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Citation: Chin. Phys. B, 2022, 31 (**2**): 020301. DOI: 10.1088/1674-1056/ac140b
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Time evolution law of a two-mode squeezed light field passing through twin diffusion channels

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(Received 11 March 2021; revised manuscript received 7 July 2021; accepted manuscript online 14 July 2021)

We explore the time evolution law of a two-mode squeezed light field (pure state) passing through twin diffusion channels, and we find that the final state is a squeezed chaotic light field (mixed state) with entanglement, which shows that even though the two channels are independent of each other, since the two modes of the initial state are entangled with each other, the final state remains entangled. Nevertheless, although the squeezing (entanglement) between the two modes is weakened after the diffusion, it is not completely removed. We also highlight the law of photon number evolution. In the calculation process used in this paper, we make full use of the summation method within the ordered product of operators and the generating function formula for two-variable Hermite polynomials.

Keywords: two-mode squeezed light field, twin diffusion channels, time evolution law, entanglement

PACS: 03.65.–w, 42.50.–p, 63.20.–e **DOI:** 10.1088/1674-1056/ac140b

1. Introduction

Photon diffusion and photon dissipation are two processes in optical communications[1–4] in which decoherence usually happens.[5,6] In previous works, the diffusion and dissipation of photons have been investigated using different methods. The diffusion coefficient for photon migration should be dependent on the absorption in an absorbing medium; as a result, the absorption-dependent diffusion coefficient has been inferred from diffusion theory.[7] In addition, the evolution from a single-mode squeezed vacuum state to a squeezed chaotic state with decreasing squeezing and decoherence in an amplitude-dissipative channel has been studied.[8] Exploration has also been made of the evolution of a two-mode squeezed vacuum when it passes through a two-mode amplitude dissipative channel, with the same decay rate κ, using the continuous-variable entangled state approach.[9–12]

In this paper, we study the time evolution of a two-mode squeezed state,[13,14] which can be generated in a parametric amplifier, passing through twin diffusion channels, which means that each mode independently passes through a single-mode diffusion channel. It should be noted that a two-mode squeezed state is, itself, an entangled state, which involves the idle mode and the signal mode, so it is interesting to investigate how these modes evolve and how the photon number in these modes varies.[15–17] In the following, we make full use of the summation method within the ordered product of operators and the generating function formula of two-variable Hermite polynomials.

Theoretically, a two-mode squeezed state is expressed by

$$\exp\left[\theta\left(a^{\dagger}b^{\dagger}-ab\right)\right]|00\rangle=\operatorname{sech}\theta\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)|00\rangle, \quad (1)$$

where $\tanh\theta$ is a squeezing parameter and $a,a^{\dagger}$; $b,b^{\dagger}$ are the two modes' photon annihilation and creation operators, which obey $[a,a^{\dagger}]=[b,b^{\dagger}]=1$. The density operator of a two-mode squeezed state is

$$\rho_0=\operatorname{sech}^2\theta\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)|00\rangle\langle 00|\exp\left(ab\tanh\theta\right), \quad (2)$$

which is a pure state. To calculate the a-mode photon number (the b-mode is the same), we set

$$\operatorname{Tr}(\rho_0 a^{\dagger}a)=x. \quad (3)$$

Then, from

$$\begin{aligned} &a\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)|00\rangle\\ &=b^{\dagger}\tanh\theta\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)|00\rangle, \end{aligned} \quad (4)$$

we have

$$\begin{aligned} x&=\operatorname{sech}^2\theta\langle 00|\exp\left(ab\tanh\theta\right)a^{\dagger}a\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)|00\rangle\\ &=\operatorname{sech}^2\theta\tanh^2\theta\langle 00|\exp\left(ab\tanh\theta\right)bb^{\dagger}\\ &\quad\times\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)|00\rangle\\ &=\tanh^2\theta\operatorname{sech}^2\theta\langle 00|\exp\left(ab\tanh\theta\right)\left(b^{\dagger}b+1\right)\\ &\quad\times\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)|00\rangle\\ &=(x+1)\tanh^2\theta. \end{aligned} \quad (5)$$

Thus, the a-mode photon number is

$$\operatorname{Tr}\left(\rho_0 a^{\dagger}a\right)=x=\sinh^2\theta, \quad (6)$$

which only relies on the squeezing parameter. In the following we discuss the time evolution law of a two-mode squeezed state passing through twin diffusion channels, and we find that it evolves into a two-mode squeezed chaotic state, after which, the photon number variation can be derived correspondingly.

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2. Time evolution of the density operator of a two-mode squeezed state in twin diffusion channels

It is known that the time evolution equation for a density operator in a single-mode diffusion channel is[18,19]

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -\kappa\left(a^{\dagger}a\rho + \rho a a^{\dagger} - a\rho a^{\dagger} - a^{\dagger}\rho a\right). \tag{7}$$

Here, κ is the diffusion coefficient. Using an entangled state representation[20–23] and integration within the ordered product of operators (IWOP),[24,25] we can obtain the solution in an infinitive summation form[26]

$$\rho(t) = \sum_{m,n=0}^{\infty} M_{m,n}\rho_0 M_{m,n}^{\dagger}. \tag{8}$$

Here, ρ_0 is the initial density operator and $M_{m,n}$ is

$$M_{m,n} = \sqrt{\frac{1}{m!n!}\frac{(\kappa t)^{m+n}}{(\kappa t+1)^{m+n+1}}}\,a^{\dagger m}\left(\frac{1}{1+\kappa t}\right)^{a^{\dagger}a} a^{n}, \tag{9}$$

which must satisfy $\sum_{m,n=0}^{\infty} M_{m,n}^{\dagger}M_{m,n} = 1$ to guarantee $\mathrm{Tr}\rho(t) = 1$.

Another equation describing the evolution of b-mode diffusion channels is

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -\kappa\left(b^{\dagger}b\rho + \rho b b^{\dagger} - b\rho b^{\dagger} - b^{\dagger}\rho b\right). \tag{10}$$

For both the a mode and the b mode, the density operator solution to the diffusion equation is

$$\rho(t) = \sum_{m,n,m',n'=0}^{\infty} M_{m',n'}M_{m,n}\rho_0 M_{m,n}^{\dagger}M_{m',n'}^{\dagger}. \tag{11}$$

By substituting Eq. (2) into Eq. (11) and using Eq. (9), we obtain

$$\begin{aligned}\rho(t) = {} & \mathrm{sech}^2\theta \sum_{m,n,m',n'=0}^{\infty} \frac{1}{m!n!m'!n'!}\frac{(\kappa t)^{m+n+m'+n'}}{(\kappa t+1)^{m+n+2+m'+n'}} a^{\dagger m}b^{\dagger m'}\left(\frac{1}{1+\kappa t}\right)^{a^{\dagger}a+b^{\dagger}b} \\ & \times a^{n}b^{n'}\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)|00\rangle\langle 00|\exp\left(ab\tanh\theta\right)a^{\dagger n}b^{\dagger n'}\left(\frac{1}{1+\kappa t}\right)^{a^{\dagger}a+b^{\dagger}b} a^{m}b^{m'}.\end{aligned} \tag{12}$$

It is difficult to perform summation over the operators, since creation operators do not commute with annihilation operators. Thus, we should adopt the method of summation within the ordered product of the operators,[27,28] because within an ordered product of operators, permutation is possible. Let us first examine the part of Eq. (12)

$$\begin{aligned} & a^{n}b^{n'}\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)|00\rangle \\ = {} & a^{n}\left(a^{\dagger}\tanh\theta\right)^{n'}\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)|00\rangle, \end{aligned} \tag{13}$$

where we have used

$$\left[b, f\left(b^{\dagger}\right)\right] = \frac{\partial}{\partial b^{\dagger}} f\left(b^{\dagger}\right). \tag{14}$$

Due to $a|z\rangle = z|z\rangle$, we can employ the over-completeness relation of the coherent state $\int(\mathrm{d}^2 z/\pi)|z\rangle\langle z| = 1$ and the IWOP method, which leads to

$$a^{n}a^{\dagger n'} = (-\mathrm{i})^{n+n'} : H_{n',n}\left(\mathrm{i}a^{\dagger}, \mathrm{i}a\right) :, \tag{15}$$

here

$$\begin{aligned} H_{n',n}\left(\mathrm{i}a^{\dagger}, \mathrm{i}a\right) = {} & \sum_{l=0}^{\min(n,n')} \frac{n'!n!(-1)^{l}}{l!(n'-l)!(n-l)!} \\ & \times\left(\mathrm{i}a^{\dagger}\right)^{n'-l}(\mathrm{i}a)^{n-l}. \end{aligned} \tag{16}$$

The $H_{n',n}\left(\mathrm{i}a^{\dagger}, \mathrm{i}a\right)$ is a two-variable Hermite polynomial.[29–31] It then follows from Eq. (13) that

$$\begin{aligned} & a^{n}b^{n'}\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)|00\rangle \\ = {} & \tanh^{n'}\theta\,(-\mathrm{i})^{n+n'} : H_{n',n}\left(\mathrm{i}a^{\dagger}, \mathrm{i}a\right) : \exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)|00\rangle \\ = {} & \tanh^{n'}\theta \sum_{l=0}^{\min(n,n')} \frac{n'!n!a^{\dagger(n'-l)}a^{n-l}}{l!(n'-l)!(n-l)!} \\ & \times\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)|00\rangle. \end{aligned} \tag{17}$$

Further, using the operator identity

$$\begin{aligned} \mathrm{e}^{A}B\mathrm{e}^{-A} = {} & B + [A,B] + \frac{1}{2!}[A,[A,B]] \\ & + \frac{1}{3!}[A,[A,[A,B]]] + \cdots, \end{aligned} \tag{18}$$

we can deal with the terms in Eq. (17) and obtain

$$\begin{aligned} & a^{\dagger(n'-l)}a^{n-l}\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)|00\rangle \\ = {} & a^{\dagger(n'-l)}\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)\exp\left(-a^{\dagger}b^{\dagger}\tanh\theta\right) \\ & \times a^{n-l}\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)|00\rangle \\ = {} & a^{\dagger(n'-l)}\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)\cdot\left(b^{\dagger}\tanh\theta\right)^{n-l}|00\rangle, \end{aligned} \tag{19}$$

therefore

$$\begin{aligned} & a^{n}b^{n'}\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)|00\rangle \\ = {} & \tanh^{n'}\theta\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right) \end{aligned}$$

$$\times \sum_{l=0}^{\min(n,n')} \frac{n'!n!a^{\dagger(n'-l)}}{l!(n'-l)!(n-l)!}\left(b^{\dagger}\tanh\theta\right)^{n-l}|00\rangle$$
$$= (-\mathrm{i})^{n+n'}\tanh^{n'}\theta H_{n',n}\left(\mathrm{i}a^{\dagger},\mathrm{i}b^{\dagger}\tanh\theta\right)$$
$$\times\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)|00\rangle. \tag{20}$$

Going back to examine $\rho(t)$ in Eq. (12) and using

$$\left(\frac{1}{1+\kappa t}\right)^{a^{\dagger}a+b^{\dagger}b} = \exp\left[\left(a^{\dagger}a+b^{\dagger}b\right)\ln\frac{1}{1+\kappa t}\right] \tag{21}$$

to operate on Eq. (20), we obtain

$$\left(\frac{1}{1+\kappa t}\right)^{a^{\dagger}a+b^{\dagger}b} a^{n}b^{n'}\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)|00\rangle$$
$$= \exp\left[\left(a^{\dagger}a+b^{\dagger}b\right)\ln\frac{1}{1+\kappa t}\right](-\mathrm{i})^{n+n'}\tanh^{n'}\theta$$
$$\times H_{n',n}\left(\mathrm{i}a^{\dagger},\mathrm{i}b^{\dagger}\tanh\theta\right)\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)|00\rangle$$
$$= (-\mathrm{i})^{n+n'}\tanh^{n'}\theta H_{n',n}\left(\frac{\mathrm{i}a^{\dagger}}{1+\kappa t},\frac{\mathrm{i}b^{\dagger}\tanh\theta}{1+\kappa t}\right)$$
$$\times\exp\left[\frac{1}{(1+\kappa t)^{2}}a^{\dagger}b^{\dagger}\tanh\theta\right]|00\rangle. \tag{22}$$

With this result and using the new generating function formula[32] for $H_{m,n}(x,y)$:

$$\sum_{m,n=0}^{\infty}\frac{s^{m}t^{n}}{m!n!}H_{m,n}(x,y)H_{m,n}\left(x',y'\right)$$
$$= \frac{1}{1-st}\exp\left[\frac{sxx'+tyy'-st\left(xy+x'y'\right)}{1-st}\right], \tag{23}$$

we can convert Eq. (12) into

$$\rho(t) = \operatorname{sech}^{2}\theta\sum_{m,m'=0}^{\infty}\frac{a^{\dagger m}b^{\dagger m'}}{m'!m!}\left(\frac{\kappa t}{\kappa t+1}\right)^{m+m'}a^{\dagger m}b^{\dagger m'}$$
$$\times\sum_{n,n'=0}^{\infty}\frac{1}{n'!n!}\frac{(\kappa t)^{n+n'}}{(\kappa t+1)^{n+n'+2}}$$
$$\times\left(\frac{1}{1+\kappa t}\right)^{a^{\dagger}a+b^{\dagger}b}a^{n}b^{n'}\exp\left(a^{\dagger}b^{\dagger}\tanh\theta\right)|00\rangle$$
$$\times\langle 00|\exp\left(ab\tanh\theta\right)a^{n}b^{n'}\left(\frac{1}{1+\kappa t}\right)^{a^{\dagger}a+b^{\dagger}b}a^{m}b^{m'}$$
$$= \operatorname{sech}^{2}\theta\sum_{m,m'=0}^{\infty}\frac{a^{\dagger m}b^{\dagger m'}}{m'!m!}\left(\frac{\kappa t}{\kappa t+1}\right)^{m+m'}$$
$$\times\sum_{n,n'=0}^{\infty}\frac{1}{n'!n!}\frac{(\kappa t)^{n+n'}}{(\kappa t+1)^{n+n'+2}}$$
$$\times(-\mathrm{i})^{n+n'}\tanh^{n'}\theta H_{n',n}\left(\frac{\mathrm{i}a^{\dagger}}{1+\kappa t},\frac{\mathrm{i}b^{\dagger}\tanh\theta}{1+\kappa t}\right)$$
$$\times\exp\left[\frac{1}{(1+\kappa t)^{2}}a^{\dagger}b^{\dagger}\tanh\theta\right]|00\rangle$$
$$\times\langle 00|\exp\left[\frac{1}{(1+\kappa t)^{2}}ab\tanh\theta\right]$$
$$\times H_{n',n}\left(\frac{-\mathrm{i}a}{1+\kappa t},\frac{-\mathrm{i}b\tanh\theta}{1+\kappa t}\right)$$
$$\times\tanh^{n'}\theta\,(i)^{n+n'}a^{m}b^{m'}. \tag{24}$$

Remembering the vacuum projector $|00\rangle\langle 00| =: \exp\left(-a^{\dagger}a-b^{\dagger}b\right):$, we see

$$\rho(t) = \operatorname{sech}^{2}\theta\sum_{m,m'=0}^{\infty}\frac{a^{\dagger m}b^{\dagger m'}}{m'!m!}\left(\frac{\kappa t}{\kappa t+1}\right)^{m+m'}$$
$$\times\sum_{n,n'=0}^{\infty}\frac{(\kappa t+1)^{-2}}{n'!n!}\left(\frac{\kappa t\tanh^{2}\theta}{\kappa t+1}\right)^{n'}\left(\frac{\kappa t}{\kappa t+1}\right)^{n}$$
$$\times:H_{n',n}\left(\frac{\mathrm{i}a^{\dagger}}{1+\kappa t},\frac{\mathrm{i}b^{\dagger}\tanh\theta}{1+\kappa t}\right)$$
$$\times H_{n',n}\left(\frac{-\mathrm{i}a}{1+\kappa t},\frac{-\mathrm{i}b\tanh\theta}{1+\kappa t}\right)$$
$$\times\exp\left[\frac{\tanh\theta}{(1+\kappa t)^{2}}\left(a^{\dagger}b^{\dagger}+ab\right)\right]$$
$$\times\exp\left(-a^{\dagger}a-b^{\dagger}b\right):a^{m}b^{m'}, \tag{25}$$

where the summation over n,n' is now within normal ordering; therefore, the creation operators can be permuted with the annihilation operators within : :, and the summation over n,n' is feasible. Using Eq. (23), we obtain

$$\rho(t) = \frac{\operatorname{sech}^{2}\theta(\kappa t+1)^{-2}}{1-\frac{(\kappa t)^{2}\tanh^{2}\theta}{(\kappa t+1)^{2}}}\sum_{m,m'=0}^{\infty}\frac{a^{\dagger m}b^{\dagger m'}}{m'!m!}\left(\frac{\kappa t}{\kappa t+1}\right)^{m+m'}$$
$$\times:\exp\left\{\frac{1}{(\kappa t)^{2}\operatorname{sech}^{2}\theta+2\kappa t+1}\frac{\kappa t\tanh^{2}\theta}{\kappa t+1}\right.$$
$$\left.\times\left[\left(a^{\dagger}a+b^{\dagger}b\right)+\frac{\kappa t\tanh\theta}{1+\kappa t}\left(a^{\dagger}b^{\dagger}+ab\right)\right]\right\}$$
$$\times\exp\left[\frac{\tanh\theta}{(1+\kappa t)^{2}}\left(a^{\dagger}b^{\dagger}+ab\right)\right]$$
$$\times\exp\left(-a^{\dagger}a-b^{\dagger}b\right):a^{m}b^{m'}. \tag{26}$$

Noticing that the right-hand side is still in normal ordering, the summation over m,m' can be performed as well. The result is

$$\rho(t) = \frac{\operatorname{sech}^{2}\theta}{(\kappa t)^{2}\operatorname{sech}^{2}\theta+2\kappa t+1}$$
$$\times:\exp\left[\left(\frac{-1}{\kappa t+1}\right)\left(a^{\dagger}a+b^{\dagger}b\right)\right]$$
$$\times\exp\left[\frac{\tanh\theta}{(\kappa t+1)^{2}}\left(a^{\dagger}b^{\dagger}+ab\right)\right]$$
$$\times\exp\left\{\frac{1}{(\kappa t)^{2}\operatorname{sech}^{2}\theta+2\kappa t+1}\frac{\kappa t\tanh^{2}\theta}{1+\kappa t}\right.$$
$$\left.\times\left[\left(a^{\dagger}a+b^{\dagger}b\right)+\frac{\kappa t\tanh^{2}\theta}{1+\kappa t}\left(a^{\dagger}b^{\dagger}+ab\right)\right]\right\}:$$

$$= \frac{\operatorname{sech}^2\theta}{(\kappa t)^2\operatorname{sech}^2\theta+2\kappa t+1} \times :\exp\left\{\frac{1}{(\kappa t)^2\operatorname{sech}^2\theta+2\kappa t+1}\left[\left(a^\dagger b^\dagger + ab\right)\tanh\theta - \left(\kappa t\operatorname{sech}^2\theta+1\right)\left(a^\dagger a+b^\dagger b\right)\right]\right\}:. \quad (27)$$

This result indicates that after passing through twin diffusion channels, the initial two-mode squeezed state ρ_0 (pure state) evolves into the squeezed chaotic state (a mixed state) described by Eq. (27),[33] where the exponential term involving $\left(a^\dagger a+b^\dagger b\right)$ embodies the chaos. Clearly, when $t=0$, Eq. (27) reduces to Eq. (2).

On the other hand, from Eq. (27), we can see that the original squeezing parameter $\tanh\theta$ has changed to

$$\frac{\tanh\theta}{(\kappa t)^2\operatorname{sech}^2\theta+2\kappa t+1}, \quad (28)$$

and due to

$$(\kappa t)^2\operatorname{sech}^2\theta+2\kappa t+1=(\kappa t+1)^2-(\kappa t)^2\tanh^2\theta>1, \quad (29)$$

we conclude that after passing through the twin diffusion channels, the squeezing becomes weaker. Nevertheless, even though the squeezing (entanglement) between the two modes is weakened after the diffusion, the entanglement is not completely removed.

We let $A=(\kappa t)^2\operatorname{sech}^2\theta+2\kappa t+1$; we can now put the final density operator into a neater form

$$\rho(t) = \frac{\operatorname{sech}^2\theta}{A}:\exp\left\{\frac{1}{A}\left[\left(a^\dagger b^\dagger+ab\right)\tanh\theta - \left(\kappa t\operatorname{sech}^2\theta+1\right)\left(a^\dagger a+b^\dagger b\right)\right]\right\}:, \quad (30)$$

and it is necessary to check

$$\begin{aligned}
&\operatorname{Tr}\rho(t)\\
&= \frac{\operatorname{sech}^2\theta}{A}\int\frac{\mathrm{d}^2z_1\,\mathrm{d}^2z_2}{\pi^2}\langle z_1,z_2|:\exp\left\{\frac{1}{A}\left[\left(a^\dagger b^\dagger+ab\right)\tanh\theta-\left(\kappa t\operatorname{sech}^2\theta+1\right)\left(a^\dagger a+b^\dagger b\right)\right]\right\}:|z_1,z_2\rangle\\
&= \frac{\operatorname{sech}^2\theta}{A}\int\frac{\mathrm{d}^2z_1\,\mathrm{d}^2z_2}{\pi^2}\exp\left[\frac{1}{A}\left(z_1^*z_2^*+z_1z_2\right)\tanh\theta-\frac{1}{A}\left(\kappa t\operatorname{sech}^2\theta+1\right)\left(|z_1|^2+|z_2|^2\right)\right]\\
&= \frac{\operatorname{sech}^2\theta}{A}\int\frac{\mathrm{d}^2z_2}{\pi}\frac{A}{\kappa t\operatorname{sech}^2\theta+1}\exp\left[-\frac{A}{\kappa t\operatorname{sech}^2\theta+1}|z_2|^2\frac{\tanh^2\theta}{A^2}\right]\exp\left[-\frac{1}{A}\left(\kappa t\operatorname{sech}^2\theta+1\right)|z_2|^2\right]\\
&= \frac{\operatorname{sech}^2\theta}{\kappa t\operatorname{sech}^2\theta+1}\int\frac{\mathrm{d}^2z_2}{\pi}\exp\left(-\frac{\operatorname{sech}^2\theta}{\kappa t\operatorname{sech}^2\theta+1}|z_2|^2\right)=1,
\end{aligned} \quad (31)$$

which confirms the correctness of our calculations.

3. The photon number in the final state

After performing a partial trace over the b mode for Eq. (27), we derive

$$\begin{aligned}
\operatorname{Tr}_b\rho(t) &= \frac{\operatorname{sech}^2\theta}{A}\int\frac{\mathrm{d}^2z_2}{\pi}\langle z_2|:\exp\left\{\frac{1}{A}\left[\left(a^\dagger b^\dagger+ab\right)\tanh\theta-\left(\kappa t\operatorname{sech}^2\theta+1\right)\left(a^\dagger a+b^\dagger b\right)\right]\right\}:|z_2\rangle\\
&= \frac{\operatorname{sech}^2\theta}{A}\int\frac{\mathrm{d}^2z_2}{\pi}:\exp\left[\frac{1}{A}\left(a^\dagger z_2^*+az_2\right)\tanh\theta-\frac{1}{A}\left(\kappa t\operatorname{sech}^2\theta+1\right)\left(a^\dagger a+|z_2|^2\right)\right]:\\
&= \frac{\operatorname{sech}^2\theta}{\kappa t\operatorname{sech}^2\theta+1}:\exp\left(-\frac{\operatorname{sech}^2\theta}{\kappa t\operatorname{sech}^2\theta+1}a^\dagger a\right):,
\end{aligned} \quad (32)$$

which results in a chaotic field in the a mode; we then deduce the a-mode photon number in the final state

$$\begin{aligned}
\operatorname{Tr}_a\left[\operatorname{Tr}_b\rho(t)a^\dagger a\right] &= \operatorname{Tr}_a\left[\operatorname{Tr}_b\rho(t)aa^\dagger\right]-1\\
&= \operatorname{Tr}_a\left[\operatorname{Tr}_b\rho(t)a\int\frac{\mathrm{d}^2z_2}{\pi}|z_1\rangle\langle z_1|a^\dagger\right]-1\\
&= \frac{\operatorname{sech}^2\theta}{\kappa t\operatorname{sech}^2\theta+1}\int\frac{\mathrm{d}^2z_1}{\pi}\langle z_1|:\exp\left(-\frac{\operatorname{sech}^2\theta}{\kappa t\operatorname{sech}^2\theta+1}a^\dagger a\right):|z_1|^2|z_1\rangle-1\\
&= \frac{\operatorname{sech}^2\theta}{\kappa t\operatorname{sech}^2\theta+1}\int\frac{\mathrm{d}^2z_1}{\pi}|z_1|^2:\exp\left(-\frac{\operatorname{sech}^2\theta}{\kappa t\operatorname{sech}^2\theta+1}|z_1|^2\right):-1\\
&= \frac{\kappa t\operatorname{sech}^2\theta+1}{\operatorname{sech}^2\theta}-1=\kappa t+\sinh^2\theta.
\end{aligned} \quad (33)$$

By comparison with the initial photon number in Eq. (6), we find that in the final state, the a-mode photon number has increased by an amount κt, which is proportional to the diffusion parameter. The b-mode photon number also increases by the same amount κt.

4. Conclusion

We have discussed the time evolution law of a two-mode squeezed state passing through twin diffusion channels, and we find that it evolves into a two-mode squeezed chaotic state. The photon number in each mode increases by an amount κt. In the calculation process used in this paper, we have made full use of the summation method within the ordered product of operators and the generating function formula of two-variable Hermite polynomials. We also pointed out the law of photon number evolution.

Acknowledgments

Project supported by the National Natural Science Foundation of China (Grant No. 11775208), the Foundation for Young Talents in College of Anhui Province, China (Grant No. gxyq2019077), and the Natural Science Foundation of the Anhui Higher Education Institutions of China (Grant Nos. KJ2019A0688 and KJ2020A0638).

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