\



Cascading failures of overload behaviors using a new coupled network model between edges

Yu-Wei Yan(严玉为), Yuan Jiang(蒋沅), Rong-Bin Yu(余荣斌), Song-Qing Yang(杨松青), and Cheng Hong(洪成)
Citation: Chin. Phys. B, 2022, 31 (**1**): 018901. DOI: 10.1088/1674-1056/ac1337
Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Network analysis and spatial agglomeration of China's high-speed rail: A dual network approach

Wei Wang(王微), Wen-Bo Du(杜文博), Wei-Han Li(李威翰), Lu (Carol) Tong(佟路), and Jiao-E Wang(王姣娥)
Chin. Phys. B, 2021, 30 (**1**): 018901. DOI: 10.1088/1674-1056/abaee3

Modularity-based representation learning for networks

Jialin He(何嘉林), Dongmei Li(李冬梅), and Yuexi Liu(刘阅希)
Chin. Phys. B, 2020, 29 (**12**): 128901. DOI: 10.1088/1674-1056/abbbec

Manufacturing enterprise collaboration network: An empirical research and evolutionary model

Ji-Wei Hu(胡辑伟), Song Gao(高松), Jun-Wei Yan(严俊伟), Ping Lou(娄平), Yong Yin(尹勇)
Chin. Phys. B, 2020, 29 (**8**): 088901. DOI: 10.1088/1674-1056/ab96a8

Asynchronism of the spreading dynamics underlying the bursty pattern

Tong Wang(王童), Ming-Yang Zhou(周明洋), Zhong-Qian Fu(付忠谦)
Chin. Phys. B, 2020, 29 (**5**): 058901. DOI: 10.1088/1674-1056/ab81fe

Theoretical analyses of stock correlations affected by subprime crisis and total assets: Network properties and corresponding physical mechanisms

Shi-Zhao Zhu(朱世钊), Yu-Qing Wang(王玉青), Bing-Hong Wang(汪秉宏)
Chin. Phys. B, 2019, 28 (**10**): 108901. DOI: 10.1088/1674-1056/ab3f22

Cascading failures of overload behaviors using a new coupled network model between edges

Yu-Wei Yan(严玉为), Yuan Jiang(蒋沅)†, Rong-Bin Yu(余荣斌), Song-Qing Yang(杨松青), and Cheng Hong(洪成)

School of Information Engineering, Nanchang Hangkong University, Nanchang 330063, *China*

(Received 31 May 2021; revised manuscript received 5 July 2021; accepted manuscript online 12 July 2021)

With the development of network science, the coupling between networks has become the focus of complex network research. However, previous studies mainly focused on the coupling between nodes, while ignored the coupling between edges. We propose a novel cascading failure model of two-layer networks. The model considers the different loads and capacities of edges, as well as the elastic and coupling relationship between edges. In addition, a more flexible load-capacity strategy is adopted to verify the model. The simulation results show that the model is feasible. Different networks have different behaviors for the same parameters. By changing the load parameters, capacity parameters, overload parameters, and distribution parameters reasonably, the robustness of the model can be significantly improved.

Keywords: cascade failure, overload, load distribution, edge-based

PACS: 89.75.–k, 89.75.Fb, 89.75.Hc **DOI:** 10.1088/1674-1056/ac1337

1. Introduction

Cascading failure,[1] also known as avalanche reaction, refers to the large-scale failure of one or more damaged nodes in a network due to the coupling relationship. There are many cascading failures in real life. The occurrences of these cascading failures have a negative impact on lives. For example, in 2003, a large-scale power failure took place in the United States.[2] The blackout caused huge losses to local production and life. What is worse, economic losses amounted to hundreds of millions of dollars. This blackout is listed as the most serious accident in the United States. Therefore, it is of great necessity for relevant researchers to study the robustness and vulnerability of networks.[3–16]

In previous studies, the researchers studied the robustness of a single network, such as aviation network,[17] transportation network[18] and power grid.[19] However, with the development of network science, researchers have realized that no network exists in isolation. There is a relationship between different networks, such as power-aviation networks, power-computer networks, communication-transmission networks. In 2010, Buldyrew *et al.*[20] first proposed the coupling node model. They pointed out that when a node is attacked, the impact will be transmitted to the adjacent network through the coupling relationship. After that, many models based on coupling network have been proposed. First of all, Shao *et al.*[21] established the multiple dependent relationship, and analyzed the cascading failure of networks with numerical analysis methods. They assumed that as long as the coupling edge of the node fails completely, the node will fail. Multiple interdependencies make the network transform from the first stage to the second stage. Dobson *et al.*[22] established the adjacency relationship of networks according to the similarity of networks. They noted that the dependency relationship between two kinds of nodes. One is to establish the dependency relationship between the nodes with large degrees in two networks, the other to establish the dependency relationship between the nodes with large clusters in two networks. The research shows that the connection according to the similarity of the networks can improve the networks resistance to random attacks. Muro *et al.*[23] proposed a recovery model and pointed out that the failure process and recovery process can be alternated with each other. By detecting the boundary nodes and recovering them with a certain proportion, the robustness of the networks can be effectively improved.

In a real network, the edges are also important, such as Internet lines, power lines, and transportation lines. Therefore, it is of great significance to study the cascade failure caused by edge collapse. In 2002, Kim *et al.*[24] studied cascading failures caused by edge collapse in a single network. In 2008, Chen *et al.*[25] proposed a single network load distribution model, and obtained the threshold conditions leading to cascading failures. The load of the edge is defined as $(K_iK_j)^\theta$, where K_i and K_j are the degrees of the nodes connected by edge $E_{i,j}$ and θ is a variable parameter. Motter *et al.*[26] found that by deleting some edges in scale-free networks in advance, the impact of cascading failures can be effectively reduced.

All the cited studies do not consider the coupling relationship between edges in two or more networks. Nevertheless, the coupling between edges is very common in real networks. For example, in a railway network, when one of two closely related sides is damaged, the other will be damaged as well.

†Corresponding author. E-mail: jiangyuan@nchu.edu.cn

© 2022 Chinese Physical Society and IOP Publishing Ltd

However, the failure mode of the edge is different from that of the node. When a node fails, all the edges of the node will fail. At the same time, when a node in the network leaves the most important connection block, it will also fail. The node failure is shown in Fig. 1(a). When node A1 fails, node A2 also fails because it is separated from the most important connector. However, when the edge between nodes A1 and A5 in Fig. 1(b) fails, it will not damage the overall structure of the network. Therefore, it is of great significance to study the correlation of edges in two-layer networks. In this paper, a new edge network model is proposed to study cascading failures in two-layer networks. Moreover, a more flexible load capacity strategy is adopted to verify the model. The results show that this model is feasible. By changing the load parameters, capacity parameters, overload parameters and distribution parameters reasonably, the robustness of the system can be significantly improved.

This paper is arranged as follows: In Section 2, the special model of between edges is described in detail, including load distribution process, overload behavior, residual load distribution strategy and robustness evaluation. In Section 3, analyses and simulations are presented. Finally, the conclusion is made in Section 4.

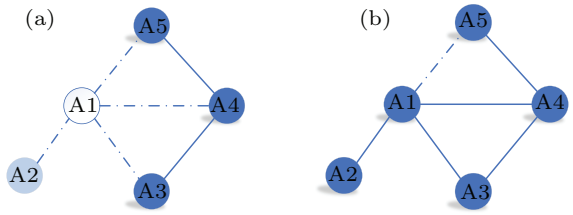


Fig. 1. Network failure diagram: (a) node failure diagram, (b) edge failure diagram.

2. Special network model between edges

In this paper, a two-layer network is established with two subnetworks denoted as networks A and B. The nodes are numbered as N_{A} and N_{B}, respectively. Therefore, the size of the network is $N = N_{\mathrm{A}} = N_{\mathrm{B}}$. A random method is adopted to connect the edges of networks A and B. Considering that the number of edges is different in networks with the same number of nodes, the minimum number of edges in the two networks is taken as the number of dependencies in the two networks. In order to verify the applicability of the proposed model, scale-free network,[27] small-world network[28] and random network model are taken as the subnetworks of the two-layer network in this research.

Figure 2(a) is a simplified diagram of the model. In Fig. 2(a), $\{\mathrm{A1}, \mathrm{A2}, \mathrm{A3}, \ldots\}$ represent the nodes of subnetwork A and $\{\mathrm{B1}, \mathrm{B2}, \mathrm{B3}, \ldots\}$ represent the nodes of subnetwork B. The solid lines represent the connection between the nodes within the subnetworks, and the dashed lines represent the connection between the edges. When an edge in subnetwork A or subnetwork B is attacked, it is the edges belonging to the maximum component of subnetwork A/B that can maintain their functions while other edges will fail. Assuming that some edges in subnetwork A fail due to the initial attack, the failed edges in subnetwork A will be transferred to the edges of subnetwork B through the coupling relationship, resulting in the failure of subnetwork B. Likewise, the invalid edges in subnetwork B will also be transferred to subnetwork A through the coupling relationship. The iteration does not end until subnetwork A and subnetwork B reach stability. For example, supposing that edges $E_{2,3}$ in subnetwork A is deliberately attacked, the failure of $E_{2,3}$ will cause the failure of $E_{2,3}$ in subnetwork B. Concurrently, the failure of edges $E_{2,3}$ in subnetwork B will cause the failure in its internal edges $E_{1,2}$, which breaks away from the giant connecting piece. The first step is shown in Fig. 2(b). The failure of edges $E_{1,2}$ in subnetwork B will cause the failure of edges $E_{4,5}$ in subnetwork A and the failure of edges $E_{4,5}$ in subnetwork A will cause the failure in its internal edges $E_{2,4}$. The second step is shown in Fig. 2(c). The failure of edge $E_{2,4}$ in subnetwork A will cause the failure of edge $E_{3,5}$ of subnetwork B. The final network is stable, and the final steady state of the overall system is shown in Fig. 2(d).

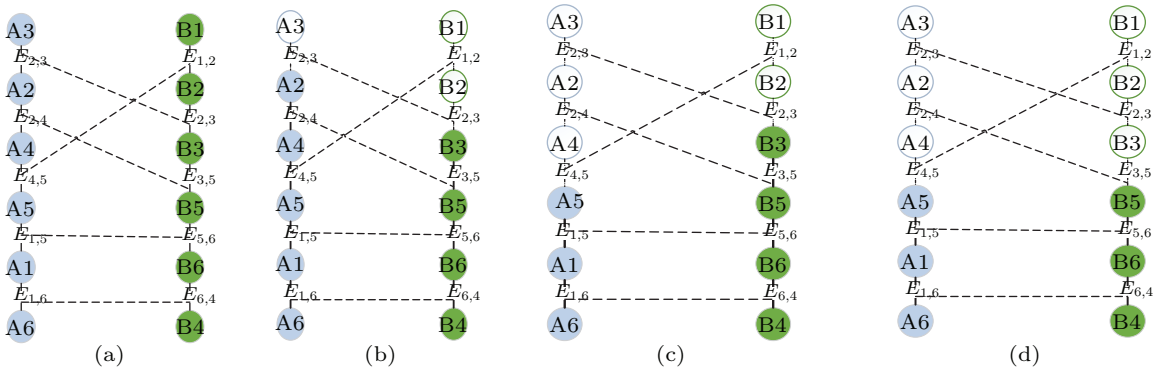


Fig. 2. The failure process of the new model between edges: (a) initial state of the network, (b) failure of the first step, (c) failure of the second step, (d) stable state of the network.

2.1. Load distribution process

In this study, it is assumed that the edge carries the load, such as the flow of road in the transportation and the electrical load in power. Based on the literature,[29] it is assumed that edge $E_{i,j}(0)$ carries initial load in the following formula:

$$L_{E_{ij}}(0) = (K_i K_j)^{\theta}, \tag{1}$$

where K_i and K_j are the degrees of the nodes connected by the edge $E_{i,j}$, and θ is a variable parameter.

In order to be closer to the actual network, a more flexible model of capacity is adopted in this research. There are two parameters in the following model:

$$C_{E_{ij}} = L_{E_{i,j}} + \beta L^{\alpha}_{E_{i,j}}, \tag{2}$$

where $\alpha \geq 0$, $\beta \geq 0$. When $\alpha = 1$, the model is transformed into the ML model.

As is shown in Fig. 3, the load of one edge $E_{i,j}$ that is attacked will be transferred to its adjacent edge $E_{i,m}$. Then edge $E_{i,j}$ transfers its load to edge $E_{i,m}$ according to the following formula:

$$\Pi^{\mathrm{A}}_{E_{i,m}} = \frac{L_{E_{i,m}}}{\sum_{\alpha \in z} L_{E_{i,a}} + \sum_{b \in k} L_{E_{j,b}}}, \tag{3}$$

where $L_{E_{i,m}}$ is the load of the edge $E_{i,m}$, Z is the neighbor nodes of node i, and K is the neighbor nodes of node j.

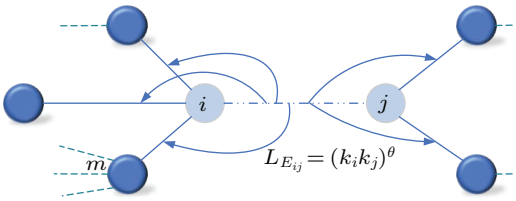


Fig. 3. The load distribution strategy when an edge fails.

As the real network has elasticity, the edge may not immediately fail when an edge exceeds the capacity. Only when the edge exceeds the maximum capacity, it will fail immediately. Therefore, the overload coefficient δ is used to express the processing ability of edge $E_{i,m}$ for additional load. When the load of edge $E_{i,m}$ exceeds $\delta C_{E_{i,m}}$, edge $E_{i,m}$ immediately fails. When edge $E_{i,m}$ is greater than $C_{E_{i,m}}$ and less than $\delta C_{E_{i,m}}$, edge $E_{i,m}$ will fail with a certain probability. We judge the state of edge $E_{i,m}$ by the following formula:

$$\begin{cases} L_{E_{i,m}} \leq C_{E_{i,m}}, & \text{normal;} \\ C_{E_{i,m}} < L_{E_{i,m}} < \delta C_{E_{i,m}},\ n_{\mathrm{random}} > P_{E_{i,m}}, & \text{overload;} \\ C_{E_{i,m}} < L_{E_{i,m}} < \delta C_{E_{i,m}},\ n_{\mathrm{random}} \leq P_{E_{i,m}}, & \text{failure;} \\ L_{E_{i,m}} \geq \delta C_{E_{i,m}}, & \text{failure;} \end{cases} \tag{4}$$

where n_{random} is a random number from 0 to 1, and $P_{E_{i,m}}$ is the failure probability of edge $E_{i,m}$.

In the actual network, the edge processing capacity for the additional load is also different. Sometimes a small-scale overload may cause the edge to fail. To describe this character, the parameter ω is introduced to characterize it. $P_{E_{i,m}}$ is calculated by the following formula:

$$P_{E_{i,m}} = \left(\frac{L_{E_{i,m}} - C_{E_{i,m}}}{\delta C_{E_{i,m}} - C_{E_{i,m}}} \right)^{\omega}. \tag{5}$$

It is generally believed that the higher the load of edge exceeds the capacity, the greater the probability of failure is. Therefore, $\omega > 0$.

In order to ensure that the non-failure overload edge can just keep to be normal, the residual load distribution strategy[30] is utilized to distribute its excess load. The overload edge $E_{j,m}$ is allocated according to the following formula:

$$\Pi_{E_{j,z}} = \frac{C_{E_{j,z}} - L_{E_{j,z}}}{\sum_{\chi \in \Gamma_j} (C_{E_{j,\chi}} - L_{E_{j,\chi}}) + \sum_{\eta \in \Gamma_m} (C_{E_{m,\eta}} - L_{E_{m,\eta}})}, \tag{6}$$

where Γ_j denotes the neighbor node of node j, and Γ_m denotes the neighbor node of node m.

2.2. Robustness of the network

To trigger the cascading failure, we attack the most loaded edge in subnet A and the most loaded edge in subnet B simultaneously. Since the largest connected piece of a network is an important indicator to measure the robustness of the network, the size of the largest connected piece is recorded as G,

$$G = \frac{N'_{E_{\mathrm{A}}} + N'_{E_{\mathrm{B}}}}{N_{E_{\mathrm{A}}} + N_{E_{\mathrm{B}}}}, \tag{7}$$

where $N_{E_{\mathrm{A}}}$ is the initial number of edges in subnetwork A, $N_{E_{\mathrm{B}}}$ represents the initial number of edges in subnetwork B, $N'_{E_{\mathrm{A}}}$ represents the number of edges after the failure in subnetwork A, and $N'_{E_{\mathrm{B}}}$ represents the number of edges after the failure in subnetwork B. The more edges the network has after being attacked, the more robust the network is.

3. Analyses of simulations

Different topologies of the network have different effects on its dynamics. In order to understand and control the effects of cascading failures on the network, we choose different topologies of the network (ER-ER edge-coupling network, WS-WS edge-coupling network, BA-BA edge-coupling network, BA-ER edge-coupling network, BA-WS edge-coupling network and WS-ER edge-coupling network) to conduct this research. At the same time, the influence of load parameter, capacity parameter, overload parameter, and distribution coefficient on the robustness of the network are investigated. Parameters about the network are listed in Table 1. The simulations run independently 500 times on average.

Table 1. Network parameters.

Network parameters	ER network	BA network	WS network
Number of network nodes	200	200	200
Average degree $\langle k \rangle$	6	6	6
Parameters of generation	$P = 0.03$	$m_0 = 3, m = 3$	$P = 0.1, K = 6$

3.1. Influence of overload parameter δ on the model

To study the influence of overload coefficient δ on the model, other parameters should not exert a massive impact on the robustness of the network. Assuming the distribution coefficient $\omega = 1$, $\alpha = 0.6$, load coefficient $\theta = 1$, the G–β curves with different values of the overload coefficient δ are shown in Fig. 4. As can be seen in Fig. 4, with the β parameter increasing, the robustness of the model gradually increases. However, in the BA-WS edge-coupling network and the BA-ER edge-coupling network, the robustness is weaker than those in other networks, which indicates that different structures will lead to different robustness in the edge-coupling network. Meanwhile, it is found that the span from $\delta = 1.3$ to $\delta = 1.4$ is not as large as the span from $\delta = 1.0$ to $\delta = 1.1$ in this model.

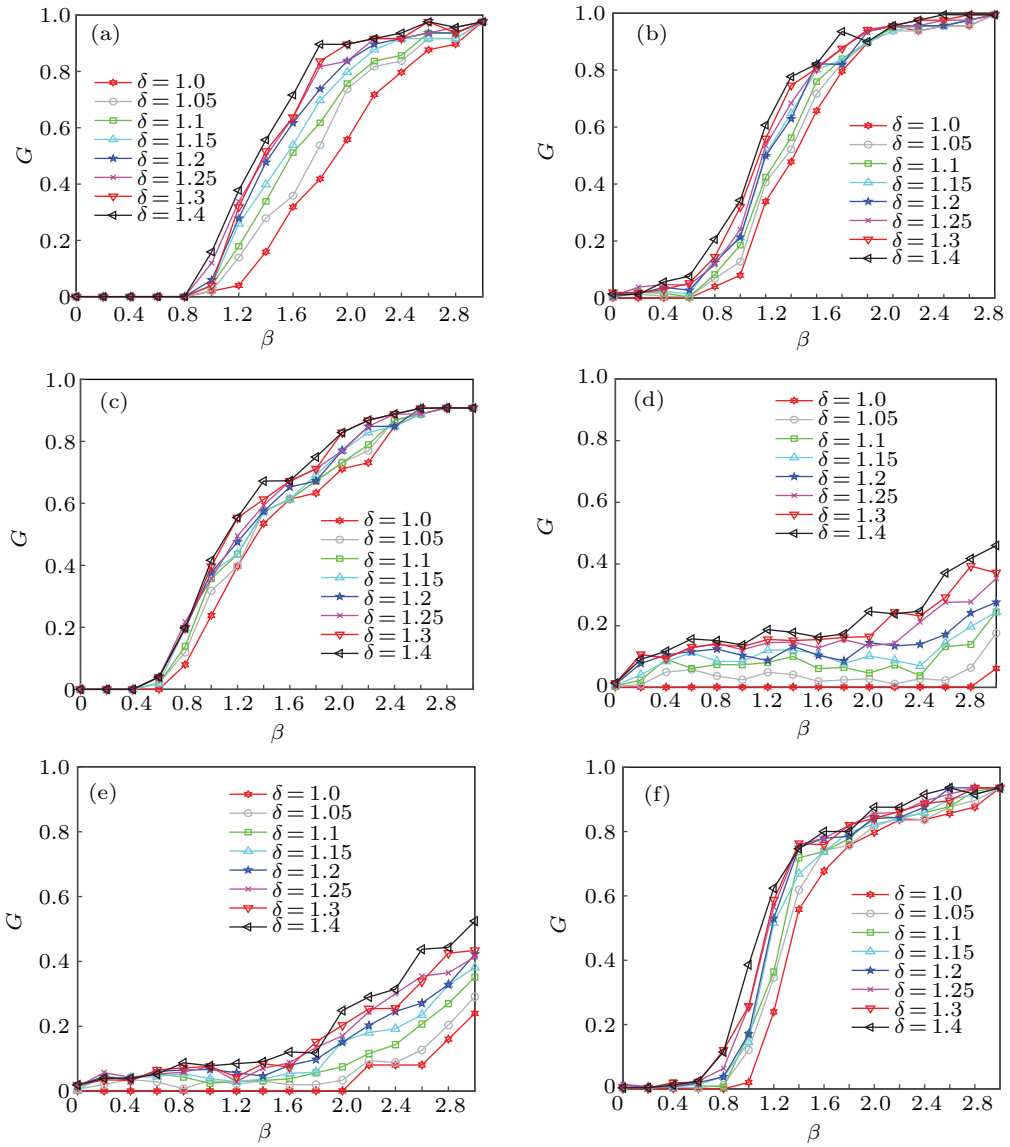


Fig. 4. Robustness of network under different overload parameters δ: (a) BA-BA network of coupled edge, (b) ER-ER network of coupled edge, (c) WS-WS network of coupled edge, (d) BA-WS network of coupled edge, (e) BA-ER network of coupled edge, (f) WS-ER network of coupled edge.

In brief, it can be found that when the overload parameter δ increases in a small range, the robustness can be improved obviously. However, when δ is increased to a certain extent, the robustness changes little. Moreover, the robustness of the BA-WS and BA-ER edge-coupling network is weak.

3.2. Influence of load coefficient θ on the model

In order to study the influence of the load coefficient θ on this model, other parameters do not have an excessive influence. Therefore, the overload coefficient $\delta = 1.15$, the distribution coefficient $\omega = 1$, and $\alpha = 0.6$. Different G–β curves for the load parameter θ are shown in Fig. 5.

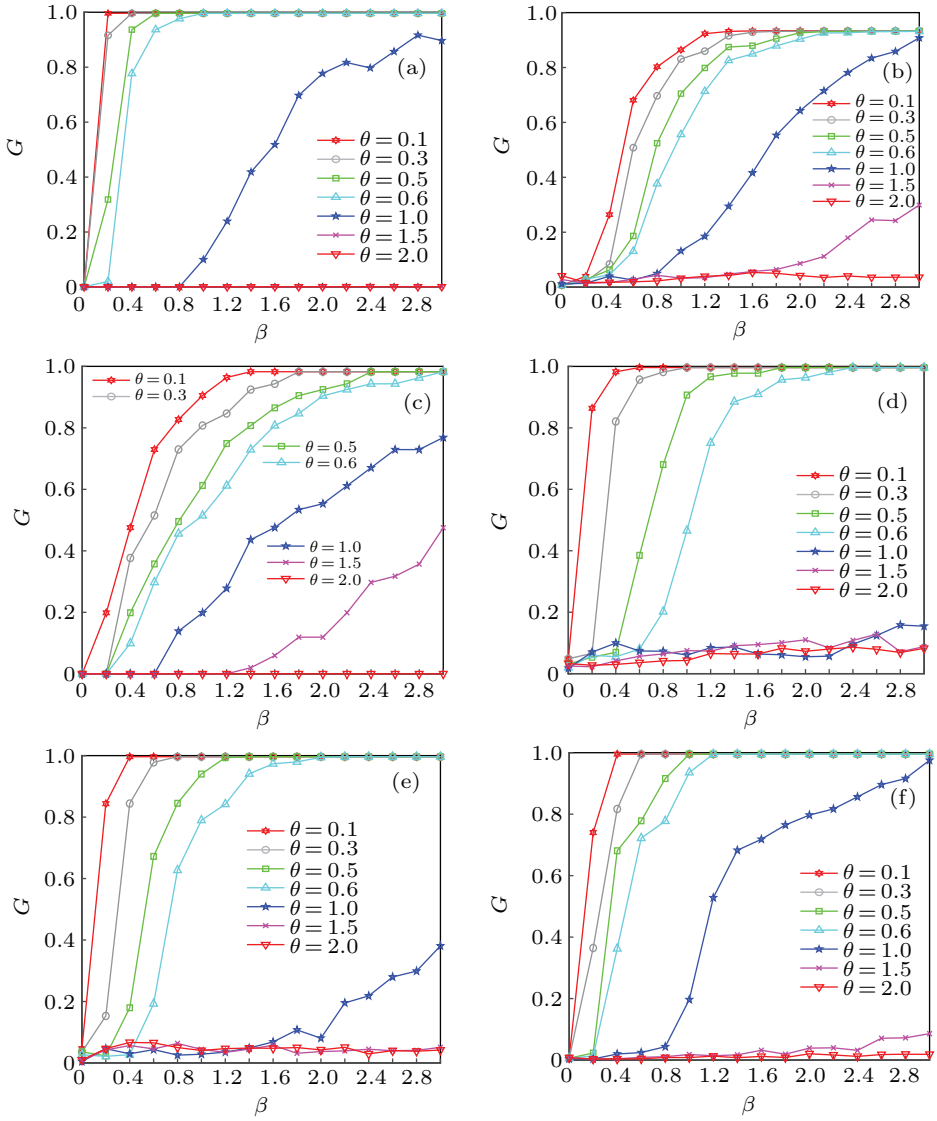


Fig. 5. Network robustness under different load coefficients θ: (a) BA-BA network of coupled edge, (b) ER-ER network of coupled edge, (c) WS-WS network of coupled edge, (d) BA-WS network of coupled edge, (e) BA-ER network of coupled edge, (f) WS-ER network of coupled edge.

Firstly, when $\theta < 1$, the robustness of different networks behaves differently with the increase of β. It can be seen that the BA-BA edge-coupling network is more sensitive to the β parameter in Fig. 5(a). The robustness of the BA-BA edge-coupling network increases in the range of β from 0 to 0.4. When $\beta = 0.8$, the network robustness is the strongest.

In Fig. 5(b), the robustness of ER-ER edge-coupling network increases in the range of $\beta = 0.4$–1.2. When $\beta = 2$, the network robustness is the strongest. In Fig. 5(c), the robustness of the WS-WS edge-coupling network becomes obvious in the range of $\beta = 0$–1.6. When $\beta = 2.8$, the network robustness is strongest. In Fig. 5(d), the robustness of the BA-WS edge-coupling network is enhanced in the range of $\beta = 0$–1.2. When $\beta = 2.4$, the network robustness is the strongest. In Fig. 5(e), the robustness of the BA-ER edge-coupling network becomes clear in the range of $\beta = 0$–0.8. When $\beta = 1.8$, the network robustness is supreme. In Fig. 5(f), the robustness of the WS-ER edge-coupling network turns remarkable in the range of $\beta = 0$–1. When $\beta = 1$, the network robustness is notable.

When $\theta = 1$, the robustness of ER-ER, BA-BA, and WS-WS edge-coupling networks rises slowly. Furthermore, it can be seen that the robustness of the BA-WS network is lower than 0.2 in Fig. 5(d). In Fig. 5(e), it can be seen that the robustness of the BA-ER edge-coupling network shows a clear upward trend, starting at $\beta = 1.6$. However, the robustness of the WS-ER edge-coupling network is the strongest.

When $\theta = 1.5$, only the robustness of the WS-WS and ER-ER edge-coupling networks shows a more obvious upward trend.

When $\theta = 2$, the robustness of all networks is terrible. It can be seen that when $\theta > 1$, its load parameter θ has a massive impact on the robustness of the networks.

3.3. Influence of the distribution coefficient ω on the model

In order to study the influence of the distribution coefficient ω on the model, it is assumed that the overload coefficient $\delta = 1.15$, $\alpha = 0.6$, and the load coefficient $\theta = 0.5$. The

G–B curves with different values of the distribution coefficient ω are shown in Fig. 6.

It can be seen in Fig. 6 that the robustness of the networks with different structures will gradually increase with the increase of parameter β. However, as the β value increases to a certain extent (turning point β_c), the change of distribution parameter ω will not cause any change in the robustness of the model. The turning point β_c is different for networks with different structures, including the BA-BA edge-coupling networks $\beta_c = 1$, the ER-ER edge-coupling networks $\beta_c = 0.9$, the WS-WS edge-coupling networks $\beta_c = 1.7$, the BA-WS edge-coupling networks $\beta_c = 1.2$, BA-ER edge-coupling networks $\beta_c = 1.1$, and the WS-ER edge-coupling networks $\beta_c = 0.9$. Therefore, finding a suitable ω is significant for controlling cascade failure in real networks.

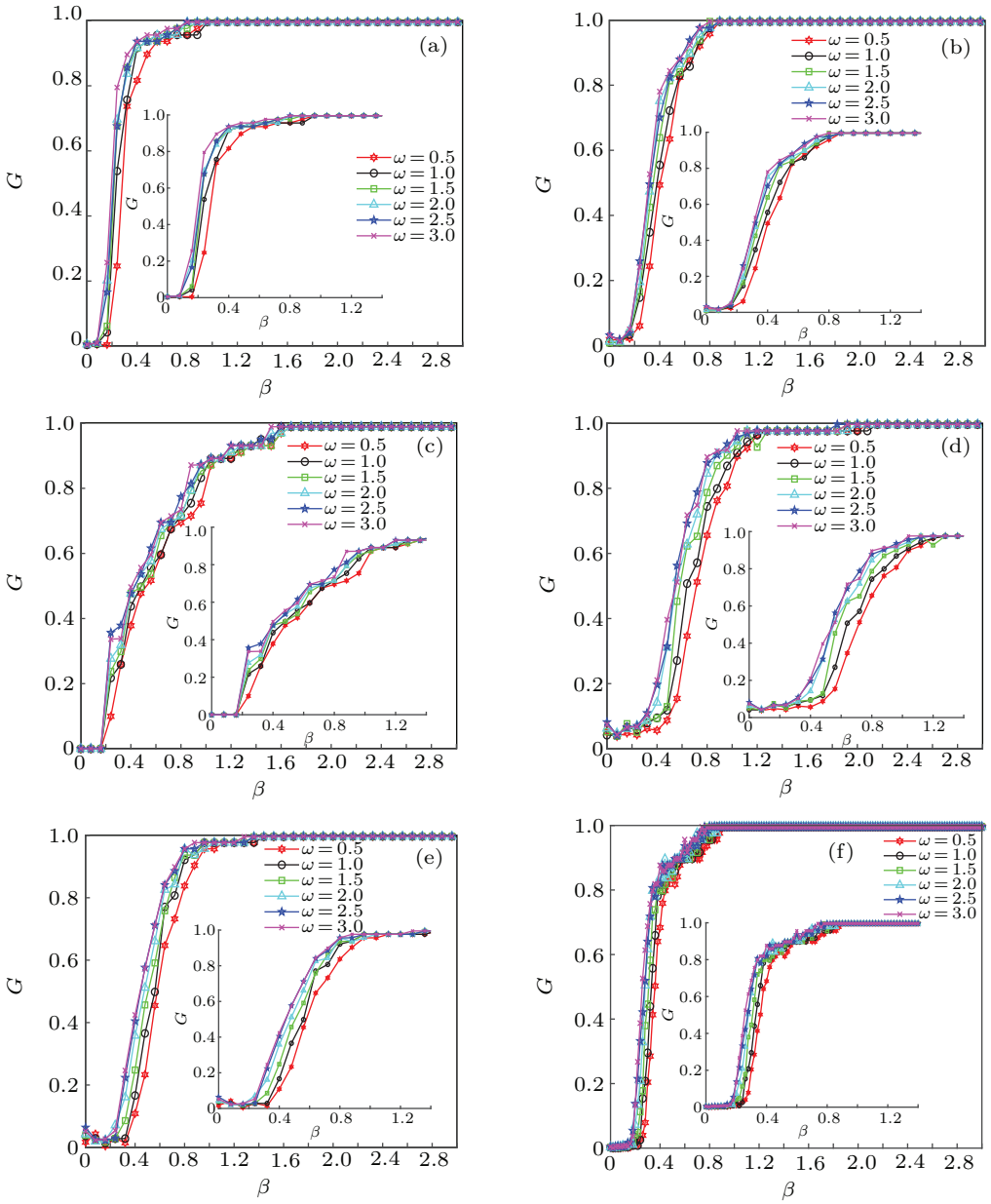


Fig. 6. Network robustness under different distribution coefficients ω (insets are the enlarged pictures): (a) BA-BA network of coupled edge, (b) ER-ER network of coupled edge, (c) WS-WS network of coupled edge, (d) BA-WS network of coupled edge, (e) BA-ER network of coupled edge, (f) WS-ER network of coupled edge.

3.4. Influence of capacity parameters α and β on the model

To study the influence of capacity parameters α and β on the model, this research is carried out with the distribution coefficients $\omega = 1$, overload coefficient $\delta = 1.15$, and load coefficient $\theta = 0.6$. The simulation results are shown in Fig. 7.

It can be seen in Fig. 7(a) that the robustness of the model gradually increases with the increase of α. However, when the critical value $\alpha = 1$, the networks of different structures show the strongest robustness. Similarly, in Fig. 7(b), when the critical value $\beta = 1.4$, the networks show the strongest robustness. It can be observed that the robustness of the BA-BA

edge-interdependence networks is stronger than those in other networks. In general, by adjusting the α and β parameters, the networks can obtain maximum robustness more quickly. Therefore, it is unwise to adjust a single parameter of α or β parameters in a real network.

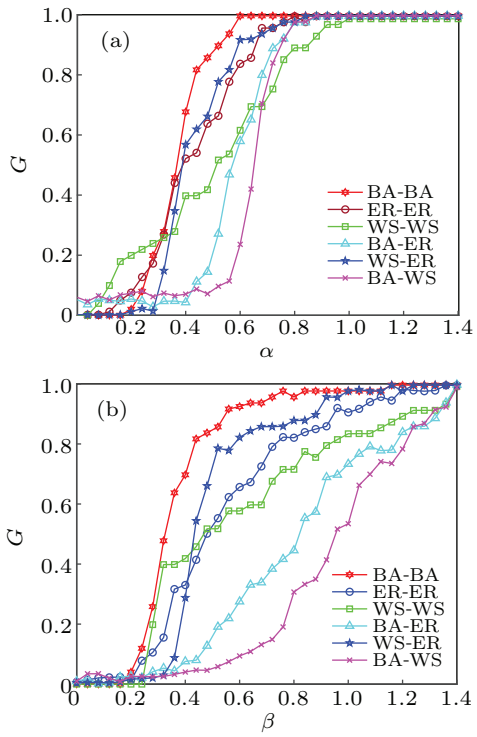


Fig. 7. Robustness of different networks under capacity parameters: (a) robustness under the condition of $\beta = 0.8$, (b) robustness under the condition of $\alpha = 0.6$.

4. Conclusion

The edge in the network not only has the function of maintaining the overall structure of the network but also serves as the medium of information dissemination. Therefore, studying the coupling on edges of the network is of great significance for studying the robustness of the network. Considering multiple parameters and apply them to BA-BA, ER-ER, WS-WS, BA-WS, BA-ER, WS-ER networks, we construct an edge-to-edge coupling model of different two-layer networks. It is found that difference in the structure of the model will lead to difference in robustness, and the span from $\delta = 1.3$ to $\delta = 1.4$ is not as large as the span from $\delta = 1.0$ to $\delta = 1.1$. Meanwhile, when $\theta > 1$, its load parameter θ leads to poor robustness of the network. In distribution parameter ω, the critical threshold β_c is explored. The turning point β_c is different in different structures. As to the capacity parameters, it is found that adjusting the capacity parameters α and β simultaneously can improve the robustness of the network significantly.

In general, this model provides a new reference for controlling and avoiding cascade failure. For further studies, we will explore the robustness of networks from the perspective of the coupling relationship between edges and that between nodes simultaneously in the future.

Acknowledgements

This work is supported by the National Natural Science Foundation of China (Grant No. 61663030), the Natural Science Foundation of Jiangxi Province, China (Grant No. 20142BAB207021), and the Innovation Fund Designated for Graduate Students of Jiangxi Province, China (Grant No. YC2021-S680).

References

[1] Baldick R, Chowdhury B, Dobson L *et al.* 2008 *IEEE Power and Energy Society General Meeting*, July 20–24, 2008, Pittsburgh, PA, USA, pp. 1–8
[2] Crucitti P, Latora V and Marchiori M 2003 *Phys. Rev. E* **4** 045104
[3] Strogatz S H 2001 *Nature* **410** 268
[4] Ren T, Wang Y F, Liu M M and Xu Y J 2016 *Chin. Phys. B* **25** 020101
[5] Chen D B, Lv L Y, Shang M S, Zhang Y C and Zhou T 2019 *Physica A* **391** 1777
[6] Wang J R, Wang J P, He Z and Xu H T 2015 *Chin. Phys. B* **24** 060101
[7] Goh K I, Kahng B and Kim D 2002 *Phys. Rev. Lett.* **88** 108701
[8] Holme P, Kim B J, Yoon C N and Han S K 2002 *Phys. Rev. E* **65** 056109
[9] Albert R, Jeong H and Albert B 2000 *Nature* **406** 387
[10] Simonsen I, Buzna L, Peters K, Bornholdt S and Helbing D 2008 *Phys. Rev. Lett.* **100** 218701
[11] Cohen R, Erez K, Ben-Avraham D and Havlin S 2000 *Phys. Rev. Lett.* **85** 4626
[12] Liu S T and Wang P 2018 *Fractal Control Theory* (Singapore: Springer Nature) pp. 9–24
[13] Liu S T, Zhang Y P, and Liu C 2020 *A Fractal Control and Its Applications* (Singapore: Springer Nature) pp. 129–163
[14] Cohen R, Erez K, Ben-Avraham D and Havlin S 2001 *Phys. Rev. Lett* **86** 3682
[15] Paul G, Tanizawa T, Havlin S and Stanley H E 2004 *Eur. Phys. J. B* **38** 187
[16] Tanizawa T, Paul G, Cohen R, Havlin S and Stanley H E 2005 *Phys. Rev. E* **71** 047101
[17] Li K Z, Zhou J, Zeng M H and Li X M 2010 *Chin. Phys. B* **19** 090510
[18] Li S M, Xu X H and Meng L H 2012 *Chin. Phys. B* **21** 088901
[19] Zhang Y and Osman Y 2016 *Sci. Rep.* **6** 27625
[20] Buldyrev S V, Parshani R, Paul G, Stanley H E and Havlin S 2010 *Nature* **464** 1025
[21] Shao J, Buldyrev S V, Havlin S and Stanley H E 2011 *Phys. Rev. E* **83** 036116
[22] Shai S and Dobson S 2012 *Phys. Rev. E* **86** 066120
[23] Muro M A D, Rocca C E, Stanley H E, Havlin S and Braunstein L A 2016 *Sci. Rep.* **6** 22834
[24] Holme P and Kim B J 2002 *Phys. Rev. E* **65** 066109
[25] Wang W X and Chen G 2008 *Phys. Rev. E* **77** 026101
[26] Motter A E 2004 *Phys. Rev. Lett.* **93** 098701
[27] Barabasi, Albert L and Reka 1999 *Science* **286** 509
[28] Watts and Duncan J 1999 *Am. J. Soc.* **105** 493
[29] Wang W X and Chen G 2008 *Phys. Rev. E* **77** 026101
[30] Hao Y C, Li C and Wei L 2018 *Syst. Eng. Electron* **40** 2282