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Modeling and character analyzing of multiple fractional-order memcapacitors in parallel connection

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Recently, the memory elements-based circuits have been addressed frequently in the nonlinear circuit theory due to their unique behaviors. Thus, the modeling and characterizing of the mem-elements become essential. In this paper, the analysis of the multiple fractional-order voltage-controlled memcapacitors model in parallel connection is studied. Firstly, two fractional-order memcapacitors are connected in parallel, the equivalent model is derived, and the characteristic of the equivalent memcapacitor is analyzed in positive or negative connection. Then a new understanding manner according to different rate factor K and fractional order α is derived to explain the equivalent modeling structure conveniently. Additionally, the negative order appears, which is a consequence of the combination of memcapacitors in different directions. Meanwhile, the equivalent parallel memcapacitance has been drawn to determine that multiple fractional-order memcapacitors could be calculated as one composite memcapacitor. Thus, an arbitrary fractional-order equivalent memcapacitor could be constructed by multiple fractional-order memcapacitors.

Keywords: memcapacitor, fractional calculus, parallel connection, equivalent model

PACS: 84.30.Bv, 07.50.Ek **DOI:** 10.1088/1674-1056/ac05b1

1. Introduction

In 1971, Leon O. Chua postulated a new electrical element called memristor, which described the missing link between the charge q and the flux φ.[1] Then the notion of memristor was extended to the memristive system in 1976[2] and generalized to inductive and capacitive elements, named as meminductor and memcapacitor in 2009.[3] As the name implicated, mem-elements have the memory function for the state of history, and their parameter values depend on the polarity, magnitude, and time of the excitation that have been applied. Since a practical memristor device was first realized by HP laboratory in 2008,[4] more researchers began to engage in the related research.[5,6]

At present, many people are working on actual devices or materials[7–10] in the field of the memristor. Although these articles are very creative, the low productivity in laboratories and the different performance of these devices block the advancement of this mem-element. As a forward-looking and instructive work, modeling of memristor can identify some of the potentials of devices that have not yet been applied and guide the research in materials science. Thus, many people have constructed different theoretical models based on the real memory memristors and analyzed the characteristics of their models.[11–14]

Similar to the memristor, memcapacitor is also difficult to manufacture. The modeling of memcapacitor is worth discussing. Up to now, the SPICE circuit of memcapacitor using essential circuit elements is realized.[15] At the same time, the design of memcapacitor emulator based on memristor[16] is proposed because there are constitutive relationships between memory devices. Recently, using the memcapacitor to implement the adaptive neuron is becoming a hot spot.[17,18] There are many related types of research based on the integer-order memcapacitor,[19–21] but few of them are based on the fractional-order memcapacitor.

Fractional calculus[22] is the generalization of integer order calculus, which extends the order of calculus functions from integer to non-integer. Due to the natural objects are generally fractional, it is more accurate than the classical integer methods by using the technique of fractional calculus to describe a real object. Memory effect is another character in fractional calculus. Growing evidence has shown that fractional differential equations can describe many dynamical systems, such as fractional order control, oscillator, chaos systems, and other applications.[23–26] As a result, fractional calculus is used to build an accurate memory device model in this paper.

Fractional-order mem-elements[27,28] are new branches deriving from the above fields. Some researchers have already done the essential work about their characters.[29,30] With the consideration of the memory effect of memory elements, the technique of fractional calculus is used to analyze the characteristics of memcapacitor. The new chaotic system is proposed recently,[31,32] which has been used as a critical component in the fractional-order memcapacitor (denoted as FOMC). Meanwhile, the dynamic characteristics of fractional-order memcapacitor are discussed.[33] Although there are many pieces of

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related research about FOMC in the neural network[34] and analogue logic circuit,[35] many important areas are still undiscovered.

When there are two or more FOMCs connected in parallel, the dynamic characters are worth considering.[36] Therefore, the model of fractional-order memcapacitors in parallel connection is investigated, and some interesting phenomena are observed in this paper.

This paper is arranged as follows: In Section 2, the basic concept of relations of circuit elements and fractional calculus are recalled. Moreover, a generalized model of the FOMC is introduced. In Section 3, the mathematical model of parallel fractional-order is deduced. In Section 4, a simple form of two FOMCs in parallel is analyzed, and a new way to analyze the FOMCs in parallel connection is proposed. In Section 5, the equivalent memcapacitor of multiple fractional-order memcapacitors in parallel connection is constructed, and the effects of ρ and θ are analyzed separately. Finally, the conclusions are given in Section 6.

2. Basic concepts

2.1. Relations of circuit elements

There are four fundamental circuit variables, namely, electric current i, voltage v, charge q, and magnetic flux φ, describing the circuit dynamics. The definition of electric current i is the time derivative of electric charge q ($i = \mathrm{d}q/\mathrm{d}t$). According to the Faraday's law, voltage v is the time derivative of magnetic flux φ ($v = \mathrm{d}\varphi/\mathrm{d}t$).

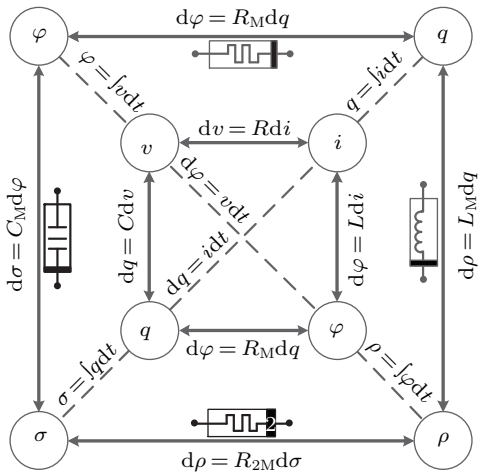


Fig. 1. Relations of the fundamental circuit variables.

As mentioned in the other research, Chua noted that another three relations connected pairs of the four fundamental circuit variables and proposed the concepts of memristor, meminductor, and memcapacitor.[1] The memristor is defined by the relationship between the charge q and the flux φ: $R_\mathrm{M} = \mathrm{d}\varphi/\mathrm{d}q$, the meminductor is defined by the relationship between the time integration of flux ($\rho = \int \varphi \mathrm{d}t$) and the charge q: $L_\mathrm{M} = \mathrm{d}\rho/\mathrm{d}q$, and the memcapacitor is defined by the relationship between the time integration of charge ($\sigma = \int q \mathrm{d}t$) and the flux: $C_\mathrm{M} = \mathrm{d}\sigma/\mathrm{d}\varphi$ as shown in Fig. 1.

The memcapacitor is very important in the circuit principle and neural network application research, but the relevant study is far from enough. This paper focuses on the modeling and characteristics of memcapacitor to fulfil imperfections in this part.

2.2. Fractional calculus

Fractional order calculus is the general expansion of the integer order calculus, extending it to an arbitrary order. Growing evidence has shown that fractional differential equations can describe many dynamical systems, for example, fractional order control, DC–DC converters, and other different applications.[13,14] In this section, the fractional-order calculus is introduced in the traditional memcapacitor to make the model more accurate. The ${}_{t_0}\mathscr{D}_t^\alpha$ is the fundamental operator used to denote α-order fractional integral or derivative, defined as

$$ {}_{t_0}\mathscr{D}_t^\alpha f(t) = \begin{cases} \frac{\mathrm{d}^\alpha}{\mathrm{d}t^\alpha} f(t), & \alpha > 0, \\ f(t), & \alpha = 0, \\ \int_{t_0}^t f(t)(\mathrm{d}\tau)^{-\alpha}, & \alpha < 0, \end{cases} \tag{1} $$

where $\alpha \in R$ is the order of the operator and t_0 is the limits of the α-order integral. When $t_0 = 0$, ${}_{t_0}\mathscr{D}_t^\alpha$ can be denoted as $\mathscr{D}_t^\alpha$.

There are several definitions of fractional calculus. The most commonly used definitions are Caputo definition ${}_{t_0}^{\mathrm{C}}\mathscr{D}_t^\alpha f(t)$, Riemann–Liouville definition ${}_{t_0}^{\mathrm{RL}}\mathscr{D}_t^\alpha f(t)$ and Grünwald–Letnikov definition ${}_{t_0}^{\mathrm{GL}}\mathscr{D}_t^\alpha f(t)$.

Riemann–Liouville definition and Grünwald–Letnikov definition are equivalent when $f(t)$ is of class C^m, where $m-1 \le \alpha < m$, $m \in N$. Grünwald–Letnikov definition is more likely a numerical method used for the calculation of Riemann–Liouville α-order fractional derivative.

The sine function is extremely important for the basic circuit theory, so the sinusoidal resource is mostly used in this paper. By considering the Riemann–Liouville definition and the mathematical method proposed by Radwan *et al.* in 2013,[38] the fractional-order sine and cosine functions could be calculated as

$$ \begin{cases} \mathscr{D}_t^{-\alpha} \sin(2\pi n f t) = \frac{2\pi n f t^{\alpha+1} {}_1F_2\left[1; 1+\frac{\alpha}{2}, \frac{3+\alpha}{2}; -\pi^2 n^2 f^2 t^2\right]}{\Gamma(\alpha+2)}, \\ \mathscr{D}_t^{-\alpha} \cos(2\pi n f t) = \frac{t^\alpha {}_1F_2\left[1; \frac{\alpha+1}{2}, 1+\frac{\alpha}{2}; -\pi^2 n^2 f^2 t^2\right]}{\Gamma(\alpha+1)}, \\ {}_1F_2\left[a; (b_1, b_2); z\right] = \sum_{n=0}^{\infty} \frac{(a)_n}{(b_1)_n (b_2)_n} \frac{z^n}{n!}; \left\langle (x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} \right\rangle. \end{cases} \tag{2} $$

When $t \gg 0$, both sides of the formula have changed from an intermediate state to a steady state,[39] and Eq. (2) could be considered as

$$ \begin{cases} \mathscr{D}_t^{-\alpha} \sin(\omega t) \approx \omega^{-\alpha} \sin(\omega t - \alpha\frac{\pi}{2}), \\ \mathscr{D}_t^{-\alpha} \cos(\omega t) \approx \omega^{-\alpha} \cos(\omega t - \alpha\frac{\pi}{2}). \end{cases} \tag{3} $$

When time t tends to infinity, the system is in a stable state, and the sign of approximately equal approaches to the equal.

2.3. Modelling of fractional-order memcapacitor

Since the memcapacitor could not be physically implemented with stability and reliability, many emulating methods have been presented to study their dynamic behaviors. In general, mathematical modeling is the most effective method to analyze the circuit based on mem-elements. Figure 2 shows a voltage-controlled FOMC model.

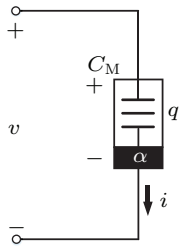


Fig. 2. A voltage-controlled fractional-order memcapacitor model.

According to the generalized model of FOMC proposed in Ref. [28], the state equation of the voltage-controlled FOMC model is given by

$$\begin{cases} q(t) = C_{\mathrm{M}}(x)v(t), \\ C_{\mathrm{M}}(x) = C_{\min} + C_{\mathrm{d}}x(t), \\ x = x_0 + \eta k D_t^{-\alpha} v, \end{cases} \tag{4}$$

where $q(t)$ is the accumulated charge on the memcapacitor at time t, and $v(t)$ is the corresponding voltage. $C_{\mathrm{M}}(x)$ is the memcapacitance, which depends on the state of the system, $C_{\mathrm{d}} = C_{\mathrm{M}\max} - C_{\mathrm{M}\min}$ ($C_{\mathrm{M}\max}$ and $C_{\mathrm{M}\min}$ are the boundaries of the memcapacitance), $x(t)$ represents the state variable of the memcapacitor with $x_0 = (C_0 - C_{\mathrm{M}\min})/C_{\mathrm{d}}$, where C_0 is the initial value of capacitance and x_0 is the initial value of x. $\eta = \pm 1$ represents the connection way with the applied source.

Accordingly, Eq. (4) could also be expressed as

$$\begin{cases} q(t) = C_{\mathrm{M}} v(t), \\ C_{\mathrm{M}} = C_{\min} + C_{\mathrm{d}}(x_0 + \eta k D_t^{-\alpha} v) \\ \qquad = (C_{\min} + C_{\mathrm{d}} x_0) + \eta k C_{\mathrm{d}} D_t^{-\alpha} v \\ \qquad = C_0 + \eta K D_t^{-\alpha} v, \end{cases} \tag{5}$$

where $C_0 = C_{\min} + C_{\mathrm{d}} x_0$, and $K = kC_{\mathrm{d}}$. Thus, the FOMC model could be fully described with four parameters, which can be denoted as $(C_0, K, \alpha, \eta)_{C_{\mathrm{M}}}$.

In this paper, the parallel-connected FOMC has been analyzed with sinusoidal excitation, the voltage across the memcapacitor is $v(t) = V_{\mathrm{m}} \sin(\omega t)$, and the ω in this paper is limited to 1 for convenience.

Since other model parameters, such as the frequency and amplitude of excitation, have already been discussed in other papers,[29] there will be no more detailed analysis about them in this paper.

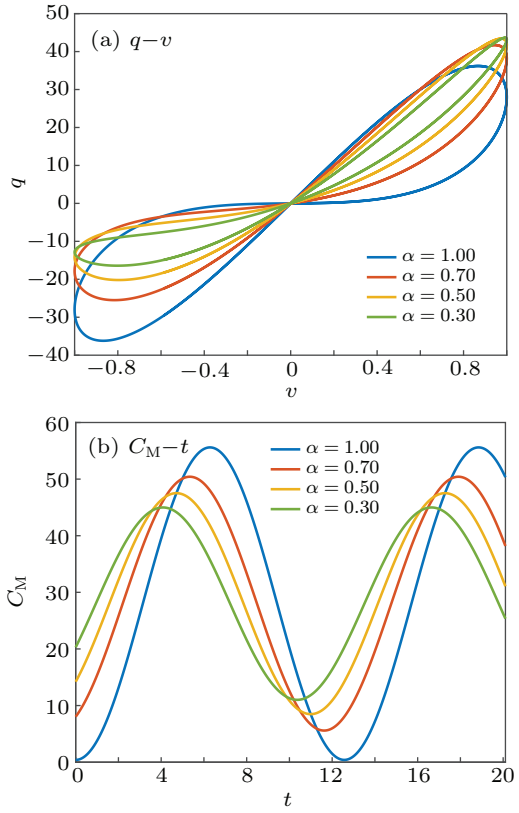


Fig. 3. Character of fractional-order memcapacitor under sinusoidal signal with different order, $\alpha = 0.3, 0.5, 0.7, 1$.

The voltage-controlled memcapacitor model has the characteristic of zero-crossing and hysteresis loop (charge-voltage hysteretic curve) when a sinusoidal voltage is applied, as shown in Fig. 3. The pinch hysteresis loop is an essential characteristic for the memory elements.[37] Figure 3(a) shows the influence of the order α on the charge-voltage hysteretic curves of the voltage-controlled memcapacitor model. It is found that the curve is bent upward as the parameter order α decreases, and the hysteresis curves distorted, which causes it to be asymmetric about the origin.

3. Modeling of two FOMCs in parallel

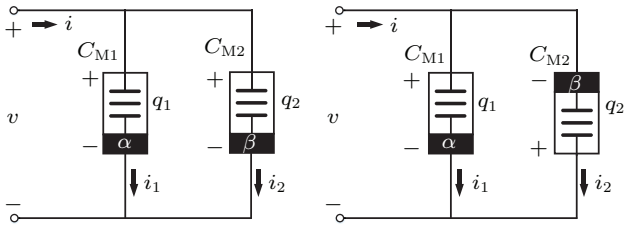
The $\eta_{\mathrm{c}} = 1$ represents the identical connection, and $\eta_{\mathrm{c}} = -1$ represents the opposite connection of two FOMCs in parallel. Figure 4 shows the different connections of FOMCs in parallel.

The mathematic model of single voltage-controlled FOMC is given by Eq. (3). With the consideration of two FOMCs connected in parallel, the generalized mathematic model can be obtained as

$$\begin{cases} q(t) = C_{\mathrm{Ms}} v(t), \\ C_{\mathrm{Ms}} = C_{\mathrm{M1}} + C_{\mathrm{M2}}, \\ C_{\mathrm{M1}} = C_{01} + \eta_1 K_1 D_t^{-\alpha_1} v, \\ C_{\mathrm{M2}} = C_{02} + \eta_2 K_2 D_t^{-\alpha_2} v. \end{cases} \tag{6}$$

Conventionally, the sinusoidal signal is the key in the analysis of any periodic signal. The response of circuit elements is extremely important in many applications. Thus, in

this section, the effect of changing the fractional order of the FOMCs will be studied when a single tone voltage is applied and given by $v(t) = V_{\rm M}\sin\omega t$ with $V_{\rm M} = 1$.



(a) Negetiveconnection (b) Positiveconnection

Fig. 4. Different connections of FOMCs in parallel.

Naturally, when two FOMCs are connected in parallel, the $C_{\rm Ms}$ can be presented as

$$\begin{aligned} C_{\rm Ms} &= C_{\rm M1} + C_{\rm M2} \\ &= C_{01} + \eta_1 K_1 D_t^{-\alpha_1} v + C_{02} + \eta_2 K_2 D_t^{-\alpha_2} v \\ &= C_{01} + \eta_1 K_1 \omega^{-\alpha_1} \sin\left(\omega t - \alpha_1 \frac{\pi}{2}\right) \\ &\quad + C_{02} + \eta_2 K_2 C_{d2} \omega^{-\alpha_2} \sin\left(\omega t - \alpha_2 \frac{\pi}{2}\right) \\ &= (C_{01} + C_{02}) \\ &\quad + \eta_1 \sin(\omega t)\left[K_1 \omega^{-\alpha_1}\cos\alpha_1\frac{\pi}{2} + \frac{\eta_2}{\eta_1} K_2 \omega^{-\alpha_2}\cos\alpha_2\frac{\pi}{2}\right] \\ &\quad - \eta_1 \cos(\omega t)\left[K_1 \omega^{-\alpha_1}\sin\alpha_1\frac{\pi}{2} + \frac{\eta_2}{\eta_1} K_2 \omega^{-\alpha_2}\sin\alpha_2\frac{\pi}{2}\right]. \end{aligned} \tag{7}$$

To simplify Eq. (7), A and B are defined as

$$\begin{cases} A = K_1 \omega^{-\alpha_1}\cos\alpha_1\frac{\pi}{2} + \eta_1\eta_2 K_2 \omega^{-\alpha_2}\cos\alpha_2\frac{\pi}{2}, \\ B = K_1 \omega^{-\alpha_1}\sin\alpha_1\frac{\pi}{2} + \eta_1\eta_2 K_2 \omega^{-\alpha_2}\sin\alpha_2\frac{\pi}{2}, \end{cases} \tag{8}$$

where $\eta_2/\eta_1 = \eta_1\eta_2$.

Then, the connection of two FOMCs can be rewritten as follows:

$$\begin{aligned} C_{\rm Ms} &= (C_{01}+C_{02}) + \eta_1\left[\sin(\omega t)A - \cos(\omega t)B\right] = (C_{01}+C_{02}) + \eta_1\frac{|A|}{A}\left[\sin(\omega t)|A| - \cos(\omega t)B\frac{|A|}{A}\right] \\ &= (C_{01}+C_{02}) + \eta_1\frac{|A|}{A}\sqrt{A^2+B^2}\sin\left(\omega t - \arctan\frac{B}{A}\right) = C_{0\rm s} + \eta_{\rm s}K_{\rm s}\omega^{-\alpha_{\rm s}}\sin\left(\omega t - \alpha_{\rm s}\frac{\pi}{2}\right), \end{aligned} \tag{9}$$

And the corresponding parameters of $C_{\rm Ms}$ are given as

$$\begin{cases} C_{0\rm s} = C_{01} + C_{02}, \quad \eta_{\rm s} = \eta_1\frac{|A|}{A}, \quad \alpha_{\rm s} = \arctan\frac{B}{A}\Big/\frac{\pi}{2} = \frac{2}{\pi}\arctan\frac{K_1\omega^{-\alpha_1}\sin\alpha_1\frac{\pi}{2} + \eta_1\eta_2 K_2\omega^{-\alpha_2}\sin\alpha_2\frac{\pi}{2}}{K_1\omega^{-\alpha_1}\cos\alpha_1\frac{\pi}{2} + \eta_1\eta_2 K_2\omega^{-\alpha_2}\cos\alpha_2\frac{\pi}{2}}, \\ K_{\rm s} = \frac{\sqrt{A^2+B^2}}{\omega^{-\alpha_{\rm s}}} = \omega^{\alpha_{\rm s}}\sqrt{(K_1\omega^{-\alpha_1})^2 + (K_2\omega^{-\alpha_2})^2 + 2K_1K_2\omega^{-(\alpha_1+\alpha_2)}\eta_1\eta_2\cos(\alpha_1-\alpha_2)\frac{\pi}{2}}. \end{cases} \tag{10}$$

The memcapacitance of the FOMC is consisted of $\{C_0, K, \alpha, \eta\}$. If these FOMCs are denoted as $(C_0, K, \alpha, \eta)_{C_{\rm M}}$, this equivalent process could be denoted as

$$\begin{aligned} &(C_{01}, K_1, \alpha_1, \eta_1)_{C_{\rm M1}} // (C_{02}, K_2, \alpha_2, \eta_2)_{C_{\rm M2}} \\ &\xrightarrow{(\omega)} (C_{0\rm s}, K_{\rm s}, \alpha_{\rm s}, \eta_{\rm s})_{C_{\rm Ms}}. \end{aligned} \tag{11}$$

As a result, the FOMC consisted of two memcapacitors would be a new memcapacitor, whose parameters can all be calculated according to Eq. (10).

4. Character analyzing of FOMCs in parallel connection

4.1. Two typical FOMCs connected in parallel with identical polarities

After discussing the mathematical formula, a proper model is introduced to explain the simplification of multiple FOMCs connected in parallel. This section will describe some basic phenomena based on the theory proposed above.

It is assumed that the two devices are identical to make the analysis easier. When $K_1\omega^{-\alpha_1} = K_2\omega^{-\alpha_2} = K_{\rm C}$, we have

$$\begin{cases} A = K_{\rm C}\left(\cos\alpha_1\frac{\pi}{2} + \eta_1\eta_2\cos\alpha_2\frac{\pi}{2}\right), \\ B = K_{\rm C}\left(\sin\alpha_1\frac{\pi}{2} + \eta_1\eta_2\sin\alpha_2\frac{\pi}{2}\right). \end{cases} \tag{12}$$

Then $\alpha_{\rm s}$ and $K_{\rm s}$ could be derived by

$$\begin{cases} \alpha_{\rm s}\frac{\pi}{2} = \arctan\frac{\sin\alpha_1\frac{\pi}{2} + \eta_1\eta_2\sin\alpha_2\frac{\pi}{2}}{\cos\alpha_1\frac{\pi}{2} + \eta_1\eta_2\cos\alpha_2\frac{\pi}{2}}, \\ K_{\rm s}\omega^{-\alpha_{\rm s}} = \sqrt{2}K_1\omega^{-\alpha_1}\sqrt{1+\cos\left((\alpha_1-\alpha_2)\frac{\pi}{2}\right)}. \end{cases} \tag{13}$$

When connecting in positive and parameter $\eta = \eta_1\eta_2 = 1$, $\alpha_{\rm s}$ and $K_{\rm s}$ can be calculated as

$$\begin{cases} \alpha_{\rm s}\frac{\pi}{2} = \arctan\frac{2\sin\frac{\alpha_1+\alpha_2}{2}\frac{\pi}{2}\cos\frac{\alpha_1-\alpha_2}{2}\frac{\pi}{2}}{2\cos\frac{\alpha_1+\alpha_2}{2}\frac{\pi}{2}\cos\frac{\alpha_1-\alpha_2}{2}\frac{\pi}{2}} = \frac{\alpha_1+\alpha_2}{2}\frac{\pi}{2}, \\ K_{\rm s}\omega^{-\alpha_{\rm s}} = K_{\rm C}\sqrt{2+2\cos\left((\alpha_1-\alpha_2)\frac{\pi}{2}\right)} \\ \qquad\qquad = 2K_{\rm C}\cos\frac{\alpha_1-\alpha_2}{2}\frac{\pi}{2}. \end{cases} \tag{14}$$

The process of parallel connection is shown in Fig. 5. Figure 5(a) represents the pinched hysteresis loop of two FOMCs, and Fig. 5(b) represents the memcapacitance of them.

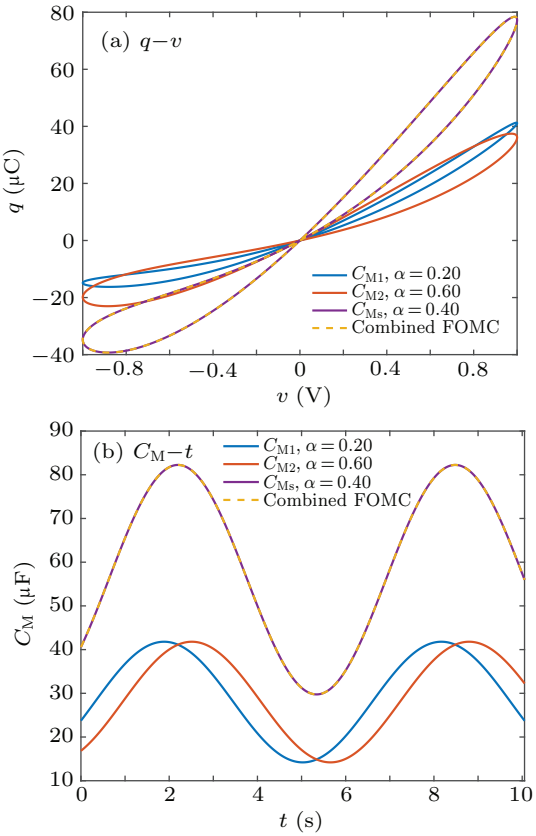


Fig. 5. Two simple FOMCs positive connected in parallel.

Table 1. A simple FOMC and its parameters.

C_M	C_0	K	α	η
C_{M1}	28	13.8	0.2	1
C_{M2}	28	13.8	0.6	1
C_{Ms}	56	26.25	0.4	1

The purple line is calculated in the normal process, and the dotted line is calculated by the above formula in Figs. 5(a) and 5(b). It can be easily seen that the result of the parallel connection matches the formula. More details of the parameters are listed in Table. 1, where K means the rate factors, η represents the different mode of connection and α represents the fractional order.

When the parameter $\eta = 1$, two FOMCs are connected in parallel with identical polarity. Parameters of the combined FOMC change according to a specific pattern. The new fractional-order α_s will be the average value of two parameters α_1 and α_2. In other words, $\alpha_s = \frac{\alpha_1+\alpha_2}{2}$ and does not change with other parameters.

When other parameters remain unchanged, K represents the fluctuation range of memcapacitance. When $\omega = 1$, Eq. (14) tend to $K_s = 2K_C \cos\frac{\alpha_1-\alpha_2}{2}\frac{\pi}{2}$. That means K_s reaches a maximum when $\alpha_1 = \alpha_2$. In other words, two parallel FOMCs of the same order α have the largest memcapacitance. This conclusion is obvious when integer order capacitors are connected in parallel.

But these conclusions cannot be easily seen in Fig. 5 and Table 1. At the same time, this description method is not accurate and will bring unintuitive changes due to parameters. There is a new manner of understanding this process, named the polar coordinate method, in which the horizontal and vertical coordinates do not have a special meaning but just a scale of length.

The FOMC has four main parameters (C_0, K, α, η). The parameters α and K influence the equivalent FOMC most when η is a constant. Their small changes lead to new behaviors and different hysteresis loops. If $K\omega^{-\alpha}$ and $\alpha\frac{\pi}{2}$ are considered as ρ and θ in polar coordinates, the combination process of $(C_{01}, K_1, \alpha_1, \eta_1)_{C_{M1}}$ and $(C_{02}, K_2, \alpha_2, \eta_2)_{C_{M2}}$ could be illustrated as shown in Fig. 6. The equivalent modeling structure could be explained conveniently, and the two parameters ρ and θ can intuitively show the effect of the combination.

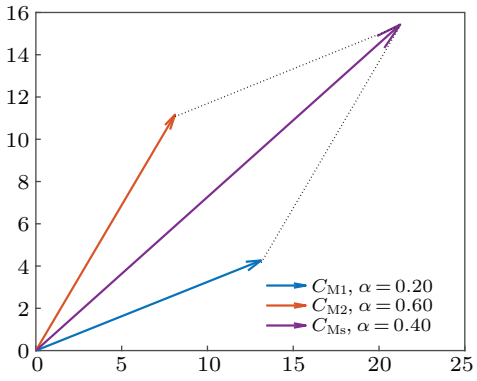


Fig. 6. Two simple FOMCs positive connected in parallel.

From Table 1 and Fig. 6, the parameters of equivalent FOMCs could be easily calculated when two FOMCs with identical polarities and $K_1\omega^{-\alpha_1} = K_2\omega^{-\alpha_2} = K_C$. α_s is precisely the mean value of α_1 and α_2. Meanwhile, the equivalent process could be calculated by equations and illustrated by the polar coordinates form with the fluctuation range and phase changes equivalent to ρ and θ.

4.2. Two typical FOMCs connected in parallel with opposite polarities

When connected in negative, which means $\eta_1\eta_2 = -1$, α_s and K_s can be obtained as follows:

$$\begin{cases} \alpha_s\frac{\pi}{2} = \arctan\dfrac{-2\sin\frac{\alpha_1-\alpha_2}{2}\frac{\pi}{2}\cos\frac{\alpha_1+\alpha_2}{2}\frac{\pi}{2}}{2\sin\frac{\alpha_1+\alpha_2}{2}\frac{\pi}{2}\sin\frac{\alpha_1-\alpha_2}{2}\frac{\pi}{2}} = \dfrac{\alpha_1+\alpha_2-2}{2}\dfrac{\pi}{2}, \\ K_s\omega^{-\alpha_s} = K_1\omega^{-\alpha_1}\sqrt{2-2\cos\left((\alpha_1-\alpha_2)\frac{\pi}{2}\right)} = 2K_1\omega^{-\alpha_1}\cos\frac{\alpha_1-\alpha_2-2}{2}\frac{\pi}{2}. \end{cases} \tag{15}$$

Then, the parameters are chosen as shown in Table 2, and the corresponding v–q and other curves are shown in Fig. 7.

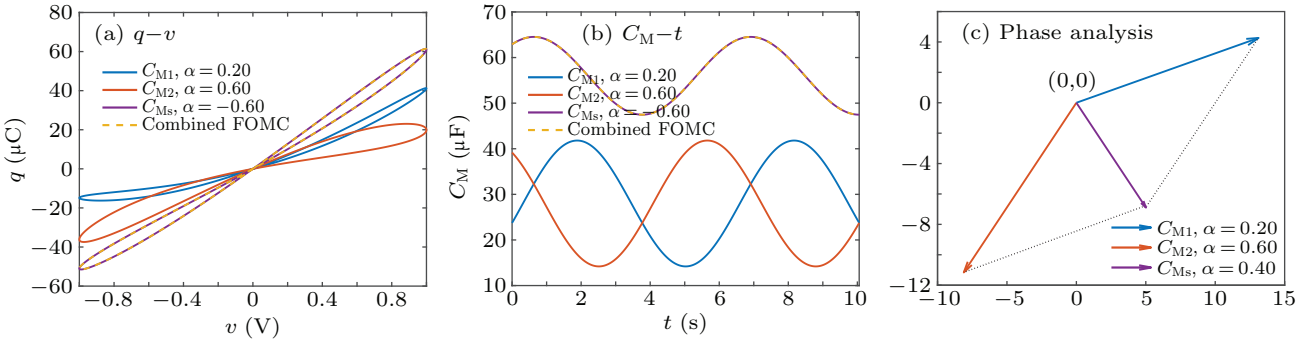


Fig. 7. The negative order of FOMC and its components's order.

Table 2. The negative order FOMC ($\alpha < 0$) and its component FOMCs ($\alpha > 0$).

C_M	C_0	K	α	η
C_{M1}	28	13.8	0.2	1
C_{M2}	28	13.8	0.6	–1
C_{Ms}	56	8.53	–0.6	1

The purple line is calculated in the normal process, and the dotted line is calculated by the above formula in Fig. 7. It can be easily seen that the result of the parallel connection matches the formula. At the same time, when $\eta = -1$, the area of the pinch hysteresis loop is minor than $\eta = 1$ in Fig. 5 with all the same parameters of others. It means the parallel directions lead to this difference. In other words, two FOMCs connected in parallel in identical polarities produce a superposition effect, and two connected in opposite polarities have a reduction effect.

Some interesting phenomena show up during the process of reconstruction. One of the most important phenomena is the equivalent negative order. As shown in Fig. 7, two general positive FOMCs connected in parallel can be constituted into an equivalent FOMC with negative order.

Figure 7 is a contrast group with $\eta = -1$, which means the polarity of FOMC is different from Fig. 5. From the comparison between Tables 2 and 1, different η_2 with all the other parameters set to be equal means the FOMCs connected reversely and cause the difference as a different sign of α_s. This negative order can be understood as the phase shift, seen in the polar coordinate figure in Fig. 7. It could also be found that when $K_1\omega^{-\alpha_1} = K_2\omega^{-\alpha_2} = K_C$ and $\alpha_1, \alpha_2 \in [0,1]$, the α_s should be strictly negative and $\alpha_s = \frac{\alpha_1+\alpha_2-2}{2}$.

When other parameters remain unchanged, K represents the fluctuation range of memcapacitance. When $\omega = 1$, Eq. (15) tend to $K_s = 2K_C \cos\frac{\alpha_1-\alpha_2-2}{2}\frac{\pi}{2}$. That means K_s reaches a minimum when $\alpha_1 = \alpha_2$. In other words, two parallel FOMCs of the opposite order α have the smallest memcapacitance.

By using polar coordinates, different C_M could be illustrated in Fig. 8. The parameters C_0 and K are set to be constants to make Fig. 8 more intuitive in showing the relationship between α, η, ρ, and θ.

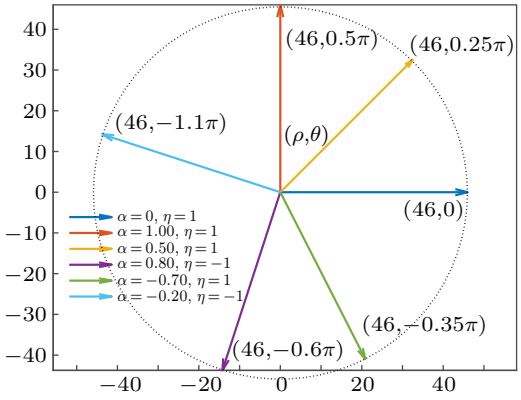


Fig. 8. Different (α, η) and corresponding polar coordinates.

Table 3. FOMCs and their corresponding (ρ, θ).

C_M	C_0	K	α	η	ρ	θ
C_{M1}	28	46	0	1	46	0
C_{M2}	28	46	1	1	46	0.5π
C_{M3}	28	46	0.5	1	46	0.25π
C_{M4}	28	46	0.8	–1	46	-0.6π
C_{M5}	28	46	–0.7	1	46	-0.35π
C_{M6}	28	46	–0.2	–1	46	-1.1π

Parameter θ rotates counterclockwise in $(0, 2\pi)$, and the parameter ρ means the length of polar coordinate, which is a constant in Fig. 8. As shown in Figs. 5–7 , parameter ρ is positively correlated with the fluctuation range of memcapacitance, and the parameter θ depends on α and η. (ρ, θ) are the critical parameters in the calculation of parallel FOMCs.

Figure 8 and Table 3 reveal the relationship between (ρ, θ) and the FOMC C_M. Each vector in Fig. 8 is corresponding with an FOMC in Table 3. The range of α is limited to $(-1, 1)$, and the corresponding range of θ is limited to $(0, 2\pi)$. At the same time, C_0 are remain unchanged because it does not affect (ρ, θ). Then the vector in Fig. 8 can be calculated when K, α and η changed. According to Eq. (10) and Fig. 8, $(\rho, \theta) = \left(K\omega^{-\alpha}, \alpha\frac{\pi}{2} + (\eta - 1)\frac{\pi}{2}\right)$, which means the

essential parameters of $C_{\rm M}$ can be changed from (C_0, K, α, η) to a new form (C_0, ρ, θ). This new form provides a new way of thinking about the FOMC by using polar coordinate to understand fractional-order mem-element. At the same time, a new conclusion can be drawn that two devices in parallel can be equivalent to the addition of two vectors.

In this section, two FOMCs with their parameters chosen appropriately can constitute a new FOMC whose order is negative. Furthermore, FOMC with negative order could be generally constituted through combining two FOMCs in opposite polarities. Also, all the $C_{\rm M}$ could be described in the form of polar coordinates, which will make the equivalent process convenient.

5. Equivalent of multi FOMCs in complex situations

After analyzing the two typical FOMCs connected in parallel with identical or opposite polarities, the combination process in some complex situations will be discussed in this section. Firstly, multiple FOMCs connected in parallel will be analyzed and illustrated in Subsection 5.1.

5.1. Illustrating of multi FOMCs connected in parallel

According to Eq. (10), it is easy to have two FOMCs in parallel together as a new equivalent FOMC, while it is hard to directly combine several FOMCs and get a result for the parameters (C_0, K, α, η) of $C_{\rm Ms}$. Combining with the conclusion that two devices in parallel can be equivalent to the addition of two vectors, it can be easily derived that many FOMCs linked in parallel can be calculated one by one as vectors. Thus, by continuously combing the equivalent FOMC called $C_{\rm Ms}$ with the new FOMC, the multiple FOMCs connected in parallel could also be easily combined as the equivalent FOMC. The parameters of the equivalent FOMC can also be calculated by Eqs. (13) and (15) step by step.

The process of combination is shown in Fig. 9, and the corresponding parameters of FOMCs are listed in Table 4 analysis of the fractional-order memcapacitor.

The purple line is calculated in the normal process, and the dotted line is calculated by the above formula in Fig. 9(a). It can be easily seen that the result of the parallel connection matches the formula. $C_{\rm M1}$, $C_{\rm M2}$, and $C_{\rm M3}$ are three initial vectors whose colors are blue, orange and green, respectively. The orange dotted line is the translation of the orange vector $C_{\rm M1}$ and the green one is vector $C_{\rm M2}$. After a simple vector superposition, the combined FOMC named $C_{\rm Ms}$ is shown as the purple vector. The parameters of $C_{\rm Ms}$ are calculated by Eqs. (13) and (15) and shown in Table 4. In this way, the difficult-to-calculate fractional problem is easily converted into vector addition, and the same result is obtained. The polar coordinates diagram intuitively shows the parallel process of the FOMCs comparing with the traditional hysteresis loop diagram.

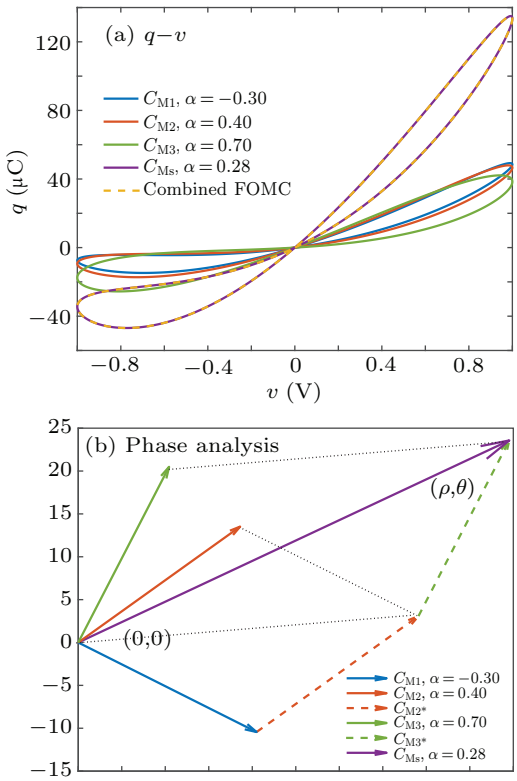


Fig. 9. Multi FOMCs connected in parallel.

Table 4. Multi FOMCs and their corresponding $C_{\rm Ms}$.

$C_{\rm M}$	C_0	K	α	η	ρ	θ
$C_{\rm M1}$	28	23	–0.3	1	23	-0.15π
$C_{\rm M2}$	28	23	0.4	1	23	0.2π
$C_{\rm M3}$	28	23	0.7	1	23	0.35π
$C_{\rm Ms}$	84	54.9	0.28	1	54.8	0.14π

As shown in Fig. 9(b), the combination process for multi FOMCs is much easier to understand as polar coordinates compared with using Eq. (10). The equivalent $C_{\rm Ms}$ could be constituted by connecting polar coordinates form of multi FOMCs one after another. Besides combining two FOMCs, multiple FOMCs could also be constituted into an equivalent form with all parameters reconstructed. When considering the polar coordinates form of FOMCs, there are two crucial parameters ρ and θ. The impact of each parameter will be discussed separately in Subsections 5.2 and 5.3.

5.2. FOMCs connected with different θ

The parameters of $C_{\rm M1}$ remain consistent as ($C_{01} = 28$, $K_1 = 9.2$, $\alpha_1 = 0.3$, and $\eta_1 = 1$). The different $C_{\rm M2}$ with distinctive θ are connected with $C_{\rm M1}$. The corresponding curves are shown in Fig. 10(b).

The $C_{\rm M2-1}$, $C_{\rm M2-2}$, and $C_{\rm M2-3}$ are obtained by only changing θ when the other parameters are consistent. The orange, green, and yellow dotted lines in Fig. 10 represent these

three vectors, respectively. $C_{\rm Ms}$ is an equivalent combination of $C_{\rm M1}$ and $C_{\rm M2}$. The combination of $C_{\rm M1}$ and $C_{\rm M2}$ named $C_{\rm Ms1}$, $C_{\rm Ms2}$, and $C_{\rm Ms3}$ are drawn as orange, green, and yellow vectors, respectively. Similarly, $C_{\rm Ms}$ is calculated using the method in section 5.1 and shown as Fig. 10(b).

Figure 10(a) is the intrinsic characteristics of mem-elements–hysteresis loop, and Fig. 10(b) is the polar diagram. When multiple FOMCs are involved in parallel connection, the hysteresis loop is complicated and challenging to feel intuitive. The comparison of Figs. 10(a) and 10(b) shows the simplicity and clarity of the polar coordinates in the parallel connection calculation.

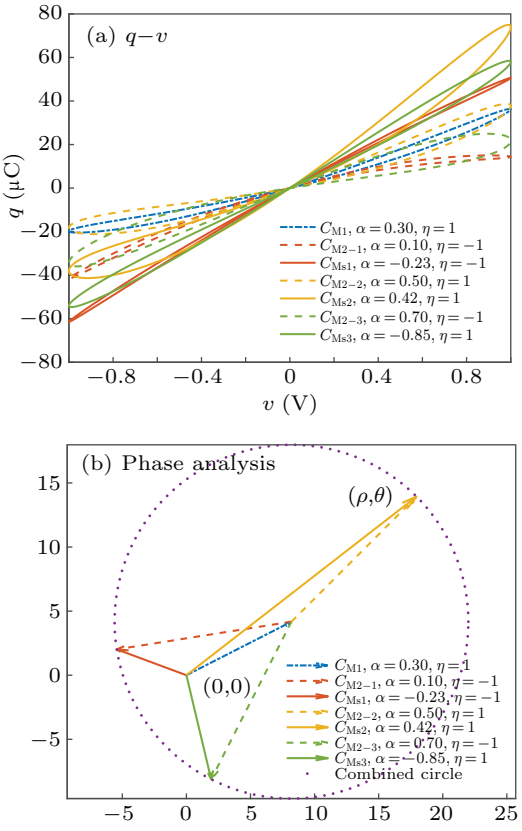


Fig. 10. FOMCs connected with different θ.

Table 5. The different parameters α and η of $C_{\rm Ms}$ caused by the change of $C_{\rm M2}$.

$C_{\rm Ms}$	C_0	K	α	η
$C_{\rm Ms1}$	56	5.80	–0.23	–1
$C_{\rm Ms2}$	56	22.93	0.36	1
$C_{\rm Ms3}$	56	8.35	–0.85	1

When changing θ with other parameters unchanged, vector $C_{\rm M2}$ will move in a circle whose radius is ρ. At the same time, θ means different parameters α and η in $C_{\rm Ms}$ shown in Table 5.

The parameters $C_{02}=28$ and $K_2=13.8$ remain consistent with $\alpha_2=\{0.1,0.4,0.7\}$ and $\eta_2=\{-1,1,-1\}$, which means θ of $C_{\rm M2}$ varies. At the same time, the parameters of $C_{\rm Ms}$ vary as Table 5 list. The process of combination are shown in Fig. 10(a).

When the polar coordinates form of $C_{\rm M1}$ drawn located in the combined circle, that is, $\rho_1<\rho_2$, as shown in Fig. 10(b), the value of $\rho_{\rm s}$ or $K_{\rm s}$ varies between $[\rho_2-\rho_1,\rho_2+\rho_1]$ and the value of $\theta_{\rm s}$ could range the whole 2π period, behaving as different $\alpha_{\rm s}$ orders and $\eta_{\rm s}$, which means the negative order could undoubtedly appear.

Whereas when $\rho_1>\rho_2$, there have a chance that the sign of $\eta_{\rm s}$ and α keeps unchanged if the parameters of $C_{\rm M1}$ and $C_{\rm M2}$ are appropriately chosen. It could also be easily found that using the polar coordinates form and drawing the combined circle is a very convenient way to analyze the effect of θ. As the $\theta_{1,2}$ changes, $\theta_{\rm s}$ and $\rho_{\rm s}$ change together with the sign of $\alpha_{\rm s}$ and $\eta_{\rm s}$. The use of the polar coordinate method can offer a new angle to understand the fractional-order mem-elements.

5.3. FOMCs connected with different ρ and the constitution of arbitrary order FOMCs

Since the combination of order $\alpha_{1,2}\in(0,1)$ is already appeared and analyzed several times in the above paragraphs, the fractional-order α_1 and α_2 are chosen as $\alpha_1=0$ and $\alpha_2=1$ to make sure that θ is unchanged and the impact of different ρ will be discussed here, which means

$$\begin{aligned}&(C_{01},K_1,\alpha_1=0,\eta_1)_{C_{\rm M1}}//(C_{02},K_2,\alpha_2=1,\eta_2)_{C_{\rm M2}}\\&\xrightarrow{(\omega)}(C_{0\rm s},K_{\rm s},\alpha_{\rm s},\eta_{\rm s})_{C_{\rm Ms}}.\end{aligned}\tag{16}$$

Then according to Eq. (10), the following equation could be derived with $\alpha_1=0$, and $\alpha_2=1$:

$$\begin{cases}C_{0\rm s}=C_{01}+C_{02},\quad \eta_{\rm s}=\eta_1,\\ \alpha_{\rm s}=\eta_1\eta_2\frac{2}{\pi}\arctan\frac{K_2}{\omega K_1},\\ K_{\rm s}=\frac{\sqrt{A^2+B^2}}{\omega^{-\alpha_{\rm s}}}=\omega^{\alpha_{\rm s}}\sqrt{(K_1)^2+\left(\frac{K_2}{\omega}\right)^2},\\ A=K_1,\quad B=\eta_1\eta_2\frac{K_2}{\omega}.\end{cases}\tag{17}$$

The parameters of $C_{\rm M1}$ remain consistent as ($C_{01}=41.8$, $K_1=23$, $\alpha_1=1$, and $\eta_1=1$). C_{02} and α_2 of $C_{\rm M2}$ also remain as $C_{02}=41.8$, and $\alpha_2=0$. Other different parameters of $C_{\rm M2}$ and $C_{\rm Ms}$ are shown in Table 6. Then the corresponding curves are shown in Fig. 11.

Figure 11(a) is still a hysteresis loop diagram, which is more complicated and can explain that devices are mem-elements. Figure 11(b) is the polar diagram of $C_{\rm M1}$, $C_{\rm M2}$, and $C_{\rm Ms}$ when θ are unchanged. In Fig. 11(b), other parameters of $C_{\rm M1}$ and $C_{\rm M2}$ are unchanged when ρ and η are variable. In other words, the fractional-order are chosen as $\alpha_1=0$, $\alpha_2=1$, $\eta_1=\pm1$, and $\eta_2=\pm1$. According to the fact that any two-dimensional vectors can be obtained by two vectors with different directions in this flat. $C_{\rm Ms}$ can be FOMC whose parameters are arbitrarily specified.

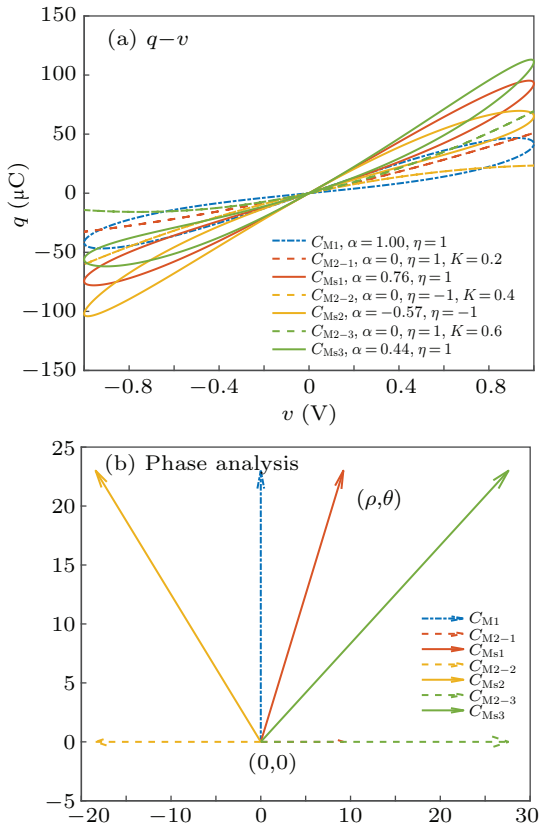


Fig. 11. Multi FOMCs connected in parallel.

Table 6. Different ρ and corresponding $C_{\rm Ms}$.

$C_{\rm Ms}$	ρ_2	η_2	$\alpha_{\rm s}$	$\eta_{\rm s}$	$K_{\rm s}$
$C_{\rm Ms1}$	9.2	1	0.76	1	24.78
$C_{\rm Ms2}$	18.4	–1	–0.57	–1	29.45
$C_{\rm Ms3}$	27.6	1	0.44	1	35.93

In Fig. 11, the orange, green, and yellow dotted lines represent $C_{\rm M2-1}$, $C_{\rm M2-2}$, $C_{\rm M2-3}$, respectively, and the blue one represents $C_{\rm M1}$.

The combination of $C_{\rm M1}$ and $C_{\rm M2}$ named $C_{\rm Ms1}$, $C_{\rm Ms2}$, and $C_{\rm Ms3}$ are drawn as orange, green, and yellow solid vectors, respectively. It could be easily observed that two integral orders are combined to constitute an equivalent fractional order, that is to say, $0<|\alpha_{\rm s}|<1$ being a fractional-order whereas the $\alpha_1=1$ and $\alpha_2=0$ being integral orders. In this case, the equivalent method establishes the relationship between FOMC and general mem-elements.

As shown in Fig. 11, different ρ_2 or K_2 leads different $\rho_{\rm s}$ and $\theta_{\rm s}$, where the value of $\theta_{\rm s}$ is located between the θ_1 and θ_2 and closer to the FOMC with the bigger ρ. The FOMC with a bigger ρ means it has a more significant weight coefficient in the equivalent parameter.

Although several different pairs of $C_{\rm M1}$ and $C_{\rm M2}$ could combine equivalent $C_{\rm Ms}$, a simple method to constitute a specified $C_{\rm Ms}$ could be employed according to Eq. (17) and Fig. 11. Also, by letting $\alpha_1=0$ and $\alpha_2=1$, other parameters of $C_{\rm M1}$ and $C_{\rm M2}$ could be calculated by Eq. (18), which is directly derived from Eq. (17) when the parameters of $C_{\rm Ms}$ are already specified.

$$\begin{cases} \eta_1=\eta_{\rm s}, \\ \eta_2=\dfrac{|\alpha_{\rm s}|}{\alpha_{\rm s}}\eta_{\rm s}, \\ C_{01}=C_{02}=C_{0{\rm s}}/2, \\ K_1=\dfrac{K_{\rm s}}{\omega^{\alpha_{\rm s}}}\cos\dfrac{\pi}{2}\alpha_{\rm s}, \\ K_2=\dfrac{\omega K_{\rm s}}{\omega^{\alpha_{\rm s}}}\sin\dfrac{\pi}{2}|\alpha_{\rm s}|. \end{cases} \tag{18}$$

In Subsection 5.3, it is found that two integral order memcapacitor could be connected in parallel to constitute an FOMC. Therefore it leads to a simple way to constitute an order-specified $C_{\rm Ms}$ with Eq. (18). Meanwhile, it could be more available since the device of integral order memcapacitor is much easier to be obtained. The parameters' values of $C_{\rm M1}$ and $C_{\rm M2}$ could be easily calculated, and the process of combination is straightforward with Eq. (18).

6. Conclusion

This paper discussed the generalized model of voltage-controlled fractional-order memcapacitor and mostly considered the parallel-connected FOMCs. The behavior of parallel-connected FOMCs is explored, and some conclusions are made. Two fractional-order memcapacitors are connected in parallel with identical and opposite polarities, the equivalent model is derived, and the characteristic of the equivalent order is analyzed.

For general FOMCs, the fractional-order $\alpha\in(0,1)$. FOMCs connected in parallel with identical polarities produce superimposing effect when rate factor K increase and the amplitude fluctuation range increase with K. FOMCs connected in parallel with opposite polarities produce a cancelation effect when the rate factor K decreases and the amplitude fluctuation range decreases with K. Additionally, the negative order appears, and the situations when it happens are discussed clearly.

This paper proposes a new simplified calculation method based on the new generalized model and the most basic sine wave compared with the previous highly complicated fractional-order calculations of other investigations. This method can solve the problem in a stable system that satisfies the conditions in Subsection 2.2. Then a new understanding manner by using polar coordinates is derived to explain the equivalent modelling structure conveniently. The new polar coordinates method is relatively concise and clear to illustrate the fractional-order change and amplitude fluctuations. Accordingly, multiple fractional-order memcapacitors could be calculated into one composite memcapacitor with its key parameters (C_0,α,η) reconstructed by Eqs. (13) and (15). Meanwhile, two integral order memcapacitors could be connected in parallel to constitute an FOMC, which means the fractional orders memcapacitors could be constructed into ar-

bitrary orders by the method referred to in Subsection 5.3. Additionally, in some oscillator circuits, this method of designing FOMC parameters is also advantageous.

In future work, this research will be extended to all fractional-order mem-elements and hybrid connection methods.

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References

[1] Chua L O 1971 *IEEE Trans. Circuit Theory* **18** 507
[2] Chua L O and Kang S M 1976 *Proc. IEEE* **64** 209
[3] Ventra M D, Pershin Y V and Chua L O 2009 *Proc. IEEE* **97** 1717
[4] Strukov D B, Snider G S, Stewart D R and Williams R S 2008 *Nature* **80** 453
[5] Emara A A M, Aboudina M M and Fahmy H A H 2017 *Microelectronics J* **39** 64
[6] Yin Z Y, Tian H, Chen G H and Chua L O 2015 *IEEE T Circuits-II* **62** 402
[7] Yang J J, Pickett M D, Li X, Ohlberg D, Stewart D R and Williams R S 2008 *Nat. Nanotechnol.* **3** 429
[8] Jo S H, Chang T, Ebong I, Bhadviyan B, Mazumder P and Lu W 2010 *Nano Lett.* **10** 1297
[9] Chanthbouala A, Garcia V, Cherifi R O, Bouzehouane K, Fusil S, Moya X, Xavier S, Yamada H, Deranlot C, Mathur N, Bibes M, Barthélémy A and Grollier J 2012 *Nat. Mater.* **11** 860
[10] Pershin Y V and Ventra M D 2011 *Adv. Phys.* **60** 145
[11] Kavehei O, Iqbal A, Kim Y S, Eshraghian K, Al-Sarawi S F and Abbott D 2010 *P. Roy. Soc. A-Math. Phy.* **466** 2175
[12] Kvatinsky S, Friedman E G, Kolodny A and Weiser U C 2012 *IEEE. T. Circuits-I.* **60** 211
[13] Prodromakis T, Peh B P, Papavassiliou C and Toumazou C 2011 *IEEE. T. Electron. Dev.* **58** 3099
[14] Wang Y, Sun J, Yao L and Zhang X 2016 *IET. Circ. Device. Syst.* **10** 244
[15] Biolek D, Biolek Z and Biolkova V 2010 *Electron. Lett.* **46** 520
[16] Biolek D and Biolkova V 2010 *Electron. Lett.* **46** 1428
[17] Martinez J, Di Ventra M and Pershin Y V 2010 *Phys Rev B.* **81** 19
[18] Feali M S, Ahmadi A and Hayati M 2018 *Neurocomputing.* **309** 157
[19] Wang W, Xu H, Hou Y W and Liu H J 2014 *Appl. Mech. Mater.* **644** 3426
[20] Di V M and Pershin Y V 2010 *Electron. Lett.* **46** 517
[21] Iu H C, Fitch A L Wang X Y and Qi W G 2012 *Phys. Lett. A* **376** 394
[22] Sun H G, Zhang Y, Baleanu D, Chen W and Chen Y Q 2018 *Commun. Nonlinear Sci.* **64** 213
[23] Zou Q, Jin Q B and Zhang R D 2016 *Chemometr. Intell. Lab.* **152** 34
[24] Radwan A G, Elwakil A S and Soliman A M 2008 *IEEE T. Circuits-I.* **55** 2051
[25] Wu X J and Shen S L 2009 *Int. J. Comput. Math.* **86** 1274
[26] Cao Y P and Bai C Z 2014 *Abstr. Appl. Anal.* **2014** 1
[27] Petrá I and Chen Y Q 2012 *Proceedings of the 13th International Carpathian Control Conference*, May 28–31, 2012, High Tatras, Slovaki
[28] Guo Z, Si G Q, Xu X, Qu K and Li S 2019 *Nonlinear Dyn.* **95** 101
[29] Fouda M E and Radwan A G 2014 *Microelectronics J.* **45** 1372
[30] Maan A K, Jayadevi D A and James A P 2016 *IEEE Netw.* **28** 1734
[31] Gu M Y, Liu J B, Wang G Y, Liang Y and Liu F P 2019 *Acta Phys. Sin.* **68** 228401 (in Chinese)
[32] Rajagopal K, Karthikeyan and Srinivasan A 2017 *Nonlinear Dyn.* **91** 1491
[33] Jo S H, Chang T, Ebong I, Bhadviya B B, Mazumder P and Lu W 2010 *Nano Lett.* **10** 1297
[34] Pu Y F and Yuan X 2016 *IEEE Access* **4** 1872
[35] Radwan A G, Emira A A, Abdelaty A M and Azar A T 2017 *ISA T.* **82** 184
[36] Zhou Z,Yu D S and Wang X Y 2017 *Chin. Phys B.* **26** 120701
[37] Biolkova V, Biolek D and Biolek Z 2011 *Electron. Lett.* **47** 1385
[38] Fouda M E and Radwan A G 2015 *Circuits, Syst. Signal Process.* **34** 961
[39] Yu Y J and Wang Z H 2015 *Acta Phys. Sin.* **64** 238401 (in Chinese)