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Mobility edges and reentrant localization in one-dimensional dimerized non-Hermitian quasiperiodic lattice*

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The mobility edges and reentrant localization transitions are studied in one-dimensional dimerized lattice with non-Hermitian either uniform or staggered quasiperiodic potentials. We find that the non-Hermitian uniform quasiperiodic disorder can induce an intermediate phase where the extended states coexist with the localized ones, which implies that the system has mobility edges. The localization transition is accompanied by the $\mathcal{PT}$ symmetry breaking transition. While if the non-Hermitian quasiperiodic disorder is staggered, we demonstrate the existence of multiple intermediate phases and multiple reentrant localization transitions based on the finite size scaling analysis. Interestingly, some already localized states will become extended states and can also be localized again for certain non-Hermitian parameters. The reentrant localization transitions are associated with the intermediate phases hosting mobility edges. Besides, we also find that the non-Hermiticity can break the reentrant localization transition where only one intermediate phase survives. More detailed information about the mobility edges and reentrant localization transitions are presented by analyzing the eigenenergy spectrum, inverse participation ratio, and normalized participation ratio.

Keywords: Anderson localization, non-Hermitian quasiperiodic lattice, mobility edges

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1. Introduction

Anderson localization is an important research topic in condensed matter physics and has been extensively studied since the pioneer work of Anderson.[1] The phenomenon of localization induced by random disorder depends crucially on the dimensionality. For example, it is found that any finite disorder can localize all single-particle states in one- or two-dimensional noninteracting disordered systems.[2–4] In three dimension, the system can undergo a metal–insulator transition with the increase of disorder and forms a regime hosting both extended and localized states. The key feature of this hybrid regime is the existence of the mobility edges, which divide the states into extended or localized states by a critical energy. The quasiperiodic systems could display localization transitions at a finite quasiperiodic potential strength in one dimension. A typical quasiperiodic example is the Aubry–André–Harper (AAH) model,[5,6] which exhibits a self-dual symmetry and has an energy-independent localization transition, i.e., all the states are completely extended (or localized) if the quasiperiodic potential strength is smaller (or greater) than the critical value. Therefore, mobility edges do not exist for the original AAH model. However, with the introduction of long-range hopping terms or modified quasiperiodic potentials, many generalized AAH models tends to have exact energy-dependent mobility edges.[7–12] These models with exact mobility edges are beneficial to provide a deeper understanding of Anderson localization in one-dimensional quasicrystals.

Recently, an important progress is that the one-dimensional dimerized lattice with Hermitian staggered quasiperiodic potential can hold two localization transitions with the increase of disorder strength.[13] Generally, all the states remain localized after the localization transition. However, it is not always true in the one-dimensional dimerized quasiperiodic systems. In Ref. [13], the authors find that the competition between the dimerization and quasiperiodic disorder leads to a reentrant localization transition. The reentrant localization transition is defined as follows. After the first localization transition, some already localized states become extended for a range of intermediate disorder strength. With further increase of the quasiperiodic disorder strength, second localization transition arises where all the states become localized again. The reentrant localization transition is associated with intermediate phases hosting the mobility edges. This nontrivial reentrant localization can also be induced by the anisotropic dipole exchange in two-dimensional

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disordered dipolar quantum systems.[14]

In recent years, the Anderson localization and topological phase transitions in various non-Hermitian disordered systems have attracted a lot of attention.[15–27] In general, the non-Hermiticity comes from the nonreciprocal hopping processes or complex onsite potentials, which may induce exotic phenomena without Hermitian counterparts, such as the exceptional points,[28] $\mathcal{PT}$ symmetry breaking,[29] non-Hermitian skin effects,[30–32] as well as the fantastic dynamics and transport properties.[33–35] In the study of Hatano–Nelson model,[36,37] it is found that the asymmetric hopping introduced in one-dimensional Anderson model results in a localization-delocalization transition and the existence of mobility edges. Besides the researches on non-Hermitian Anderson models, the non-Hermitian extension of the AAH models have also been studied in the contexts.[15–21,38] It is shown that the localization transition is accompanied by a topological phase transition in the non-Hermitian AAH models.[15,17,20] The non-Hermitian generalized AAH models with exact mobility edges are also studied.[39–43] However, discussions of the non-Hermitian effect on the reentrant localization in one-dimensional dimerized quasiperiodic systems are still absent.

In this paper, we study the mobility edges and the reentrant localization transitions in one-dimensional dimerized lattice with non-Hermitian either uniform or staggered quasiperiodic potentials. Based on the analysis of energy spectrum, inverse participation ratio (IPR) and normalized participation ratio (NPR), the phase diagrams of the system is obtained. For the non-Hermitian uniform quasiperiodic potentials, the results show that there exists an intermediate phase with mobility edges because of the competition between the dimerization and the quasiperiodic disorder. From the behavior of mobility edges induced by the non-Hermitian potentials and comparing it with the Hermitian case, we find that the present regime of extended phase is reduced while that of localized phase is enlarged, and the localization transition is accompanied by the $\mathcal{PT}$ symmetry breaking transition. For the non-Hermitian staggered quasiperiodic potentials, there exist multiple reentrant localization transitions in a range of disorder strength for some suitable dimerization and non-Hermitian parameters. With the model parameters presented here, the system has three intermediate phases where the localized states coexist with the extended states. Therefore, after the first reentrant of localization transition, subsequent reentrant arises with the further increase of non-Hermitian parameter. The transitions are associated with separate intermediate regimes hosting mobility edges, and the critical regimes of multiple reentrant localization transitions are achieved. The results also indicate that the non-Hermitian parameter can break the reentrant localization transitions if it is large enough.

The rest of the paper is organized as follows. In Section 2, the non-Hermitian extension of one-dimensional dimerized lattice with uniform or staggered quasiperiodic potentials is introduced. In Section 3, the mobility edges in the system with non-Hermitian uniform quasiperiodic disorder are calculated, and the phase diagram is obtained and analyzed. In Section 4, the localization properties of the dimerized lattice with non-Hermitian staggered quasiperiodic potentials are studied, and the multiple reentrant localization transitions are interpreted. The corresponding phase diagram is given. The main results are summarized in Section 5.

2. The model Hamiltonian

Considering a one-dimensional dimerized lattice with non-Hermitian on-site quasiperiodic potential, the model Hamiltonian reads

$$\begin{aligned} H &= t_1 \sum_{n=1}^{N} (c_{n,B}^{\dagger} c_{n,A} + \text{H.c.}) + t_2 \sum_{n=1}^{N-1} (c_{n+1,A}^{\dagger} c_{n,B} + \text{H.c.}) \\ &\quad + \sum_{n=1}^{N} V_A n_{n,A} \cos\left[2\pi(2n-1)\alpha + \mathrm{i}h\right] \\ &\quad + \sum_{n=1}^{N} V_B n_{n,B} \cos\left[2\pi(2n)\alpha + \mathrm{i}h\right]. \end{aligned} \tag{1}$$

In Hamiltonian (1), the boundary condition is the periodic one and the length of the chain is $L = 2N$, which is divided into N unit cells. Denote the two sites in one unit cell as A and B. $c_{n,A}^{\dagger}$ [or $c_{n,B}^{\dagger}$] and $c_{n,A}$ [or $c_{n,B}$] are the creation and annihilation operators on the site (n,A) [or (n,B)], respectively. $n_{n,A}$ and $n_{n,B}$ are the particle number operators on the related sites. The intra- and intercell hopping strength are denoted by t_1 and t_2, respectively. We set the intracell hopping $t_1 = 1$ as the energy unit and define the quantity $\delta = t_2/t_1$ as the hopping dimerization hereinafter. H.c. denotes the Hermitian conjugate terms. $\mathrm{i}h$ is a complex phase factor and $\alpha = (\sqrt{5}-1)/2$ is an irrational number. The strength of the non-Hermitian on-site quasiperiodic potentials at the subsites A and B are characterized by the quantities V_A and V_B, respectively. Since the complex on-site potential $V_n = V_{A/B}\cos(2\pi n\alpha + \mathrm{i}h)$ is quasiperiodic for the site number n and satisfies $V_n = V_{-n}^{*}$, the Hamiltonian (1) possesses $\mathcal{PT}$ symmetry. If the system is in the $\mathcal{PT}$ symmetric phase, the energy spectrum of the Hamiltonian are real. As the non-Hermitian parameter h increases, the $\mathcal{PT}$ symmetry will be broken and complex energies appeared. The quasiperiodic potentials are uniform if $V_A = V_B$ or staggered if $V_A = -V_B$. In the clean limit $V_A = V_B = 0$ and the Hermitian limit $h = 0$, the system (1) reduces to the paradigmatic Su–Schrieffer–Heeger (SSH) model,[44] which exhibits a topological nontrivial-to-trivial phase transition at the critical point $\delta = 1$.

In this paper, the irrational number α is defined as $\alpha = \lim_{m\to\infty}(F_{m-1}/F_m)$, where $\{F_m\}$ are the Fibonacci numbers. The system size is set as $L = F_m$. In the numerical simulation, we

let $L = 1974$ thus there are $N = 987$ unit cells. And we consider the dimerization parameter $\delta = 0.5$ and $\delta = 2.2$ separately.

In order to study the localization properties of the eigenstates of model (1), we introduce two physical quantities: inverse participation ratio (IPR) and normalized participation ratio (NPR).[45,46] For the i-th eigenstate $\psi^i = (\psi_1^i, \psi_2^i, \cdots, \psi_L^i)^{\mathrm{T}}$ where T means the transpose, the IPR and NPR are defined as

$$\mathrm{IPR}^i = \frac{\sum_{n=1}^{L} |\psi_n^i|^4}{\left(\sum_{n=1}^{L} |\psi_n^i|^2\right)^2},$$
$$\mathrm{NPR}^i = \left(L \sum_{n=1}^{L} |\psi_n^i|^4\right)^{-1}. \tag{2}$$

If ψ^i is localized, IPR^i is finite and NPR^i is proportional to $1/L$ which goes to zero at a limit of large L. If ψ^i is extended, $\mathrm{IPR}^i \sim 1/L$ and NPR^i is finite instead. These discussions indicate that IPR^i is sensitive to the localized states while the inverse of NPR^i is sensitive to the extended states. We further average these two quantities over all the eigenstates and obtain

$$\langle \mathrm{IPR} \rangle = \frac{1}{L} \sum_{i=1}^{L} \mathrm{IPR}^i,$$
$$\langle \mathrm{NPR} \rangle = \frac{1}{L} \sum_{i=1}^{L} \mathrm{NPR}^i. \tag{3}$$

$\langle \mathrm{IPR} \rangle$ and $\langle \mathrm{NPR} \rangle$ carry complete information about the extended or localized properties of all the eigenstates of the system. If $\langle \mathrm{IPR} \rangle \sim L^{-1}$ and $\langle \mathrm{NPR} \rangle$ is finite, the system is in the extended phase; If $\langle \mathrm{IPR} \rangle$ is finite and $\langle \mathrm{NPR} \rangle \sim L^{-1}$, the system is in the localized phase; If both $\langle \mathrm{IPR} \rangle$ and $\langle \mathrm{NPR} \rangle$ are finite, some eigenstates are extended and some are localized, which corresponds to the intermediate phase.

$\langle \mathrm{IPR} \rangle$ and $\langle \mathrm{NPR} \rangle$ can be combined as[13,46]

$$\eta = \log_{10} [\langle \mathrm{IPR} \rangle \langle \mathrm{NPR} \rangle], \tag{4}$$

which helps distinguish intermediate phases from fully extended or localized phases clearly. This is because that η is larger than $-\log_{10} L$ in intermediate phases and smaller than $-\log_{10} L$ in fully extended or localized phases. Now, we are ready to compute the non-Hermitian effects and phase diagrams of system (1).

3. Uniform quasiperiodic disorder

We first consider the system with non-Hermitian uniform quasiperiodic disorder, i.e., $V_A = V_B = V$. In the limit of $\delta = 1$, model (1) reduces to the non-Hermitian extension of AAH model which exhibits localization transition without any mobility edges. Considering the general case and focusing on the non-Hermitian effects, we solve system (1) numerically. When the dimerization parameter $\delta = 0.5$, the real parts of energy spectrum versus the strength of quasiperiodic potentials V are shown in Fig. 1(a), where different colors denote

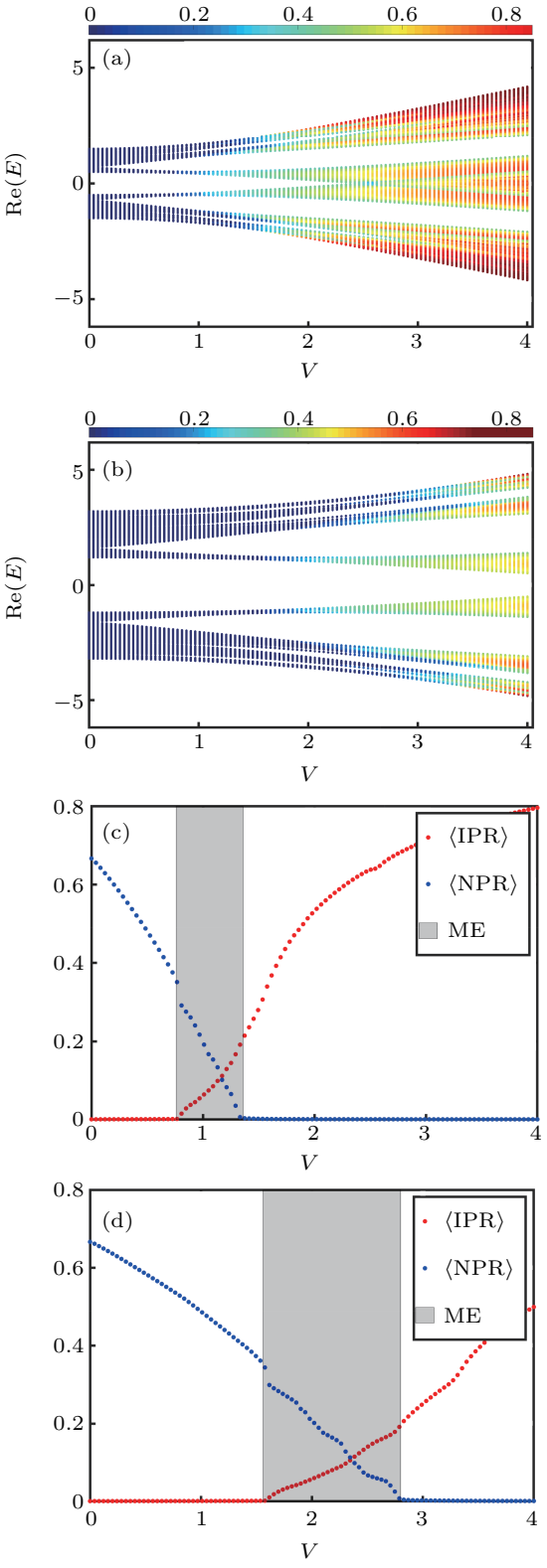


Fig. 1. (a), (b) The real parts of energy spectrum $\mathrm{Re}(E)$ of the system with non-Hermitian uniform disorder versus the strength of quasiperiodic potential V with $\delta = 0.5$ and 2.2, respectively, where different colors denote different values of $\{\mathrm{IPR}^i\}$ for each associated eigenstate. (c), (d) $\langle \mathrm{IPR} \rangle$ (red dots) and $\langle \mathrm{NPR} \rangle$ (blue dots) as the functions of V with $\delta = 0.5$ and 2.2, respectively. The shaded regimes are the intermediate phases with mobility edges. Here the non-Hermitian parameter is $h = 0.05$.

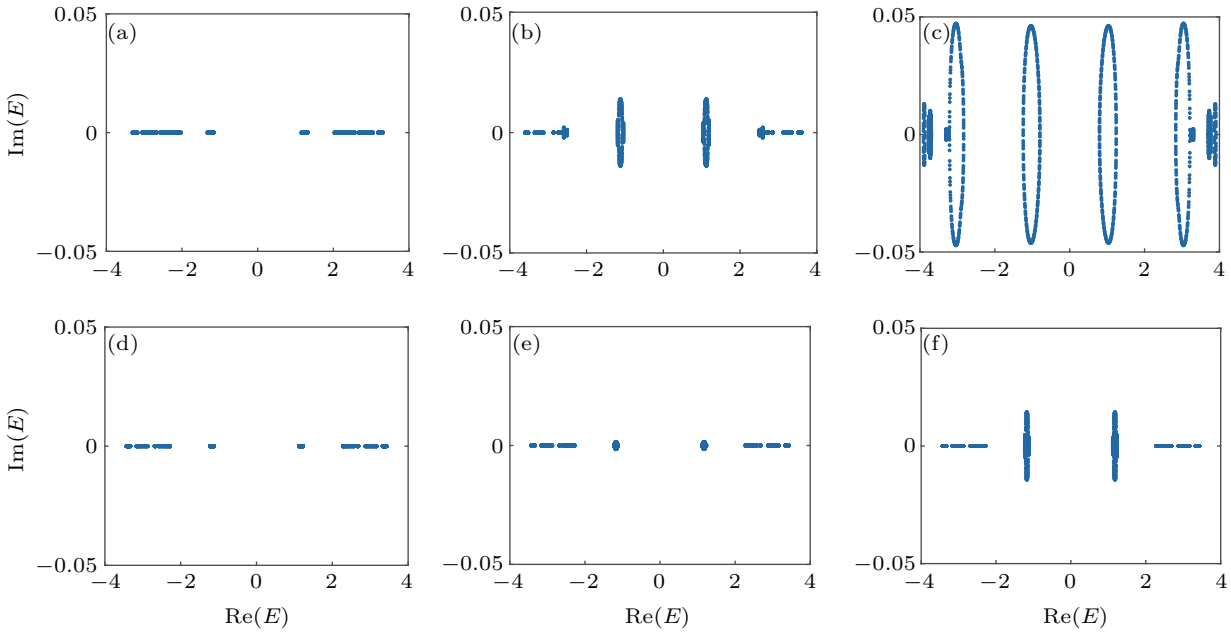


Fig. 2. The energy spectrum of the Hamiltonian (1) with non-Hermitian uniform quasiperiodic potential on the complex plane: (a) $V = 1.0$, (b) $V = 2.0$, and (c) $V = 3.0$ for $h = 0.05$, $\delta = 2.2$; (d) $h = 0.05$, (e) $h = 0.125$, and (f) $h = 0.2$ for $V = 1.5$, $\delta = 2.2$.

different values of $\{\text{IPR}^i\}$ for each associated eigenstate and the non-Hermitian parameter $h = 0.05$. Localization transition and presence of the mobility edges can be easily seen from Fig. 1(a), as well as that all the eigenstates are extended in weak disorder. With the increase of V, some eigenstates with real middle energy spectrum become localized, while the eigenstates with the edge energy spectrum keep extended. Thus the extended states coexist with the localized ones and the system has mobility edges. With the further increase of V, all the eigenstates are localized and the system enters into the localized phase. The resultant $\langle\text{IPR}\rangle$ and $\langle\text{NPR}\rangle$ are shown in Fig. 1(c) as the red and blue dots, respectively. Therefore, there exists an intermediate phase with mobility edges where both $\langle\text{IPR}\rangle$ and $\langle\text{NPR}\rangle$ are finite (shaded regime for $0.76 < V < 1.36$). As can be clearly seen from Fig. 1(c), the system is in the extended phase when $V < 0.76$, while for an intermediate disorder strength $0.76 < V < 1.36$ the system is in the intermediate phase and has both extended and localized states with mobility edges. As the quasiperiodic potential strength $V > 1.36$, the system undergoes a localization transition where all the states become localized. Then we consider the case of $\delta = 2.2$. The real parts of energy spectrum and $\{\text{IPR}^i\}$ as the functions of V are presented in Fig. 1(b), and the corresponding $\langle\text{IPR}\rangle$ and $\langle\text{NPR}\rangle$ are shown in Fig. 1(d). It can be seen that the mobility edges have a shift and the present range is $1.56 < V < 2.80$, which implies that the competition between the dimerization and non-Hermitian uniform quasiperiodic disorder leads to the existence of intermediate phases with mobility edges.

The $\mathcal{PT}$ symmetry of system (1) can be broken by the disorder, as can be seen from the energy spectrum of the model with different disorder strengths shown in Fig. 2. All the energies presented in Figs. 2(a) and 2(d) are real, which means that the system is in the $\mathcal{PT}$ symmetric phase as well as the extended phase. However, from Figs. 2(b) and 2(e), we find with the increase of disorder strength V and non-Hermitian parameter h, some energies become complex thus the system is in the $\mathcal{PT}$ symmetry-broken phase as well as the intermediate phase, which implies that there must be a $\mathcal{PT}$ phase transition at certain critical value of V or h. On further increasing V or h, it can be seen from Figs. 2(c) and 2(f) that the eigenenergies are complex and the system is still in the $\mathcal{PT}$ symmetry-broken phase. However, this $\mathcal{PT}$ symmetry-broken phase is localized. Therefore, it can be stated that localization transition in the system is accompanied by the $\mathcal{PT}$ symmetry-breaking transition.

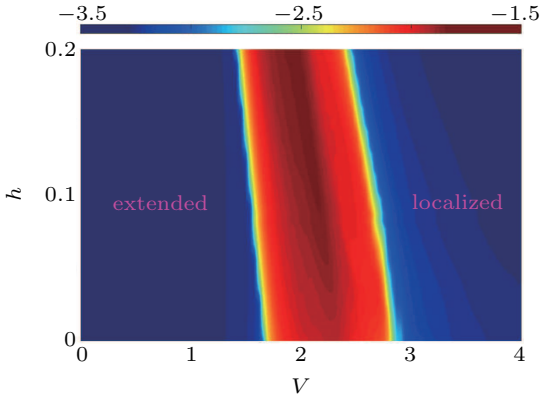


Fig. 3. Phase diagram of the system with non-Hermitian uniform disorder in V–h plane with dimerization parameter $\delta = 2.2$. Different colors indicate different values of η. The dark blue regimes are extended phase and localized phase, while the dark red regime is the intermediate phase with mobility edges.

Complete phase diagram of the system in V–h plane with $\delta = 2.2$ is obtained by the computation of quantity η given by Eq. (4). As shown in Fig. 3, we find that system (1) has

three different phases such as extended, intermediate, and localized phases. Note that all the eigenstates are extended in completely extended phase while localized in completely localized phase. The intermediate phase, where extended states coexist with localized states, has mobility edges. With the increase of non-Hermitian parameter h, the extended regime decreases while the localized regime increases. It is found that the localization transition arises readily in contrast with the Hermitian case.

4. Staggered quasiperiodic disorder

In this section, we focus on the mobility edges and reentrant localization in system (1) with non-Hermitian staggered quasiperiodic disorder, i.e., $V_A = -V_B = V$. Figure 4 presents the real parts of energy spectrum, $\{\text{IPR}^i\}$ for each eigenstate, $\langle\text{IPR}\rangle$ and $\langle\text{NPR}\rangle$ versus different values of V with the model parameter $\delta = 0.5$ and 2.2. We first analyse the case of $\delta = 0.5$. It is seen from Figs. 4(a) and 4(c) that there are two transition points $V_1 = 0.40$ and $V_2 = 1.40$ due to the competition between the dimerization and the staggered disorder. For weak disorder strength $V < V_1$, all the eigenstates are extended and the system has real eigenvalues. If the disorder strength is larger than V_1 and smaller than V_2, i.e., $V_1 < V < V_2$, the eigenstates of the edge spectrum gradually become localized while the eigenstates of the middle spectrum are still extended, which means that extended states and localized states can coexist within a finite interval of V. If the disorder strength is very large, i.e., $V > V_2$, all the eigenstates become localized and the system enters into the localized phase. From the above analysis, it can be concluded that the system is in extended phase if $V < V_1$, or in intermediate phase if $V_1 < V < V_2$, or in localized phase if $V > V_2$.

The most interesting thing happens when δ is set as some suitable values. For example, when $\delta = 2.2$ and $h = 0.05$, the real part of energy spectrum is shown in Fig. 4(b) and the corresponding $\langle\text{IPR}\rangle$ and $\langle\text{NPR}\rangle$ are shown in Fig. 4(d). From Fig. 4(d), we find that there is more than one localization transition with the increase of disorder strength V. For $V < 0.84$, the energy spectrum is real and the system is in the extended phase. For $0.84 < V < 1.64$, the edge spectrum is complex and the associated edge eigenstates are localized. Thus the system is in the intermediate phase. For $1.64 < V < 2.24$, all the eigenenergies are complex and the system is in the fully localized phase. For $2.24 < V < 2.40$, some eigenenergies in the middle spectrum are real and the system is in the intermediate phase. For $2.40 < V < 2.48$, all the eigenenergies are complex and the system is in the fully localized phase. For $2.48 < V < 2.84$, some eigenenergies in the middle spectrum are real and the corresponding states are extended. For $V > 2.84$, all the eigenenergies are complex and the states are fully localized again.

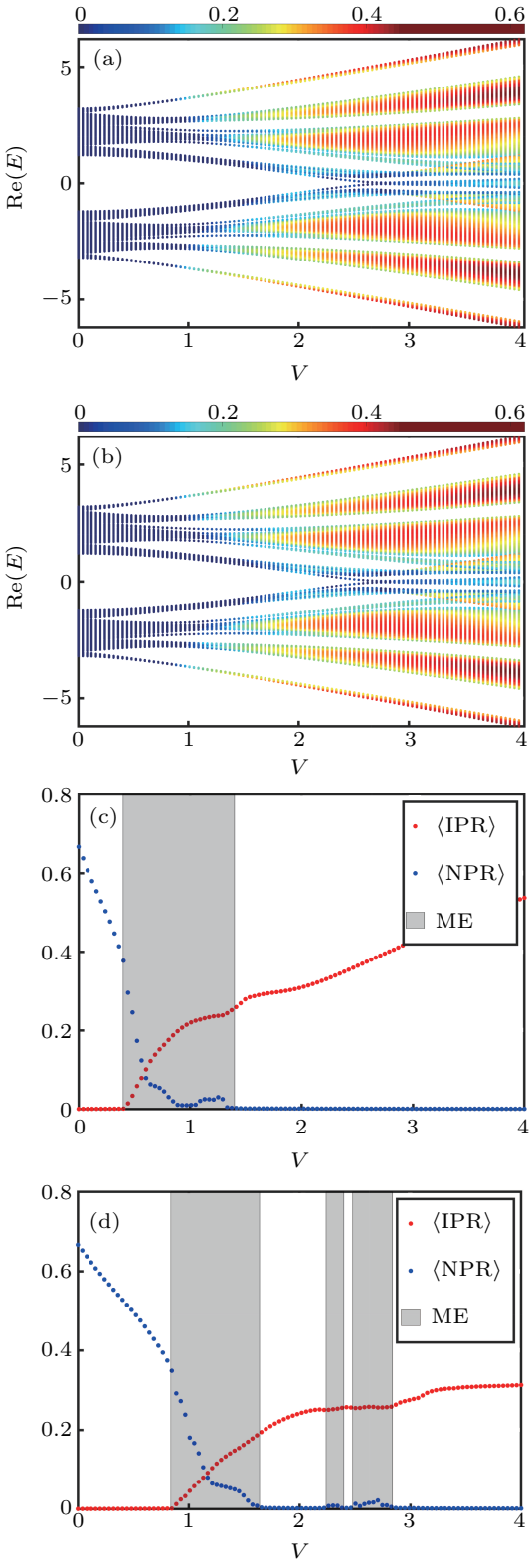


Fig. 4. (a), (b) The real parts of energy spectrum $\text{Re}(E)$ of the system with non-Hermitian staggered disorder versus the strength of quasiperiodic potential V with $\delta = 0.5$ and 2.2, respectively, where different colors denote different values of $\{\text{IPR}^i\}$ for each associated eigenstate. (c), (d) $\langle\text{IPR}\rangle$ (red dots) and $\langle\text{NPR}\rangle$ (blue dots) as the functions of V with $\delta = 0.5$ and 2.2, respectively. The shaded regimes are the intermediate phases with mobility edges. Here the non-Hermitian parameter is $h = 0.05$.

Then it can be concluded that there exist three localization transitions induced by the competition between the dimerization and staggered disorder. These reentrant localization

transitions are associated with the intermediate phases hosting mobility edges. In the regimes of intermediate phases, i.e., for $0.84 < V < 1.64$, $2.24 < V < 2.40$, and $2.48 < V < 2.84$, both the $\langle \text{IPR} \rangle$ and $\langle \text{NPR} \rangle$ are finite. Note that these three localization transitions belong to multiple reentrant localization transitions. The reentrant localization transition refers to that the already localized states first become extended and then become localized again. The localization transitions which can be reentered again are commonly called multiple reentrant localization transitions. Therefore, we demonstrate the existence of multiple reentrant localization transitions in the present non-Hermitian system. It should be remarked that multiple reentrant localization transitions are quite different from those in Hermitian case ($h = 0$) which has only two intermediate phases and exhibits one reentrant localization transition. Multiple reentrant localization transitions are also different from those in the non-dimerization quasiperiodic system ($\delta = 1$) where the states remain localized once the disorder strength V passes through the transition point.

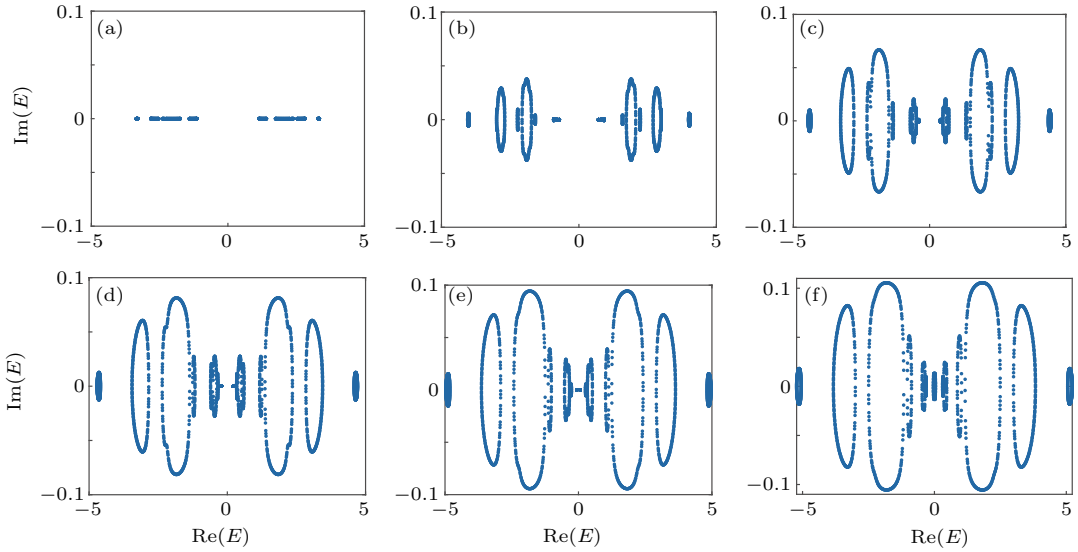


Fig. 5. The energy spectrum of the Hamiltonian (1) with non-Hermitian staggered quasiperiodic potential on the complex plane: (a) $V = 0.5$, (b) $V = 1.5$, (c) $V = 2.0$, (d) $V = 2.3$, (e) $V = 2.6$, (f) $V = 2.9$. Here the model parameters are $h = 0.05$ and $\delta = 2.2$.

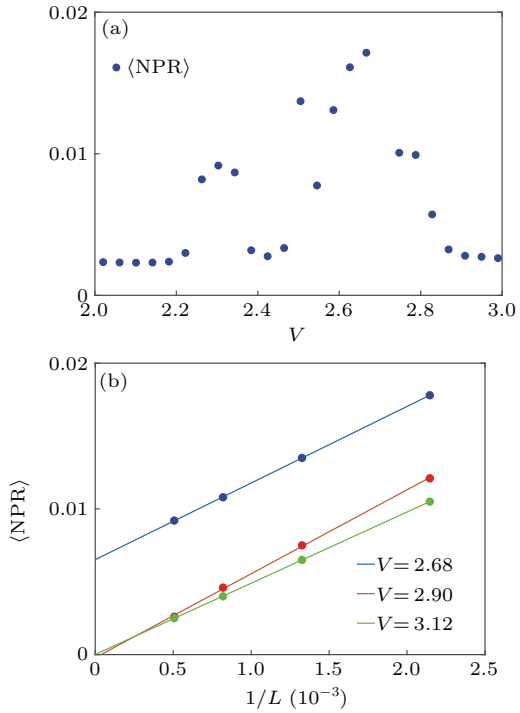


Fig. 6. (a) The values of $\langle \text{NPR} \rangle$ for $2.0 < V < 3.0$ with the model parameters $\delta = 2.2$ and $h = 0.05$. (b) The finite size scaling behaviors of $\langle \text{NPR} \rangle$. The data can be fitted as $\langle \text{NPR} \rangle = 5.76L^{-1}$ for $V = 2.90$, $\langle \text{NPR} \rangle = 5.25L^{-1} + 0.0065$ for $V = 2.68$, and $\langle \text{NPR} \rangle = 4.88L^{-1}$ for $V = 3.12$. At the thermodynamic limit, $\langle \text{NPR} \rangle \neq 0$ only in the case of $V = 2.68$.

In order to double check the above conclusion, the energy spectrum is calculated for specific values of disorder strength V in the regime of each phase, and the results are shown in Fig. 5. It is clear that all results presented in Figs. 5(a)–5(f) are consistent with those obtained from Fig. 4(d).

We should remark that the third intermediate regime is narrow and the corresponding values of $\langle \text{NPR} \rangle$ are small, and detailed values of $\langle \text{NPR} \rangle$ for $2.0 < V < 3.0$ are shown in Fig. 6(a). Obviously, the values are finite and much larger than the required $1/L \approx 0.0005$. In order to confirm the nonzero value of $\langle \text{NPR} \rangle$ at the thermodynamic limit $L \to \infty$, we perform the finite size scaling analysis of $\langle \text{NPR} \rangle$. The values of $\langle \text{NPR} \rangle$ in different system sizes with $V = 2.68$, 2.90, and 3.12 are indicated in Fig. 6(b). As can be seen, $\langle \text{NPR} \rangle = 0$ at the thermodynamic limit for $V = 2.90$ and 3.12, which means that the system is in the localized phase. The value of $\langle \text{NPR} \rangle$ keeps finite at the thermodynamic limit in the case of $V = 2.68$, which means that the system is in the intermediate phase. These results demonstrate that there exists the third stable intermediate phase.

By the computation of η, we obtain the phase diagram of the system in V–h plane shown in Fig. 7. As can be seen from Fig. 7, if the non-Hermitian parameter $h = 0$, system (1)

degenerates into the Hermitian version which has two stable intermediate phases. If $h < 0.025$, there are two intermediate regimes and the system has reentrant localization. However, compared with the Hermitian case, the extended regime is smaller and localized regime is larger. If $h = 0.05$, there exist three intermediate phases and the system undergoes three localization transitions. With the increase of h, the extended regime decreases while the localized regime increases. The behavior of first intermediate regime is similar to that of the system with non-Hermitian uniform quasiperiodic potential, while the other intermediate phases regimes gradually disappear. If h is large enough, only one intermediate phase will survive and there is no reentrant localization transition. Note that three intermediate phases and multiple reentrant localization transitions are missing in the Hermitian case.

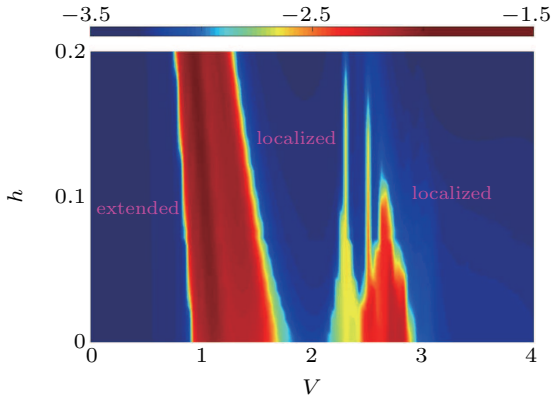


Fig. 7. The phase diagram of the system with non-Hermitian staggered disorder in V–h plane with the dimerization parameter $\delta = 2.2$. Different colors indicate different values of η. The dark blue regimes are extended or localized phases, while the remanent regimes are the intermediate phases with mobility edges.

5. Summary

In this paper, we have studied the mobility edges and the localization transitions in one-dimensional dimerized lattice with non-Hermitian quasiperiodic disorders. In order to avoid perturbation of the boundary effects, we consider the system with periodic boundary condition. Based on the analysis of energy spectrum, $\langle \mathrm{IPR} \rangle$ and $\langle \mathrm{NPR} \rangle$, we obtain the phase diagrams of the system, as shown in Figs. 3 and 7.

If the non-Hermitian quasiperiodic potentials are uniform, the system has three different phases such as extended, intermediate, and localized phases. We find that there is only one intermediate phase which has the mobility edges. With the increase of non-Hermitian parameter h, the extended regime decreases while the localized regime increases. Thus the localization transition arises more readily than the Hermitian case. It should be noted that the localization transition is accompanied by the $\mathcal{PT}$ symmetry-breaking transition.

When the non-Hermitian quasiperiodic disorder potentials are staggered, we find that besides the extended and localized phases, the system also has three intermediate phases hosting mobility edges for certain values of non-Hermitian parameters. The behavior of first intermediate phase is similar to that of uniform case. Because the regimes of the second and third intermediate phases are small, based on the finite size scaling analysis, we demonstrate that these two phases indeed exist and the corresponding $\langle \mathrm{NPR} \rangle \neq 0$ at the thermodynamic limit with system size $L \to \infty$. The system undergoes three localization transitions through three intermediate phases. Note that these three localization transitions belong to the so-called multiple reentrant localization transitions demonstrated in the present non-Hermitian system. It should be remarked that the multiple reentrant localization transitions are stable and will arise within a finite interval of disorder strength V with suitable dimerization and non-Hermitian parameters. It is also found that the non-Hermitian parameter can break the reentrant localization transition, which is due to the fact that if h is large enough, only one intermediate phase survives and no reentrant localization transition arises. These results are quite different from those in the Hermitian case.

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