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Probabilistic resumable quantum teleportation in high dimensions

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Teleportation is a quantum information process without classical counterparts, in which the sender can disembodiedly transfer unknown quantum states to the receiver. In probabilistic teleportation through a partial entangled quantum channel, the transmission is exact (with fidelity 1), but may fail in a probability and the initial state is destroyed simultaneously. We propose a scheme for nondestructive probabilistic teleportation of high-dimensional quantum states. With the aid of an ancilla in the hands of the sender, the initial quantum information can be recovered when teleportation fails. The ancilla acts as a quantum apparatus to measure the sender's subsystem. Erasing the information recorded in it can resume the initial state.

Keywords: probabilistic teleportation, entanglement, successful probability

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1. Introduction

Quantum teleportation is one of the processes unique to the quantum information, which serves as an important example for the most intriguing uses of entanglement.[1–4] It has been widely studied both theoretically and experimentally since it was put forwards.[5–24] One reason for it gains so much attraction is that, the original scheme and its various extensions play key roles in several contexts in quantum communication, including quantum repeaters, quantum networks and cryptographic conferences.[25–29]

In the original and also the simplest version of teleportation,[2] Alice (the sender) can transfer an unknown state from a qubit to Bob (the receiver), with fidelity 1 and successful probability 1, without physical transmission of the qubit itself. The key ingredient in the protocol is a two-qubit Bell state shared by them as a quantum channel. Alice makes a joint measurement on the two qubits in her hands, projecting them onto one of the four Bell states with equal probability. Through a classical channel, Alice sends Bob two classical bits to inform him of her outcome. According to the classical information, Bob can perform appropriate unitary operations on his qubit to rebuild Alice's initial state, and thereby accomplishes the teleportation.

A variant of the process called probabilistic teleportation was proposed based on the consideration that a quantum channel may be prepared in a partially entangled pure state in practice.[5–9] Such teleportation is exact (with fidelity 1), but may sometimes fail as the price to pay for the fidelity. In principle, Alice's joint measurement is destructive, and it is generally assumed that the information encoded in the state to be teleported is lost when the teleportation fails. In their recent work,[9] Roa and Groiseau presented a nondestructive scheme for probabilistic teleportation by introducing an ancillary qubit. This avoids losing the initial information and offers the chance to repeat the teleportion process. The nondestructive scheme has been extended in many branches, including bidirectional teleportation,[30] teleportation of an entangled state[31] and multihop scenario.[32,33]

In this work, we present a general protocol for nondestructive probabilistic teleportation of high-dimensional quantum states, in which Alice can resume her initial state when the teleportation fails. The motivation for this work is not only to extend the study of Roa and Groiseau to high-dimensional systems, but also based on the following considerations. In theory, high-dimensional channels, especially the partially entangled ones, have more rich entanglement properties. Our protocol provides a sample to study the cooperative relationship among the entanglement, joint measurement and classical information in a quantum information process. In practice, teleporting high-dimensional states is required in the task to completely rebuild the quantum states of a real particle remotely. And, recent experiment progresses[16,17] show the possibility of implementing our present protocol in optical systems. We make a remark here that, although Fu *et al.*[32,33] give a version of nondestructive probabilistic teleportation in high dimension using an auxiliary particle in Bob's hand, we maintain the approach in Ref. [9] with an ancilla belonging to Alice. In this case, only the sender, Alice, is required to have the ability of bipartite operations, such as generation and measurement of entangled states.

In addition, one can notice the similarity of our protocol and the quantum two-path interference experiments.[34] In

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the latter, interference fringes vanish when the which-path information is acquired by a detector, while reappear when the detector returns the information to the particle. The ancilla in our protocol can be regarded as a quantum apparatus to measure Alice's system. The initial state is recovered by erasing the information recorded after an unambiguous quantum state discrimination.

2. Teleportation of a qutrit

Let us start with the teleportation of a qutrit (a three-level quantum system) . Suppose Alice wishes to teleport to Bob an arbitrary qutrit state as

$$|\phi\rangle_1 = \alpha_0|0\rangle_1 + \alpha_1|1\rangle_1 + \alpha_2|2\rangle_1, \tag{1}$$

with $|\alpha_0|^2 + |\alpha_1|^2 + |\alpha_2|^2 = 1$. The participants share a two-qutrit entangled state as the quantum channel,

$$|\Phi\rangle_{23} = b_0|00\rangle_{23} + b_1|11\rangle_{23} + b_2|22\rangle_{23}, \tag{2}$$

with $|b_0|^2 + |b_1|^2 + |b_2|^2 = 1$. Without loss of generality, we assume that the Schmidt coefficients are non-negative real numbers and $b_0 \le b_1 \le b_2$. Here, the three qutrits are identified by the subscripts $1,2$ and 3, respectively, and qutrits 1 and 2 belong to Alice while qutrit 3 is in the hands of Bob. The total state of the three-qutrit system can be written as

$$\begin{aligned}
|\Psi\rangle_{123} &= |\phi\rangle_1 \otimes |\Phi\rangle_{23} \\
&= \frac{1}{3}\big[\, |\psi_{00}\rangle_{12}(\alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle)_3 \\
&\quad + |\psi_{10}\rangle_{12}(\alpha_0|0\rangle + \alpha_1\omega^2|1\rangle + \alpha_2\omega|2\rangle)_3 \\
&\quad + |\psi_{20}\rangle_{12}(\alpha_0|0\rangle + \alpha_1\omega|1\rangle + \alpha_2\omega^2|2\rangle)_3 \\
&\quad + |\psi_{01}\rangle_{12}(\alpha_1|0\rangle + \alpha_2|1\rangle + \alpha_0|2\rangle)_3 \\
&\quad + |\psi_{11}\rangle_{12}(\alpha_1|0\rangle + \alpha_2\omega^2|1\rangle + \alpha_0\omega|2\rangle)_3 \\
&\quad + |\psi_{21}\rangle_{12}(\alpha_1|0\rangle + \alpha_2\omega|1\rangle + \alpha_0\omega^2|2\rangle)_3 \\
&\quad + |\psi_{02}\rangle_{12}(\alpha_2|0\rangle + \alpha_0|1\rangle + \alpha_1|2\rangle)_3 \\
&\quad + |\psi_{12}\rangle_{12}(\alpha_2|0\rangle + \alpha_0\omega^2|1\rangle + \alpha_1\omega|2\rangle)_3 \\
&\quad + |\psi_{22}\rangle_{12}(\alpha_2|0\rangle + \alpha_0\omega|1\rangle + \alpha_1\omega^2|2\rangle)_3\,\big],
\end{aligned} \tag{3}$$

where $\omega = e^{i\frac{2\pi}{3}}$ is the triple root, and $|\psi_{nm}\rangle_{12}$ are nine linearly independent two-qutrit states, equivalent to channel (2) under local unitary transformations, as follows:

$$\begin{aligned}
|\psi_{00}\rangle_{12} &= (b_0|00\rangle + b_1|11\rangle + b_2|22\rangle)_{12}, \\
|\psi_{10}\rangle_{12} &= (b_0|00\rangle + b_1\omega|11\rangle + b_2\omega^2|22\rangle)_{12}, \\
|\psi_{20}\rangle_{12} &= (b_0|00\rangle + b_1\omega^2|11\rangle + b_2\omega|22\rangle)_{12}, \\
|\psi_{01}\rangle_{12} &= (b_0|10\rangle + b_1|21\rangle + b_2|02\rangle)_{12}, \\
|\psi_{11}\rangle_{12} &= (b_0|10\rangle + b_1\omega|21\rangle + b_2\omega^2|02\rangle)_{12}, \\
|\psi_{21}\rangle_{12} &= (b_0|10\rangle + b_1\omega^2|21\rangle + b_2\omega|02\rangle)_{12}, \\
|\psi_{02}\rangle_{12} &= (b_0|20\rangle + b_1|01\rangle + b_2|12\rangle)_{12}, \\
|\psi_{12}\rangle_{12} &= (b_0|20\rangle + b_1\omega|01\rangle + b_2\omega^2|12\rangle)_{12}, \\
|\psi_{22}\rangle_{12} &= (b_0|20\rangle + b_1\omega^2|01\rangle + b_2\omega|12\rangle)_{12}.
\end{aligned} \tag{4}$$

The vector of qutrit 3 in each term (each line) of the total state (3) is equivalent to the initial state (1) under a unitary transformation. However, the corresponding states of qutrits 1 and 2 $|\psi_{nm}\rangle_{12}$ are not orthogonal to each other. To teleportate the initial state exactly, an option is that, Alice performs an unambiguous quantum state discrimination process,[6,7,12,13] to distinguish the nine states with no error but a probability of failures. As in the standard teleportation through a maximal entangled quantum channel,[16,17] Bob can rebuild the initial state on his qutrit by performing appropriate unitary operations according to Alice's outcome. In another scheme, Alice still performs a joint measurement in nine maximally entangled basis, while Bob needs an extracting quantum state process.[5] Here, we follow the former scheme, which only requires Alice's ability to manipulate two or more particles.

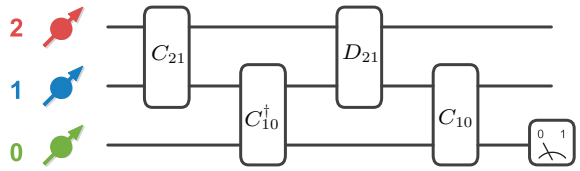


Fig. 1. Alice's sequential operations on her subsystems: (1) The GCNOT gate C_{21} factorizes the states of 1 and 2 to be discriminated. (2) The apparatus 0 measures 1 by using $C_{10}^{\dagger}$. (3) Alice performs a joint transformation D_{21} on 1 and 2 to discriminate the states of 2. (4) The GCNOT gate C_{10} erases the information of 1 recorded in 0. (5) Alice measures 0 to divide the procedure into success and failure parts.

For the sake of brevity, before putting forward our final total state in this section, we only show the states in the hands of Alice. We show the sequential operations of Alice in Fig. 1. To discriminate the nine two-qutrit states (4), Alice factorizes the states by applying a generalized controlled-NOT (GCNOT) gate onto her bipartite system and obtain

$$\mathcal{C}_{21}|\psi_{nm}\rangle_{12} = |m\rangle_1(b_0|0\rangle + b_1\omega^n|1\rangle + b_2\omega^{2n}|2\rangle)_2, \tag{5}$$

with $m,n = 0,1,2$. Here, we define the GCNOT gate C_{ij} acting on qutrits i and j as

$$\mathcal{C}_{ij} = |0\rangle_i\langle 0| \otimes I_j + |1\rangle_i\langle 1| \otimes \mathcal{V}_j + |2\rangle_i\langle 2| \otimes \mathcal{V}_j^{\dagger}, \tag{6}$$

where I_j is the identity of j and $\mathcal{V}_j = |0\rangle_j\langle 1| + |1\rangle_j\langle 2| + |2\rangle_j\langle 0|$. It shifts the target j clockwise or anticlockwise when the control qutrit i is a $|1\rangle$ or $|2\rangle$.

To distinct the nine states in Eq. (5), Alice can perform a von Neumann measurement on qutrit 1 followed by an unambiguous quantum state discrimination on qutrit 2. However, these operations destroy the initial state of qutrit 1 even though the discrimination fails. Here, following the protocol of Roa and Groiseau,[9] we introduce an extra auxiliary qutrit 0 which acts as a quantum apparatus to measure qutrit 1. The key point is that, when we erase the information of qutrit 1 recorded in the ancilla, the initial state of qutrit 1 recoveries when discrimination fails. Alice applies the inverse of the GCNOT gate on

the ancilla initial in $|0\rangle_0$ and qutrit 1, and obtain the nine states in her hands as

$$\mathcal{C}_{10}^{\dagger}|0\rangle_0|m\rangle_1|\tau_n\rangle_2 = |m\rangle_0|m\rangle_1|\tau_n\rangle_2, \tag{7}$$

where $|\tau_n\rangle_2 = b_0|0\rangle_2 + b_1\omega^n|1\rangle_2 + b_2\omega^{2n}|2\rangle_2$ and $m,n = 0,1,2$. Then, qutrit 1 serves as an ancilla in the unambiguous quantum state discrimination of qutrit 2. Let us define two unitary transformations on qutrit 2 as

$$\begin{aligned}\mathcal{U}_1 &= \frac{b_0}{b_1}I_1 + \sqrt{1-\left(\frac{b_0}{b_1}\right)^2}\mathcal{V}_1,\\ \mathcal{W}_1 &= \frac{b_0}{b_2}I_1 + \sqrt{1-\left(\frac{b_0}{b_2}\right)^2}\mathcal{V}_1,\end{aligned} \tag{8}$$

and a controlled-unitary operation

$$\mathcal{D}_{21} = |0\rangle_2\langle 0|\otimes I_1 + |1\rangle_2\langle 1|\otimes\mathcal{U}_1 + |2\rangle_2\langle 2|\otimes\mathcal{W}_1. \tag{9}$$

Alice performs it on qutrits 1 and 2 and obtains

$$\begin{aligned}&\mathcal{D}_{21}|m\rangle_0|m\rangle_1|\tau_n\rangle_2\\ =\ &|m\rangle_0\big[\sqrt{3}b_0|m\rangle_1|\varkappa_n\rangle_2 + |m\oplus_3 2\rangle_1\\ &\times(\sqrt{b_1^2-b_0^2}\omega^n|1\rangle_2 + \sqrt{b_2^2-b_0^2}\omega^{2n}|2\rangle_2)\big],\end{aligned} \tag{10}$$

where $\oplus_3$ denotes modulo 3 addition, and $|\varkappa_n\rangle = \frac{1}{\sqrt{3}}(|0\rangle_2 + \omega^n|1\rangle_2 + \omega^{2n}|2\rangle_2)$. In the above form, the nine states $|m\rangle_0|m\rangle_1|\varkappa_n\rangle_2$ corresponding to successful discrimination are orthogonal to each other, and orthogonal to the states with qutrits 0 and 1 in $|m\rangle_0|m\oplus_3 2\rangle_1$. The latter nine are for the failure of discrimination as they are not linearly independent.

The final step of Alice's unitary operations is to erase the information measured by qutrit 0. Applying the GCNOT gate $\mathcal{C}_{10}$, Alice can obtain

$$\begin{aligned}&\mathcal{C}_{10}|m\rangle_0|m\rangle_1 = |0\rangle_0|m\rangle_1,\\ &\mathcal{C}_{10}|m\rangle_0|m\oplus_3 2\rangle_1 = |1\rangle_0|m\oplus_3 2\rangle_1.\end{aligned} \tag{11}$$

The information recorded on the apparatus qutrit 0 can be erased partially, as it returns $|0\rangle_0$ in the terms corresponding to successful discrimination but becomes $|1\rangle_0$ for the case of failure. This divides the total four-qutrit state into two parts as

$$\begin{aligned}|\Delta\rangle\ =\ &\frac{b_0}{\sqrt{3}}|0\rangle_0\big[|0\rangle_1(|\varkappa_0\rangle|\phi_{00}\rangle + |\varkappa_1\rangle|\phi_{10}\rangle + |\varkappa_2\rangle|\phi_{20}\rangle)_{23}\\ &+|1\rangle_1(|\varkappa_0\rangle|\phi_{01}\rangle + |\varkappa_1\rangle|\phi_{11}\rangle + |\varkappa_2\rangle|\phi_{21}\rangle)_{23}\\ &+|2\rangle_1(|\varkappa_0\rangle|\phi_{02}\rangle + |\varkappa_1\rangle|\phi_{12}\rangle + |\varkappa_2\rangle|\phi_{22}\rangle)_{23}\big]\\ &+|1\rangle_0\big[\sqrt{b_1^2-b_0^2}|\phi_{02}\rangle|1\rangle|1\rangle\\ &+\sqrt{b_2^2-b_0^2}|\phi_{00}\rangle|2\rangle|2\rangle\big]_{123}.\end{aligned} \tag{12}$$

Here $|\phi_{nm}\rangle$ are the states of qutrit 3 multiplied to $|\psi_{nm}\rangle_{12}$ in Eq. (3) as

$$\begin{aligned}|\phi_{00}\rangle\ &=\ (\alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle),\\ |\phi_{10}\rangle\ &=\ (\alpha_0|0\rangle + \alpha_1\omega^2|1\rangle + \alpha_2\omega|2\rangle),\\ |\phi_{20}\rangle\ &=\ (\alpha_0|0\rangle + \alpha_1\omega|1\rangle + \alpha_2\omega^2|2\rangle),\\ |\phi_{01}\rangle\ &=\ (\alpha_1|0\rangle + \alpha_2|1\rangle + \alpha_0|2\rangle),\\ |\phi_{11}\rangle\ &=\ (\alpha_1|0\rangle + \alpha_2\omega^2|1\rangle + \alpha_0\omega|2\rangle),\\ |\phi_{21}\rangle\ &=\ (\alpha_1|0\rangle + \alpha_2\omega|1\rangle + \alpha_0\omega^2|2\rangle),\\ |\phi_{02}\rangle\ &=\ (\alpha_2|0\rangle + \alpha_0|1\rangle + \alpha_1|2\rangle),\\ |\phi_{12}\rangle\ &=\ (\alpha_2|0\rangle + \alpha_0\omega^2|1\rangle + \alpha_1\omega|2\rangle),\\ |\phi_{22}\rangle\ &=\ (\alpha_2|0\rangle + \alpha_0\omega|1\rangle + \alpha_1\omega^2|2\rangle).\end{aligned} \tag{13}$$

Then, the state to be teleported is encoded in qutrit 3 in the first part of $|\Delta\rangle$, but in qutrit 1 in the second part.

Alice performs a von Neumann measurements on qutrit 0 in the standard basis. It projects qutrit 0 to $|0\rangle_0$ in a probability $3|b_0|^2$. Then, she measures the nine orthogonal direct product states $|0\rangle_0|m\rangle_1$, and informs Bob to perform appropriate unitary operations on his qutrit 3 to rebuild the initial state, and thereby the teleportation succeeds. On the contrary, the qutrit 0 could be projected to $|1\rangle_0$ in a probability $1-3|b_0|^2$, which means the failure of the teleportation. Alice can recover the state $|\phi\rangle_1$ in qutrit 1, by performing a joint unitary transformation $\mathcal{B}_{21} = I_1\otimes(|0\rangle_2\langle 0| + |2\rangle_2\langle 2|) + \mathcal{V}_1\otimes|1\rangle_2\langle 1|$ on qutrits 1 and 2. Alternatively, Alice could perform a von Neumann measurements on qutrit 2 in the standard basis. She obtains the initial state $|\phi\rangle_1$, by using the operation $\mathcal{V}_1$ on qutrit 1 when qutrit 2 was projected to $|2\rangle_2$, while I_1 corresponding to $|0\rangle_2$ and $|1\rangle_2$.

3. General protocol

Now we turn to the general protocol for teleporting a qu*N*it (a *N*-level quantum system) through a partially entangled two-qu*N*it quantum channel. Since it is direct to extend the above scheme to *N*-level systems, we present the following results in compact general formulae in this part. Let an initial qu*N*it state to be

$$|\phi\rangle_1 = \sum_{i=0}^{N-1}\alpha_i|i\rangle_1, \tag{14}$$

where $\alpha_{i=0,1,\ldots,N-1}$ are complex numbers satisfying the normalization condition $\sum_{i=0}^{N-1}|\alpha_i|^2 = 1$. Alice and Bob share a two-qu*N*it entangled state as the quantum channel, which can be generally written in the form of the Schmidt disposition as

$$|\Phi\rangle_{23} = \sum_{j=0}^{N-1}b_j|j\rangle_2|j\rangle_3, \tag{15}$$

with the real coefficient $0\le b_0\le b_1\le\cdots\le b_{N-1}$ and $\sum_{j=0}^{N-1}b_j^2 = 1$. The total state of the tripartite system is given by

$$\begin{aligned}|\Psi\rangle_{123}\ &=\ |\phi\rangle_1|\Phi\rangle_{23}\\ &=\ \frac{1}{N}\sum_{m=0,n=0}^{N-1,N-1}|\psi_{nm}\rangle_{12}\sum_{f=0}^{N-1}\alpha_{f\oplus m}\,\mathrm{e}^{\frac{-\mathrm{i}2\pi fn}{N}}|f\rangle_3,\end{aligned} \tag{16}$$

where $\oplus$ denotes modulo N addition, and $|\psi_{nm}\rangle_{12}$ are N^2 linearly independent bipartite states

$$|\psi_{nm}\rangle_{12} = \sum_{k=0}^{N-1} b_k \mathrm{e}^{\frac{\mathrm{i}2\pi kn}{N}} |k\oplus m\rangle_1 |k\rangle_2 . \tag{17}$$

In the above form in Eq. (16), the states multiplied to $|\psi_{nm}\rangle_{12}$ are equivalent to the state (14) on qu$N$it 3 under unitary transformations as

$$\sum_{f=0}^{N-1} \alpha_{f\oplus m} \mathrm{e}^{\frac{-\mathrm{i}2\pi fn}{N}} |f\rangle_3 = U_3^{(n,m)} |\phi\rangle_3 \tag{18}$$

with

$$U_j^{(n,m)} = \sum_{f=0}^{N-1} \mathrm{e}^{\frac{-\mathrm{i}2\pi fn}{N}} |f\rangle_j \langle f\oplus m| , \tag{19}$$

and the subscript j denotes the j-th subsystem.

Alice's first two operations are to disentangle subsystems 1 and 2 in the states $|\psi_{nm}\rangle_{12}$, and to measure 2 by using an auxiliary apparatus, qu$N$it 0 initial in $|0\rangle_0$. Here, we define the N-level GCNOT gate as

$$C_{ij} = \sum_{y=0}^{N-1} |y\rangle_i \langle y| \otimes U_j^{(0,y)} . \tag{20}$$

The four-partite state becomes

$$\begin{aligned} |\Omega\rangle &= C_{10}^{\dagger}\left(|0\rangle_0 C_{21} |\Psi\rangle_{123} \right) \\ &= \frac{1}{N} \sum_{m=0,n=0}^{N-1,N-1} |mm\rangle_{01} \left(\sum_{j=0}^{N-1} b_j \mathrm{e}^{\frac{\mathrm{i}2\pi jn}{N}} |j\rangle_2 \right) U_3^{(n,m)} |\phi\rangle_3 . \end{aligned}$$

To unambiguously discriminate the states of particle 2, Alice applies a joint unitary transformation on 1 and 2

$$D_{21} = \sum_{y=0}^{N-1} |y\rangle_2 \langle y| \otimes \left(\frac{b_0}{b_y} I_1 + \sqrt{1-\left(\frac{b_0}{b_y}\right)^2} U_1^{(0,1)} \right) ,$$

and obtains the whole system in the state

$$\begin{aligned} |\Gamma\rangle &= D_{21} |\Omega\rangle \\ &= \frac{b_0}{N} \sum_{m=0,n=0}^{N-1,N-1} |mm\rangle_{01} \left(\sum_{j=0}^{N-1} \mathrm{e}^{\frac{\mathrm{i}2\pi jn}{N}} |j\rangle_2 \right) U_3^{(n,m)} |\phi\rangle_3 \\ &\quad + \sum_{m=0,j=1}^{N-1,N-1} \sqrt{b_j^2 - b_0^2} |m\rangle_0 \alpha_{j\oplus m} |m\oplus N-1\rangle_1 |jj\rangle_{23} . \end{aligned}$$

When Alice erases the information in the auxiliary apparatus, 0, the changes of the qu$N$it 1 are recorded as $|1\rangle_0$, and the total state becomes

$$\begin{aligned} |\Delta\rangle &= C_{10} |\Gamma\rangle \\ &= |0\rangle_0 \frac{b_0}{N} \sum_{m=0,n=0}^{N-1,N-1} |m\rangle_1 \left(\sum_{j=0}^{N-1} \mathrm{e}^{\frac{\mathrm{i}2\pi jn}{N}} |j\rangle_2 \right) U_3^{(n,m)} |\phi\rangle_3 \\ &\quad + |1\rangle_0 \sum_{j=1}^{N-1} \sqrt{b_j^2 - b_0^2} U_1^{(0,j\oplus 1)} |\phi\rangle_1 |jj\rangle_{23} . \end{aligned}$$

Obviously, it is divided into two parts of success and failure, which can be collapsed by Alice's measurements in the standard basis. When Alice's outcome $|0\rangle_0$ occurs in the probability Nb_0^2, the teleportation can be accomplished by two local von Neumann measurement on qu$N$its 1 and 2. One the other hand, Alice can recover the initial state by a joint unitary operation, $B_{21} = \sum_{j=0}^{N-1} U_1^{(0,j\oplus 1)\dagger} \otimes |j\rangle_2\langle j|$, on her two systems, when the task of teleportation fails in the probability $1 - Nb_0^2$.

4. Summary

We present a scheme for nondestructive probabilistic teleportation of high-dimensional quantum states. A partial entangled pure state severs as the quantum channel, whose smallest coefficient determines the successful probability of exactly teleporting a state. With the aid of an auxiliary particle, Alice can recover her initial state to be teleported when teleportation fails. Compared to the existing results,[32,33] our protocol only requires the sender, Alice, to have the ability to perform bipartite operations, while the dimension of the ancilla needs to be the same as the state to be teleported. In addition, the ancilla acts as a quantum apparatus to measure Alice's system. The process of resuming the initial state can be regarded as erasing information recorded in the ancilla.

As the following research, it is a fundamental problem to explore the roles of quantum correlations in our four-party procedure, which is a fundamental problem in quantum information. In addition, the relation between our protocol and the theory of extracting information from a quantum system by multiple observers[35] would be interesting, since the quantum correlations in the latter are studied in many works.[36–38] While we focus here on the teleportation using quantum channels with the same dimension as the state to be teleported, it is a natural extension to apply the present ideas in the case with different dimension.[14] And finally, we hope that the process can be implemented in laboratories with the help of the techniques recently developed in optical systems.[16,17]

Acknowledgements

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