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Interaction region of magnon-mediated spin torques and novel magnetic states*

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We determine the region in which the magnon-mediated spin torques exist. This region can be controlled by the spin waves. In terms of stability analysis of magnetization dynamics based on the spin-wave background, we obtain the instability conditions of spin waves. With these results, we find the relationship between unstable regions and the formation of Akhmediev breather, Kuznetsov–Ma breather and rogue waves. We establish the phase diagram of some novel magnetic excitaions.

Keywords: magnon-mediated spin torque, instability, breather, phase diagram

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1. Introduction

In recent years, there has been significant progress in solid-state random access memories and magnetic hard disk drives for storing digital information. In principle, these advances depend on new materials and a variety of nonlinear excitation states of magnetization[1,2] in magnetic devices, which can be driven by the magnetic field[3] or spin-polarized current.[4–13] In these excitation states, a domain wall can be seen as a potential hill which separates two generated magnetic states. Of great significance is the implementation of memory and logic technologies with the high density and high speed.[14–18] This progress requires reorienting the magnetization with the development of efficient mechanisms driven by the least possible current and power. There is considerable progress in this field, such as the spin Hall effect[19] in the heavy metal, and the Rashba–Edelstein effect[20,21] in the ferromagnet, which arose from the spin–orbit interactions in heavy-metal/ferromagnet bilayers.[22–26] Also, this spin-transfer torque has been studied in antiferromagnets.[27,28]

In addition, in the study of two-dimensional film systems, spin waves[29] and skyrmion[30] have also attracted much attention, which can be nucleated as a metastable state in thin films. Therefore, it provides potential application for the design of new generation memory based on skyrmion motion in nanotracks.[31–33] Recently, the destruction of spatial inversion symmetry in some materials leads to the formation of Dzyaloshinskii–Moriya interaction.[34] It also exists at the interface between a magnetic layer and an adjacent one with high spin–orbit coupling.[35–37] The Dzyaloshinskii–Moriya interaction can be regarded as the asymmetric magnetic interaction caused by the spin–orbit coupling. It tends to make the magnetization rotating around a local vector, and forms some chiral structure, such as skyrmions and helix structure.[38–42] The Dzyaloshinskii–Moriya interaction can also affect the linear and the nonlinear magnetic excitation such as spin wave, domain wall and dynamic soliton.

Although there have great achievements about this spin-polarized current, many shortcomings should be brought to the attention, such as Joule heat and corresponding power dissipation. The spin propagation length is typically on the order of nanometers,[43] which prevents transmission of spin information over long distances. Recently, the magnon current has shown even greater advantages with much less dissipation. This is because the flux carries the spin moment, rather than moving electrons.[44] It has been found that spin waves can transport spin angular momentum in ferromagnetic and antiferromagnetic insulators, and it could potentially be exploited for high-speed, low-power magnonic devices for signal transmission and magnetic logic applications. Therefore, it becomes the research hotspots for long-distance transport,[45–48] spin waves (magnons) driving magnetization dynamics,[49–53] thermally driven domain wall motion,[54–56] and magnetization switching.[57] However, it is not well clear for the interaction region of magnon-mediated spin torque, and the relation

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between the instability condition and novel excitation in ferromagnet.

In this paper, in terms of stability analysis we clarify the interaction region of magnon-mediated spin torques. By stability analysis of magnetization dynamics based on the spin wave background, we obtain the instability conditions of spin waves. With these results, we find the relationship between unstable regions and the formation of Akhmediev breather, Kuznetsov–Ma breather and rogue waves. We obtain the phase diagram of some novel magnetic excitations.

2. Linear stability analysis of spin waves

As a general discussion, we consider the dimensionless Landau–Lifshitz equation

$$\frac{\partial \boldsymbol{m}}{\partial t} = -\boldsymbol{m} \times \frac{\partial^2 \boldsymbol{m}}{\partial x^2}, \tag{1}$$

where $\boldsymbol{m}$ is the normalized magnetization with $|\boldsymbol{m}| = 1$. Obviously, Eq. (1) admits a spin-wave (SW) solution, i.e., $\boldsymbol{m}_0 \equiv (m_{01}, m_{02}, m_{03}) = \left(A_\mathrm{c}\cos\varphi_0, A_\mathrm{c}\sin\varphi_0, \sqrt{1-A_\mathrm{c}^2}\right)$ with $\varphi_0 = -k_\mathrm{c}x + \omega_\mathrm{c}t$ and the dispersion relation $\omega_\mathrm{c} = k_\mathrm{c}^2 m_{03}$. The general form of unit magnetization vector can also be expressed as $\boldsymbol{m}_0 = (\sin\theta_0\cos\varphi_0, \sin\varphi_0\sin\theta_0, \cos\theta_0)$ in polar coordinates and $\boldsymbol{m}_0 = (\hat{\boldsymbol{e}}_r, 0, 0)$ in spherical coordinates.

Although we construct the exact soliton solutions of Eq. (1) on the spin wave background,[59] the region is not clarified, in which the magnon-mediated spin torques emerge. In the following we can clarify this region by presenting the linear stability analysis,[58,60–65] which shows an approximate characterization on the stability of perturbations with spin wave background. To this purpose, we apply perturbations

$$\boldsymbol{m} = \boldsymbol{m}_0 + \delta\boldsymbol{m} = \boldsymbol{m}_0 + \left(u_0\hat{\boldsymbol{e}}_\theta + v_0\hat{\boldsymbol{e}}_\varphi\right)\mathrm{e}^{\mathrm{i}(Kx-\Omega t)}, \tag{2}$$

where u_0 and v_0 are small amplitudes, $\hat{\boldsymbol{e}}_\theta = (\cos\theta_0\cos\varphi_0, \cos\theta_0\sin\varphi_0, -\sin\theta_0)$, $\hat{\boldsymbol{e}}_\varphi = (-\sin\varphi_0, \cos\varphi_0, 0)$, and Ω is the perturbation frequency. By substituting Eq. (2) into Eq. (1), and linearizing the equations, the perturbation dispersion relation is obtained as

$$\Omega = K\Omega_V, \tag{3}$$

where $\Omega_V = 2k_\mathrm{c}m_{03} \pm \sqrt{K^2 - k_\mathrm{c}^2A_\mathrm{c}^2}$. The stability condition of the system is that the imaginary part of the frequency cannot appear. Therefore, the instability condition is $K^2 < k_\mathrm{c}^2A_\mathrm{c}^2$. We demonstrate the dispersion diagram of Eq. (3) in Fig. 1(a), in which the blue and red lines are the real component of the perturbation frequency Ω. It can be seen from the illustration that there are two special intersections $\pm k_\mathrm{c}A_\mathrm{c}$. The frequency is imaginary when K is between $+k_\mathrm{c}A_\mathrm{c}$ to $-k_\mathrm{c}A_\mathrm{c}$, but only the real part of the frequency is represented in the figure, i.e., $\Omega = 2Kk_\mathrm{c}m_{03}$. We find that when $K = 0$, the corresponding Ω is zero, and the perturbation frequency can be the same as the frequency of spin wave background, which can be clearly observed by the definition $\boldsymbol{m}_+ \equiv m_1 + \mathrm{i}m_2 = \left[A_\mathrm{c} + (u_0m_{03} + \mathrm{i}v_0)\mathrm{e}^{\mathrm{i}K(x-\Omega_V t)}\right]\mathrm{e}^{\mathrm{i}\varphi_0}$. It can generate resonance behavior which is manifested as the instability of resonance amplification. Therefore, the case $K = 0$ no longer fits the above assumption at this time, and the following discussion for the instability analysis does not include the point $K = 0$. However, the perturbation form needs to be assumed again.

We demonstrate the instability gain $\mathrm{Im}\Omega$ distributed on the $k_\mathrm{c}A_\mathrm{c}$–K plane, as shown in Fig. 1(b). It is seen that there are two distinctive regimes, namely, the stability and instability region. In general, the perturbation uncertainty and the nonlinear excitation is composed of two terms, one is the spin wave background, and the other term is the perturbation term, i.e., $\boldsymbol{m}(x,t) = \boldsymbol{m}_0(x,t) + f_\mathrm{pert}$. We can know qualitatively that in the instability mechanism f_pert is amplifying and growing, while in the stability mechanism does not grow with time. It should be noted that Eq. (3) can not accurately describe the case of $K = 0$ being the perturbation instability gain situation, and then one can define a special red line for the resonance lines. Therefore, the disturbances in different regions exhibit different nonlinear dynamic behaviors.

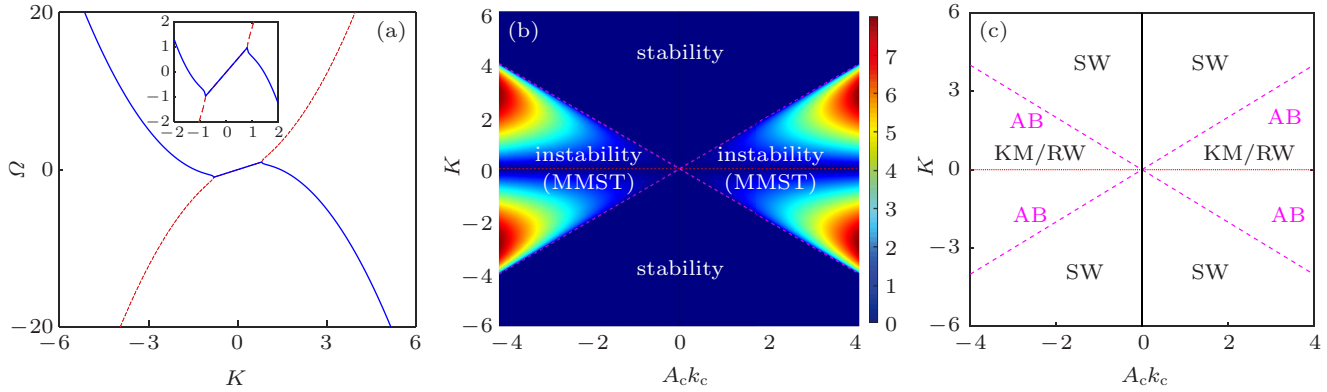


Fig. 1. Interaction region of magnon-mediated spin torque. (a) Dispersion law of perturbation waves in Eq. (3) with parameters $k_\mathrm{c} = 1$, $A_\mathrm{c} = 0.8$, and the sign $\pm$ corresponding to the red and the blue lines, respectively. (b) The stability and perturbation instability gain distributed on the product of spin wave background amplitude and wave vector $k_\mathrm{c}A_\mathrm{c}$ and perturbation wave vector K space. The instability region corresponds exactly to the interaction effect of magnon-mediated spin torque (MMST). (c) Phase diagrams of the relation of erturbation instability region and various nonliear excitations, such as spin wave (SW), Akhmediev breather (AB), Kuznetsov–Ma breather (KM), and rogue wave (RW).

From the above discussion and our early results,[59,66–68] we confirm that the region of instability corresponds to the one that the magnon-mediated spin torques appear. In this region, the interaction of spin wave and soliton forms some novel nonlinear magnetic excitations, such as breather and rogue wave solutions obtained by solving Eq. (1) with Darboux transform. Similar results are also studied in other fields.[69–75] These magnetic states can be clarified as follows, also shown in Fig. 1(c). There is a special line ($k_cA_c = 0$), which corresponds to the dark soliton solution under the ground state background.[76] For the case of $k_c = 0$ and $A_c = 0$, any perturbation on this line will not be amplified, which is consistent with the characteristic that dark solitons are stable under small perturbations, as shown in Fig. 1(c) with a solid black line.

As shown in Ref. [59], the critical point $|\mu| = k_cA_c$ forms a dividing line between the instability process ($|\mu| < k_cA_c$) and periodic process ($|\mu| > k_cA_c$) of Eq. (1), where μ is the soliton amplitude. When $|\mu| < k_cA_c$, the Akhmediev breather solution [see Eqs. (A1) and (A2)] has spatial periodicity with the period $2\pi/k_{AB}$, where $k_{AB} = \sqrt{k_c^2A_c^2 - \mu^2}$, while the temporal period is infinity. When $\mu = 0$, the wave number k_{AB} of breather reaches the maximum, i.e., $k_{AB} = \pm k_cA_c$, corresponding to two purple dotted line in Fig. 1(c). As μ increases, the corresponding k_{AB} decreases, while the corresponding spatial period of the Akhmediev breather solution increases. As μ approaches k_cA_c, and k_{AB} approaches 0, the period of space and time approaches infinity, we get the rogue wave solution.[59] It can be seen that the corresponding formation region of Akhmediev breather solution[69,70] is between the purple dotted line and the red dotted line, as shown in Fig. 1(c). When $|\mu| > k_cA_c$, we obtain the Kuznetsov–Ma breather solution [see Eqs. (A1) and (A3)], which has temporal periodicity $2\pi/(\mu\sqrt{\mu^2 - k_c^2A_c^2})$. It is spatially localized. The space period tends to be infinite and the corresponding wave number is zero. When $|\mu| \to k_cA_c$, the novel rogue wave solution emerges[59] see Eqs. (A1) and (A4). The time and space are both local, and the time and space period tend to infinity. As it happens, the wave number corresponding to the above analysis is zero. And the perturbance frequency is the same as the background frequency, thus generating resonance amplification, as shown in Fig. 1(c). It can be seen that Kuznetsov–Ma breather soliton and rogue wave correspond to the line of $K = 0$.

When $K = 0$, i.e., the resonance frequency lines shown in Fig. 1(c), the initial perturbation form proves unstable amplified and can be written as a general linear disturbance εp, where $p = u_1\hat{e}_\theta + v_1\hat{e}_\varphi$ with $u_1 \equiv 1 + f_1(x,t)$ and $v_1 \equiv 1 + f_2(x,t)$. By a tedious calculation, we obtain

$$\begin{aligned} m_+ &= [A_c + \varepsilon(u_1 m_{03} + \mathrm{i}v_1)]\, \mathrm{e}^{\mathrm{i}\varphi}, \\ m_3 &= m_{03} - \varepsilon u_1 A_c, \end{aligned} \tag{4}$$

where $f_1 = 0$, $f_2 = a_2 t + (a_2 + \xi_2)x/\xi_1$, with $\xi_1 = -2k_cm_{03}$, $\xi_2 = k_c^2A_c^2$, and a_2 being the real parameters. Eq. (4) shows the instability property of the resonant line $K = 0$ in Fig. 1(c).

It is interesting that how to control the interaction region of magnon-mediated spin torque. From the discussion we give the phase diagram of instability gain increasing with the increase of amplitude A_c in the space K–k_c, as shown in Fig. 2. The structure characteristics of Akhmediev breather changes accordingly.[59] As the amplitude A_c increases, the region of disturbance wave number that causes unstable amplification also increases. In addition, we can see the perturbation frequency increases with the increasing K, while the perturbation corresponding to the high frequency in the unstable gain diagram has no obvious excitation in the background of the spin wave. The states of this system can still be considered as the spin wave background, which is represented in Fig. 2. It is very clear that the classical resonance vibration is confirmed, which cannot be driven away from harmonic frequency to produce resonance amplification.

From the above detailed discussion we see that under the interaction of spin waves and magnetic soliton, the instability region witnesses the effect of magnon-mediated spin torque, in which the magnons transport spin angular momentum in ferromagnet. These novel magnon-mediated spin torque excite new magnetic states, including Akhmediev breather, Kuznetsov–Ma breather, and rogue waves. It also brought with the localized spatial and temporal states of magnetization, which takes the high-density magnons, and is useful microwave field for magnonic devices.

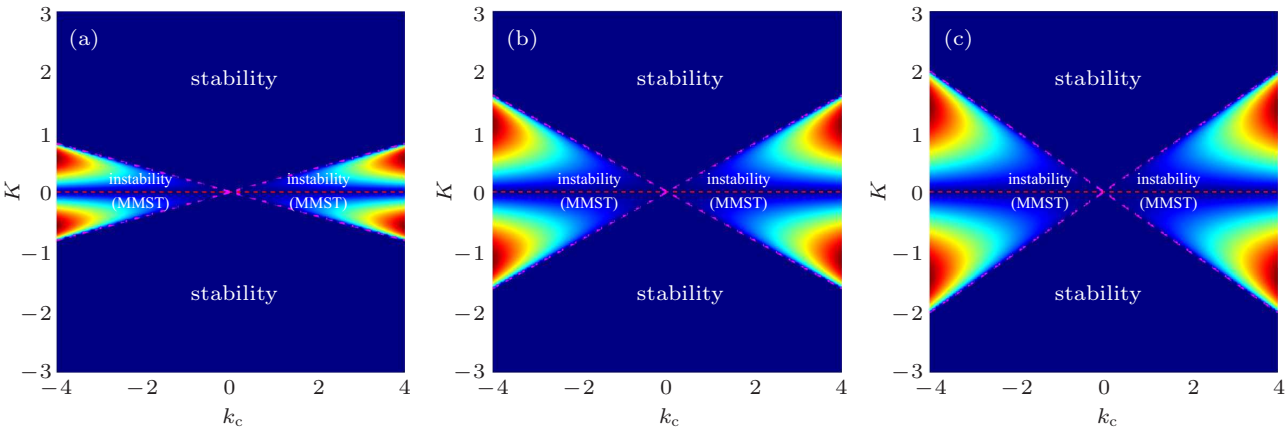


Fig. 2. Phase diagrams of the control region for the effect of magnon-mediated spin torque (MMST) by spin wave amplitude, (a) $A_c = 0.2$, (b) $A_c = 0.4$, (c) $A_c = 0.5$.

3. Conclusion

In conclusion, we investigate the instability of magnetization dynamics by linear stability analysis. With the interaction between spin wave and perturbation, we obtain the region that the magnon-mediated spin torques exist. This instability region provides the relationship between instability condition and some novel nonlinear analytical solution. The excitations of Akhmediev breather, Kuznetsov–Ma breather, and rogue waves are quantitatively located on the instability gain spectrum plane. In particular, rogue waves come from instability excitation with resonance disturbance, which are existed in different nonlinear systems. We find how to control the effect of magnon-mediated spin torque, and it is of great significance for nonlinear controllable excitation.

Appendix A

Exact breather solutions and rogue waves

We obtain the exact solutions of Eq. (1) as the form

$$\boldsymbol{m}\cdot\boldsymbol{\sigma} = \boldsymbol{F}(\boldsymbol{m}_0\cdot\boldsymbol{\sigma})\boldsymbol{F}^{-1}, \tag{A1}$$

where $\boldsymbol{\sigma}$ is Pauli matrix and the matrix $\boldsymbol{F}$ is given by

$$\boldsymbol{F} = \frac{1}{|\xi|^2(P+Q)}\begin{pmatrix} \xi^* P+\xi Q & -\mu R^* \mathrm{e}^{-\mathrm{i}\delta} \\ \mu R \mathrm{e}^{\mathrm{i}\delta} & \xi P+\xi^* Q \end{pmatrix},$$

with $\xi = \mathrm{i}\mu/2 + \nu/2$, $N = \sqrt{(k_\mathrm{c}+2\xi m_{03})^2+4\xi^2 A_\mathrm{c}^2}$, $\beta = -\mathrm{i}2\xi + \mathrm{i}m_{0,3}k_\mathrm{c}$, $P = h_{11}h_{11}^*$, $Q = h_{12}h_{12}^*$, $R = -\mathrm{i}\mathrm{e}^{\mathrm{i}\varphi_0}h_{11}^*h_{12}$, $h_{11} = \mathrm{i}(C_1\mathrm{e}^{B} - C_2\mathrm{e}^{-B})\mathrm{e}^{\mathrm{i}\varphi_0/2}$, $h_{12} = (C_1\mathrm{e}^{-B} - C_2\mathrm{e}^{B})\mathrm{e}^{-\mathrm{i}\varphi_0/2}$, $C_1 = \sqrt{(\mu m_{03}+\mathrm{i}(A_\mathrm{c}^2k_\mathrm{c}+N))/2}$, $C_2 = \sqrt{(\mu m_{03}+\mathrm{i}(N-A_\mathrm{c}^2k_\mathrm{c}))/2}$, and $B = -\mathrm{i}N(x+\mathrm{i}\beta t)/2$.

Akhmediev breathers

In ferromagnet this ubiquitous process of magnetization dynamics can be realized by the condition $|\mu| < A_\mathrm{c}k_\mathrm{c}$ and $\nu = -k_\mathrm{c}m_{03}$ in Eq. (A1). The parameters is given by

$$\begin{aligned} P &= -A_\mathrm{c}(k_\mathrm{c}\cosh\theta_1 + N\sinh\theta_1) - \mu\cos\phi_1 - Nm_{03}\sin\phi_1, \\ Q &= A_\mathrm{c}(-k_\mathrm{c}\cosh\theta_1 + N\sinh\theta_1) - \mu\cos\phi_1 + Nm_{03}\sin\phi_1, \\ R &= \mu\cosh\theta_1 + \mathrm{i}Nm_{03}\sinh\theta_1 \\ &\quad - A_\mathrm{c}(\mathrm{i}N\sin\phi_1 - k_\mathrm{c}\cos\phi_1), \end{aligned} \tag{A2}$$

where $\theta_1 = \mu N t$, $\phi_1 = -N(x - 2k_\mathrm{c}m_{03}T)$ with $N = \sqrt{A_\mathrm{c}^2k_\mathrm{c}^2 - \mu^2}$.

Kuznetsov–Ma soliton solution

Under the condition $|\mu| > A_\mathrm{c}k_\mathrm{c}$ and $\nu = -k_\mathrm{c}m_{03}$ we obtain the magnetic Kuznetsov–Ma soliton solution of Eq. (A1), which can be proposed as prototypes of hydrodynamic of rogue waves. This solution is characterized by the following parameters:

$$\begin{aligned} P &= \mu\cosh\theta_2 + \zeta m_{03}\sinh\theta_2 - A_\mathrm{c}(\zeta\sin\phi_2 - k_\mathrm{c}\cos\phi_2), \\ Q &= \mu\cosh\theta_2 - \zeta m_{03}\sinh\theta_2 + A_\mathrm{c}(k_\mathrm{c}\cos\phi_2 + \zeta\sin\phi_2), \\ R &= A_\mathrm{c}(\mathrm{i}\zeta\sinh\theta_2 - k_\mathrm{c}\cosh\theta_2) \\ &\quad - \mu\cos\phi_2 + \mathrm{i}\zeta m_{03}\sin\phi_2, \end{aligned} \tag{A3}$$

where $\zeta = \sqrt{\mu^2 - A_\mathrm{c}^2k_\mathrm{c}^2}$, $\theta_2 = \zeta(x - 2m_{03}k_\mathrm{c}t)$, and $\phi_2 = \mu\zeta t$.

Rogue waves

In the limit case of $|\mu| \to A_\mathrm{c}k_\mathrm{c}$, we get the novel magnetic rogue wave of Eq. (1), where the main parameters is given by

$$\begin{aligned} P &= \left(2tA_\mathrm{c}k_\mathrm{c}^2 - A_\mathrm{c}k_\mathrm{c}m_{03}x \pm 1\right)^2 \\ &\quad + A_\mathrm{c}^3k_\mathrm{c}^2\left(A_\mathrm{c}x^2 - 3A_\mathrm{c}k_\mathrm{c}^2t^2 \mp 6t\right), \\ Q &= \left(2tA_\mathrm{c}k_\mathrm{c}^2 - A_\mathrm{c}k_\mathrm{c}m_{03}x \mp 1\right)^2 \\ &\quad + A_\mathrm{c}^3k_\mathrm{c}^2\left(A_\mathrm{c}x^2 - 3t^2A_\mathrm{c}k_\mathrm{c}^2 \pm 6t\right), \\ R &= \mathrm{i}2A_\mathrm{c}^2k_\mathrm{c}\left(x - 3tk_\mathrm{c}m_{03}\right) + (P+Q)/2 - 2, \end{aligned} \tag{A4}$$

where the sign $\pm$ denotes the limit case $\mu \to \pm A_\mathrm{c}k_\mathrm{c}$, respectively.

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