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Channel parameters-independent multi-hop nondestructive teleportation

Hua-Yang Li(李华阳)[1], Yu-Zhen Wei(魏玉震)[2], Yi Ding(丁祎)[1], and Min Jiang(姜敏)[1,3,†]

[1] *School of Electronics & Information Engineering, Soochow University, Suzhou* 215006, *China*
[2] *School of Information Engineering, Huzhou University, Huzhou* 313000, *China*
[3] *Key Laboratory of System Control and Information Processing, Ministry of Education, Shanghai* 200240, *China*

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A multi-hop nondestructive teleportation scheme independent of channel parameters based on Bell pairs is presented, where the coefficients of the quantum channel are unknown to all the communication nodes. With Bell measurement and channel matching technology the unknown channel parameters can be eliminated probabilistically with the help of the intermediate nodes. Then the source node Alice can teleport an unknown state to the remote destination node Bob. In our scheme the teleportation is generalized first to the scenario independent of channel parameters, which makes the requirement of quantum channel reduced. Our scheme still preserves the initial unknown state even if this teleportation fails. Moreover, performance analysis shows that our scheme has a higher communication efficiency.

Keywords: multi-hop quantum communication, entanglement swapping, teleportation

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1. Introduction

Quantum teleportation (QT) is an important and interesting task which has recently attracted much attention because of its promising applications in the field of secure quantum communication and quantum computing. In a teleportation protocol, an unknown quantum state can be recreated at a distant location by exploiting shared entanglement and classical communication between the sender and the receiver. In 1993, Bennett *et al.*[1] first proposed the standard quantum teleportation to teleport an arbitrary single-qubit state based on the maximally entangled Bell state, which was verified experimentally by Bouwmeester *et al*.[2] After that, lots of QT schemes were presented with different quantum technologies. For example, the QT schemes based on EPR pairs,[3–5] GHZ states,[6,7] W states,[8–10] cluster states,[11–13] mixed states,[14,15] and continuous-variable states[16] have been proposed. Nevertheless, in the actual quantum communication, it is difficult to establish the direct entanglement between the sender and the remote receiver on account of inevitable losses on the quantum channel.

Recently, quantum teleportation network based on entanglement swapping,[17–19] which aims to establish the entanglement between two distant nodes via intermediate nodes, has become a research hotspot. In a multi-hop chain, there are available entangled channels between adjacent nodes. It is possible that quantum channels could be built to transmit the unknown state between the source and the destination node. In 2013, Yu *et al.* proposed a routing protocol for a wireless ad hoc quantum communication network.[20] In 2014, Wang *et al*. devised a quantum wireless multi-hop teleportation scheme based on arbitrary Bell pairs.[21] In 2015, Shi *et al.* proposed a quantum wireless multi-hop network in which an arbitrary single qubit could be teleported hop by hop based on shared Werner states.[22] In 2016, Xiong *et al*. proposed a multi-hop teleportation protocol of partially entangled GHZ state, with the help of auxiliary particles.[23] In 2017, Zou *et al*. presented a multi-hop teleportation scheme of two-qubit state via the composite GHZ-Bell channel.[24] Unfortunately, owing to the inevitable couplings between the quantum states and their surroundings, the maximally entangled channel readily evolved into the non-maximally entangled channel,[25,26] which brings the risk of data loss.[27] In order to avoid possibly destructing the unknown state to be teleported, some improved schemes were reported by establishing the faithful channel beforehand.[28–31]

However, in the above QT schemes, the partially entangled state is used most widely as the quantum channel. In these schemes, the sender or the receiver needs to know the parameter information of the entangled channel completely. Then he introduces an auxiliary particle and adopts a joint unitary transformation which is related to the channel parameters to reconstruct the initial state. Considering the fact that the quantum teleportation may occur in the situation where the channel parameters cannot be known to all communication participants, we firstly propose a multi-hop nondestructive teleportation scheme independent of the channel parameters based on Bell pairs, where the coefficients of quantum channel are unknown to all communication participants to reduce the requirements for quantum channel. With the Bell measurement performed by the intermediate nodes, the unknown parame-

†Corresponding author. E-mail: jiangmin08@suda.edu.cn
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ters of the Bell pairs can be eliminated probabilistically. Then the source node and the destination node can establish a maximally entangled Bell channel. The unknown state can be recovered by the destination node Bob or preserved by source node Alice. Besides the performance analysis shows that our scheme has a higher communication efficiency.

The rest of this paper is organized as follows. In Section 2 a basic two-hop nondestructive teleportation scheme is described based on two Bell pairs where the coefficients of quantum channel are unknown to all communication participants. Then the construction rule of the direct Bell channel shared by two communication nodes in a multi-hop chain composed of Bell pairs is introduced. In Section 3 a multi-hop nondestructive teleportation scheme independent of channel parameters based on channel matching is presented. In Section 4, the performance of our scheme is analyzed. Finally, some conclusions are drawn in Section 5.

2. Preliminaries

In this section, a basic two-hop nondestructive teleportation scheme is first introduced based on the two Bell pairs where the coefficients of quantum channel are unknown to all communication participants, which is described in Subsection 2.1. Then the construction rule of the direct Bell channel finally shared by two communication nodes in a chain composed of Bell pairs is discussed in Subsection 2.2.

2.1. Two-hop nondestructive teleportation scheme

Without loss of generality, the source node Alice is assumed to want to teleport an unknown arbitrary single-qubit $|\gamma\rangle_t$ to the destination node Bob who does not share direct entanglement with Alice and $|\gamma\rangle_t$ can be written as

$$|\gamma\rangle_t = a_1|0\rangle + a_2|1\rangle, \tag{1}$$

where a_1 and a_2 are complex and satisfy the normalization condition $|a_1|^2 + |a_2|^2 = 1$.

The intermediate node Cindy shares one Bell pair $|\varphi\rangle_{A_1B_1}$ with Alice and another one $|\varphi\rangle_{A_2B_2}$ with Bob, which can be expressed as

$$|\varphi\rangle_{A_iB_i} = b_1|00\rangle_{A_iB_i} + b_2|11\rangle_{A_iB_i} \quad (i=1,2), \tag{2}$$

where b_1 and b_2 are the channel parameters that satisfy the normalization condition $|b_1|^2 + |b_2|^2 = 1$, and they are unknown to all communication participants.

As shown in Fig. 1, Alice owns particles t and A_1. Cindy owns particles B_1 and A_2, and Bob owns particle B_2. The initial quantum state of the whole system reads

$$\begin{aligned}|\varphi\rangle_{tA_1B_1A_2B_2} &= |\gamma\rangle_t \otimes |\varphi\rangle_{A_1B_1} \otimes |\varphi\rangle_{A_2B_2} \\ &= (a_1|0\rangle + a_2|1\rangle)_t \otimes (b_1|00\rangle + b_2|11\rangle)_{A_1B_1} \\ &\quad \otimes (b_1|00\rangle + b_2|11\rangle)_{A_2B_2}.\end{aligned} \tag{3}$$

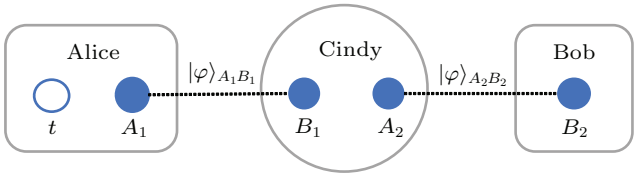


Fig. 1. Qubit distribution of two-hop nondestructive teleportation.

To teleport $|\gamma\rangle_t$ from Alice to Bob, three participants will perform a series of operations in three steps, as shown in Fig. 2.

Step 1 Alice performs one CNOT gate on her own particles t and A_1, and then performs one H gate on her particle t. The CNOT gate can be expressed as

$$\mathrm{CNOT}|mn\rangle = |m\rangle|(m+n) \bmod 2\rangle. \tag{4}$$

The H gate can be described as

$$H = ((|0\rangle + |1\rangle)\langle 0| + (|0\rangle - |1\rangle)\langle 1|)/\sqrt{2}. \tag{5}$$

Now the state $|\varphi\rangle_{tA_1B_1A_2B_2}$ of the whole system will turn into

$$\begin{aligned}|\varphi'\rangle_{tA_1B_1A_2B_2} &= \frac{1}{\sqrt{2}}(a_1b_1(|000\rangle + |100\rangle)_{tA_1B_1} \\ &\quad + a_1b_2(|011\rangle + |111\rangle)_{tA_1B_1} \\ &\quad + a_2b_1(|010\rangle - |110\rangle)_{tA_1B_1}) \\ &\quad + a_2b_2(|001\rangle - |101\rangle)_{tA_1B_1}) \\ &\quad \otimes (b_1|00\rangle + b_2|11\rangle)_{A_2B_2}.\end{aligned} \tag{6}$$

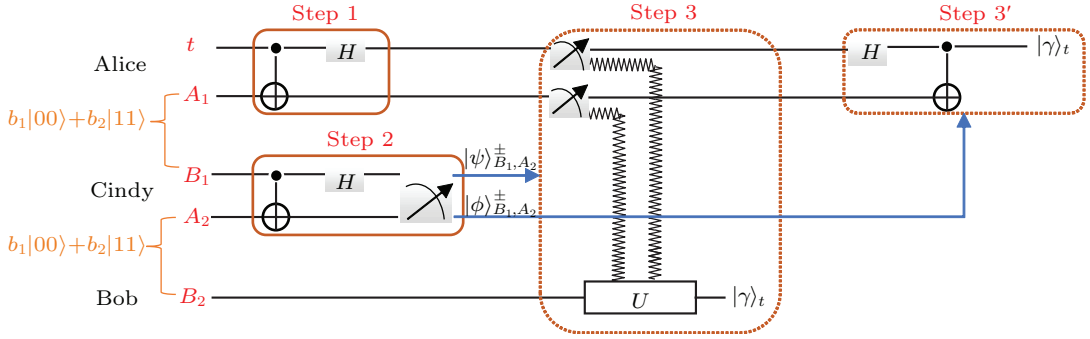


Fig. 2. Quantum circuit for two-hop nondestructive teleportation with two Bell channels.

Step 2 The state $|\varphi'\rangle_{tA_1B_1A_2B_2}$ in Eq. (6) can be written as

$$\begin{aligned}|\varphi'\rangle_{tA_1B_1A_2B_2} &= \frac{1}{\sqrt{2}}\big(a_1b_1^2(|00000\rangle+|10000\rangle)+a_1b_1b_2(|00011\rangle+|10011\rangle)+a_1b_1b_2(|01100\rangle+|11100\rangle)\\ &\quad +a_1b_2^2(|01111\rangle+|11111\rangle)+a_2b_1^2(|01000\rangle-|11000\rangle)+a_2b_1b_2(|01011\rangle-|11011\rangle)\\ &\quad +a_2b_1b_2(|00100\rangle-|10100\rangle)+a_2b_2^2(|00111\rangle-|10111\rangle)\big)_{tA_1B_1A_2B_2}\\ &= \sqrt{\frac{b_1^4+b_2^4}{2}}\cdot|\varphi\rangle^{\pm}_{B_1A_2}\otimes\frac{1}{\sqrt{2\left(b_1^4+b_2^4\right)}}\big(a_1b_1^2(|000\rangle+|100\rangle)_{tA_1B_2}\pm a_1b_2^2(|011\rangle+|111\rangle)_{tA_1B_2}\\ &\quad +a_2b_1^2(|010\rangle-|110\rangle)_{tA_1B_2}\pm a_2b_2^2(|001\rangle-|101\rangle)_{tA_1B_2}\big)\\ &\quad +\sqrt{b_1^2b_2^2}\cdot|\psi\rangle^{\pm}_{B_1A_2}\otimes\frac{1}{2}\big(a_1(|001\rangle\pm|010\rangle+|101\rangle\pm|110\rangle)_{tA_1B_2}\\ &\quad +a_2(|011\rangle\pm|000\rangle\pm|100\rangle-|111\rangle)_{tA_1B_2}\big),\end{aligned}\tag{7}$$

where $|\phi\rangle^{\pm}_{B_1A_2}=(|00\rangle\pm|11\rangle)/\sqrt{2}$ and $|\psi\rangle^{\pm}_{B_1A_2}=(|01\rangle\pm|10\rangle)/\sqrt{2}$.

Now Cindy performs Bell measurement on her two particles B_1 and A_2 with the basis $\{|\phi\rangle^{\pm}=(|00\rangle\pm|11\rangle)/\sqrt{2},|\psi\rangle^{\pm}=(|01\rangle\pm|10\rangle)/\sqrt{2}\}$ and informs Bob of her measurement result.

Next when the measurement result of Cindy is $|\psi\rangle^{\pm}_{B_1A_2}=(|01\rangle\pm|10\rangle)/\sqrt{2}$, the scheme will come to Step 3. Otherwise, it will come to Step 3′.

Note that Step 1 and Step 2 can be performed simultaneously.

Step 3 According to Eq. (7), if the measurement result of Cindy is $|\psi\rangle^{\pm}_{B_1A_2}=(|01\rangle\pm|10\rangle)/\sqrt{2}$, the state of the remaining particles t, A_1, and B_2 will become

$$\begin{aligned}|\varphi\rangle_{tA_1B_2} &= \frac{1}{2}\big((a_1(|001\rangle\pm|010\rangle+|101\rangle\pm|110\rangle)_{tA_1B_2}\\ &\quad +a_2(|011\rangle\pm|000\rangle\pm|100\rangle-|111\rangle)_{tA_1B_2})\big)\\ &= \frac{1}{2}\big(|00\rangle_{tA_1}\otimes(a_1|1\rangle\pm a_2|0\rangle)_{B_2}-|01\rangle_{tA_1}\\ &\quad \otimes(a_1|0\rangle\pm a_2|1\rangle)_{B_2}\\ &\quad +|10\rangle_{tA_1}\otimes(a_1|1\rangle\mp a_2|0\rangle)_{B_2}-|11\rangle_{tA_1}\\ &\quad \otimes(a_1|0\rangle\mp a_2|1\rangle)_{B_2}\big).\end{aligned}\tag{8}$$

It is obvious that no unknown channel parameters b_1 and b_2 exist in Eq. (8). Next, Alice performs one projection measurement on her two particles t and A_1 with the basis $\{|00\rangle,|01\rangle,|10\rangle,|11\rangle\}$. According to the measurement result sent by Alice, Bob can perform the corresponding unitary operation (one of four Pauli matrices: $I=|0\rangle\langle0|+|1\rangle\langle1|$, $Z=|0\rangle\langle0|-|1\rangle\langle1|$, $X=|0\rangle\langle1|+|1\rangle\langle0|$, $XZ=|0\rangle\langle1|-|1\rangle\langle0|$) on particle B_2 to recover the original unknown state. For example, when the measurement result of Cindy is $|\psi\rangle^{-}_{B_1A_2}$ and the measurement result of Alice is $|00\rangle_{tA_1}$, the state of particle B_2 will turn into $a_1|1\rangle-a_2|0\rangle$ Bob can recover the original unknown state by performing XZ unitary operation on particle B_2.

Step 3′ If the measurement result of Cindy is $|\phi\rangle^{\pm}_{B_1A_2}=(|00\rangle\pm|11\rangle)/\sqrt{2}$, the state of the remaining particles t, A_1, and B_2 will be

$$\begin{aligned}|\varphi'\rangle_{tA_1B_2} &= \big(a_1b_1^2(|000\rangle+|100\rangle)\pm a_1b_2^2(|011\rangle+|111\rangle)\\ &\quad +a_2b_1^2(|010\rangle-|110\rangle)\\ &\quad \pm a_2b_2^2(|001\rangle-|101\rangle)\big)_{tA_1B_2}/\sqrt{2\left(b_1^4+b_2^4\right)}.\end{aligned}\tag{9}$$

In this case, the unknown channel parameters cannot be eliminated and Bob cannot recover the original qubit state $|\gamma\rangle_t$. Then Alice performs one H gate operation on particle t and one CNOT gate operation on particles t and A_1. The state of $|\varphi'\rangle_{tA_1B_2}$ will transform into

$$\begin{aligned}&(\mathrm{CNOT})_{t,A_1}(H)_t|\varphi'\rangle_{tA_1B_2}\\ &=(a_1|0\rangle+a_2|1\rangle)_t\otimes\frac{1}{\sqrt{b_1^4+b_2^4}}\left(b_1^2|00\rangle\pm b_2|11\rangle\right)_{A_1,B_1}.\end{aligned}\tag{10}$$

Although the teleportation fails in this case, according to Eq. (10), we can know that the unknown state $|\gamma\rangle_t$ is still preserved in qubit t. Hence, the integrity of the unknown state $|\gamma\rangle_t$ can be maintained. In this way, the next teleportation can restart.

In this subsection, we present a basic two-hop nondestructive teleportation scheme based on two Bell pairs where the coefficients of quantum channel are unknown to all communication participants. In this scheme, with some simple and feasible operations, the unknown channel parameters can be eliminated probabilistically. Then the teleportation can succeed in being independent of the parameters of the Bell pairs. Even if this teleportation fails, the unknown state can still be preserved by the sender Alice, which could be retransmitted in the next teleportation. Therefore, our scheme ensures the security of communication.

2.2. Construction of direct Bell channel in chain composed of Bell pairs

Considering that the above scheme may be generalized to multi-hop nondestructive teleportation based on Bell pairs where the coefficients of quantum channel are unknown to all the communication nodes. In a multi-hop quantum chain, n ($n \geq 2$) Bell pairs are shared among the source node Alice, $n-1$ intermediate nodes $\text{Cindy}_1, \text{Cindy}_2, \ldots, \text{Cindy}_{n-1}$, and the destination node Bob.

As shown in Fig. 3, Alice possesses particles t and A_1. Bob owns particle B_n. Cindy_i has particles B_i and A_{i+1}, $i = 1, 2, \ldots, n-1$. The whole quantum channel can be given by

$$|\varphi\rangle = \left\{(b_1|00\rangle + b_2|11\rangle)^{\otimes n}\right\}_{A_1B_1A_2B_2,\ldots,B_{n-1}A_nB_n}. \quad (11)$$

Like the above two-hop nondestructive teleportation, each intermediate node Cindy_i performs Bell one measurement on their own two particles B_i and A_{i+1} and informs Bob of the measurement outcome ($i = 1, 2, \ldots, n-1$). After all the Bell measurements are performed, Alice and Bob will share one direct Bell channel $|\phi\rangle_{A_1B_n}$.

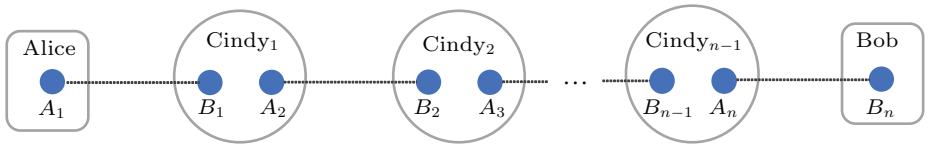


Fig. 3. Qubit distribution in a quantum chain composed of Bell pairs, with dotted line denoting Bell pair.

We first consider that the relation between the measurement results of all the intermediate nodes and the Bell channel $|\varphi\rangle_{A_1B_n}$. Now we label the measurement outcome of Cindy_i as p_i ($p_i = 0$ denotes the outcome $|\phi\rangle^{\pm}_{B_iA_{i+1}} = (|00\rangle \pm |11\rangle)_{B_iA_{i+1}}/\sqrt{2}$, $p_i = 1$ represents the result $|\psi\rangle^{\pm}_{B_iA_{i+1}} = (|01\rangle \pm |10\rangle)_{B_iA_{i+1}}/\sqrt{2}$, $i = 1, 2, \ldots, n-1$). The relationship between Bell measurement results of $n-1$ intermediate nodes and the direct Bell channel $|\varphi\rangle_{A_1B_n}$ which is finally shared by Alice and Bob can be expressed as follows.[32]

If the number of $p_i = 0$ among all the intermediate nodes is odd, then

$$\begin{aligned}|\varphi\rangle_{A_1,B_n} = {} & b_1 \prod_{i=1}^{n-1} b_1^{\overline{\oplus_{j=1}^{i} p_j}} \cdot b_2^{\oplus_{j=1}^{i} p_j} |00\rangle_{A_1,B_n} \\ & \pm b_2 \prod_{i=1}^{n-1} b_2^{\overline{\oplus_{j=1}^{i} p_j}} \cdot b_1^{\oplus_{j=1}^{i} p_j} |11\rangle_{A_1,B_n}. \end{aligned} \quad (12)$$

If the number of $p_i = 0$ among all the intermediate nodes is even, then

$$\begin{aligned}|\varphi\rangle_{A_1,B_n} = {} & b_1 \prod_{i=1}^{n-1} b_1^{\overline{\oplus_{j=1}^{i} p_j}} \cdot b_2^{\oplus_{j=1}^{i} p_j} |01\rangle_{A_1,B_n} \\ & \pm b_2 \prod_{i=1}^{n-1} b_2^{\overline{\oplus_{j=1}^{i} p_j}} \cdot b_1^{\oplus_{j=1}^{i} p_j} |10\rangle_{A_1,B_n}. \end{aligned} \quad (13)$$

Here the normalization condition of amplitude is ignored, and $\oplus_{j=1}^{i} p_j$ denotes the modulo 2 addition of $p_1, p_2, \ldots, p_i$.

According to Eqs. (12), (13) and the Bell measurement results from all the intermediate nodes, Bob can obtain the direct Bell channel $|\varphi\rangle_{A_1B_n}$. If $|\varphi\rangle_{A_1B_n}$ is the maximally entangled state, Alice can teleport an unknown state to Bob similar to two-hop nondestructive teleportation. Otherwise, Alice cannot teleport the unknown state.

3. The multi-hop nondestructive teleportation scheme

In this section, the source node Alice wishes to teleport the unknown single-qubit state $|\gamma\rangle_t$ to the destination node Bob through $2n-1$ ($n \geq 2$) intermediate nodes $\text{Cindy}_1, \text{Cindy}_2, \ldots, \text{Cindy}_{2n-1}$ as shown in Fig. 4. Each intermediate node shares the Bell pair $b_1|00\rangle + b_2|11\rangle$ with his neighboring nodes where the coefficients b_1 and b_2 are unknown to all communication participants.

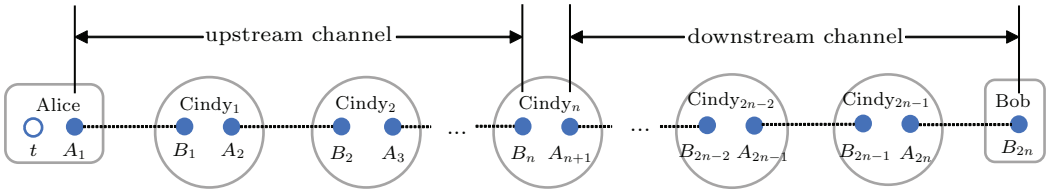


Fig. 4. Qubit distribution of multi-hop nondestructive teleportation based on channel matching, with dotted line denoting Bell pair.

The initial quantum state of the whole system can be written as

$$|\varphi\rangle = (a_1|0\rangle + a_2|1\rangle)_t \otimes \left\{(b_1|00\rangle + b_2|11\rangle)^{\otimes 2n}\right\}_{A_1B_1A_2B_2\ldots A_{2n}B_{2n}}. \quad (14)$$

Here Alice possesses particles t and A_1. Bob owns particle B_{2n}. Cindy_i owns particles B_i and A_{i+1}, $i =$

$1,2,\ldots,2n-1$. Cindy$_n$ divides the whole quantum channel $\{(b_1|00\rangle+b_2|11\rangle)^{\otimes 2n}\}_{A_1B_1A_2B_2,\ldots,A_{2n}B_{2n}}$ into one upstream channel $\{(b_1|00\rangle+b_2|11\rangle)^{\otimes n}\}_{A_1B_1\ldots A_nB_n}$ and one downstream channel $\{(b_1|00\rangle+b_2|11\rangle)^{\otimes n}\}_{A_{n+1}B_{n+1}\ldots A_{2n}B_{2n}}$.

Our scheme can be completed in the following four steps as shown in Fig. 5.

Step 1 Intermediate nodes Cindy$_1$, Cindy$_2$, ..., Cindy$_{n-1}$ in upstream channel, and Cindy$_{n+1}$, Cindy$_2$, ..., Cindy$_{2n-1}$ in downstream channel respectively perform one Bell measurement on their own two particles simultaneously and send the measurement result to the node Cindy$_n$. Then according to Eq. (12), Eq. (13), and $2n-2$ measurement results from intermediate nodes, Cindy$_n$ can obtain one upstream direct channel as $|\varphi\rangle_{A_1B_n}$ and one downstream direct channel as $|\varphi\rangle_{A_{n+1}B_{2n}}$. Here $|\varphi\rangle_{A_1B_n}$ is shared by Alice and Cindy$_n$. $|\varphi\rangle_{A_{n+1}B_{2n}}$ is shared by Cindy$_n$ and Bob.

Step 2 Now the channel matching is ready to begin according to the state $|\varphi\rangle_{A_1B_n}$ and the state $|\varphi\rangle_{A_{n+1}B_{2n}}$. For simplicity, the amplitude parameters of the state $|\varphi\rangle_{A_1B_n}$ are assumed to be $\pm c_1$ and $\pm c_2$, and the amplitude parameters of the state $|\varphi\rangle_{A_{n+1}B_{2n}}$ are assumed to be $\pm d$ and $\pm d_2$ (where $|c_1|^2+|c_2|^2=1$ and $|d_1|^2+|d_2|^2=1$).

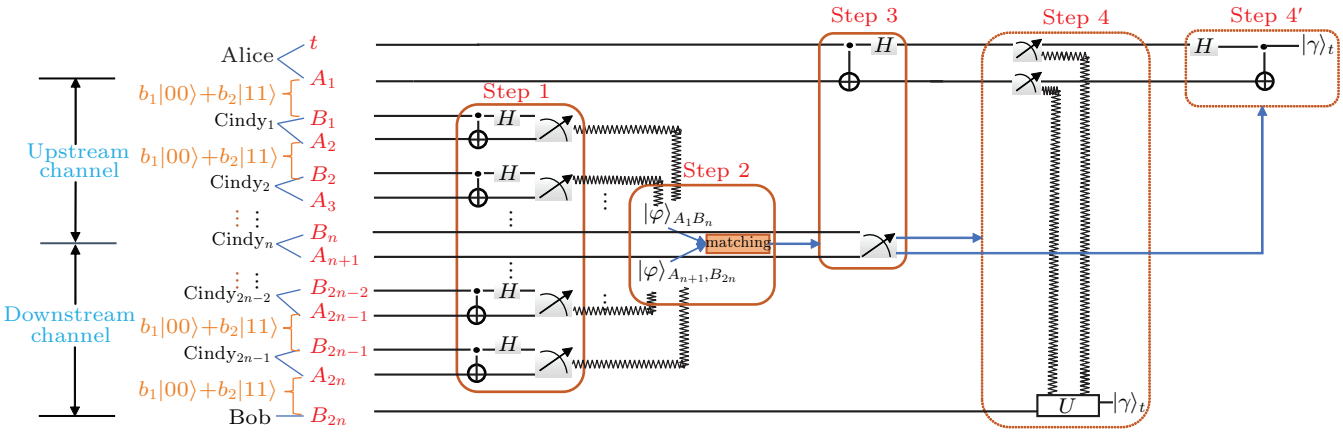


Fig. 5. Quantum circuit for multi-hop nondestructive teleportation scheme based on channel matching.

If the amplitude parameters of both states $|\varphi\rangle_{A_1B_n}$ and $|\varphi\rangle_{A_{n+1}B_{2n}}$ satisfy $|c_1|=|d_1|$ or $|c_1|=|d_2|$, then the channel matching between the upstream channel and the downstream channel succeeds. For convenience, we take for example the state $|\varphi\rangle_{A_1B_n}=c_1|00\rangle+c_2|11\rangle$ and the state $|\varphi\rangle_{A_{n+1}B_{2n}}=c_2|01\rangle+c_1|10\rangle$ to explain the matching principle. Now the entire system will become

$$\begin{aligned}|\varphi\rangle_{tA_1B_nA_{n+1}B_{2n}} &= |\gamma\rangle_t\otimes|\varphi\rangle_{A_1B_n}\otimes|\varphi\rangle_{A_{n+1}B_{2n}}\\ &= (a_1|0\rangle+a_2|1\rangle)_t\otimes(c_1|00\rangle+c_2|11\rangle)_{A_1B_n}\\ &\quad\otimes(c_2|01\rangle+c_1|10\rangle)_{A_{n+1}B_{2n}}.\end{aligned}\tag{15}$$

According to Eqs. (15) and (6), we can know that the multi-hop nondestructive teleportation turns into two-hop nondestructive teleportation among Alice, Cindy$_n$, and Bob now. In this case, Alice can teleport the unknown state to Bob. And the channel matching between upstream channel and downstream channel succeeds. Our scheme will come to Step 3.

However, when the amplitude parameters of both states $|\varphi\rangle_{A_1B_n}$ and $|\varphi\rangle_{A_{n+1}B_{2n}}$ satisfy $|c_1|\neq|d_1|$ and $|c_1|\neq|d_2|$, the channel matching between upstream channel and downstream channel fails. For instance, if the state $|\varphi\rangle_{A_1B_n}$ is $c_1|00\rangle_{A_1B_n}-c_2|11\rangle_{A_1B_n}$ and the state $|\varphi\rangle_{A_{n+1}B_{2n}}$ is $d_1|01\rangle_{A_{n+1}B_{2n}}-d_2|10\rangle_{A_{n+1}B_{2n}}$, then the entire system will become

$$\begin{aligned}|\varphi\rangle_{A_1B_nA_{n+1}B_{2n}} &= (a_1|0\rangle+a_2|1\rangle)_t\otimes(c_1|00\rangle-c_2|11\rangle)_{A_1B_n}\otimes(d_1|01\rangle-d_2|10\rangle)_{A_{n+1}B_{2n}}\\ &= (a_1|0\rangle+a_2|1\rangle)_t\otimes\frac{1}{\sqrt{2}}\left(\sqrt{c_1^2d_1^2+c_2^2d_2^2}\cdot|\varphi\rangle^{\pm}_{B_nA_{n+1}}\otimes\frac{(c_1d_1|01\rangle\pm c_2d_2|10\rangle)_{A_1B_{2n}}}{\sqrt{c_1^2d_1^2+c_2^2d_2^2}}\right.\\ &\quad\left.-\sqrt{c_1^2d_2^2+c_2^2d_1^2}\,|\psi\rangle^{\pm}_{B_nA_{n+1}}\otimes\frac{(c_1d_2|00\rangle\pm c_2d_1|11\rangle)_{A_1B_{2n}}}{\sqrt{c_1^2d_2^2+c_2^2d_1^2}}\right).\end{aligned}\tag{16}$$

Now Cindy$_n$ performs one Bell measurement on her two particles B_i and A_{i+1}. From Eq. (16), it is obvious that no matter what the Bell measurement result of Cindy$_n$ is, Alice and Bob finally cannot share a maximally entangled state. Then this multi-hop teleportation fails. Fortunately, the unknown state is still preserved by the sender Alice. It can be teleported in the next teleportation.

The entangled state $|\varphi\rangle_{A_1B_n}$ and $|\varphi\rangle_{A_{n+1}B_{2n}}$ in other forms

can similarly be analyzed.

Step 3 The scheme to be adopted below is similar to the above two-hop nondestructive teleportation scheme. Alice performs one CNOT gate on her own particles t and A_1, and then performs one H gate on her particle t. The state $|\varphi\rangle_{tA_1B_nA_{n+1}B_{2n}}$ in Eq. (15) will become

$$\begin{aligned}|\varphi'\rangle_{tA_1B_nA_{n+1}B_{2n}} = & \frac{1}{\sqrt{2}}\big(a_1c_1\left(|000\rangle+|100\rangle\right)_{tA_1B_n}+a_1c_2\left(|011\rangle+|111\rangle\right)_{tA_1B_n}+a_2c_1\left(|010\rangle-|110\rangle\right)_{tA_1B_n}\\ &+a_2c_2\left(|001\rangle-|101\rangle\right)_{tA_1B_n}\big)\otimes\left(c_2|01\rangle+c_1|10\rangle\right)_{A_{n+1}B_{2n}}.\end{aligned}\tag{17}$$

Meanwhile, the state $|\varphi'\rangle_{tA_1B_nA_{n+1}B_{2n}}$ can be written as

$$\begin{aligned}|\varphi'\rangle_{tA_1B_nA_{n+1}B_{2n}} = & \frac{1}{\sqrt{2}}\left(a_1c_1c_2\left(|00001\rangle+|10001\rangle\right)+a_1c_1^2\left(|00010\rangle+|10010\rangle\right)+a_1c_2^2\left(|01101\rangle+|11101\rangle\right)\right.\\ &+a_1c_1c_2\left(|01110\rangle+|11110\rangle\right)+a_2c_1c_2\left(|01001\rangle-|11001\rangle\right)+a_2c_1^2\left(|01010\rangle-|11010\rangle\right)\\ &\left.+\ a_2c_2^2\left(|00101\rangle-|10101\rangle\right)+a_2c_1c_2\left(|00110\rangle-|10110\rangle\right)\right)_{tA_1B_nA_{n+1}B_{2n}}\\ = & \sqrt{c_1^2c_2^2}\cdot|\phi\rangle^{\pm}_{B_nA_{n+1}}\otimes\frac{1}{2}\big(\left(a_1\left(|001\rangle\pm|010\rangle+|101\rangle\pm|110\rangle\right)_{tA_1B_{2n}}\right.\\ &\left.+a_2\left(|011\rangle\pm|000\rangle\pm|100\rangle-|111\rangle\right)_{tA_1B_{2n}}\right)\big)\\ &+\sqrt{\frac{c_1^4+c_2^4}{2}}\cdot|\psi\rangle^{\pm}_{B_nA_{n+1}}\otimes\frac{1}{\sqrt{2(c_1^4+c_2^4)}}\left(a_1c_1^2\left(|000\rangle+|100\rangle\right)\pm a_1c_2^2\left(|011\rangle+|111\rangle\right)\right.\\ &\left.+a_2c_1^2\left(|010\rangle-|110\rangle\right)\pm a_2c_2^2\left(|001\rangle-|101\rangle\right)\right)_{tA_1B_{2n}}.\end{aligned}\tag{18}$$

Then Cindy$_n$ performs one Bell measurement on her two particles B_i and A_{i+1} with the basis $\{|\phi\rangle^{\pm}=(|00\rangle\pm|11\rangle)/\sqrt{2},|\psi\rangle^{\pm}=(|01\rangle\pm|10\rangle)/\sqrt{2}\}$, and informs Alice and Bob of her measurement result.

Next, when the measurement outcome of Cindy$_n$ is $|\varphi\rangle^{\pm}_{B_nA_{n+1}}$, the scheme will come to Step 4. Otherwise, it will come to Step 4$'$.

Step 4 According to Eq. (18), if the Bell measurement result of Cindy$_n$ is $|\varphi\rangle^{\pm}_{B_nA_{n+1}}$, the state of the remaining particles t, A_1, and B_{2n} will become

$$\begin{aligned}|\varphi\rangle_{tA_1B_{2n}} = & \frac{1}{2}\big(\left(a_1\left(|001\rangle\pm|010\rangle+|101\rangle\pm|110\rangle\right)_{tA_1B_{2n}}\right.\\ &\left.+a_2\left(|011\rangle\pm|000\rangle\pm|100\rangle-|111\rangle\right)_{tA_1B_{2n}}\right)\big)\\ = & \frac{1}{2}\big(|00\rangle_{tA_1}\otimes\left(a_1|1\rangle\pm a_2|0\rangle\right)_{B_{2n}}-|01\rangle_{tA_1}\\ &\otimes\left(a_1|0\rangle\pm a_2|1\rangle\right)_{B_{2n}}\\ &+|10\rangle_{tA_1}\otimes\left(a_1|1\rangle\mp a_2|0\rangle\right)_{B_{2n}}-|11\rangle_{tA_1}\\ &\otimes\left(a_1|0\rangle\mp a_2|1\rangle\right)_{B_{2n}}\big).\end{aligned}\tag{19}$$

The next operations are similar to the operations in above two-hop nondestructive teleportation scheme. Alice performs one projection measurement on her two particles t and A_1 with the basis $\{|00\rangle,|01\rangle,|10\rangle,|11\rangle\}$ and informs Bob of her measurement result.

Finally, according to Alice's measurement result, Bob performs corresponding local unitary operation on his particle B_{2n} to recover the unknown single-qubit state $|\gamma\rangle_t$.

Step 4$'$ A scheme to be used below is similar to the above two-hop nondestructive teleportation scheme as indicated in Subsection 2.1, Alice performs one extra H gate operation on her particle t and then performs one CNOT gate operation on her two particles t and A_1. Then the unknown qubit state $|\gamma\rangle_t$ can be preserved on particle t.

In this section, we present a multi-hop nondestructive teleportation independent of channel parameters based on $2n$ Bell pairs, where the coefficients of quantum channel are unknown to all the communication nodes. By using the Bell measurement, a series of local unitary operations, and channel matching technology, the source node Alice can efficiently teleport an unknown qubit state to the destination node Bob with the help of $2n-1$ intermediate nodes. Even if this teleportation fails, our scheme can preserve the initial unknown state $|\gamma\rangle_t$. It can continue to be retransmitted in the next teleportation.

4. Performance analysis

In this section, we discuss the performance of our scheme in terms of classical communication cost and quantum communication delay. Besides, we compare our scheme with some previous teleportation schemes.

4.1. Classical communication cost

The quantity of classical data that need to be transmitted can be defined as the classical communication cost in the scheme.[17] In our scheme, the classical data which need to be transmitted, are the Bell measurement data from intermediate nodes, and two-qubit projection measurement data from the

source node. Both Bell measurement and two-qubit projection measurement has four possible results, which can be denoted by 2-bit classical information 00, 01, 10, 11. Then the total communication cost is the product of the bit number of each measurement result and the node number that need to transmit measurement result in the protocol.[17] Therefore, the classical communication cost in our scheme is

$$C_{\text{total}}^{\text{c-bit}} = 2(N-1)+2 = 2N \ \ (\text{bits}), \tag{20}$$

where N is the number of hop counts in the multi-hop nondestructive teleportation scheme.

4.2. Quantum communication delay

In the quantum network, the delay introduced by measurements, unitary operations and the transmission of measurement results via classical channels is inevitable. It should be kept as low as possible due to the limited decoherence time in quantum memory and quality of service. It is assumed that each intermediate node takes D_{BM} seconds to complete the Bell measurement and takes D_{trans1} seconds to transmit measurement outcome results. The source node Alice takes D_{PM} seconds to finish the two-qubit projection measurement and takes D_{trans2} seconds to transmit measurement result . The unitary operations of our scheme consist of CNOT gate, H gate, and Pauli operators (including I, Z, X, XZ operators). The corresponding unitary operation delays can be given by D_{CN}, D_{H}, and D_{Pau} respectively.

The delay of the channel matching is negligible. Thus, our scheme can be divided into three parts. The first part delay D_1, including D_{BM}, D_{trans1}, can be expressed as

$$D_1 = D_{\text{BM}} + D_{\text{trans1}}. \tag{21}$$

The second part is that after the channel matching between upstream channel and downstream channel succeeds, Alice performs one CNOT gate on her own particles t and A_1, and then performs one H gate on her particle t. Meanwhile, Cindy$_n$ needs to perform Bell measurement on her two particles B_i and A_{i+1}. Thus, the second part delay D_2 can be written as

$$D_2 = \max\{D_{\text{CN}} + D_{\text{H}}, D_{\text{BM}} + D_{\text{trans1}}\}. \tag{22}$$

The third part is that according to the measurement outcome of Cindy$_n$, Alice will perform projection measurement on her two particles t and A_1. Next, Bob will perform corresponding local unitary operation on his particle B_{2n} to recover the unknown single-qubit state according to Alice's measurement result. Thus, the third part delay D_3 can be given by

$$D_3 = D_{\text{PM}} + D_{\text{trans2}} + D_{\text{Pau}}. \tag{23}$$

Therefore, the total delay of successful teleportation of our scheme is

$$\begin{aligned} D_{\text{total}} &= D_1 + D_2 + D_3 \\ &= \max\{D_{\text{CN}} + D_{\text{H}}, D_{\text{BM}} + D_{\text{trans1}}\} \\ &\quad + D_{\text{BM}} + D_{\text{trans1}} + D_{\text{PM}} + D_{\text{trans2}} + D_{\text{Pau}}. \end{aligned} \tag{24}$$

Besides, with the channel matching technology, the intermediate node Cindy$_n$ can probabilistically eliminate the unknown channel parameters according to the Bell measurement results from upstream channel and downstream channel. Then Alice and Bob can share one maximally entangled Bell channel. The teleportation can succeed in being independent of the parameters of the Bell pairs. Once this teleportation fails, the unknown state can still be preserved by Alice. The integrity of the information can be preserved. Hence, our scheme ensures the security of communication.

5. Comparison

In previous nondestructive teleportation schemes,[29,31] it is necessary to use the known quantum channels, introduce auxiliary particles and perform two-qubit unitary operations. However, in our protocol, the teleportation can succeed in being independent of the channel parameters without extra auxiliary particles or two-qubit unitary operations. Therefore, it reduces the requirements for the quantum channel itself and makes the experiment implemented more conveniently.

In addition, we also compare the efficiency of our protocol with those of previous multi-hop teleportation schemes in two aspects: communication cost and communication delay, as demonstrated in Table 1. Owing to the channel matching technology and extra one Bell measurement performed by the intermediate node, the communication delay of our scheme is higher than those of the protocols in Refs. [10,21]. However, with the simultaneous entanglement swapping technology in both the upstream channel and downstream channel, the total communication delay of our scheme is lower than those in Refs. [10,24].

Table 1. Comparisons of efficiency among previous QT protocols. N is the number of hop counts in the quantum network.

Protocol	Channel type	Communication cost	Communication delay
Ref. [7]	GHZ states	$2N$	$N(D_{\text{BM}}+D_{\text{trans1}})+D_{\text{Pau}}$
Ref. [10]	W states	$3N$	$D_{\text{BM}}+D_{\text{trans1}}+D_{\text{Pau}}$
Ref. [21]	Bell pairs	$2N$	$D_{\text{BM}}+D_{\text{trans1}}+D_{\text{Pau}}$
Ref. [24]	GHZ-Bell states	$4N$	$N(D_{\text{BM}}+D_{\text{trans1}})+D_{\text{Pau}}$
Our protocol	unknown Bell pairs	$2N$	D_{total}

6. Conclusions

In this paper, we present a multi-hop nondestructive teleportation scheme based on the Bell pairs, where the coefficients of quantum channel are unknown to all the communication nodes. With basic Bell measurement, single-qubit unitary operations and channel matching technology, it is possible for the sender and the receiver to share one maximally entangled Bell channel. Then the teleportation can succeed in being independent of the parameters of the Bell pairs. Our scheme

preserves the initial unknown state even if the teleportation fails. Thus, it ensures the security of communication. Besides, our scheme firstly generalizes the teleportation to the scenario independent of channel parameters and auxiliary particles and multi-qubit unitary operations are avoided, which reduces the quantum channel requirement and enhances the flexibility of implementation.

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