al.* have completed a particular implementation of quantum network coding using measurement-based quantum computation on IBM Q processors. Pathumsoot *et al.* have demonstrated how a quantum computer can be used to model an entangling quantum network and experimentally confirmed the operation of quantum network coding in a superconducting device. This shows that it has taken a step from the theory of quantum network coding toward practical use on real devices. At present, quantum network coding in the form of quantum multi-unicast communication[12,16–24] emerges in endlessly, while quantum network coding in the form of quantum multicast communication[27–30] is relatively less.

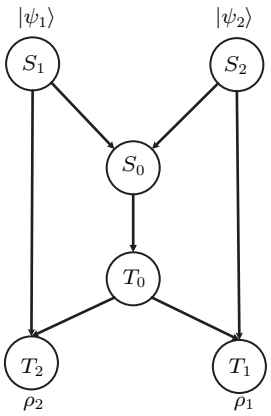


Fig. 2. Quantum network coding with universal cloning.

In 2006, Shi and Soljanin[27] began to study the multicast problem in quantum networks. Through quantum lossless compression and decompression technology, quantum multicast communication can be completed in quantum networks. In 2009, Li *et al.*[28] proposed a new optimization quantum-inspired evolutionary algorithm (QEA) to solve the multicast routing problems. In 2010, Xing *et al.*[29] proposed an improved quantum-inspired evolutionary algorithm for coding resource optimization based network coding multicast scheme. In 2015, Xu *et al.*[30] proposed a quantum cooperative multicast coding scheme. In Ref. [30], Xu *et al.* gave the explicit definition of quantum cooperative multicast, and designed an efficient scheme for cooperative multicast of two-level quantum states over the butterfly network. Quantum multicast communication solves the one-to-many quantum communication problem, which is a great significance to the practical application of quantum communication networks[31–35] in the future.

We study quantum network coding and quantum multicast communication. In Ref. [36], Li *et al.* designed universal quantum operations by applying local operation and classical communication,[38] and proposed a perfect quantum multiple-unicast network coding protocol. In Ref. [37], Pati proposed assisted cloning and orthogonal complementing of unknown quantum state. Without violating the quantum non-cloning theorem, by applying quantum non-demolitional measurement (QND),[39–41] the assisted clone and orthogonal complementation of an unknown quantum state can be achieved with the help of the quantum and classical channels. Subsequently, a number of perfect assisted cloning schemes[42–44] have been proposed.

In this paper, we propose a quantum multicast communication with network coding, which achieves quantum 2-pair multicast communication over the butterfly network. Then we extend quantum 2-pair multicast communication to the extended butterfly network, and quantum k-pair multicast communication is proposed over the extended butterfly network. Firstly, an EPR state is shared between each adjacent repeater of butterfly network, and use the quantum operations introduced in Subsection 2.1 to generate entanglement relationship between non-adjacent senders and receivers. Secondly, after each sender adds auxiliary particles according to the multicast number k , the added auxiliary particles and the EPR state form a multi-particle entangled state through controlled NOT gates. Thirdly, we use Bell basis and single-particle basis to measure multi-particle entangled state, in which Bell basis measurement is quantum non-demolition measurement. Finally, by designing encoding and decoding strategies, quantum multicast communication can be completed with the help of classical and quantum channels.

2. Preliminaries

2.1. Quantum complementary state

In our scheme, Victor prepares an arbitrary quantum state $|\psi\rangle_v = \alpha|0\rangle_v + \beta|1\rangle_v$, where $\alpha = \cos(\theta/2)$, $\beta = \mathrm{e}^{\mathrm{i}\phi}\sin(\theta/2)$, just like representing an arbitrary quantum state on the bloch sphere. The normalization condition $|\alpha|^2 + |\beta|^2 = 1$ is satisfied. We define that $|\psi_\perp\rangle_v$ is quantum complementary state of $|\psi\rangle_v$, in which $|\psi_\perp\rangle_v = \alpha|1\rangle_v - \beta^*|0\rangle_v$.

In order to achieve quantum multicast communication, Victor needs to perform the special single particle basis. We assume the special single particle basis $\{|m\rangle, |n\rangle\}$ is

$$|m\rangle = \alpha|0\rangle + \beta|1\rangle,$$
$$|n\rangle = \beta^*|0\rangle - \alpha|1\rangle.$$

The normalization and orthogonalization relationships are satisfied between the set of basis.

2.2. Quantum operations

We refer to the encoding and decoding operations in Refs. [21,36] to design quantum operations over the extended butterfly network. The extended butterfly network is shown in Fig. 3.

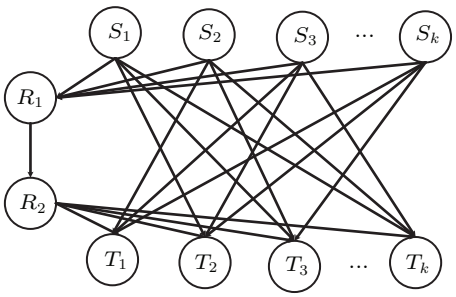


Fig. 3. The extended butterfly network.

In our scheme, each node is regarded as a quantum repeater on the butterfly network, and a pair of maximum entangled state $|\Psi^+\rangle = (1/\sqrt{2})(|00\rangle + |11\rangle)$ is shared between adjacent repeaters. The first step in our scheme is to establish the entangled relationship between non-adjacent senders and receivers. We partition the extended butterfly network and design quantum operations. The details are shown in the following figures and tables.

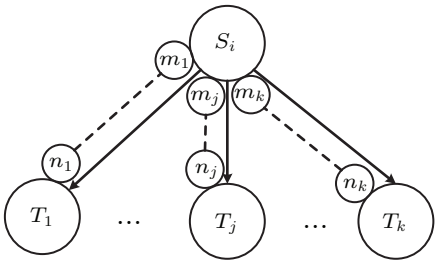


Fig. 4. One source node to multiple target nodes: EPR pairs are shared between a source node and multiple target nodes, Connection operation $\mathrm{Con}^{m_1}_{m_2\to n_2,\ldots,m_k\to n_k}$ is applied as shown in Table 1.

Table 1. $\mathrm{Con}^{m_1}_{m_2\to n_2,\ldots,m_k\to n_k}$.

1 Quantum repeater S_i performs $\mathrm{CNOT}^{m_1}_{m_2},\ldots,\mathrm{CNOT}^{m_1}_{m_k}$ successively.
2 Quantum repeater S_i uses $\{\|0\rangle,\|1\rangle\}$ to measure particles $m_2,\ldots,m_k$ and let $g=(g_2,\ldots,g_k)$ be measurement result, where $g_i\in\{0,1\}$.
3 Quantum repeater S_i sends $g_2,\ldots,g_k$ to quantum repeater T_j $(2\le j\le k)$ through the classic channel, if g_j $(2\le j\le k)=1$, then T_j applies an X to particle n_j.
Here, the particles $m_1,m_2,\ldots,m_k$ are owned by quantum repeater S_i; the particles $n_1,n_2,\ldots,n_k$ are owned by quantum repeater T_j, respectively; and X is Pauli operator.

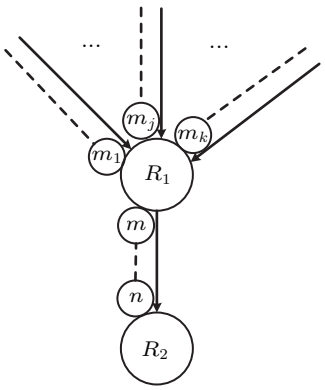


Fig. 5. Multiple sources node to intermediate node: EPR pairs are shared between multiple source nodes and an intermediate node, Connection operation $\mathrm{Con}^{m_1\oplus m_2\oplus\cdots\oplus m_k}_{m\to n}$ is applied as shown in Table 2.

Table 2. $\mathrm{Con}^{m_1\oplus m_2\oplus\cdots\oplus m_k}_{m\to n}$.

1 Quantum repeater R_1 performs $\mathrm{CNOT}^{m_1\oplus m_2\oplus\cdots\oplus m_k}_{m}$.
2 Quantum repeater R_1 uses $\{\|0\rangle,\|1\rangle\}$ to measure particle m and let $\lambda\in(0,1)$ be measurement result.
3 Quantum repeater R_1 sends λ to quantum repeater R_2 via classic channel, if λ=1, then R_2 applies an X to particle n.
Here, the particles m, m_1, $m_2,\ldots,m_k$ are owned by quantum repeater R_1; the particle n is owned by quantum repeater R_2; and X is Pauli operator.

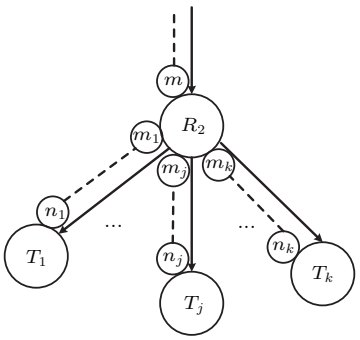


Fig. 6. Intermediate node to multiple target nodes: EPR pairs are shared between an intermediate node and multiple target nodes, Connection operation $\mathrm{Con}^{m}_{m_1\to n_1,m_2\to n_2,\ldots,m_k\to n_k}$ is applied as shown in Table 3.

Table 3. $\mathrm{Con}^{m}_{m_1\to n_1,m_2\to n_2,\ldots,m_k\to n_k}$.

1 Quantum repeater R_2 performs $\mathrm{CNOT}^{m}_{m_1},CNOT^{m}_{m_2},\ldots,CNOT^{m}_{m_k}$ successively.
2 Quantum repeater R_2 uses $\{\|0\rangle,\|1\rangle\}$ to measure particles $m_1,m_2,\ldots,m_k$ and let $g=(g_1,g_2,\ldots,g_k)$ be measurement result, where $g_i\in\{0,1\}$.
3 Quantum repeater R_2 sends $g_1,g_2,\ldots,g_k$ to quantum repeater T_j $(1\le j\le k)$ via classic channel, if g_j $(1\le j\le k)$=1, then T_j applies an X to particle n_j.
Here, the particles $m,m_1,m_2,\ldots,m_k$ are owned by quantum repeater R_2; the particle n_j is owned by quantum repeater T_j $(1\le j\le k)$; and X is the Pauli operator.

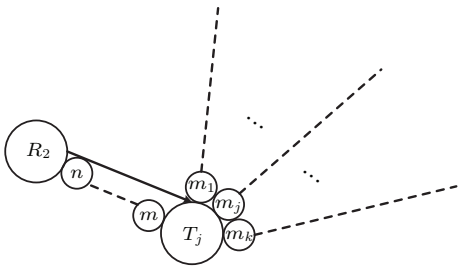


Fig. 7. Intermediate node to one target node: an EPR pair is shared between an intermediate node and a target node, Controlled-NOT gate $\mathrm{CNOT}^{m_1\oplus m_2\oplus\cdots\oplus m_k}_{m}$ is applied as shown in Table 4.

Table 4. $\mathrm{CNOT}^{m_1\oplus m_2\oplus\cdots\oplus m_k}_{m}$.

1 An XOR operation $m_1\oplus m_2\oplus\ldots\oplus m_k$ is performed to get classical result λ at quantum repeater T_j.
2. A controlled Not gate $CNOT^{\lambda}_{m}$ is performed at quantum repeater T_j, where λ is the control qubit and m is the target qubit.
Here, the particles $m,m_1,m_2,\ldots,m_k$ are owned by quantum repeater T_j; the particle n is owned by quantum repeater R_2.

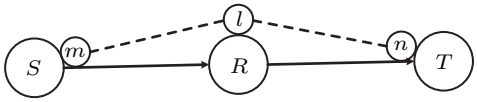


Fig. 8. Remove operation: delete the extra particles through Remove operation $\mathrm{Rem}_{l\to n}$ as shown in Table 5.

Table 5. $\mathrm{Rem}_{l\to n}$.

1 Quantum repeater R applies the Hadamard gate to particle l.
2 Quantum repeater R uses $\{\|0\rangle,\|1\rangle\}$ to measure particle l and let $\lambda\in(0,1)$ be measurement result.
3 Quantum repeater R sends λ to quantum repeater T via classic channel, if $\lambda=1$, then quantum repeater T applies an Z to particle n.
Here, the particle m is owned by quantum repeater S; the particle l is owned by quantum repeater R; the particle n is owned by quantum repeater T; and Z is Pauli operator.

Through the above connection (Figs. 4–7) and remove (Fig. 8) operations, the entanglement relationship can be established in the first step of our scheme.

3. Our works

3.1. Quantum 2-pair multicast communication

We propose a quantum 2-pair multicast communication scheme over the butterfly network. As shown in Fig. 9, Victor V prepares two arbitrary quantum states $|\psi_1\rangle_1=\alpha_1|0\rangle_1+\beta_1|1\rangle_1$, $|\psi_2\rangle_2=\alpha_2|0\rangle_2+\beta_2|1\rangle_2$, sends $|\psi_1\rangle_1$ to S_1 and $|\psi_2\rangle_2$ to S_2. Where $\alpha_1=\cos(\theta_1/2)$, $\alpha_2=\cos(\theta_2/2)$, $\beta_1=\mathrm{e}^{\mathrm{i}\phi_1}\sin(\theta_1/2)$, $\beta_2=\mathrm{e}^{\mathrm{i}\phi_2}\sin(\theta_2/2)$. It satisfies the normalization condition $|\alpha_1|^2+|\beta_1|^2=1$, $|\alpha_2|^2+|\beta_2|^2=1$. On the butterfly network, we allow either one qubit transmission in quantum channel or no more than two-bit classical communication in classical channel. Our scheme is divided into four stages: establishing entanglement relationship, adding auxiliary particles, quantum measurements and encoding process, and transmission and decoding process.

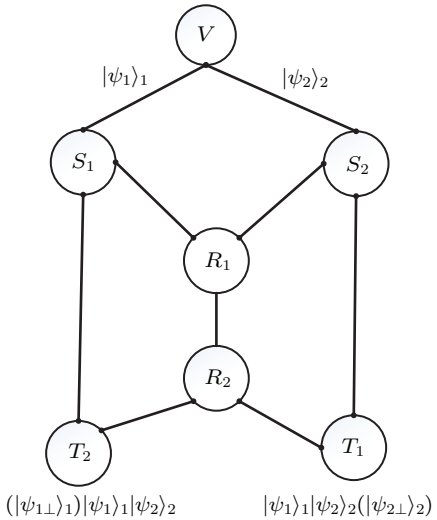


Fig. 9. The butterfly network with preparer.

Firstly, establishing entanglement relationship. As shown in Fig. 10, the Bell state $|\Psi^+\rangle=(1/\sqrt{2})(|00\rangle+|11\rangle)$ is shared between adjacent repeaters. The initial state of quantum system is

$$|\Gamma\rangle = |\Psi^+\rangle_{s_{11}r_{11}}|\Psi^+\rangle_{s_{12}t_{21}}|\Psi^+\rangle_{s_{22}r_{12}}| \otimes|\Psi^+\rangle_{s_{21}t_{12}}|\Psi^+\rangle_{r_1r_2}|\Psi^+\rangle_{r_{21}t_{11}}|\Psi^+\rangle_{r_{22}t_{22}}. \quad (1)$$

According to quantum operations introduced in Subsection 2.1, the repeaters S_1, S_2 apply connection operations $\mathrm{Con}^{s_{11}}_{s_{12}\to t_{21}}$, $\mathrm{Con}^{s_{22}}_{s_{21}\to t_{12}}$ respectively. The quantum state $|\Gamma_1\rangle$ is obtained as

$$\begin{aligned}|\Gamma_1\rangle = {}& \frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)_{r_{11}s_{11}t_{21}}\\ &\otimes\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)_{r_{12}s_{22}t_{12}}|\Psi^+\rangle_{r_1r_2}\\ &\otimes|\Psi^+\rangle_{r_{21}t_{11}}|\Psi^+\rangle_{r_{22}t_{22}}.\end{aligned} \quad (2)$$

Repeater R_1 applies connection operation $\mathrm{Con}^{r_{11}\oplus r_{12}}_{r_1\to r_2}$. The quantum state $|\Gamma_2\rangle$ is obtained as

$$\begin{aligned}|\Gamma_2\rangle = {}& \frac{1}{2}(|000000\rangle_{r_{11}s_{11}t_{21}r_{12}s_{22}t_{12}}|0\rangle_{r_2}\\ &+|000111\rangle_{r_{11}s_{11}t_{21}r_{12}s_{22}t_{12}}|1\rangle_{r_2}\\ &+|111000\rangle_{r_{11}s_{11}t_{21}r_{12}s_{22}t_{12}}|1\rangle_{r_2}\\ &+|111111\rangle_{r_{11}s_{11}t_{21}r_{12}s_{22}t_{12}}|0\rangle_{r_2})\\ &\otimes|\Psi^+\rangle_{r_{21}t_{11}}|\Psi^+\rangle_{r_{22}t_{22}}.\end{aligned} \quad (3)$$

Repeater R_2 applies connection operations $\mathrm{Con}^{r_2}_{r_{21}\to t_{11}}$, $\mathrm{Con}^{r_2}_{r_{22}\to t_{22}}$. We can deduce the quantum state $|\Gamma_3\rangle$,

$$\begin{aligned}|\Gamma_3\rangle = {}& \frac{1}{2}(|000000\rangle_{r_{11}s_{11}t_{21}r_{12}s_{22}t_{12}}|000\rangle_{r_2t_{11}t_{22}}\\ &+|000111\rangle_{r_{11}s_{11}t_{21}r_{12}s_{22}t_{12}}|111\rangle_{r_2t_{11}t_{22}}\\ &+|111000\rangle_{r_{11}s_{11}t_{21}r_{12}s_{22}t_{12}}|111\rangle_{r_2t_{11}t_{22}}\\ &+|111111\rangle_{r_{11}s_{11}t_{21}r_{12}s_{22}t_{12}}|000\rangle_{r_2t_{11}t_{22}}).\end{aligned} \quad (4)$$

Repeaters T_1 and T_2 apply controlled NOT gates $\mathrm{CNOT}^{t_{12}}_{t_{11}}$, $\mathrm{CNOT}^{t_{21}}_{t_{22}}$ respectively. The quantum state $|\Gamma_4\rangle$ is obtained as

$$\begin{aligned}|\Gamma_4\rangle = {}& \frac{1}{2}(|000000\rangle_{r_{11}s_{11}t_{21}r_{12}s_{22}t_{12}}|000\rangle_{r_2t_{11}t_{22}}\\ &+|000111\rangle_{r_{11}s_{11}t_{21}r_{12}s_{22}t_{12}}|101\rangle_{r_2t_{11}t_{22}}\\ &+|111000\rangle_{r_{11}s_{11}t_{21}r_{12}s_{22}t_{12}}|110\rangle_{r_2t_{11}t_{22}}\\ &+|111111\rangle_{r_{11}s_{11}t_{21}r_{12}s_{22}t_{12}}|011\rangle_{r_2t_{11}t_{22}}).\end{aligned} \quad (5)$$

In order to remove excess particle states, the repeaters T_1, T_2, R_1 and R_2 apply Remove operations $\mathrm{Rem}_{t_{12}\to r_2}$, $\mathrm{Rem}_{t_{21}\to r_2}$, $\mathrm{Rem}_{r_2\to r_{11}}$, $\mathrm{Rem}_{r_{11}\to s_{11}}$, $\mathrm{Rem}_{r_{12}\to s_{22}}$. We get the quantum state $|\Gamma_5\rangle$,

$$|\Gamma_5\rangle = \tfrac{1}{\sqrt{2}}(|00\rangle+|11\rangle)_{s_{11}t_{11}}\otimes\tfrac{1}{\sqrt{2}}(|00\rangle+|11\rangle)_{s_{22}t_{22}}. \quad (6)$$

Secondly, adding auxiliary particles. In our scheme, auxiliary particles are added according to the multicast number k. The number of auxiliary particles is $(k-2)\times 2$. According to the formula, there is no need to add auxiliary particles for quantum 2-pair multicast communication. Go directly to quantum measurements and encoding process. It should be noted that quantum 2-pair multicast communication is the only case which no auxiliary particles are added.

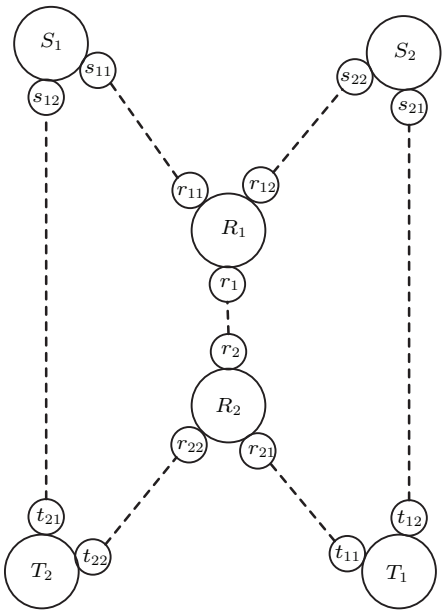


Fig. 10. The butterfly network with entanglement.

Thirdly, quantum measurements and encoding process. The combined state of $|\psi_i\rangle_i$ and $|\Psi^+\rangle_{s_{ii}t_{ii}}$ at each sender S_i ($i \in (1,2)$) is

$$\begin{aligned}|\Upsilon_i\rangle &= (\alpha_i|0\rangle_i+\beta_i|1\rangle_i)\otimes\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)_{s_{ii}t_{ii}}\\ &= \frac{1}{2}(|\Psi^+\rangle_{is_{ii}}|\psi_i\rangle_{t_{ii}}+|\Psi^-\rangle_{is_{ii}}\sigma_z|\psi_i\rangle_{t_{ii}}\\ &\quad+|\Phi^+\rangle_{is_{ii}}\sigma_x|\psi_i\rangle_{t_{ii}}+|\Phi^-\rangle_{is_{ii}}\sigma_z\sigma_x|\psi_i\rangle_{t_{ii}})\\ &= \frac{1}{2\sqrt{2}}[(|m\rangle_i\sigma_z\sigma_x|\psi_\perp\rangle_{s_{ii}}+|n\rangle_i\sigma_z\sigma_x|\psi\rangle_{s_{ii}})|\psi\rangle_{t_{ii}}\\ &\quad+(|m\rangle_i\sigma_x|\psi_\perp\rangle_{s_{ii}}+|n\rangle_i\sigma_x|\psi\rangle_{s_{ii}})\sigma_z|\psi\rangle_{t_{ii}}\\ &\quad-(|m\rangle_i\sigma_z|\psi_\perp\rangle_{s_{ii}}+|n\rangle_i\sigma_z|\psi\rangle_{s_{ii}})\sigma_x|\psi\rangle_{t_{ii}}\\ &\quad+(|m\rangle_i|\psi_\perp\rangle_{s_{ii}}+|n\rangle_i|\psi\rangle_{s_{ii}})\sigma_z\sigma_x|\psi\rangle_{t_{ii}}]. \end{aligned} \tag{7}$$

Each sender S_i uses Bell basis $\{|\Psi^+\rangle,|\Psi^-\rangle,|\Phi^+\rangle,|\Phi^-\rangle\}$ to measure particles i, s_{ii}, which is a quantum non-demolition measurement. The measurement results of Bell basis are encoded into 2-bit X_i according to Table 6.

Table 6. Bell measurements coding table.

Encoding (X_i)	Unitary operation $U(X_i)$	Measurement result
00	I	$\|\Psi^+\rangle$
01	σ_x	$\|\Phi^+\rangle$
10	σ_z	$\|\Psi^-\rangle$
11	$\sigma_x\sigma_z$	$\|\Phi^-\rangle$

After sender S_i performs Bell basis measurement, follow by sending particle i to Victor. Victor uses the single particle basis $\{|m\rangle_i,|n\rangle_i\}$ to measure particle i, and encodes the measurement result with 1-bit Y_i as shown in Table 7, where $Y_{i,j}$ represents the encoding result of the j-the single particle basis measurement applied by Victor to the sender S_i. After Victor performs the single particle basis measurement, sender S_i will get quantum state $U(Y_i)|\psi\rangle_{s_{ii}}$ or quantum complementary state $U(Y_i)|\psi_\perp\rangle_{s_{ii}}$. According to the Y_i received from Victor, sender S_i applies recovery operation $U(Y_i)$ to obtain $|\psi\rangle_{s_{ii}}$ or $|\psi_\perp\rangle_{s_{ii}}$ with equal probability.

Table 7. Single particle basis coding table.

Bell basis	Measurement result	Encoding (Y_i)	Get state	Recovery operation $U(Y_{i,j})$
$\|\Psi^+\rangle$	$\|m\rangle_i$	0	$\|\psi_\perp\rangle_{s_{ii}}$	$\sigma_x\sigma_z$
	$\|n\rangle_i$	1	$\|\psi\rangle_{s_{ii}}$	$\sigma_x\sigma_z$
$\|\Psi^-\rangle$	$\|m\rangle_i$	0	$\|\psi_\perp\rangle_{s_{ii}}$	σ_x
	$\|n\rangle_i$	1	$\|\psi\rangle_{s_{ii}}$	σ_x
$\|\Phi^+\rangle$	$\|m\rangle_i$	0	$\|\psi_\perp\rangle_{s_{ii}}$	σ_z
	$\|n\rangle_i$	1	$\|\psi\rangle_{s_{ii}}$	σ_z
$\|\Phi^-\rangle$	$\|m\rangle_i$	0	$\|\psi_\perp\rangle_{s_{ii}}$	I
	$\|n\rangle_i$	1	$\|\psi\rangle_{s_{ii}}$	I

Finally, transmission and decoding process. Sender S_1 sends X_1 to intermediate node R_1 and receiver T_2 via classic channels $C(S_1,R_1)$ and $C(S_1,T_2)$, respectively. Sender S_2 sends X_2 to intermediate node R_1 and receiver T_1 via classic channels $C(S_2,R_1)$ and $C(S_2,T_1)$, respectively. An XOR operation is performed to get $X_1\oplus X_2$ at R_1, and $X_1\oplus X_2$ is transmitted to next node R_2. At R_2, $X_1\oplus X_2$ is copied and transmitted to receivers T_1 and T_2 via classic channels $C(R_2,T_1)$ and $C(R_2,T_2)$, respectively. Receiver T_1 performs the XOR operation $(X_1\oplus X_2)\oplus X_2$ to obtain X_1, and applies unitary operation $U(X_1)^{-1}$ to quantum state $U(X_1)|\psi_1\rangle_{t_{11}}$ to get quantum state $|\psi_1\rangle_{t_{11}}$. Receiver T_2 performs the XOR operation $(X_1\oplus X_2)\oplus X_1$ to obtain X_2, and applies unitary operation $U(X_2)^{-1}$ to quantum state $U(X_2)|\psi_2\rangle_{t_{22}}$ to get quantum state $|\psi_2\rangle_{t_{22}}$. Sender S_1 sends quantum state $|\psi_1\rangle$ or quantum complement state $|\psi_{1\perp}\rangle$ to receiver T_2 through quantum channel $Q(S_1,T_2)$. Sender S_2 sends quantum state $|\psi_2\rangle$ or quantum complement state $|\psi_{2\perp}\rangle$ to receiver R_1 through quantum channel $Q(S_2,T_1)$.

Through the above four stages, quantum state $|\psi_1\rangle$ with

probability 1 and quantum state $|\psi_2\rangle$ with probability $1/2$ or quantum complement state $|\psi_{2\perp}\rangle$ with probability $1/2$ can be obtained at receiver T_1. Quantum state $|\psi_2\rangle$ with probability 1 and quantum state $|\psi_1\rangle$ with probability $1/2$ or quantum complement state $|\psi_{1\perp}\rangle$ with probability $1/2$ can be obtained at receiver T_2. It should be noted that if the amplitudes of the transmitted quantum states $|\psi_1\rangle_1$ and $|\psi_2\rangle_2$ are real numbers, Victor will apply the special single particle basis $\{|m\rangle = \alpha|0\rangle + \beta|1\rangle, |n\rangle = \beta|0\rangle - \alpha|1\rangle\}$to perform quantum measurements, then each receiver T_1, T_2 receives both quantum states $|\psi_1\rangle$ and $|\psi_2\rangle$ with probability 1. Thereby, quantum 2-pair multicast communication can be completed over the butterfly network.

3.2. Quantum *k*-pair multicast communication scheme

We extend quantum 2-pair multicast communication to extended butterfly network, and propose quantum k-pair multicast communication scheme over the extended butterfly network.

As shown in Fig. 11, Victor prepares k arbitrary quantum states $\sum_{i=1}^{k} |\psi_i\rangle_i = \alpha_i|0\rangle_i + \beta_i|1\rangle_i$, sends them to corresponding sender S_i, where $\alpha_i = \cos(\theta_i/2)$, $\beta_i = \mathrm{e}^{\mathrm{i}\phi_i}\sin(\theta_i/2)$, and satisfies normalization condition $|\alpha_i|^2 + |\beta_i|^2 = 1$, $i \in \{1,\ldots,k\}$. On quantum k-pair network, we allow either one qubit transmission in quantum channel or no more than two-bit classical communication in classical channel. Similarly, quantum k-pair multicast communication scheme is divided into four stages, namely establishing entanglement relationship, adding auxiliary particles, quantum measurements and encoding process, and transmission and decoding process.

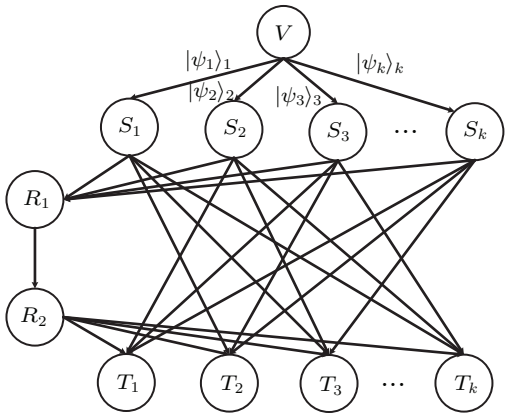


Fig. 11. The extended butterfly network with preparer V.

Firstly, establishing entanglement relationship. The initial entanglement relationship over the extended butterfly network is shown in Fig. 12. By applying quantum operations introduced in Subsection 2.2, the entangled relationship is established between sender S_i and receiver T_i, $i \in \{1,\ldots,k\}$.

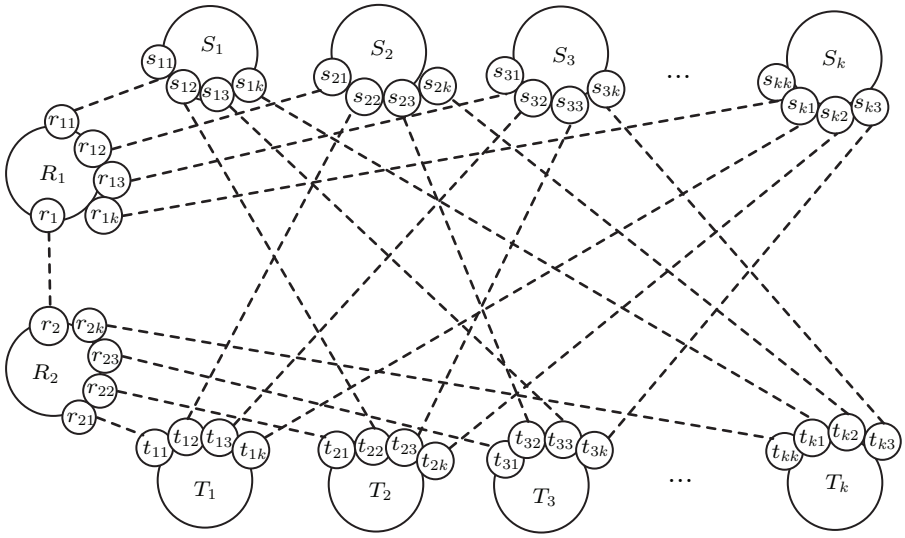


Fig. 12. The extended butterfly network with entanglement.

The initial state of quantum system is

$$\begin{aligned}|\Gamma\rangle \;=\; & |\Psi^+\rangle_{s_{11}r_{11}}|\Psi^+\rangle_{s_{12}t_{21}}|\Psi^+\rangle_{s_{13}t_{31}}\otimes\ldots\otimes|\Psi^+\rangle_{s_{1k}t_{k1}}\otimes\ldots\\ & \otimes|\Psi^+\rangle_{s_{kk}r_{1k}}\otimes|\Psi^+\rangle_{s_{k1}t_{1k}}|\Psi^+\rangle_{s_{k2}t_{2k}}\otimes\ldots\otimes|\Psi^+\rangle_{r_1r_2}\\ & \otimes|\Psi^+\rangle_{r_{21}t_{11}}|\Psi^+\rangle_{r_{22}t_{22}}\otimes\ldots\otimes|\Psi^+\rangle_{r_{2k}t_{kk}}. \qquad (8)\end{aligned}$$

Each repeater S_i applies a series of Connection operations $\otimes_{j=1,j\neq i}^{k}\mathrm{Con}_{s_{ij}\to t_{ji}}^{s_{ii}}$. The quantum state $|\Gamma_1\rangle$ is obtained as

$$\begin{aligned}|\Gamma_1\rangle \;=\; & \frac{1}{\sqrt{2^k}}\otimes_{j=1}^{k}\sum_{x_j=0}^{1}(|x_j\rangle_{r_1j}|x_j\rangle_{s_{jj}}\\ & \otimes_{i=1,i\neq j}^{k}|x_j\rangle_{t_{ij}})\otimes|\Psi^+\rangle_{r_1r_2}\\ & \otimes|\Psi^+\rangle_{r_{21}t_{11}}|\Psi^+\rangle_{r_{22}t_{22}}\otimes\ldots\otimes|\Psi^+\rangle_{r_{2k}t_{kk}}. \qquad (9)\end{aligned}$$

Repeater R_1 applies Connection operation $\mathrm{Con}_{r_1\to r_2}^{r_{11}\oplus r_{12}\oplus\ldots\oplus r_{1k}}$.

The quantum state $|\Gamma_1\rangle$ becomes

$$|\Gamma_2\rangle = \frac{1}{\sqrt{2^k}}[\otimes_{j=1}^{k}\sum_{x_j=0}^{1}(|x_j\rangle_{r_{1j}}|x_j\rangle_{s_{jj}} \otimes_{i=1,i\neq j}^{k}|x_j\rangle_{t_{ij}})]\otimes|N\rangle_{r_2} \otimes|\Psi^+\rangle_{r_{21}t_{11}}|\Psi^+\rangle_{r_{22}t_{22}}\ldots|\Psi^+\rangle_{r_{2k}t_{kk}}. \quad (10)$$

Here, N is the result of XOR of particles $\sum_{j=1}^{k}\oplus r_{1j}$. Repeater R_2 applies a series of connection operations $\otimes_{i=1}^{k}\mathrm{Con}_{r_{2i}\to t_{ii}}^{r_2}$. We can deduce the quantum state $|\Gamma_3\rangle$,

$$|\Gamma_3\rangle = \frac{1}{\sqrt{2^k}}\left[\otimes_{j=1}^{k}\sum_{x_j=0}^{1}(|x_j\rangle_{r_{1j}}|x_j\rangle_{s_{jj}} \otimes_{i=1,i\neq j}^{k}|x_j\rangle_{t_{ij}})|N\rangle_{r_2}\right]\otimes_{m=1}^{k}|N\rangle_{t_{mm}}. \quad (11)$$

Each repeater T_i applies controlled NOT gate $\mathrm{CNOT}_{t_{ii}}^{\sum_{j=1,j\neq i}^{k}\oplus t_{ij}}$. The quantum state $|\Gamma_4\rangle$ is obtained as

$$|\Gamma_4\rangle = \frac{1}{\sqrt{2^k}}\otimes_{j=1}^{k}\sum_{x_j=0}^{1}(|x_j\rangle_{r_{1j}}|x_j\rangle_{s_{jj}} \otimes_{i=1,i\neq j}^{k}|x_j\rangle_{t_{ij}})|N\rangle_{r_2}\otimes_{m=1}^{k}\sigma_x^M|N\rangle_{t_{mm}}. \quad (12)$$

Here, M is the result of XOR of particles $\sum_{j=1,j\neq i}^{k}\oplus t_{ij}$ at each receiver T_i, and $M\in\{0,1\}$.

Each repeater T_i applies a series of remove operations $\otimes_{j=1,j\neq i}^{k}\mathrm{Rem}_{t_{ij}\to r_2}$. repeater R_1 applies a Remove operation $\mathrm{Rem}_{r_2\to r_{11}}$. Each repeater S_i applies a series of Remove operations $\otimes_{i=1}^{k}\mathrm{Rem}_{r_{1i}\to s_{ii}}$. The final state of quantum system is obtained as

$$|\Gamma_5\rangle = \frac{1}{\sqrt{2^k}}\otimes_{i=1}^{k}(|00\rangle+|11\rangle)_{s_{ii}t_{ii}}. \quad (13)$$

Secondly, adding auxiliary particles. According to the multicast number k, auxiliary particles are added to each sender of extended butterfly network. Each sender S_i introduces $(k-2)\times 2$ extra auxiliary qubits, which are initialized to $\otimes_{j=1}^{(k-2)\times 2}|0\rangle_{\mathrm{Aux}_{i,j}}$. After adding auxiliary particles, the state of quantum system is

$$|\Omega\rangle = \frac{1}{\sqrt{2^k}}\otimes_{i=1}^{k}(|00\rangle+|11\rangle)_{s_{ii}t_{ii}}\otimes_{j=1}^{(k-2)\times 2}|0\rangle_{\mathrm{Aux}_{i,j}} \quad (14)$$

Here, particle $\mathrm{Aux}_{i,j}$ represents the j-th auxiliary particle added by sender S_i. Each sender S_i performs $(k-2)\times 2$ controlled NOT gates $\mathrm{CNOT}_{\mathrm{Aux}_{i,j}}^{s_{ii}}$, where $j\in(1,\ldots,(k-2)\times 2)$. We get the quantum state $|\Omega_1\rangle$,

$$|\Omega_1\rangle = \frac{1}{\sqrt{2^k}}\otimes_{i=1}^{k}(|00\rangle_{s_{ii}t_{ii}}\otimes_{j=1}^{(k-2)\times 2}|0\rangle_{\mathrm{Aux}_{i,j}} +|11\rangle_{s_{ii}t_{ii}}\otimes_{j=1}^{(k-2)\times 2}|1\rangle_{\mathrm{Aux}_{i,j}}). \quad (15)$$

Thirdly, quantum measurements and encoding process. Before the quantum measurements, the state of the whole quantum system is

$$|\Omega_2\rangle = \frac{1}{\sqrt{2^k}}\otimes_{i=1}^{k}[(\alpha_i|0\rangle_i+\beta_i|1\rangle_i)(|00\rangle_{s_{ii}t_{ii}} \otimes_{j=1}^{(k-2)\times 2}|0\rangle_{\mathrm{Aux}_{i,j}}+|11\rangle_{s_{ii}t_{ii}}\otimes_{j=1}^{(k-2)\times 2}|1\rangle_{\mathrm{Aux}_{i,j}})]. \quad (16)$$

Each sender S_i applies Bell-basis to measure particle pairs (i,s_{ii}), $(\mathrm{Aux}_{i,1},\mathrm{Aux}_{i,2})$, …, $(\mathrm{Aux}_{i,(k-2)\times 2-1}$, $\mathrm{Aux}_{i,(k-2)\times 2})$ successively, which is a quantum non-demolition measurement. After Bell-basis measurements, each receiver T_i will get one of the quantum states $\{|\psi_i\rangle_{t_{ii}},\sigma_z|\psi_i\rangle_{t_{ii}},\sigma_x|\psi_i\rangle_{t_{ii}},\sigma_z\sigma_x|\psi_i\rangle_{t_{ii}}\}$. Each sender S_i sends particles i, $\mathrm{Aux}_{i,1}$, …, $\mathrm{Aux}_{i,(k-2)\times 2-1}$ to Victor. After receiving the single particles, Victor applies the special single particle basis $\{|m\rangle,|n\rangle\}$ to measure the received particles i, $\mathrm{Aux}_{i,1}$, …, $\mathrm{Aux}_{i,(k-2)\times 2-1}$. Here Victor will perform $k-1$ single particle basis measurements. The basis $\{|m\rangle,|n\rangle\}$ is

$$|m\rangle = \alpha|0\rangle+\beta|1\rangle,\quad |n\rangle = \beta^*|0\rangle-\alpha|1\rangle. \quad (17)$$

Victor encodes each measurement result with 1-bit Y_i according to Table 7, and transmits $(k-1)$ bits Y_i to sender S_i. Each sender S_i receives $(k-1)$ bits Y_i from Victor. When the Y_i is 0, quantum complement state is obtained at sender, otherwise quantum state is obtained. After Victor performs the special single particle measurement, The state of particles s_{ii}, $\mathrm{Aux}_{i,2}$, …, $\mathrm{Aux}_{i,(k-2)\times 2}$ will be obtained at each sender S_i in the following form:

$$\begin{cases} |\Psi^+\rangle_{is_{ii}} = \frac{1}{\sqrt{2}}(|m\rangle_i\sigma_z\sigma_x|\psi_{i\perp}\rangle_{s_{ii}}+|n\rangle_i\sigma_z\sigma_x|\psi_i\rangle_{s_{ii}}),\\ |\Psi^+\rangle_{\mathrm{Aux}_{i,1}(\mathrm{Aux}_{i,2})} = \frac{1}{\sqrt{2}}(|m\rangle_{\mathrm{Aux}_{i,1}}\sigma_z\sigma_x|\psi_{i\perp}\rangle_{\mathrm{Aux}_{i,2}}+|n\rangle_{\mathrm{Aux}_{i,1}}\sigma_z\sigma_x|\psi_i\rangle_{\mathrm{Aux}_{i,2}}),\\ \ldots,\\ |\Psi^+\rangle_{\mathrm{Aux}_{i,(k-2)\times 2-1}(\mathrm{Aux}_{i,(k-2)\times 2})} = \frac{1}{\sqrt{2}}(|m\rangle_{\mathrm{Aux}_{i,(k-2)\times 2-1}}\sigma_z\sigma_x|\psi_{i\perp}\rangle_{\mathrm{Aux}_{i,(k-2)\times 2}}+|n\rangle_{\mathrm{Aux}_{i,(k-2)\times 2-1}}\sigma_z\sigma_x|\psi_i\rangle_{\mathrm{Aux}_{i,(k-2)\times 2}}), \end{cases} \quad (18)$$

$$\begin{cases} |\Psi^-\rangle_{is_{ii}} = \frac{1}{\sqrt{2}}(|m\rangle_i\sigma_x|\psi_{i\perp}\rangle_{s_{ii}}+|n\rangle_i\sigma_x|\psi_i\rangle_{s_{ii}}),\\ |\Psi^-\rangle_{\mathrm{Aux}_{i,1}(\mathrm{Aux}_{i,2})} = \frac{1}{\sqrt{2}}(|m\rangle_{\mathrm{Aux}_{i,1}}\sigma_x|\psi_{i\perp}\rangle_{\mathrm{Aux}_{i,2}}+|n\rangle_{\mathrm{Aux}_{i,1}}\sigma_x|\psi_i\rangle_{\mathrm{Aux}_{i,2}}),\\ \ldots,\\ |\Psi^-\rangle_{\mathrm{Aux}_{i,(k-2)\times 2-1}(\mathrm{Aux}_{i,(k-2)\times 2})} = \frac{1}{\sqrt{2}}(|m\rangle_{\mathrm{Aux}_{i,(k-2)\times 2-1}}\sigma_x|\psi_{i\perp}\rangle_{\mathrm{Aux}_{i,(k-2)\times 2}}+|n\rangle_{\mathrm{Aux}_{i,(k-2)\times 2-1}}\sigma_x|\psi_i\rangle_{\mathrm{Aux}_{i,(k-2)\times 2}}), \end{cases} \quad (19)$$

$$\begin{cases} |\Phi^+\rangle_{is_{ii}} = \frac{1}{\sqrt{2}}(|m\rangle_i\sigma_z|\psi_{i\perp}\rangle_{s_{ii}} + |n\rangle_i\sigma_z|\psi_i\rangle_{s_{ii}}), \\ |\Phi^+\rangle_{\mathrm{Aux}_{i,1}(\mathrm{Aux}_{i,2})} = \frac{1}{\sqrt{2}}(|m\rangle_{\mathrm{Aux}_{i,1}}\sigma_z|\psi_{i\perp}\rangle_{\mathrm{Aux}_{i,2}} + |n\rangle_{\mathrm{Aux}_{i,1}}\sigma_z|\psi_i\rangle_{\mathrm{Aux}_{i,2}}), \\ \ldots, \\ |\Phi^+\rangle_{\mathrm{Aux}_{i,(k-2)\times 2-1}(\mathrm{Aux}_{i,(k-2)\times 2})} = \frac{1}{\sqrt{2}}(|m\rangle_{\mathrm{Aux}_{i,(k-2)\times 2-1}}\sigma_z|\psi_{i\perp}\rangle_{\mathrm{Aux}_{i,(k-2)\times 2}} + |n\rangle_{\mathrm{Aux}_{i,(k-2)\times 2-1}}\sigma_z|\psi_i\rangle_{\mathrm{Aux}_{i,(k-2)\times 2}}), \end{cases} \tag{20}$$

$$\begin{cases} |\Phi^-\rangle_{is_{ii}} = \frac{1}{\sqrt{2}}(|m\rangle_i|\psi_{i\perp}\rangle_{s_{ii}} + |n\rangle_i|\psi_i\rangle_{s_{ii}}), \\ |\Phi^-\rangle_{\mathrm{Aux}_{i,1}(\mathrm{Aux}_{i,2})} = \frac{1}{\sqrt{2}}(|m\rangle_{\mathrm{Aux}_{i,1}}|\psi_{i\perp}\rangle_{\mathrm{Aux}_{i,2}} + |n\rangle_{\mathrm{Aux}_{i,1}}|\psi_i\rangle_{\mathrm{Aux}_{i,2}}), \\ \ldots, \\ |\Phi^-\rangle_{\mathrm{Aux}_{i,(k-2)\times 2-1}(\mathrm{Aux}_{i,(k-2)\times 2})} = \frac{1}{\sqrt{2}}(|m\rangle_{\mathrm{Aux}_{i,(k-2)\times 2-1}}|\psi_{i\perp}\rangle_{\mathrm{Aux}_{i,(k-2)\times 2}} + |n\rangle_{\mathrm{Aux}_{i,(k-2)\times 2-1}}|\psi_i\rangle_{\mathrm{Aux}_{i,(k-2)\times 2}}). \end{cases} \tag{21}$$

As shown in Eqs. (18)–(21), when Bell basis measures $|\Psi^+\rangle$, sender S_i performs unitary operation $\sigma_x\sigma_z$ onto the particle $s_{ii}(\mathrm{Aux}_{i,2}, \ldots, \mathrm{Aux}_{i,(k-2)\times 2})$. Similarly, when Bell basis measures $|\Psi^-\rangle$, $|\Phi^+\rangle$ or $|\Phi^+\rangle$, sender S_i performs unitary operations σ_x, σ_Z or I onto the particles $s_{ii}(\mathrm{Aux}_{i,2}, \ldots, \mathrm{Aux}_{i,(k-2)\times 2})$, respectively. Each sender S_i will get $(k-1)$ quantum states $|\psi_i\rangle$ or quantum complementary states $|\psi_{i\perp}\rangle$.

In our scheme, after each sender S_i applies Bell basis measurements, each receiver T_i will get one of quantum states $\{|\psi_i\rangle_{t_{ii}}, \sigma_z|\psi_i\rangle_{t_{ii}}, \sigma_x|\psi_i\rangle_{t_{ii}}, \sigma_z\sigma_x|\psi_i\rangle_{t_{ii}}\}$. Each sender S_i encodes quantum operation that each receiver T_i needs to recover quantum state $|\psi_i\rangle_{t_{ii}}$ into 2-bit X_i. Each receiver T_i recovers quantum state $|\psi_i\rangle_{t_{ii}}$ according to 2-bit X_i received, namely,

(i) When the X_i received by receiver is 00, quantum operation I is applied to restore the initial state.

(ii) When the X_i received by receiver is 01, quantum operation σ_x is applied to restore the initial state.

(iii) When the X_i received by receiver is 10, quantum operation σ_z is applied to restore the initial state.

(iv) When the X_i received by receiver is 11, quantum operation $\sigma_x\sigma_z$ is applied to restore the initial state.

Finally, transmission and decoding process. Each sender S_i sends X_i to intermediate node R_1 and each receiver T_j via classic channels $C(S_i, R_1)$ and $C(S_i, T_j)(j \neq i)$. An XOR operation is performed to obtain $\oplus_{i=1}^k X_i$ at R_1. The 2-bit $\oplus_{i=1}^k X_i$ is transmitted to R_2 via classic channel $C(R_1, R_2)$. The 2-bit $\oplus_{i=1}^k X_i$ is copied and transmitted to each receiver T_i via classic channel $C(R_2, T_i)$. Each receiver T_i can obtain X_i by performing XOR operation on $\oplus_{i=1}^k X_i$ and $\oplus_{j=1,j\neq i}^k X_j$. Each receiver T_i applies unitary operation $U(X_i)^{-1}$ onto quantum state $U(X_i)|\psi_i\rangle_{t_{ii}}$ to obtain quantum state $|\psi_i\rangle_{t_{ii}}$.

Each sender S_i sends quantum states $|\psi_i\rangle_{s_{ii}}$, $|\psi_i\rangle_{\mathrm{Aux}_{i,2}}$, …, $|\psi_i\rangle_{\mathrm{Aux}_{i,(k-2)\times 2}}$ or quantum complement states $|\psi_{i\perp}\rangle_{s_{ii}}$, $|\psi_{i\perp}\rangle_{\mathrm{Aux}_{i,2}}$, …, $|\psi_{i\perp}\rangle_{\mathrm{Aux}_{i,(k-2)\times 2}}$ to each receiver T_j through quantum channel $Q(S_i, T_j)(j \neq i)$. In this way, each receiver T_i can receive one quantum state $|\psi_i\rangle_{t_{ii}}$ and $k-1$ quantum states $\sum_{j=1,j\neq i}^{k} |\psi_{[i+j]_k}\rangle$ or quantum complement states $\sum_{j=1,j\neq i}^{k} |\psi_{[i+j]_k\perp}\rangle$, among $[i+j]_k = (i+j) \bmod k$.

Through the above four stages, quantum k-pair multicast communication can be completed over the extended butterfly network. It should be noted that if the amplitudes α_i and β_i of quantum states $\sum_{i=1}^k |\psi_i\rangle_i$ are real numbers, quantum states $\sum_{i=1}^k |\psi_i\rangle_i$ are obtained at each receiver T_j, where $j \in \{1, \ldots, k\}$.

4. Conclusion

In this paper, we consider Bell states, auxiliary particles, and free classical channels as auxiliary resources to complete quantum multicast communication, which solve the bottleneck problem and improve the communication efficiency. Although auxiliary resources are consumed, quantum channels are used only once at most in our scheme. It should be noted that if the amplitudes of transmitted quantum states are real numbers, each receiver can receive k quantum states.

In our scheme, no matter how many times Bell basis measurements are applied, the receivers use only one of the following four unitary operations to recover the quantum state, that is, $\{I, \sigma_x, \sigma_z, \sigma_x\sigma_z\}$. Therefore, the capacity of classical channel is no more than 2 bits in our scheme. The maximum capacity of classical channels will not be increased with the increase of multicast number k, which saves the utilization of classical channels to some extent.

At present, quantum multicast communication is in a booming stage. We are also the first attempt to study quantum multicast communication by adding auxiliary resources. Next, we will consider how to save resources to complete quantum multicast communication over the butterfly network, which is also the task we have been trying to solve. In the future, the establishment and application of quantum network[45–47] cannot be separated from quantum multicast communication.

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