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Cross correlation mediated by distant Majorana zero modes with no overlap

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Existing studies via shot noise calculation conclude that the cross correlation between the currents in the two leads connected by a pair of Majorana zero modes (MZMs) vanishes when their coupling energy $\epsilon_{\rm M} \to 0$. Motivated by the intrinsic nature of nonlocality of the MZMs, we revisit this important problem and propose an experimental scheme to demonstrate the nonvanishing cross correlation even at the limit $\epsilon_{\rm M} \to 0$. The proposed scheme employs the Andreev-process-associated branch circuit currents, which are theoretically obtained by applying a decomposition analysis for the total currents while are accessible directly in practical measurement. For different bias voltage setup, we find intriguing results of both negative and positive correlations and carry out simple physical understanding using a quantum jump technique. Importantly, combining together with the evidence of the zero-bias-peak of conductance, the nonlocal cross correlation predicted in this work can help to confirm the existence of the nonlocal MZMs.

Keywords: Majorana zero modes, cross correlation, Andreev currents

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1. Introduction

The nonlocal nature of the Majorana zero modes (MZMs) and the obeying non-Abelian braiding statistics promise a sound potential for topological quantum computation.[1–4] Throughout the past decade, the search for reliable MZMs has become a major theme in condensed matter physics.[5–11] In order to identify the Majorana zero modes (MZMs), rich transport phenomena have been proposed, such as the fractional Josephson effects,[12–15] peculiar noise behaviors,[16–25] and the famous Majorana quantized conductance $2e^2/h$.[20,22,26–28] Recent interests also include the nonlocal transport conductances[29–37] and the proposals to distinguish the nonlocal MZMs from the topologically trivial Andreev states.[38–43]

The most direct evidences of the genuinely nonlocal nature of the MZMs should be the Majorana teleportation[44–46] and/or the cross correlation of two remote Majoranas (γ_1 and γ_2).[16–21] Both the phenomena of Majorana teleportation and nonlocal cross correlation are rather transparent by using the low energy effective Hamiltonian description, especially within the framework of "second quantization" in terms of the occupation-number-states $|0\rangle$ and $|1\rangle$ of the MZMs associated regular fermion (the f quasiparticle). Consider injecting an electron from outside into the superconductor through, for instance, the left MZM γ_1. The occupation of the nonlocal f quasiparticle should allow for extracting a particle (either an electron or a hole) through the other distant right-side MZM γ_2, thus expecting the so-called teleportation phenomenon. The same reason allows us to expect nonlocal cross correlation between the currents in the distant leads. More specifically, consider a two-lead (three-terminal) setup.[16–18,33–37] Can we find nonlocal cross correlation between the currents in different leads? A few studies showed that the cross correlation vanishes at the limit $\epsilon_{\rm M} \to 0$.[16–18]

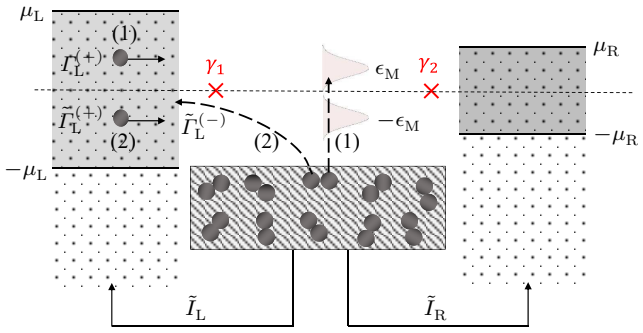


Fig. 1. Schematic circuit diagram for the Andreev reflection (AR) and inverse AR processes in the two-lead (three-terminal) device under consideration. In the AR process, taking the left lead as example, two electrons in the lead enter the superconductor, successively and coherently, and form a Cooper pair. The two electrons are of energy in resonance with (have Lorentzian centers at) $\epsilon_{\rm M}$ and $-\epsilon_{\rm M}$, respectively, as seen in Eqs. (3) and (4) in the text. The inverse AR process splits a Cooper pair and generates the excitation of an f quasiparticle in the superconductor, meanwhile, sends an electron back into the lead. In the plot, we only show the left-side current and the local Andreev process, while the right-side current and the crossed Andreev process can be similarly understood. In the diagram we also schematically show the net branch circuit currents $\tilde{I}_{\rm L}$ and $\tilde{I}_{\rm R}$, flowing back to the leads.

In this work, applying the master equation approach (and using the occupation-number-state representation), we show that it is possible by extracting component currents from the total lead-currents to expose the intrinsic nonlocal cross correlation between them, even at the limit $\epsilon_{\rm M} \to 0$, owing to the nonlocal nature of the MZMs associated f quasiparticle. Moreover, the component currents extracted from the total

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ones are nothing but the branch circuit currents flowing back to the individual leads from the superconductor, which are measurable in experiment, as schematically shown in Fig. 1. By combining together with the evidence of the zero-bias-peak of conductance (ZBPC), this nonlocal cross correlation should allow us to definitely confirm the existence of the nonlocal MZMs. This is the most important issue at present stage in the Majorana community.

2. Master equation approach

For completeness, we briefly outline the Majorana master equation (MME) approach based on Refs. [47,48], which deals with a pair of MZMs embedded in a two-lead (three-terminal) nonlocal transport setup. The low-energy effective description for a pair of MZMs can be commonly formulated by the Hamiltonian $H_{\rm M}=\mathrm{i}\epsilon_{\rm M}\gamma_1\gamma_2$, where $\epsilon_{\rm M}$ is the coupling energy of the MZMs γ_1 and γ_2. The Majorana operators are related to the regular complex fermion through the transformation of $\gamma_1=f+f^\dagger$ and $\gamma_2=-\mathrm{i}(f-f^\dagger)$. Through using the complex fermion representation, for a two-lead setup of transport, the tunnel-coupling of the MZMs to the leads is given by[19]

$$H'=\sum_{\alpha=\rm L,R}\sum_k t_{\alpha k}\left[(b_{\alpha k}^\dagger f+(-1)^{\tilde\alpha+1}b_{\alpha k}^\dagger f^\dagger)+\text{H.c.}\right], \quad (1)$$

where $b_{\alpha k}^\dagger$ ($b_{\alpha k}$) are the creation (annihilation) operators of electrons in the leads, while the leads are described by $H_{\rm leads}=\sum_{\alpha=\rm L,R}\sum_k\epsilon_{\alpha k}b_{\alpha k}^\dagger b_{\alpha k}$. It should be noted that in H' the tunneling terms only conserve charges modulo $2e$, which actually correspond to the well known Andreev reflection (AR) process. In Eq. (1), the sign factor is determined by assigning $\tilde\alpha=1$ and 2, corresponding to $\alpha=$ L and R, respectively.

The master equation approach to quantum transport simply follows the theoretical treatment of quantum dissipation, where the system-of-interest is coupled to an environment usually via energy exchange, but not involving the particle exchange. For quantum transport, the system-of-interest is the central device, while the environment is the transport leads. In the transport problem, particle exchange/tunneling between the leads and the central device is seemingly causing certain difficulty to describe the central device by using the density matrix. However, after introducing the occupied and unoccupied probabilities of states, the fundamental requirement of $\mathrm{Tr}\rho=1$ is guaranteed for the density matrix operator ρ. The transport master equation is thus well justified and has been applied broadly in practice.

Formally, the normal tunneling terms in Eq. (1) (e.g., $b_{\alpha k}^\dagger f$) correspond to the rotating-wave-approximation terms in quantum optics, while the Andreev process terms (e.g., $b_{\alpha k}^\dagger f^\dagger$) correspond to the counter-rotating-wave terms. In quantum optics, in the presence of the latter type of terms, the master equation has been well established, especially under the Born–Markov approximation.[49] Following such type of results, and in particular the specific Refs. [50,51], we construct the MME as[47,48]

$$\dot\rho=-\frac{\mathrm{i}}{\hbar}[H_{\rm M},\rho]+\sum_{\alpha=\rm L,R}\left(\Gamma_\alpha^+\mathcal{D}[f^\dagger]\rho+\Gamma_\alpha^-\mathcal{D}[f]\rho\right)+\sum_{\alpha=\rm L,R}\left(\widetilde\Gamma_\alpha^+\mathcal{D}[f]\rho+\widetilde\Gamma_\alpha^-\mathcal{D}[f^\dagger]\rho\right). \quad (2)$$

Here the Lindblad superoperator is defined by $\mathcal{D}[A]\rho=A\rho A^\dagger-\frac{1}{2}\{A^\dagger A,\rho\}$ and the tunnel-coupling rates are introduced as

$$\Gamma_\alpha^\pm=\Gamma_\alpha^{\rm e}N_\alpha^{(\pm)},\ N_\alpha^{(\pm)}=\int\mathrm{d}\omega n_\alpha^{(\pm)}(\omega)\widetilde\delta(\omega-\epsilon_{\rm M}), \quad (3a)$$

$$\widetilde\Gamma_\alpha^\pm=\Gamma_\alpha^{\rm h}\widetilde N_\alpha^{(\pm)},\ \widetilde N_\alpha^{(\pm)}=\int\mathrm{d}\omega n_\alpha^{(\pm)}(\omega)\widetilde\delta(\omega+\epsilon_{\rm M}). \quad (3b)$$

The superscripts "e" and "h" of the rates denote the coupling of the f quasiparticle to the leads via "electron" and "hole" components, respectively. We have also denoted the Fermi occupied function by $n_\alpha^{(+)}$ and the empty function by $n_\alpha^{(-)}=1-n_\alpha^{(+)}$. The spectral function $\widetilde\delta(\omega\mp\epsilon_{\rm M})$ is a generalization of the Dirac δ-function

$$\widetilde\delta(\omega\mp\epsilon_{\rm M})=\frac{1}{\pi}\frac{\Gamma}{(\omega\mp\epsilon_{\rm M})^2+\Gamma^2}, \quad (4)$$

where the broadening is given by $\Gamma=\sum_\alpha(\Gamma_\alpha^{\rm e}+\Gamma_\alpha^{\rm h})/2$. This type of generalization in terms of the Lorentzian spectral function properly accounts for the level broadening effect, which directly follows the spirit of the self-consistent Born approximation[51] and works perfectly well for transport through single-level systems (such as the single-level quantum dot). This generalization makes the transport master equation applicable under small bias voltage, while it is well known that the usual Born–Markov–Lindblad master equation is applicable only under large bias limit (with voltage much larger than the level broadening widths). Here, in the Majorana case, the subgap f quasiparticle is simply a single-level system within the low-energy effective description. The only unique feature is the presence of the Andreev-process associated counter-rotating-wave terms, which results in the two terms in the second round brackets in Eq. (2), while the two Lindblad terms in the first round brackets in Eq. (2) describe the normal tunneling process. We finally mention that, for the two types of process, the energy conservation is manifested differently in the rate expressions, i.e., by the different centers of the Lorentzian spectral functions $\widetilde\delta(\omega\mp\epsilon_{\rm M})$, see also illustration in Fig. 1.

3. Transient current and its decomposition

The MME can be straightforwardly solved using the number-state basis $\{|0\rangle,|1\rangle\}$ of the complex fermion (the f particle). Let us denote the solution of the density matrix as $\rho=p_0|0\rangle\langle0|+p_1|1\rangle\langle1|$. As an example, the left-lead current,

$I_{\rm L}=I_{\rm L}^{(1)}+I_{\rm L}^{(2)}$, can be calculated as[48]

$$\begin{aligned} I_{\rm L}^{(1)} &= \frac{e}{\hbar}\left(\Gamma_{\rm L}^{+}p_0-\Gamma_{\rm L}^{-}p_1\right), \\ I_{\rm L}^{(2)} &= \frac{e}{\hbar}\left(\widetilde{\Gamma}_{\rm L}^{+}p_1-\widetilde{\Gamma}_{\rm L}^{-}p_0\right). \end{aligned} \tag{5}$$

Physically, $I_{\rm L}^{(1)}$ is contributed by the conventional tunneling process and $I_{\rm L}^{(2)}$ is from the Andreev process, including also the crossed AR (CAR) process.

Noting that $\Gamma_{\rm L}^{+}+\Gamma_{\rm L}^{-}=\Gamma_{\rm L}^{\rm e}$ and $\widetilde{\Gamma}_{\rm L}^{+}+\widetilde{\Gamma}_{\rm L}^{-}=\Gamma_{\rm L}^{\rm h}$, we reexpress the total left-lead current as

$$\begin{aligned} I_{\rm L} &= I_{\rm L}^{(1)}+I_{\rm L}^{(2)} \\ &= \frac{e}{\hbar}\left[\left(\Gamma_{\rm L}^{+}(1-p_1)-\Gamma_{\rm L}^{-}p_1\right)+\left(\widetilde{\Gamma}_{\rm L}^{+}p_1-\widetilde{\Gamma}_{\rm L}^{-}(1-p_1)\right)\right] \\ &= \frac{e}{\hbar}\left[\Gamma_{\rm L}^{+}-\widetilde{\Gamma}_{\rm L}^{-}-p_1(\Gamma_{\rm L}^{\rm e}-\Gamma_{\rm L}^{\rm h})\right]. \end{aligned} \tag{6}$$

In the case of $\epsilon_{\rm M}=0$ or small $\epsilon_{\rm M}$, the difference of $\Gamma_{\rm L}^{\rm e}$ and $\Gamma_{\rm L}^{\rm h}$ is zero or negligibly small. We thus find that the total current $I_{\rm L}$ (and also $I_{\rm R}$) does not depend on time, even in the transient process with time dependent $p_1(t)$. This unusual feature is rooted in the unique Andreev process involved here. That is, entering of the "first" electron is conditioned on the unoccupied state of the the f quasiparticle ($n_f=0$), while entering of the "second" electron is conditioned on its occupied state ($n_f=1$). As a consequence, the total current does not depend on the occupation probability p_1. We may point out that this conclusion holds under the Markovian approximation, which guarantees the use of the rate equation and is valid on timescale longer than the so-called memory time of the reservoir (transport lead) electrons.

The feature that $I_{\rm L}$ and $I_{\rm R}$ are independent of the occupation probability $p_1(t)$ is seemingly raising an obstacle for us to investigate the cross correlation between them, since the current correlation function simply characterizes a disturbance of the occupation probability and the subsequent response associated with two successive current measurements. That is, the correlation function of the total currents $I_{\rm L}$ and $I_{\rm R}$ cannot reveal the intrinsic nature of the dynamical disturbance-and-response process. However, as to be clear soon, we can consider the cross correlation of the partial components of the currents. To achieve this, below we analyze firstly the "inside structure" of the entire lead current, which is connected with the total occupation probability of the f quasiparticle.

For the brevity of notation, let us introduce the f particle's excitation rate $r_1=\sum_{\alpha={\rm L,R}}(\Gamma_\alpha^{+}+\widetilde{\Gamma}_\alpha^{-})$, its deexcitation rate $r_2=\sum_{\alpha={\rm L,R}}(\Gamma_\alpha^{-}+\widetilde{\Gamma}_\alpha^{+})$, and the total rate $r_1+r_2=\sum_{\alpha={\rm L,R}}(\Gamma_\alpha^{\rm e}+\Gamma_\alpha^{\rm h})=2\Gamma$.

3.1. Unoccupied initial condition

Let us consider firstly the time-dependent solution under the initial condition of empty occupation of the f particle state ($n_f=0$), which reads

$$\begin{aligned} p_1 &= \frac{r_1}{2\Gamma}G(t), \\ p_0 &= \frac{r_2}{2\Gamma}G(t)+[1-G(t)]. \end{aligned} \tag{7}$$

Here we introduced $G(t)=1-{\rm e}^{-2\Gamma t}$. We organize the solution in this form for a particular advantage. It shows that the probability evolution contains two channels: one is the gradual formation of the steady-state occupation channel associated with $G(t)$ (from 0 to 1), and another one is the gradual decay of the empty occupation channel described by $1-G(t)$ (from 1 to 0). Of great interest is that we can treat the evolution of these two channels being independent, as to be clarified with details in the following.

The steady-state-occupation-channel would lead to the stationary probabilities $\bar{p}_1=\frac{r_1}{2\Gamma}$ and $\bar{p}_0=\frac{r_2}{2\Gamma}$. Substituting them into Eq. (5), we obtain the steady-state currents $I_{\rm L,st}^{(1)}$ and $I_{\rm L,st}^{(2)}$. Let us consider firstly the current $I_{\rm L,st}^{(1)}$, which can be decomposed into three components as

$$\begin{aligned} I_{\rm L,st}^{(1)}(A_1) &= \frac{e}{\hbar}\left(\Gamma_{\rm L}^{+}\widetilde{\Gamma}_{\rm L}^{+}-\Gamma_{\rm L}^{-}\widetilde{\Gamma}_{\rm L}^{-}\right)/2\Gamma, \\ I_{\rm L,st}^{(1)}(A_2) &= \frac{e}{\hbar}\left(\Gamma_{\rm L}^{+}\widetilde{\Gamma}_{\rm R}^{+}-\Gamma_{\rm L}^{-}\widetilde{\Gamma}_{\rm R}^{-}\right)/2\Gamma, \\ I_{\rm L,st}^{(1)}(A_3) &= \frac{e}{\hbar}\left(\Gamma_{\rm L}^{+}\Gamma_{\rm R}^{-}-\Gamma_{\rm L}^{-}\Gamma_{\rm R}^{+}\right)/2\Gamma. \end{aligned} \tag{8}$$

Using Eq. (3), we further reexpress them as

$$\begin{aligned} I_{\rm L,st}^{(1)}(A_1) &= \frac{e}{\hbar}\frac{\Gamma_{\rm L}^{\rm e}\Gamma_{\rm L}^{\rm h}}{2\Gamma}\left(N_{\rm L}^{(+)}-\widetilde{N}_{\rm L}^{(-)}\right), \\ I_{\rm L,st}^{(1)}(A_2) &= \frac{e}{\hbar}\frac{\Gamma_{\rm L}^{\rm e}\Gamma_{\rm R}^{\rm h}}{2\Gamma}\left(N_{\rm L}^{(+)}-\widetilde{N}_{\rm R}^{(-)}\right), \\ I_{\rm L,st}^{(1)}(A_3) &= \frac{e}{\hbar}\frac{\Gamma_{\rm L}^{\rm e}\Gamma_{\rm R}^{\rm e}}{2\Gamma}\left(N_{\rm L}^{(+)}-N_{\rm R}^{(+)}\right). \end{aligned} \tag{9}$$

We can easily convert them into the usual results from the scattering matrix or nonequilibrium Green's function approach as[48]

$$\begin{aligned} I_{\rm L,st}^{(1)}(A_1) &= \frac{e}{h}\int_{-\mu_{\rm L}}^{\mu_{\rm L}}{\rm d}\omega\mathcal{T}_{\rm LLA}^{\rm eh}(\omega), \\ I_{\rm L,st}^{(1)}(A_2) &= \frac{e}{h}\int_{-\mu_{\rm R}}^{\mu_{\rm L}}{\rm d}\omega\mathcal{T}_{\rm LRA}^{\rm eh}(\omega), \\ I_{\rm L,st}^{(1)}(A_3) &= \frac{e}{h}\int_{\mu_{\rm R}}^{\mu_{\rm L}}{\rm d}\omega\mathcal{T}_{\rm LR}^{\rm ee}(\omega). \end{aligned} \tag{10}$$

Here, the three transport coefficients (and the associated currents) correspond to, respectively, the local AR at the left lead, the crossed AR between the two leads, and the electron transmission from the left to the right lead. More explicitly, substituting Eqs. (3) and (4) into Eq. (9), we identify the transport coefficients as

$$\begin{aligned} \mathcal{T}_{\rm LLA}^{\rm eh}(\omega) &= \Gamma_{\rm L}^{\rm e}\Gamma_{\rm L}^{\rm h}/|Z|^2, \\ \mathcal{T}_{\rm LRA}^{\rm eh}(\omega) &= \Gamma_{\rm L}^{\rm e}\Gamma_{\rm R}^{\rm h}/|Z|^2, \\ \mathcal{T}_{\rm LR}^{\rm ee}(\omega) &= \Gamma_{\rm L}^{\rm e}\Gamma_{\rm R}^{\rm e}/|Z|^2, \end{aligned} \tag{11}$$

where $|Z|^2=(\omega-\epsilon_{\rm M})^2+\Gamma^2$. Actually, one can check

that these coefficients fall into the well-known form obtained from the nonequilibrium Green's function formalism or the S matrix scattering approach, i.e., $\mathcal{T}_{\alpha\beta}(\omega) = \mathrm{Tr}[\Gamma_\alpha G^{\mathrm{r}}(\omega)\Gamma_\beta G^{\mathrm{a}}(\omega)]$, where the retarded/advanced Green's function reads $G^{\mathrm{r(a)}}(\omega) = (\omega - \epsilon_{\mathrm{M}} \pm \mathrm{i}\Gamma)^{-1}$.

Substituting the time-dependent solutions of $p_1(t)$ and $p_0(t)$ given by Eq. (7) into the current expression Eq. (5), we obtain the transient current $I_{\mathrm{L}}^{(1)}(t)$ as the sum from the two channels

$$\begin{aligned} I_{\mathrm{L}}^{(1)}(A) &= I_{\mathrm{L,st}}^{(1)}(A)G(t), \\ I_{\mathrm{L}}^{(1)}(B) &= \Gamma_{\mathrm{L}}^{+}[1-G(t)]. \end{aligned} \tag{12}$$

Here $I_{\mathrm{L}}^{(1)}(A)$ is the transient current associated with the steady-state-occupation-channel, while $I_{\mathrm{L}}^{(1)}(B)$ is the current associated with the empty-occupation-channel. Remarkably, we stress that here $I_{\mathrm{L,st}}^{(1)}(A) = I_{\mathrm{L,st}}^{(1)}(A_1) + I_{\mathrm{L,st}}^{(1)}(A_2) + I_{\mathrm{L,st}}^{(1)}(A_3)$, which allows us to obtain the individual transient components.

Let us now consider further the current $I_{\mathrm{L}}^{(2)}(t)$, i.e., the current contribution of the "second" electron in the Andreev process. Similar analysis can be performed along the line for the current $I_{\mathrm{L}}^{(1)}(t)$. Firstly, for the steady-state current, we have

$$\begin{aligned} I_{\mathrm{L,st}}^{(2)}(A_1) &= \frac{e}{\hbar}\left(\widetilde{\Gamma}_{\mathrm{L}}^{+}\Gamma_{\mathrm{L}}^{+} - \widetilde{\Gamma}_{\mathrm{L}}^{-}\Gamma_{\mathrm{L}}^{-}\right)/2\Gamma, \\ I_{\mathrm{L,st}}^{(2)}(A_2) &= \frac{e}{\hbar}\left(\widetilde{\Gamma}_{\mathrm{L}}^{+}\Gamma_{\mathrm{R}}^{+} - \widetilde{\Gamma}_{\mathrm{L}}^{-}\Gamma_{\mathrm{R}}^{-}\right)/2\Gamma, \\ I_{\mathrm{L,st}}^{(2)}(A_3) &= \frac{e}{\hbar}\left(\widetilde{\Gamma}_{\mathrm{L}}^{+}\widetilde{\Gamma}_{\mathrm{R}}^{-} - \widetilde{\Gamma}_{\mathrm{L}}^{-}\widetilde{\Gamma}_{\mathrm{R}}^{+}\right)/2\Gamma. \end{aligned} \tag{13}$$

We notice that $I_{\mathrm{L,st}}^{(2)}(A_1) = I_{\mathrm{L,st}}^{(1)}(A_1)$, which simply means the equal contribution of the two electrons involved in the local AR process, to the left-lead current. The other two components can be reexpressed as[48]

$$\begin{aligned} I_{\mathrm{L,st}}^{(2)}(A_2) &= \frac{e}{h}\int_{-\mu_{\mathrm{L}}}^{\mu_{\mathrm{R}}} \mathrm{d}\omega \mathcal{T}_{RLA}^{\mathrm{eh}}(\omega), \\ I_{\mathrm{L,st}}^{(2)}(A_3) &= \frac{e}{h}\int_{-\mu_{\mathrm{L}}}^{-\mu_{\mathrm{R}}} \mathrm{d}\omega \mathcal{T}_{RL}^{\mathrm{hh}}(\omega). \end{aligned} \tag{14}$$

It is clear that $I_{\mathrm{L,st}}^{(2)}(A_2)$ is the crossed AR process current associated with a right-lead electron and a left-lead hole, while $I_{\mathrm{L,st}}^{(2)}(A_3)$ is the current owing to hole–hole tunneling from the right lead to the left one. Similar as Eq. (11), here the two transport coefficients are obtained as $\mathcal{T}_{RLA}^{\mathrm{eh}}(\omega) = \Gamma_{\mathrm{R}}^{\mathrm{e}}\Gamma_{\mathrm{L}}^{\mathrm{h}}/|Z|^2$ and $\mathcal{T}_{RL}^{\mathrm{hh}}(\omega) = \Gamma_{\mathrm{R}}^{\mathrm{h}}\Gamma_{\mathrm{L}}^{\mathrm{h}}/|Z|^2$.

Next, the transient current $I_{\mathrm{L}}^{(2)}(t)$ can be similarly obtained and separated into two parts

$$\begin{aligned} I_{\mathrm{L}}^{(2)}(A) &= I_{\mathrm{L,st}}^{(2)}(A)G(t), \\ I_{\mathrm{L}}^{(2)}(B) &= -\widetilde{\Gamma}_{\mathrm{L}}^{-}[1-G(t)]. \end{aligned} \tag{15}$$

Here the steady-state current $I_{\mathrm{L,st}}^{(2)}(A)$ is the sum of the three components given above, i.e., $I_{\mathrm{L,st}}^{(2)}(A) = I_{\mathrm{L,st}}^{(2)}(A_1) + I_{\mathrm{L,st}}^{(2)}(A_2) + I_{\mathrm{L,st}}^{(2)}(A_3)$. $I_{\mathrm{L}}^{(2)}(B)$ is the transient current owing to the gradual decay of the empty-occupation-channel, see Fig. 2(a) for a schematic illustration.

3.2. Occupied initial condition

In this subsection we briefly show the results for the initial condition of occupied state of the f particle ($n_f = 1$). The time-dependent solution of the master equation reads

$$\begin{aligned} p_1 &= \frac{r_1}{2\Gamma}G(t) + [1-G(t)], \\ p_0 &= \frac{r_2}{2\Gamma}G(t). \end{aligned} \tag{16}$$

This solution holds also the two-channel interpretation, i.e., the gradual formation of the steady-state-occupation-channel associated with $G(t)$ (from 0 to 1), and the gradual decay of the initially-occupied-channel described by $1-G(t)$ (from 1 to 0). Therefore, under the occupied initial condition, the components of transient current $I_{\mathrm{L}}^{(1)}(A)$ and $I_{\mathrm{L}}^{(2)}(A)$ associated with the formation of the steady-state-occupation-channel are the same as those under the empty initial condition. However, the transient components $I_{\mathrm{L}}^{(1)}(B)$ and $I_{\mathrm{L}}^{(2)}(B)$, owing to the decay of the initially-occupied-channel, have different results, which read

$$\begin{aligned} I_{\mathrm{L}}^{(1)}(B) &= -\Gamma_{\mathrm{L}}^{-}[1-G(t)], \\ I_{\mathrm{L}}^{(2)}(B) &= \widetilde{\Gamma}_{\mathrm{L}}^{+}[1-G(t)]. \end{aligned} \tag{17}$$

In Fig. 2(b), we illustrate the process of $I_{\mathrm{L}}^{(2)}(B)$ for the occupied initial condition.

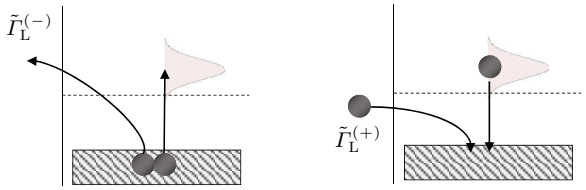


Fig. 2. Schematic diagram for transient contribution to the branch circuit current $\widetilde{I}_{\mathrm{L}}$, owing to the decay of the initial-occupation-channels. (a) The process contributing to $I_{\mathrm{L}}^{(2)}(B)$ given by Eq. (15) for the unoccupied initial condition, while $I_{\mathrm{L}}^{(1)}(B)$ given by Eq. (12) is not relevant to the branch circuit current. (b) The process contributing to $I_{\mathrm{L}}^{(2)}(B)$ given by Eq. (17) for the occupied initial condition, while $I_{\mathrm{L}}^{(1)}(B)$ is similarly not relevant to the branch circuit current.

4. Cross correlation of Andreev current fluctuations

As pointed out at the beginning of the previous section, within the formulation of rate equation (under the Markovian approximation), the total current, either I_{L} or I_{R}, is not time dependent even in the transient process of the occupation probability $p_1(t)$. However, as analyzed later in detail, we know that the individual components of the current are time dependent and can show transient behaviors. Therefore, we may consider the cross correlation between the currents $\widetilde{I}_{\mathrm{L}}$ and $\widetilde{I}_{\mathrm{R}}$, by deducting, from the total currents I_{L} and I_{R}, the components of the electron–electron and hole–hole transmission between

the left and right leads. More explicitly, we introduce $\widetilde{I}_{\rm L}(t) = I_{\rm L} - I_{\rm L}^{(1)}(A_3) - I_{\rm L}^{(2)}(A_3)$ and $\widetilde{I}_{\rm R}(t) = I_{\rm R} - I_{\rm R}^{(1)}(A_3) - I_{\rm R}^{(2)}(A_3)$, and consider their correlation. It is clear that, as shown schematically in Fig. 1, $\widetilde{I}_{\rm L}$ and $\widetilde{I}_{\rm R}$ are, respectively, the branch circuit currents flowing back to the left and right leads from the superconductor, being associated with the Andreev process. In this work, we may thus term $\widetilde{I}_{\rm L}$ and $\widetilde{I}_{\rm R}$ Andreev currents.

Following the standard formulation, let us consider the correlator of current fluctuations, $S_{\rm LR}(t) = \langle \delta\widetilde{I}_{\rm L}(t)\delta\widetilde{I}_{\rm R}(0)\rangle_{\bar\rho}$, where $\delta\widetilde{I}_\alpha(t) = \widetilde{I}_\alpha(t) - \langle\widetilde{I}_\alpha\rangle_{\bar\rho}$. $\langle\cdots\rangle_{\bar\rho}$ means making average over the steady state $\bar\rho$. Actually, we know that $S_{\rm LR}(t) = \langle\widetilde{I}_{\rm L}(t)\widetilde{I}_{\rm R}(0)\rangle_{\bar\rho} - \langle\widetilde{I}_{\rm L}\rangle_{\bar\rho}\langle\widetilde{I}_{\rm R}\rangle_{\bar\rho}$. We thus focus on calculating the correlator $\widetilde{S}_{\rm LR}(t) = \langle\widetilde{I}_{\rm L}(t)\widetilde{I}_{\rm R}(0)\rangle_{\bar\rho}$. Applying the quantum jump approach,[52] we know that after measuring the current $\widetilde{I}_{\rm R}$ at the first time ($t = 0$), the steady state is changed to be

$$\widetilde{\rho}(0) = a|0\rangle\langle 0| + b|1\rangle\langle 1|, \tag{18}$$

while the two coefficients read

$$\begin{aligned} a &= \left[\widetilde{\Gamma}_{\rm R}^{+}(\Gamma_{\rm R}^{+}+\Gamma_{\rm L}^{+}) - \Gamma_{\rm R}^{-}(\widetilde{\Gamma}_{\rm R}^{-}+\widetilde{\Gamma}_{\rm L}^{-})\right]/2\Gamma, \\ b &= \left[\Gamma_{\rm R}^{+}(\widetilde{\Gamma}_{\rm R}^{+}+\widetilde{\Gamma}_{\rm L}^{+}) - \widetilde{\Gamma}_{\rm R}^{-}(\Gamma_{\rm R}^{-}+\Gamma_{\rm L}^{-})\right]/2\Gamma. \end{aligned} \tag{19}$$

Now, $\widetilde{\rho}(0)$ is no longer the steady state. Its evolution before the second measurement of $\widetilde{I}_{\rm L}$ at time t is formally given by

$$\widetilde{\rho}(t) = {\rm e}^{\mathcal{L}t}(a|0\rangle\langle 0| + b|1\rangle\langle 1|), \tag{20}$$

where the propagator (in terms of the Liouvillian superoperator) stands for the evolution governed by the MME. After knowing this result, the correlator can be computed straightforwardly through $\widetilde{S}_{\rm LR}(t) = \langle\widetilde{I}_{\rm L}\rangle_{\widetilde{\rho}(t)}$, which contains two parts, $\widetilde{S}_{\rm LR}(t) = \widetilde{S}_{\rm LR}^{\rm (I)}(t) + \widetilde{S}_{\rm LR}^{\rm (II)}(t)$. The first part is contributed by the disturbed evolution associated with the formation of the steady-state-occupation-channel, which is given by

$$\widetilde{S}_{\rm LR}^{\rm (I)}(t) = \frac{e}{\hbar}(a+b)\left(\widetilde{I}_{\rm L,st}^{(1)} + \widetilde{I}_{\rm L,st}^{(2)}\right)G(t), \tag{21}$$

where the two components of the steady-state current flowing back from the superconductor to the left lead, i.e., $\widetilde{I}_{\rm L,st}^{(1)} = I_{\rm L,st}^{(1)}(A_1) + I_{\rm L,st}^{(1)}(A_2)$ and $\widetilde{I}_{\rm L,st}^{(2)} = I_{\rm L,st}^{(2)}(A_1) + I_{\rm L,st}^{(2)}(A_2)$, read

$$\begin{aligned} \widetilde{I}_{\rm L,st}^{(1)} &= \frac{e}{\hbar}\left[\Gamma_{\rm L}^{+}(\widetilde{\Gamma}_{\rm L}^{+}+\widetilde{\Gamma}_{\rm R}^{+}) - \Gamma_{\rm L}^{-}(\widetilde{\Gamma}_{\rm L}^{-}+\widetilde{\Gamma}_{\rm R}^{-})\right]/2\Gamma, \\ \widetilde{I}_{\rm L,st}^{(2)} &= \frac{e}{\hbar}\left[\widetilde{\Gamma}_{\rm L}^{+}(\Gamma_{\rm L}^{+}+\Gamma_{\rm R}^{+}) - \widetilde{\Gamma}_{\rm L}^{-}(\Gamma_{\rm L}^{-}+\Gamma_{\rm R}^{-})\right]/2\Gamma. \end{aligned} \tag{22}$$

Let us denote $\widetilde{I}_{\rm L,st}^{(1)} + \widetilde{I}_{\rm L,st}^{(2)} = \langle\widetilde{I}_{\rm L}\rangle_{\bar\rho}$. One can also easily check that $\frac{e}{\hbar}(a+b) = \langle\widetilde{I}_{\rm R}\rangle_{\bar\rho}$. Thus, we arrive at a compact expression

$$\widetilde{S}_{\rm LR}^{\rm (I)}(t) = \langle\widetilde{I}_{\rm L}\rangle_{\bar\rho}\langle\widetilde{I}_{\rm R}\rangle_{\bar\rho}G(t). \tag{23}$$

Moreover, the second part of the correlator, $\widetilde{S}_{\rm LR}^{\rm (II)}(t)$, which is associated with the decay of the initial-occupation-channels (a statistical mixture of the occupied and unoccupied states of the f particle, see Fig. 2), can be obtained as

$$\widetilde{S}_{\rm LR}^{\rm (II)}(t) = \left(\frac{e}{\hbar}\right)^2\left(b\widetilde{\Gamma}_{\rm L}^{+} - a\widetilde{\Gamma}_{\rm L}^{-}\right)[1-G(t)]. \tag{24}$$

Finally, we obtain the current-fluctuation-correlator as

$$\begin{aligned} S_{\rm LR}(t) &= \widetilde{S}_{\rm LR}(t) - \langle\widetilde{I}_{\rm L}\rangle_{\bar\rho}\langle\widetilde{I}_{\rm R}\rangle_{\bar\rho} \\ &= \left[\left(\frac{e}{\hbar}\right)^2\left(b\widetilde{\Gamma}_{\rm L}^{+} - a\widetilde{\Gamma}_{\rm L}^{-}\right) - \langle\widetilde{I}_{\rm L}\rangle_{\bar\rho}\langle\widetilde{I}_{\rm R}\rangle_{\bar\rho}\right]{\rm e}^{-2\Gamma t} \\ &\equiv C_{\rm LR}\,{\rm e}^{-2\Gamma t}. \end{aligned} \tag{25}$$

Here, we introduced the cross-correlation-factor $C_{\rm LR}$, which characterizes the essential correlation property, by noting that the time dependent behavior is relatively simple, and it simply leads to a Lorentzian lineshape $\sim(\omega^2 + 4\Gamma^2)^{-1}$ in frequency domain. The most important point we would like to emphasize here is that $C_{\rm LR} \neq 0$, even when the Majorana coupling energy $\epsilon_{\rm M} \to 0$.

We notice that in Refs. [16–18] it was found that the cross correlation between the total currents in the different leads vanishes at the limit $\epsilon_{\rm M} \to 0$. Here, despite using the total-occupation-probability-description based master equation approach, we have successfully isolated the component currents (i.e., the branch circuit currents) from the total ones, and found nonzero cross correlation even at the limit $\epsilon_{\rm M} \to 0$. In the occupation-number-state representation, the picture of nonzero cross correlation appears clear. Owing to the nonlocal nature of the MZMs associated f quasiparticle, it is natural to expect that the disturbance at one side should influence the electron-hole excitation at the other side, leading thus to the nonlocal crossed AR process. In certain sense, the nonlocal f particle is quite similar as the single-level electron in a quantum dot which, obviously, can correlate the currents in the two leads of transport. In the Majorana case, the left-side sum of the crossed AR and local AR currents, i.e., the branch circuit current considered in this work, is similarly correlated with the branch circuit current on the right side, through the nonlocal f particle.

We show in Fig. 3 the numerical results of the cross correlation factor $C_{\rm LR}$ as a function of the equally biased voltage considered in Ref. [17], i.e., $\mu_{\rm L} = \mu_{\rm R} = eV$ with respect to the chemical potential of the superconductor ($\epsilon_{\rm F} = 0$). Indeed, even in the case of ideally uncoupled MZMs (with $\epsilon_{\rm M} = 0$), the cross correlation is found nonzero. More specifically, we find $C_{\rm LR}$ is negative and can understand it as follows. Let us imagine a "quantum jump" associated with confirming the formation of a Cooper pair in the superconductor. The observation of this specific event allows us to define the current $\widetilde{I}_{\rm R}$ which must be larger than the average current $\langle\widetilde{I}_{\rm R}\rangle_{\bar\rho}$. We then have $\delta\widetilde{I}_{\rm R} > 0$ at the right side. After observing this event, the state would jump to $\widetilde{\rho}(0) = |0\rangle\langle 0|$, owing to the fact that the formation of a Cooper pair must annihilate the f quasiparticle. Using this disturbed new state, we calculate the left side current and obtain

$$\langle\widetilde{I}_{\rm L}\rangle_{\widetilde{\rho}(0)} = -\frac{e}{\hbar}\widetilde{\Gamma}_{\rm L}^{-}. \tag{26}$$

This current, generated by the inverse AR process, is negative. We then have $\delta\widetilde{I}_{\rm L} < 0$ and arrive at the negative correlation as shown in Fig. 3(a), by noting that the steady state current $\langle\widetilde{I}_{\rm L}\rangle_{\bar{\rho}}$ is positive.

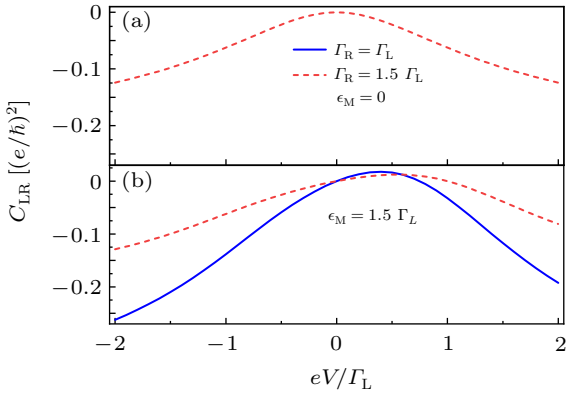


Fig. 3. Cross correlation factor $C_{\rm LR}$ as a function of the equally biased voltage considered in Ref. [17], i.e., $\mu_{\rm L}=\mu_{\rm R}=eV$ with respect to the chemical potential of the superconductor ($\epsilon_F=0$). In (a) and (b), results for the ideal ($\epsilon_{\rm M}=0$) and nonideal ($\epsilon_{\rm M}\neq 0$) Majorana cases are shown, together with the symmetric ($\Gamma_{\rm L}=\Gamma_{\rm R}$) and asymmetric ($\Gamma_{\rm L}\neq\Gamma_{\rm R}$) coupling to the leads.

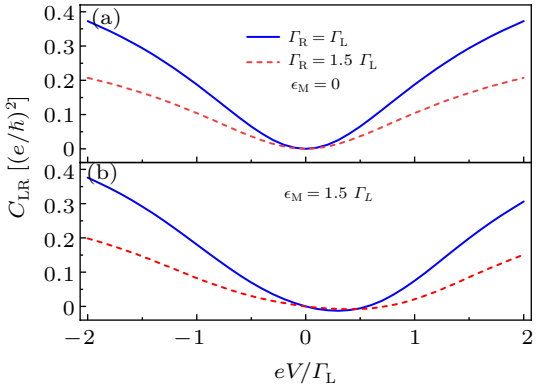


Fig. 4. Cross correlation factor $C_{\rm LR}$ as a function of the anti-symmetrically biased voltage, i.e., $\mu_{\rm L}=-\mu_{\rm R}=eV$ with respect to the chemical potential of the superconductor. Other parametric explanation is the same as in Fig. 3.

For the bias voltage dependence, the reason of the symmetric feature is relatively simple. That is, changing the bias voltage from positive to negative, the Andreev process is reversed. Then, the directions of both currents are reversed and the correlation of the current fluctuations keeps the sign unchanged. In Fig. 3, the effect of asymmetry of coupling to the leads is also shown, for both the ideal ($\epsilon_{\rm M}=0$) and nonideal ($\epsilon_{\rm M}\neq 0$) cases.

In Fig. 4, we show the results for the setup of anti-symmetrically biased voltage, i.e., $\mu_{\rm L}=-\mu_{\rm R}=eV$ with respect to the chemical potential of the superconductor. Here, it is interesting to find that the correlation is positive, rather than being negative as seen in Fig. 3. For this bias setup (e.g., $eV>0$), the inverse Andreev process on the right side leads to a negative current of $\widetilde{I}_{\rm R}$. Now, let us consider observing the "jump" event of splitting a Cooper pair. This allows us to determine the current which must be larger than the average current, i.e., $-\widetilde{I}_{\rm R} > -\langle\widetilde{I}_{\rm R}\rangle_{\bar{\rho}}$, thus $\delta\widetilde{I}_{\rm R}<0$. After observing this "jump" event, the state is updated as $\widetilde{\rho}(0)=|1\rangle\langle 1|$, owing to the fact that splitting a Cooper pair must generate the f quasiparticle. Using this disturbed new state, we calculate the left side current and obtain

$$\langle\widetilde{I}_{\rm L}\rangle_{\widetilde{\rho}(0)}=\frac{e}{\hbar}\widetilde{\Gamma}_{\rm L}^{+}. \tag{27}$$

Moreover, we can straightforwardly examine that $\frac{e}{\hbar}\widetilde{\Gamma}_{\rm L}^{+}-\langle\widetilde{I}_{\rm L}\rangle_{\bar{\rho}}<0$, under the bias voltage $eV>0$. We can thus conclude the positive correlation shown in Fig. 4. Other behaviors, such as the bias voltage and coupling asymmetry dependence, can be understood similarly as above for the results of Fig. 3.

5. Summary and discussion

Instead of the cross correlation between the total lead currents in a two-lead (three-terminal) Majorana device, we have analyzed the cross correlation of the Andreev-process-associated branch circuit currents. Despite applying a master equation approach, which deals with the total occupation probability of the MZMs associated f quasiparticle, we have been able to successfully extract out the component currents from the total ones and computed the cross correlation by means of the quantum jump technique. Importantly, we found that the cross correlation does not vanish even when the Majorana coupling energy $\epsilon_{\rm M}\to 0$, owing to the nonlocality nature of the MZMs. For different setup of bias voltage, we also found intriguing results of both negative and positive correlations and carried out simple physical understanding.

The proposed cross correlation can be measured as the usual power spectrum of current fluctuations, e.g., the noise spectrum calculated in Refs. [16–18], with the only difference by replacing the currents in the transport leads with the branch circuit currents flowing back from the superconductor to the individual leads, as schematically shown in Fig. 1. The branch circuit currents are accessible, making thus the proposed cross correlation measurable in experiment. Combining with the evidence of the zero-bias-peak of conductance, the nonlocal cross correlation predicted in this work can help to confirm the nonlocal MZMs.

At present stage, the urgent challenge for Majorana confirmation is to distinguish the MZMs from the accidental near-zero-energy Andreev bound states. It seems that this challenge largely exists in the single-lead local conductance measurement, since both states would result in similar zero-bias-peak feature in the tunneling conductance. The difficulty seems improved in the two-lead cross correlation measurement. The key conclusion of this work is the nonvanishing cross correlation at the limit $\epsilon_{\rm M}\to 0$, i.e., with no spatial overlap between the two Majorana modes. We emphasize that the result will be drastically different for the case of Andreev bound states.

This can be understood by applying the simplest model for the realization of a pair of Andreev bound states, following Ref. [43], where four MZMs are considered to form two regular fermions (f_1 and f_2) at the two ends of a quantum wire. It is clear that the sub-gap state transport cannot support any cross correlation in the two-lead measurement, owing to the absence of any transmission channel between the left and right sides because of no coupling between the f_1 and f_2 quasiparticles (Andreev bound states). For the case of Andreev bound states, there exists only local Andreev process associated with, respectively, the local f_1 and f_2 quasiparticles, which makes the Andreev-bound-state difficult to be ruled out in the single-lead local measurement for Majoarana confirmation. However, in the two-lead transport measurement, both cases have qualitatively different results, say, zero or nonzero cross correlation.

Finally, from the result of Eq. (25), we see that the behavior of time dependence is relatively simple. The exponentially decaying function in time domain does not involve any quantum coherent oscillations. In practice, in order to be able to measure such type of quantum oscillations (e.g., the Rabi oscillations of a qubit), the measurement speed should be faster than the quantum oscillations, which sets thus a requirement for the bandwidth of the measurement circuit. However, in our present case, the absence of quantum oscillations would relax the requirement for fast speed of measurement. Actually, from Eq. (25), we know that the transient process is slow, since the sub-gap state transport requires $(\Gamma, eV) \ll \Delta$, i.e., the tunnel coupling rate and bias voltage much smaller than the proximity effect induced superconducting gap. This slow transient process will make its measurement not more difficult than the measurements of current correlation functions in other contexts.

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