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Dynamics of a stochastic rumor propagation model incorporating media coverage and driven by Lévy noise*

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With the development of information technology, rumors propagate faster and more widely than in the past. In this paper, a stochastic rumor propagation model incorporating media coverage and driven by Lévy noise is proposed. The global positivity of the solution process is proved, and further the basic reproductive number R_0 is obtained. When $R_0 < 1$, the dynamical process of system with Lévy jump tends to the rumor-free equilibrium point of the deterministic system, and the rumor tends to extinction; when $R_0 > 1$, the rumor will keep spreading and the system will oscillate randomly near the rumor equilibrium point of the deterministic system. The results show that the oscillation amplitude is related to the disturbance of the system. In addition, increasing media coverage can effectively reduce the final spread of rumors. Finally, the above results are verified by numerical simulation.

Keywords: rumor propagation, stochastic process, Lévy jump, media coverage

PACS: 02.30.Jr, 02.50.Ey, 02.50.Fz, 02.30.−f **DOI:** 10.1088/1674-1056/ac0423

1. Introduction

Rumor refers to the speech that has no corresponding factual basis but is fabricated and promoted to spread through certain means. The large-scale spread of rumors can manipulate local public opinion and even cause social panic.[1–4] Especially, when information technology is more and more developed, people can get and propagate information from multiple channels more quickly, so that rumors may cause more serious negative effects in a shorter time. Therefore, the accurate grasp of the characteristics of the rumor propagation process is helpful to provide corresponding countermeasures to curb the rumor propagation, which is of important research significance.[5]

Many scholars have made research on the dynamic process of rumor propagation, and the most classic one is the Daley–Kendal (DK) model.[6] In the DK model, people are divided into three categories: ignorant, spreader and stifler. Rumors are spread through connections between the spreader and others in the crowd. As a modification of the DK model, Maki–Thompson model[7] proposed that when a rumor spreader came into contacting with another spreader, only the former spreader would stop spreading rumors. Zanette[8,9] first applied complex network theory to rumor propagation. Subsequently, Moreno *et al.*[10] developed the DK rumor propagation model and derived the dynamic average field equation describing the rumor propagation process occurring on the complex heterogeneous network. Then with the rapid development of information technology, media coverage is playing an increasingly important role in the rumor spread process, which has attracted the attention of many scholars, and they used a deterministic model to study the influence of media coverage on transmission dynamics by considering the influencing factors in the process of rumor propagation.[11–13] Yao *et al.*[14] considered the filtering function of online social media and used a mathematical model named as SDILR to explore the phenomenon of continuous rumor propagation on social media. Huo *et al.*[15] proposed a nonlinear rumor mathematical model to describe the influence of cognitive items of media reports on rumor propagation, proving that there is a nonlinear proportional relationship between ignorant individuals and media reports. In the process of rumor propagation, media coverage can play an important role in promoting social harmony and stability by influencing people's behaviors.[16–19] The media can help the government authorities prevent and intervene the spread of rumors, and timely provide scientific and correct information to clarify the rumors, so as correct the direction of public opinion and avoid the social panic caused by the rumor spread.

Most of the above researchers regard the system approximation as deterministic. According to the law of large numbers, they assume that the population is large enough and the behavior of the system presents relatively stable statistical rules, so as to establish a deterministic model. However, this method does not take into account of the random factors in the real environment. When an emergency occurs in the real world, the spreading process of rumors is often af-

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fected by the complex and changeable external environment. For example, when the Covid-19 outbreak broke out, a lot of rumors were created and spread. The spreading process of these rumors were affected by various random factors such as the development of the epidemic situation, the degree of media involvement, and the psychological factors of the public. In this case, the stochastic model can better describe these environmental disturbance factors than the deterministic model, which is more practical. In recent years, the stochastic rumor propagation model has also attracted more and more attention. Dauhoo *et al.*[20] transformed the deterministic rumor model into a stochastic rumor model by using the stochastic coefficient representing the influence in the system, and proved the existence and uniqueness of global and local solutions. Jia *et al.*[21] studied a stochastic rumor propagation model and obtained that the threshold of the stochastic model affected by white noise was less than the threshold of the deterministic model. Li *et al.*[22] studied a stochastic model to describe the interaction between tumor cells and immune system under external fluctuations and periodic treatment. Ankur *et al.*[23] introduced a stochastic rumor model in a social network containing expert intervention, and got the conclusion that noise might be one of the reasons for the persistence of rumors under stochastic circumstances.

Compared with the deterministic model, the stochastic model takes into account of the ambient noise on the rumor propagation process and can more accurate depict the influence of environmental factors. In addition, there are many uncertain factors and great volatility in emergency events when rumors spread wildly. Lévy process is a stochastic process in continuous time with independent stable increments in the left limit and right continuity: it represents the motion of a point whose continuous displacement is random and independent and statistically identical at different time intervals of the same length.[24–26] Therefore, in order to describe the propagation law of rumors more accurately, we adopt a stochastic rumor propagation model with Lévy process. The Lévy process is the best choice to describe the stochastic fluctuation of rumor propagation.

Based on the above analysis, the stochastic rumor propagation model with Lévy process can more accurately describe the dynamic process of rumor propagation. Some researchers mainly aim at the deterministic model, but few stochastic aspects. So this article uses the stochastic model with jump Lévy noise and media coverage, and the concrete research content is as follows: first, we introduce SIR model with Lévy process, and Section 3 will prove the existence and uniqueness of global positive solutions. In Sections 4 and 5, the relationship between the solution orbits of ISR systems with stochastic hops and their deterministic systems is studied under two interference conditions. Finally, the results are verified by numerical simulation.

2. The stochastic rumor propagation model with media coverage

In the traditional rumor propagation model,[27] the nodes in the social network are divided into three types: ignorant (I), spreader (S) and stifler (R). In social networks, rumors spread among people according to the rules of mass behavior. Ignorant nodes represent those who have never been exposed to rumors, and become spreaders with a certain probability through contact with the spreader. Spreaders represent people who have accepted and propagated rumors. The stiflers represent those who know the rumor but stop propagating it.

Contracts between different types of nodes are governed by the following rules:

(1) Individuals enter a specific community at a constant rate Λ.

(2) Susceptible individuals have a certain immunity rate of θ. Under the influence of the government's popular science education, some ignorant people θI have their own judgment on rumors.

(3) When an ignorant people contacts with a spreader, the ignorant becomes a spreader at a rate β_1.

(4) Media coverage can inhibit the rumor propagation. The maximum influence of media on the propagation probability when ignorant individuals contact the spreaders is β_2.

(5) Considering that media coverage can slow down the spread of rumor, but cannot completely prevent its propagation, it is assumed that $\beta_1 > \beta_2$. Media coverage will make corresponding changes with the change of stochastic environmental disturbances.

(6) Each type of individual has moved out of the group for some reason at a removal rate μ.

(7) When a spreader contacts another spreader or a stifler, the original spreader becomes a stifler at a rate λ.

The values of the above parameters range from 0 to 1.

The contact between individuals of different types leads to the dissemination and diffusion of rumor, which leads to the change of individual types, as shown in Fig. 1.

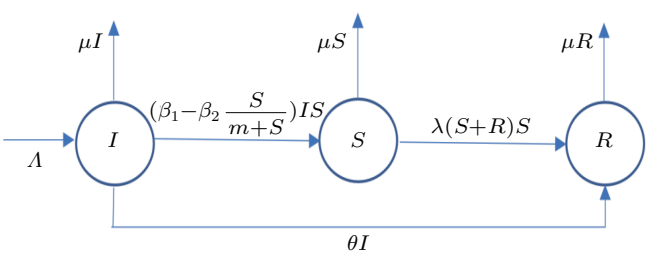


Fig. 1. Schematic diagram of rumor propagation under the influence of popular science education and media coverage.

In this case, the nonlinear dynamic model influenced by popular science education and media coverage can be repre-

sented by a common differential equation system

$$\begin{aligned}\frac{\mathrm{d}I}{\mathrm{d}t} &= \Lambda-(\beta_1-\beta_2\frac{S}{m+S})IS-\theta I-\mu I,\\ \frac{\mathrm{d}S}{\mathrm{d}t} &= (\beta_1-\beta_2\frac{S}{m+S})IS-\lambda S(S+R)-\mu S,\\ \frac{\mathrm{d}R}{\mathrm{d}t} &= \lambda S(S+R)+\theta I-\mu R,\end{aligned}\tag{1}$$

where I, S and R are all positive. $\beta_1 SI$ denotes some ignorant people choosing to believe the spreader's message and entering the spreader's group. $\beta_2\frac{S}{m+S}$ stands for the probability of media coverage obstructing rumor propagation when the ignorant I contacts the spreader S. With the increase in the number of spreaders, the media increases the intensity of reporting, so that the media's control over rumor propagation increases to a maximum. m represents the saturation coefficient, which measures the impact of media coverage on propagation, and its value range is between 0 and 1. The basic reproduction number for this model is given by $R_0=\frac{\beta_1\Lambda\mu-\mu^3}{\theta(\lambda\Lambda+\mu^2)}$, which is a threshold quantity, determining whether the rumor happens or simply dies.

In the process of rumor propagation, relevant media will adjust their reporting strategies with the change of events and specifically deny rumors. In addition, rumor propagation will be disturbed by many stochastic external factors, so the influence of stochastic interference on the system should be considered. For example, when COVID-19 broke out in Wuhan, China, a variety of rumors were generated due to the development of the epidemic at any time. The response of individuals to the rumors was affected by a variety of factors, and media reports would also be adjusted accordingly. In an emergency event, especially one that propagates very quickly and has a very wide influence, this interference is sufficient to destroy the continuity of the solution of the differential equation. Therefore, in order to make it more realistic, we can consider using the stochastic model with Lévy hop to study the propagation law of rumors. White noise represents its continuous part, and Lévy noise represents its jumping part. The stochastic model is shown below:

$$\begin{aligned}\mathrm{d}I &= \left[\Lambda-\left(\beta_1-\beta_2\frac{S}{m+S}\right)IS-\theta I-\mu I\right]\mathrm{d}t\\ &\quad+\delta_1 I\mathrm{d}W_1(t)+\int_Y q_1(y)I(t-)\tilde{N}(\mathrm{d}t,\mathrm{d}y),\\ \mathrm{d}S &= \left[\left(\beta_1-\beta_2\frac{S}{m+S}\right)IS-\lambda S(S+R)-\mu S\right]\mathrm{d}t\\ &\quad+\delta_2 S\mathrm{d}W_2(t)+\int_Y q_2(y)S(t-)\tilde{N}(\mathrm{d}t,\mathrm{d}y),\\ \mathrm{d}R &= [\lambda S(S+R)+\theta I-\mu R]\,\mathrm{d}t+\delta_3 R\mathrm{d}W_3(t)\\ &\quad+\int_Y q_3(y)R(t-)\tilde{N}(\mathrm{d}t,\mathrm{d}y).\end{aligned}\tag{2}$$

Here $W_1(t), W_2(t)$, and $W_3(t)$ represents the standard Brownian motion; δ_1, δ_2, and δ_3 are normal numbers and represents the intensity of Brownian motion, $q_i(y)>-1$ $(i=1,2,3)$ represents the intensity of jump, $\tilde{N}(\mathrm{d}t,\mathrm{d}y)=N(\mathrm{d}t,\mathrm{d}y)-v(\mathrm{d}y)$, $N(\mathrm{d}t,\mathrm{d}y)$ is the Poisson stochastic measure, $v(\mathrm{d}y)$is the compensation measure of $N(\mathrm{d}t,\mathrm{d}y)$, and $v(Y)<\infty(Y\in[0,+\infty))$.

We define an operator that is related to the following three-dimensional stochastic differential equation (SDE):

$$\begin{aligned}\mathrm{d}X &= F(t,X(t))\,\mathrm{d}t+G(t,X(t))\,\mathrm{d}W(t)\\ &\quad+\int_Y H(t,y)\tilde{N}(\mathrm{d}t,\mathrm{d}y),\\ LX(t-) &= \sum_{i=1}^{3}\frac{\partial X(t-)}{\partial x^i}F^i(t,X)\\ &\quad+\frac{1}{2}\sum_{i,j=1}^{3}\frac{\partial^2 X(t-)}{\partial x^i\partial x^j}[G^{\mathrm{T}}(t,X)G(t,X)]_{ij}\\ &\quad+\int_Y\Big[(X(t-)+H(t,y))-X(t-)\\ &\quad-\frac{\partial X(t-)}{\partial x^i}H(t,y)\Big]v(\mathrm{d}y),\end{aligned}$$

where $i=1,2,3$. If L operates on $\Phi\in C^{1,2}\left(R_+\times R^3\right)$, we have

$$\begin{aligned}L\Phi(X(t)) &= \Phi_x(X(t-))F(t,x)\\ &\quad+\frac{1}{2}\mathrm{trace}\left[G^{\mathrm{T}}(t,X(t-))\,\Phi_{xx}G(t,X(t-))\right]\\ &\quad+\int_Y\big[(\Phi(X(t-))+H(t,y))-\Phi(X(t-))\\ &\quad-\Phi_x(X(t))H(t,y)\big]v(\mathrm{d}y),\end{aligned}$$

where $\Phi_x=(\frac{\partial\Phi}{\partial x^1},\frac{\partial\Phi}{\partial x^2},\frac{\partial\Phi}{\partial x^3})$, $\Phi_{xx}=(\frac{\partial^2\Phi}{\partial x^i\partial x^j})_{3\times3}$.

According to Itô's formula, we get

$$\begin{aligned}\mathrm{d}\Phi(X(t)) &= L\Phi(X(t-))\mathrm{d}t+\Phi_x(X(t-))G(t,x)\,\mathrm{d}W_s\\ &\quad+\int_Y\big[(\Phi(X(t-))+H(t,y))\\ &\quad-\Phi(X(t-))\big]\tilde{N}(\mathrm{d}s,\mathrm{d}y).\end{aligned}$$

For the Itô's formula with jumping semimartingales, we refer to Refs. [28–30] for more details.

3. Global positive solution

In this section, we will prove the existence and uniqueness of the global positive solution. We will use Lyapunov analysis method[31,32] to prove that system (2) has local positive solutions.

Theorem 1 For any given initial value $(I(0),S(0),R(0))\in R_3^+$, $R_3^+=\{I(t),S(t),R(t)|0<I(t),S(t),R(t)<N,I(t)+S(t)+R(t)=N\}$, the model (2) has a unique global solution$(I(t),S(t),R(t))\in R_3^+$ for all $t\geq0$ almost surely.

Proof Since drift and diffusion being locally Lipschitz, $t\in[0,\tau_{\mathrm{e}})$, then there is a unique local solution $(I(t),S(t),R(t))$ for any given initial value $(I(0),S(0),R(0))\in R_3^+$, where τ_{e} is the explosion time. If we want to show this solution is global, we need to demonstrate $\tau_{\mathrm{e}}=\infty$. First of all, we prove that $I(t)$, $S(t)$, and $R(t)$ do not explode to infinity in a finite time. Let

$m_0 > 0$ be sufficiently large so that $(I(0), S(0), R(0))$ lie within the interval $[1/m_0, m_0]$. For each integer $m \geq m_0$, we define the stopping time

$$\tau_m = \inf\big\{t \in [0, \tau_e) : I(t) \notin (1/m, m) \text{ or } S(t) \notin (1/m, m) \text{ or } R(t) \notin (1/m, m)\big\},$$

where τ_m is increasing as $m \to \infty$, set $\tau_\infty = \lim\limits_{m\to\infty} \tau_m$, and we also set $\inf\emptyset = \infty$, whence $\tau_\infty \leq \tau_e$. If the hypothesis that $\tau_\infty = \infty$ is true, then $\tau_e = \infty$. So all we need to show is $\tau_\infty = \infty$. Assume contrarily that $\tau_\infty = \infty$, there is a pair of constants $T > 0$ and $\varepsilon \in (0,1)$, such that $P\{\tau_\infty \leq T\} > \varepsilon$. Thus there exists an integer $k_1 \geq k_0$ such that

$$P\{\tau_m \leq T\} \geq \varepsilon, \ \ \forall m \geq m_1.$$

Consider a function $V(x)$: $R_3^+ \to R_+ (R_+ = \{V | V \geq 0, V \in R\})$

$$\begin{aligned} V_1(I(t), S(t), R(t)) \ =\ & I - a_1 - a_1 \log \frac{I}{a_1} + S - b_1 \\ & - b_1 \log \frac{S}{b_1} + R - \log R. \end{aligned} \tag{3}$$

Based on the Itô's formula, we get

$$\begin{aligned} \mathrm{d}V_1 \ =\ & LV_1 \,\mathrm{d}t + \delta_1 (I - a_1)\,\mathrm{d}W_1 \\ & + \delta_2 (S - b_1)\,\mathrm{d}W_2 + \delta_3 (R-1)\,\mathrm{d}W_3 \\ & + \int_Y [q_1(y) I - a_1 \log(1 + q_1(y))]\,\tilde{N}(\mathrm{d}t, \mathrm{d}y) \\ & + \int_Y [q_2(y) S - b_1 \log(1 + q_2(y))]\,\tilde{N}(\mathrm{d}t, \mathrm{d}y) \\ & + \int_Y [q_3(y) R - \log(1 + q_3(y))]\,\tilde{N}(\mathrm{d}t, \mathrm{d}y), \end{aligned} \tag{4}$$

$$\begin{aligned} LV_1 \ =\ & \left(1 - \frac{a_1}{I}\right)\left[\Lambda - \left(\beta_1 - \beta_2 \frac{S}{m+S}\right) SI - \theta I - \mu I\right] \\ & + \frac{a_1 \delta_1^2}{2} + \left(1 - \frac{b_1}{S}\right)\left[\left(\beta_1 - \beta_2 \frac{S}{m+S}\right) SI \right. \\ & \left. - \lambda (S+R) S - \mu S\right] + \frac{b_1 \delta_2^2}{2} + \left(1 - \frac{1}{R}\right)[\lambda (S+R) S \\ & + \theta I - \mu R] + \frac{\delta_3^2}{2} + \int_Y [q_3(y) - \log(1 + q_1(y))]\nu(\mathrm{d}y) \\ & + \int_Y \big[a_1 q_1(y) - a_1 \log(1 + q_1(y)) + b_1 q_2(y) \\ & - b_1 \log(1 + q_2(y))\big]\nu(\mathrm{d}y) \\ =\ & \Lambda - \mu I - \frac{a_1 \Lambda}{I} + a_1 \left(\beta_1 - \beta_2 \frac{S}{m+S}\right) S + a_1(\theta + \mu) \\ & - \mu S - b_1 \left(\beta_1 - \beta_2 \frac{S}{m+S}\right) I + b_1 \lambda (S+R) + b_1 \mu \\ & - \mu R - \frac{\lambda (S+R) S}{R} - \frac{\theta I}{R} + \mu + \frac{a_1 \delta_1^2}{2} + \frac{b_1 \delta_2^2}{2} \\ & + \frac{\delta_3^2}{2} + \int_Y [q_3(y) - \log(1 + q_1(y))]\nu(\mathrm{d}y) \\ & + \int_Y \big[a_1 q_1(y) - a_1 \log(1 + q_1(y)) + b_1 q_2(y) \\ & - b_1 \log(1 + q_2(y))\big]\nu(\mathrm{d}y) \\ \leq\ & \Lambda + (a_1 \beta_1 - \mu) S + a_1(\theta + \mu) + (b_1 \beta_2 - \mu) I \\ & + b_1 \mu + b_1 \frac{\lambda \Lambda}{\mu} + \frac{a_1 \delta_1^2}{2} + \frac{b_1 \delta_2^2}{2} + \frac{\delta_3^2}{2} \\ & + \int_Y [q_3(y) - \log(1 + q_1(y))]\nu(\mathrm{d}y) \\ & + \int_Y \big[a_1 q_1(y) - a_1 \log(1 + q_1(y)) + b_1 q_2(y) \\ & - b_1 \log(1 + q_2(y))\big]\nu(\mathrm{d}y). \end{aligned}$$

When $a_1 = \mu/\beta_1$ and $b_1 = \mu/\beta_2$, we have

$$\begin{aligned} LV_1 \ \leq\ & \Lambda + \frac{\mu}{\beta_1}\theta + \frac{\mu^2}{\beta_1} + \frac{\mu^2}{\beta_2} + \frac{\lambda \Lambda}{\beta_2} + \frac{a_1 \delta_1^2}{2} + \frac{b_1 \delta_2^2}{2} \\ & + \frac{\delta_3^2}{2} + \int_Y [q_3(y) - \log(1 + q_1(y))]\nu(\mathrm{d}y) \\ & + \int_Y \left[\frac{\mu}{\beta_1} q_1(y) - \frac{\mu}{\beta_1} \log(1 + q_1(y)) \right. \\ & \left. + \frac{\mu}{\beta_2} q_2(y) - \frac{\mu}{\beta_2} \log(1 + q_2(y))\right]\nu(\mathrm{d}y) = M. \end{aligned}$$

We take the integral between 0 and $\tau_m \wedge T$ of both sides of Eq. (3) and then we take the expectation and get

$$\begin{aligned} 0 \ \leq\ & E[V(I(\tau_m \wedge T), S(\tau_m \wedge T), R(\tau_m \wedge T))] \\ \leq\ & V(I(0), S(0), R(0)) + MT. \end{aligned}$$

We can define that $H(h) := \inf\{V(x), x_i \geq h \text{ or } x_i \leq 1/h, \ \ i = 1,2,3\}$ for each $h > 0$, where $x_1 = I$, $x_2 = S$, $x_3 = R$, and $\lim\limits_{h\to\infty} H(h) = \infty$.

So when $m \to \infty$, we have $\infty > V(I(0), S(0), R(0)) + MT = \infty$ is a contradiction. Therefore $\tau_\infty = \infty$ is proved, and our model has a unique global solution $(I(t), S(t), R(t)) \in R_3^+$.

4. The dynamic property around the rumor-free equilibrium

In this section, we will study the behavior of global solution $(I(t), S(t), R(t))$ in the rumor-free equilibrium $E_0 = \left(\frac{\Lambda}{\theta+\mu}, 0, \frac{\theta\Lambda}{\mu(\theta+\mu)}\right)$. Assume that $R_0 \leq 1$, and set

$$\begin{aligned} k_1 \ =\ & 2\mu - 2\delta_1^2 - 6\int_Y q_1^2(y)\nu(\mathrm{d}y), \\ k_2 \ =\ & 2\mu + \frac{4\mu\lambda}{\beta_1} - \delta_2^2 - \frac{2\mu(\lambda - \beta_1)}{\beta_1} \\ & - \frac{2\lambda\Lambda}{\theta+\mu} - 3\int_Y q_2^2(y)\nu(\mathrm{d}y), \\ k_3 \ =\ & 2\mu + \frac{4\mu^2}{\theta} - 2\left(1 + \frac{2\mu}{\theta}\right)\delta_3^2 - \frac{2\mu(\lambda - \beta_1)}{\beta_1} \\ & - \frac{2\lambda\Lambda}{\theta+\mu} - 6\int_Y q_3^2(y)\nu(\mathrm{d}y) - \frac{4\mu}{\theta}\int_Y q_3^2(y)\nu(\mathrm{d}y), \\ K_1 \ =\ & \left[2\left(1 + \frac{2\mu}{\theta}\right)\delta_3^2 + 6\int_Y q_3^2(y)\nu(\mathrm{d}y) \right. \\ & \left. + \frac{4\mu}{\theta}\int_Y q_3^2(y)\nu(\mathrm{d}y)\right]\left(\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)^2 \end{aligned}$$

$$+\left[2\delta_1^2+6\int_Y q_1^2(y)v(\mathrm{d}y)\right]\left(\frac{\Lambda}{\theta+\mu}\right)^2.$$

Theorem 2 When the following inequalities are satisfied:

$$\begin{aligned}
2\mu &\geq 2\delta_1^2+6\int_Y q_1^2(y)v(\mathrm{d}y),\\
2\mu+\frac{4\mu\lambda}{\beta_1} &\geq \delta_2^2+\frac{2\mu(\lambda-\beta_1)}{\beta_1}+\frac{2\lambda\Lambda}{\theta+\mu}+3\int_Y q_2^2(y)v(\mathrm{d}y),\\
2\mu+\frac{4\mu^2}{\theta} &\geq 2(1+\frac{2\mu}{\theta})\delta_3^2+\frac{2\mu(\lambda-\beta_1)}{\beta_1}+\frac{2\lambda\Lambda}{\theta+\mu}\\
&\quad +6\int_Y q_3^2(y)v(\mathrm{d}y)+\frac{4\mu}{\theta}\int_Y q_3^2(y)v(\mathrm{d}y),
\end{aligned}$$

then we have

$$\limsup_{\tau\to\infty}\frac{1}{\tau}E\int_0^\tau\left[\left(I(t)-\frac{\Lambda}{\theta+\mu}\right)^2+S^2(t)+\left(R(t)-\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)^2\right]\mathrm{d}t\leq\frac{K_1}{l_1},$$

for any given initial condition $(I(0),S(0),R(0))\in R_3^+$, where $l_1=\min\{k_1,k_2,k_3\}$.

Proof Let $x(t)=I(t)-\frac{\Lambda}{\theta+\mu}$, $y(t)=S(t)$, $z(t)=R(t)-\frac{\theta\Lambda}{\mu(\theta+\mu)}$, then we have

$$\begin{aligned}
\mathrm{d}x=&\left[\Lambda-\left(\beta_1-\beta_2\frac{y}{m+y}\right)\left(xy+\frac{\Lambda y}{\theta+\mu}\right)-\theta\left(x+\frac{\Lambda}{\theta+\mu}\right)-\mu\left(x+\frac{\Lambda}{\theta+\mu}\right)\right]\mathrm{d}t\\
&+\delta_1\left(x+\frac{\Lambda}{\theta+\mu}\right)\mathrm{d}W_1(t)+\int_Y q_1(y)\left(x+\frac{\Lambda}{\theta+\mu}\right)\tilde{N}(\mathrm{d}t,\mathrm{d}y),\\
\mathrm{d}y=&\left[\left(\beta_1-\beta_2\frac{y}{m+y}\right)\left(xy+\frac{\Lambda y}{\theta+\mu}\right)-\lambda y\left(y+z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)-\mu y\right]\mathrm{d}t+\delta_2 y\mathrm{d}W_2(t)\\
&+\int_Y q_2(y)y\tilde{N}(\mathrm{d}t,\mathrm{d}y),\\
\mathrm{d}z=&\left[\lambda y\left(y+z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)+\theta\left(x+\frac{\Lambda}{\theta+\mu}\right)-\mu\left(z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)\right]\mathrm{d}t+\delta_3\left(z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)\mathrm{d}W_3(t)\\
&+\int_Y q_3(y)\left(z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)\tilde{N}(\mathrm{d}t,\mathrm{d}y).
\end{aligned}$$

If we let $V_2(x,y,z)=(x+y+z)^2+ay+bz^2$, where a, b are two constants to be determined below.

Then we have

$$\begin{aligned}
\mathrm{d}V_2 =\ & LF\,\mathrm{d}t+2(x+y+z)\left[\delta_1\left(x+\frac{\Lambda}{\theta+\mu}\right)\mathrm{d}W_1(t)+\delta_2 y\mathrm{d}W_2(t)+\delta_3\left(z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)\mathrm{d}W_3(t)\right]\\
&+a\delta_2 y\mathrm{d}W_2(t)+2bz\delta_3\left(z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)\mathrm{d}W_3(t)\\
&+\int_Y[ayq_2(y)]\tilde{N}(\mathrm{d}t,\mathrm{d}y)\\
&+\int_Y\left[q_1(y)\left(x+\frac{\Lambda}{\theta+\mu}\right)+q_2(y)y+q_3(y)\left(z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)\right]^2\tilde{N}(\mathrm{d}t,\mathrm{d}y)\\
&+\int_Y 2(x+y+z)\left[q_1(y)\left(x+\frac{\Lambda}{\theta+\mu}\right)+q_2(y)y+q_3(y)\left(z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)\right]\tilde{N}(\mathrm{d}t,\mathrm{d}y)\\
&+\int_Y\left[2bzq_3(y)\left(z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)+bq_3^2(y)\left(z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)^2\right]\tilde{N}(\mathrm{d}t,\mathrm{d}y), \qquad (5)
\end{aligned}$$

$$\begin{aligned}
LV_2 =\ & 2(x+y+z)(-\mu x-\mu y-\mu z)\\
&+a\left[\left(\beta_1-\beta_2\frac{y}{m+y}\right)\left(xy+\frac{\Lambda y}{\theta+\mu}\right)-\lambda y\left(y+z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)-\mu y\right]\\
&+2bz\left[\lambda y\left(y+z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)+\theta\left(x+\frac{\Lambda}{\theta+\mu}\right)-\mu\left(z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)\right]+\delta_1^2\left(x+\frac{\Lambda}{\theta+\mu}\right)^2+\delta_2^2y^2\\
&+(1+b)\delta_3^2\left(z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)^2\\
&+\int_Y\left[q_1(y)\left(x+\frac{\Lambda}{\theta+\mu}\right)+q_2(y)y+q_3(y)\left(z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)\right]^2v(\mathrm{d}y)\\
&+\int_Y bq_3^2(y)\left(z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)^2v(\mathrm{d}y)\\
\leq\ & -2\mu x^2-(2\mu+a\lambda)y^2-2\mu(1+b)z^2+(a\beta_1-4\mu)xy\\
&+\left(a\lambda-4\mu+\frac{2b\lambda\theta\Lambda}{\mu(\theta+\mu)}\right)yz+(2b\theta-4\mu)xz\\
&+a\left(\frac{\beta_1\Lambda}{\theta+\mu}-\frac{\lambda\theta\Lambda}{\mu(\theta+\mu)}-\mu\right)y-2b\lambda xyz\\
&+\delta_1^2\left(x+\frac{\Lambda}{\theta+\mu}\right)^2+\delta_2^2y^2\\
&+(1+b)\delta_3^2\left(z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)^2\\
&+\int_Y\left[q_1(y)\left(x+\frac{\Lambda}{\theta+\mu}\right)+q_2(y)y+q_3(y)\left(z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)\right]^2v(\mathrm{d}y)\\
&+\int_Y bq_3^2(y)\left(z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)^2v(\mathrm{d}y).
\end{aligned}$$

Choosing $a=\frac{4\mu}{\beta_1}$, $b=\frac{2\mu}{\theta}$, and since for $R_0=\frac{\beta_1\Lambda\mu-\mu^3}{\theta(\lambda\Lambda+\mu^2)}\leq 1$, we have $a\left(\frac{\beta_1\Lambda}{\theta+\mu}-\frac{\lambda\theta\Lambda}{\mu(\theta+\mu)}-\mu\right)y\leq 0$.

Using the inequalities $2ab \le a^2+b^2$, $(a+b+c)^2 \le 3a^2+3b^2+3c^2$, $(a+b)^2 \le 2a^2+2b^2$, we have

$$\begin{aligned}
LV_2 \le & -2\mu x^2 - \left(2\mu + \frac{4\mu}{\beta_1}\lambda\right)y^2 - 2\mu\left(1+\frac{2\mu}{\theta}\right)z^2 \\
& + \left(\frac{2\mu(\lambda-\beta_1)}{\beta_1} + \frac{2\lambda\Lambda}{\theta+\mu}\right)(y^2+z^2) - \frac{2\mu\lambda y}{\theta}(x^2+z^2) \\
& + \delta_1^2\left(x+\frac{\Lambda}{\theta+\mu}\right)^2 + \delta_2^2 y^2 \\
& + \left(1+\frac{2\mu}{\theta}\right)\delta_3^2\left(z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)^2 \\
& + \int_Y \left[q_1(y)\left(x+\frac{\Lambda}{\theta+\mu}\right) + q_2(y)y \right. \\
& \left. + q_3(y)\left(z+\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)\right]^2 \nu(\mathrm{d}y) \\
& + \frac{2\mu}{\theta}\int_Y q_3^2(y)(z+\frac{\theta\Lambda}{\mu(\theta+\mu)})^2\nu(\mathrm{d}y) \\
\le & -\left[2\mu - 2\delta_1^2 - 6\int_Y q_1^2(y)\nu(\mathrm{d}y)\right]x^2 \\
& - \left[2\mu + \frac{4\mu\lambda}{\beta_1} - \delta_2^2 - \frac{2\mu(\lambda-\beta_1)}{\beta_1}\right. \\
& \left. - \frac{2\lambda\Lambda}{\theta+\mu} - 3\int_Y q_2^2(y)\nu(\mathrm{d}y)\right]y^2 \\
& - \left[2\mu + \frac{4\mu^2}{\theta} - 2\left(1+\frac{2\mu}{\theta}\right)\delta_3^2 - \frac{2\mu(\lambda-\beta_1)}{\beta_1}\right. \\
& \left. - \frac{2\lambda\Lambda}{\theta+\mu} - 6\int_Y q_3^2(y)\nu(\mathrm{d}y) - \frac{4\mu}{\theta}\int_Y q_3^2(y)\nu(\mathrm{d}y)\right]z^2 \\
& + \left[2\delta_1^2 + 6\int_Y q_1^2(y)\nu(\mathrm{d}y)\right]\left(\frac{\Lambda}{\theta+\mu}\right)^2 \\
& + \left[2(1+\frac{2\mu}{\theta})\delta_3^2 + 6\int_Y q_3^2(y)\nu(\mathrm{d}y)\right. \\
& \left. + \frac{4\mu}{\theta}\int_Y q_3^2(y)\nu(\mathrm{d}y)\right]\left(\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)^2.
\end{aligned}$$

Namely, $LV_2 \le -k_1x^2 - k_2y^2 - k_3z^2 + K_1$. Taking the integral between 0 and τ of both sides of Eq. (5) and taking the expectation, we can get

$$\begin{aligned}
0 \le & E\left[V_2(x(\tau),y(\tau),z(\tau))\right] \\
\le & E\left\{\int_0^\tau \left(-k_1\left(I(t)-\frac{\Lambda}{\theta+\mu}\right)^2\right.\right. \\
& \left.\left. - k_2S^2(t) - k_3\left(R(t)-\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)^2\right)\mathrm{d}t\right\} \\
& + V_2(x(0),y(0),z(0)) + K_1\tau.
\end{aligned}$$

Let $l_1 = \min\{k_1,k_2,k_3\}$, then

$$\begin{aligned}
& E\left\{\int_0^\tau \left(\left(I(t)-\frac{\Lambda}{\theta+\mu}\right)^2 + S^2(t)\right.\right. \\
& \left.\left. + \left(R(t)-\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)^2\right)\mathrm{d}t\right\} \le \frac{V_2(x(0),y(0),z(0))}{l_1} + \frac{K_1}{l_1}\tau.
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
& \limsup_{t\to\infty}\frac{1}{t}E\int_0^\tau\left[\left(I(t)-\frac{\Lambda}{\theta+\mu}\right)^2 + S^2(t)\right. \\
& \left. + \left(R(t)-\frac{\theta\Lambda}{\mu(\theta+\mu)}\right)^2\right]\mathrm{d}t \le \frac{K_1}{l_1}.
\end{aligned}$$

Remark 1 When $R_0 \le 1$ and satisfies the conditions of the theorem, we find that the solution fluctuates around the rumor-free equilibrium.

5. The dynamic property around the rumor endemic equilibrium

In this section, we are interested in the behavior of global positive solution $(I(t),S(t),R(t))$ of the model around the rumor endemic equilibrium E^*. Assume that $R_0 > 1$, and set

$$\begin{aligned}
k_4 &= \mu - \delta_1^2 - 3\int_Y q_1^2(y)\nu(\mathrm{d}y), \\
k_5 &= \mu + \lambda p + \mu q - \delta_2^2(1+q) - (3+q)\int_Y q_2^2(y)\nu(\mathrm{d}y), \\
k_6 &= \mu - \frac{q\lambda\Lambda}{\mu} - \delta_3^2(1+q) - (3+q)\int_Y q_3^2(y)\nu(\mathrm{d}y), \\
K_2 &= \delta_1^2 I^{*2} + (1+q)\delta_2^2 S^{*2} + (1+q)\delta_3^2 R^{*2} + \frac{1}{2}\delta_2^2 pS^* \\
& \quad + \int_Y [pS^*q_2(y)]\nu(\mathrm{d}y) + \int_Y\left[3q_1^2(y)I^{*2}\right. \\
& \quad + (3+q)q_2^2(y)S^{*2} + (3+q)q_3^2(y)R^{*2} \\
& \quad \left. - pS^*\log\frac{1+q_2(y)}{S^*}\right]\nu(\mathrm{d}y).
\end{aligned}$$

Theorem 3 When the following inequalities are satisfied,

$$\begin{aligned}
& \mu \ge \delta_1^2 + 3\int_Y q_1^2(y)\nu(\mathrm{d}y), \\
& \mu + \lambda p + \mu q \ge \delta_2^2(1+q) + (3+q)\int_Y q_2^2(y)\nu(\mathrm{d}y), \\
& \mu \ge \frac{q\lambda\Lambda}{\mu} + \delta_3^2(1+q) + (3+q)\int_Y q_3^2(y)\nu(\mathrm{d}y),
\end{aligned}$$

then the solution $(I(t),S(t),R(t))$ satisfies

$$\limsup_{\tau\to\infty}\frac{1}{\tau}E\int_0^\tau\left[(I-I^*)^2 + (S-S^*)^2 + (R-R^*)^2\right]\mathrm{d}t \le \frac{K_2}{l_2}$$

for any given initial condition $(I(0),S(0),R(0)) \in R_3^+$, where $l_2 = \min\{k_4,k_5,k_6\}$.

Proof Considering the following function:

$$\begin{aligned}
V_3 = & \frac{1}{2}[I-I^*+S-S^*+R-R^*]^2 + p\left(S-S^*-S^*\log\frac{S}{S^*}\right) \\
& + \frac{q}{2}(S-S^*)^2 + \frac{q}{2}(R-R^*)^2, \qquad (6)
\end{aligned}$$

where p and q are two positive constants. Using Itô's formula, we get

$$\begin{aligned}
\mathrm{d}V_3 = & LV_3\,\mathrm{d}t + (I-I^*+S-S^*+R-R^*) \\
& \times(\delta_1 I\,\mathrm{d}B_1 + \delta_2 S\,\mathrm{d}B_2 + \delta_3 R\,\mathrm{d}B_3) + p\delta_2(S-S^*)\,\mathrm{d}B_2 \\
& + q(S-S^*)\delta_2\,\mathrm{d}B_2 + q(R-R^*)\delta_3\,\mathrm{d}B_3
\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{2}\int_Y [q_1(y)I+q_2(y)S+q_3(y)R]^2\tilde{N}(\mathrm{d}t,\mathrm{d}y)\\
&+\int_Y (I-I^*+S-S^*+R-R^*)\\
&\times[q_1(y)I+q_2(y)S+q_3(y)R]\tilde{N}(\mathrm{d}t,\mathrm{d}y)\\
&+\int_Y p\left[q_2(y)S-S\log\frac{1+q_2(y)}{S^*}\right.\\
&\left.+q(S-S^*)q_2(y)S+\frac{q}{2}(q_2(y)S)^2\right]\tilde{N}(\mathrm{d}t,\mathrm{d}y)\\
&+\int_Y q(R-R^*)q_3(y)R+\frac{q}{2}[q_2(y)R]^2\tilde{N}(\mathrm{d}t,\mathrm{d}y), \quad (7)
\end{aligned}
$$

where

$$
\begin{aligned}
LV_3 =\ & [I-I^*+S-S^*+R-R^*](\Lambda-\mu I-\mu S-\mu R)\\
&+p(S-S^*)[(\beta_1-\beta_2 f(S))I-\lambda(S+R)-\mu]\\
&+q(S-S^*)[(\beta_1-\beta_2 f(S))IS-\lambda(S+R)S-\mu S]\\
&+q(R-R^*)[\lambda(S+R)S+\theta I-\mu R]+\frac{1}{2}\delta_1^2 I^2\\
&+\frac{1}{2}(1+q)\delta_2^2 S^2+\frac{1}{2}(1+q)\delta_3^2 R^2+\frac{1}{2}\delta_2^2 pS^*\\
&+\frac{1}{2}\int_Y [q_1(y)I+q_2(y)S+q_3(y)R]^2 v(\mathrm{d}y)\\
&+\int_Y \left[pS^*q_2(y)-pS^*\log\frac{1+q_2(y)}{S^*}\right]v(\mathrm{d}y)\\
&+\int_Y \left[\frac{q}{2}q_2^2(y)S^2+\frac{q}{2}q_3^2(y)R^2\right]v(\mathrm{d}y).
\end{aligned}
$$

Since the endemic equilibrium $E^*=(I^*,S^*,R^*)$ satisfies

$$
\begin{aligned}
&\Lambda-(\beta_1-\beta_2\frac{S^*}{m+S^*})I^*S^*-\theta I^*-\mu I^*=0,\\
&(\beta_1-\beta_2\frac{S^*}{m+S^*})I^*S^*-\lambda S^*(S^*+R^*)-\mu S^*=0,\\
&\lambda S^*(S^*+R^*)+\theta I^*-\mu R^*=0,
\end{aligned}
$$

we get

$$
\begin{aligned}
LV_3 =\ & -\mu(I-I^*)^2-\mu(S-S^*)^2-\mu(R-R^*)^2\\
&-2\mu(I-I^*)(S-S^*)-2\mu(I-I^*)(R-R^*)\\
&-2\mu(S-S^*)(R-R^*)-p\beta_2(f(S)-f(S^*))(S-S^*)I\\
&+p(\beta_1-\beta_2 f(S^*))(I-I^*)(S-S^*)\\
&-\lambda p(S-S^*)^2-\lambda p(S-S^*)(R-R^*)\\
&-q\beta_2(f(S)-f(S^*))(S-S^*)IS\\
&-q(\beta_1-\beta_2 f(S^*))I(S-S^*)^2\\
&-q(\beta_1-\beta_2 f(S^*))S^*(I-I^*)(S-S^*)-\lambda q(S-S^*)^2S\\
&-\mu q(S-S^*)^2-q\lambda(S^*+R^*)(S-S^*)^2\\
&+\lambda qS(R-R^*)^2+\lambda q(S^*-R^*)(S-S^*)(R-R^*)\\
&+q\theta(I-I^*)(R-R^*)-\mu q(R-R^*)^2+\frac{1}{2}\delta_1^2 I^2\\
&+\frac{1}{2}\delta_2^2(1+q)S^2+\frac{1}{2}\delta_3^2 R^2+\frac{1}{2}\delta_2^2 pS^*\\
&+\frac{1}{2}\int_Y [q_1(y)I+q_2(y)S+q_3(y)R]^2 v(\mathrm{d}y)\\
&+\int_Y \left[pS^*q_2(y)-pS^*\log\frac{1+q_2(y)}{S^*}\right]v(\mathrm{d}y)\\
&+\int_Y \left[\frac{q}{2}q_2^2(y)S^2+\frac{q}{2}q_3^2(y)R^2\right]v(\mathrm{d}y).
\end{aligned}
$$

Choosing $p=\frac{2\mu}{\beta_1-\beta_2 f(S^*)}+\frac{2\mu S^*}{\theta}$, $q=\frac{2\mu}{\theta}$ and noticing that both $S-S^*$ and $f(S)-f(S^*)$ have the same sign, where $f(S)=\frac{m}{m+S}$, we get

$$
\begin{aligned}
LV_3 \le\ & -\mu(I-I^*)^2-[\lambda p+\mu+\mu q](S-S^*)^2\\
&-\left(\mu-\frac{q\lambda\Lambda}{\mu}\right)(R-R^*)^2+\frac{1}{2}\delta_1^2 I^2\\
&+\frac{1}{2}(1+q)\delta_2^2 S^2+\frac{1}{2}(1+q)\delta_3^2 R^2+\frac{1}{2}\delta_2^2 pS^*\\
&+\int_Y \left[\frac{q}{2}q_2^2(y)S^2+\frac{q}{2}q_3^2(y)R^2\right]v(\mathrm{d}y)\\
&+\frac{1}{2}\int_Y [q_1(y)I+q_2(y)S+q_3(y)R]^2 v(\mathrm{d}y)\\
&+\int_Y \left[pS^*q_2(y)-pS^*\log\frac{1+q_2(y)}{S^*}\right]v(\mathrm{d}y).
\end{aligned}
$$

By using the inequalities $a^2\le 2(a-b)^2+2b^2(a+b+c)^2\le 3a^2+3b^2+3c^2$, we get

$$
\begin{aligned}
LV_3 \le & -\mu(I-I^*)^2-(\mu+\lambda p+\mu q)(S-S^*)^2\\
&-\left(\mu-\frac{q\lambda\Lambda}{\mu}\right)(R-R^*)^2+\delta_1^2(I-I^*)^2\\
&+\delta_1^2 I^{*2}+\delta_2^2(1+q)(S-S^*)^2+(1+q)\delta_2^2 S^{*2}\\
&+\delta_3^2(1+q)(R-R^*)^2+(1+q)\delta_3^2 R^{*2}+\frac{1}{2}\delta_2^2 pS^*\\
&+\int_Y \left[\frac{3}{2}q_1^2(y)I^2+\frac{3+q}{2}q_2^2(y)S^2+\frac{3+q}{2}q_3^2(y)R^2\right]v(\mathrm{d}y)\\
&+\int_Y \left[pS^*q_2(y)-pS^*\log\frac{1+q_2(y)}{S^*}\right]v(\mathrm{d}y)\\
\le & -\left[\mu-\delta_1^2-3\int_Y q_1^2(y)v(\mathrm{d}y)\right](I-I^*)^2+\delta_1^2 I^{*2}\\
&+(1+q)\delta_2^2 S^{*2}+(1+q)\delta_3^2 R^{*2}\\
&-\left[\mu+\lambda p+\mu q-\delta_2^2(1+q)-(3+q)\int_Y q_2^2(y)v(\mathrm{d}y)\right]\\
&\times(S-S^*)^2+\frac{1}{2}\delta_2^2 pS^*\\
&-\left[\mu-\frac{q\lambda\Lambda}{\mu}-\delta_3^2(1+q)-(3+q)\int_Y q_3^2(y)v(\mathrm{d}y)\right]\\
&\times(R-R^*)^2+\int_Y [pS^*q_2(y)]v(\mathrm{d}y)\\
&+\int_Y \left[3q_1^2(y)I^{*2}+(3+q)q_2^2(y)S^{*2}+(3+q)q_3^2(y)R^{*2}\right.\\
&\left.-pS^*\log\frac{1+q_2(y)}{S^*}\right]v(\mathrm{d}y).
\end{aligned}
$$

Taking the integral between 0 and τ of both sides of Eq. (7) and taking the expectation, we can get

$$
\begin{aligned}
0 \le\ & E[V_3(I(\tau),S(\tau),R(\tau))]\\
\le\ & E\left\{\int_0^\tau \left(-k_4(I-I^*)^2-k_5(S-S^*)^2-k_6(R-R^*)^2\right)\mathrm{d}t\right\}\\
&+V_3(I(0),S(0),R(0))+K_2\tau.
\end{aligned}
$$

Hence

$$E\left\{\int_0^\tau (k_4(I-I^*)^2+k_5(S-S^*)^2+k_6(R-R^*)^2)\,\mathrm{d}t\right\} \leq V_3(I(0),S(0),R(0))+K_2\tau,$$

then

$$E\left\{\int_0^\tau ((I-I^*)^2+(S-S^*)^2+(R-R^*)^2)\,\mathrm{d}t\right\} \leq \frac{V_3(I(0),S(0),R(0))}{l_2}+\frac{K_2\tau}{l_2},$$

and therefore

$$\lim_{\tau\to\infty}\sup\frac{1}{\tau}E\int_0^\tau[(I-I^*)^2+(S-S^*)^2+(R-R^*)^2]\mathrm{d}t \leq \frac{K_2}{l_2}.$$

Remark 2 When $R_0 > 1$and satisfies the conditions of the last theorem, we can conclude that the solution fluctuates around the endemic equilibrium.

6. Simulation results

In this part, we will conduct numerical simulation on Matlab for the above model, and prove the above conclusion by numerical solution of stochastic differential equation, which is very important in the research of rumor propagation. We take the parameters $q_i(y)=\frac{-w_i y}{1+y^2}$, $i=1,2,3$, with $y=0.5$, $w_1=0.1$, $w_2=0.2$, and $w_3=0.3$.

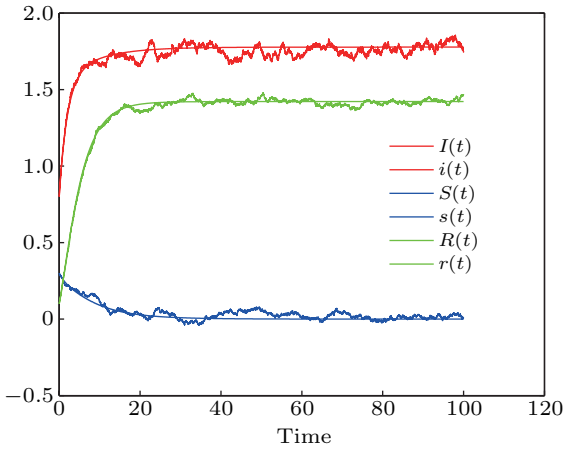


Fig. 2. Trajectories of the solutions around the rumor-free equilibrium.

As shown in Fig. 2, when the basic regenerative number $R_0 < 1$, there is rumor-free equilibrium point, and the solution process of *ISR* system with Lévy jump tends to the rumor-free equilibrium point of the deterministic system, indicating that the rumor tends to be extinct. At the local equilibrium point, that is, when $R_0 > 1$, the solution process of the system with Lévy jump makes stochastic oscillations around the rumor equilibrium point of the deterministic system, and the amplitude of the oscillations is related to the magnitude of the disturbance to the system, as shown in Fig. 3, indicating that the rumor is continuously propagating at this time.

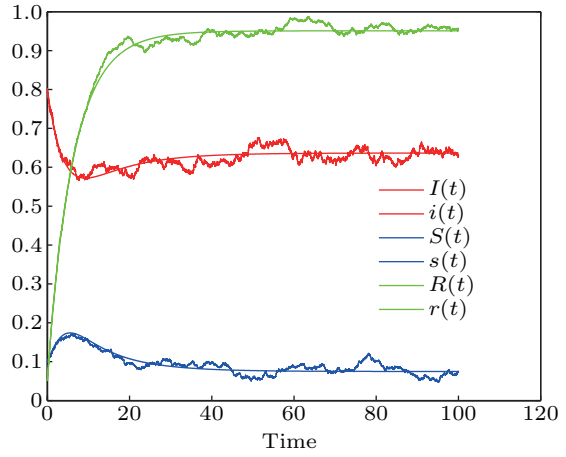


Fig. 3. Trajectories of the solutions around the endemic equilibrium.

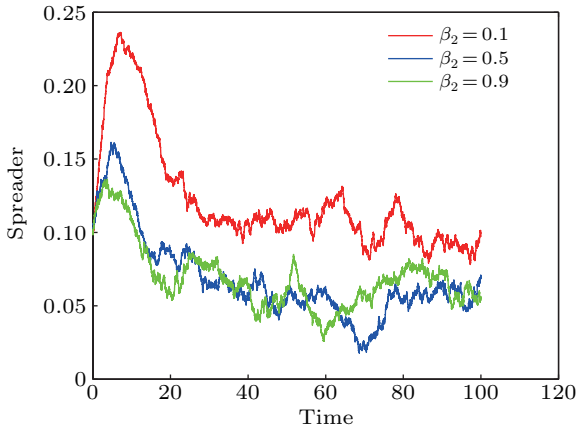


Fig. 4. Trajectories of the solutions around the endemic equilibrium with different values of β_2.

In addition, Fig. 4 is the simulation diagram in case of β_2=0.1, 0.5, and 0.9, showing the influence of different media coverage on rumor propagation scale. As shown, with the increase of media coverage, rumors spread with size reduced, to some extent, this suggests that the media reports can influence the spread of rumors and control process. Just as the derivation in the previous model, $\beta_1-\beta_2\frac{S}{m+S}$ represents the conversion rate of ignorant people accepting the rumor and entering the group of spreaders when the ignorant I contact the spreaders S. The overall conversion rate decreases as media coverage β_2 increases. For example, during the outbreak of the COVID-19 in Wuhan, China, the rumor that 'only by wearing multiple layers of masks can the virus be isolated' spread widely, which was reduced with the repeated refuting of the rumor by media coverage. Figure 5 is a simulation diagram of the case $\theta=0.1$, 0.2, and 0.3. We can see that under the influence of government science education for all, the scale of rumor propagation will decrease with the increase of individual immunization rate. For example, during the outbreak of the COVID-19, the government popularized the knowledge of the COVID-19 through the official media, making people immune to the rumor that taking antibiotics can prevent the COVID-19 infection. Therefore, we can control the spread of rumors in

emergencies by increasing popular science education and media coverage. On the one hand, the government releases the progress of emergency events and related popular science information through official channels in time to improve information transparency and citizens' ability to identify rumors. On the other hand, the media should be timely targeted to refute rumors. Increasing media coverage can change people's cognition and judgment, thus effectively reduce the number of rumor-mongers and the negative impact of rumor-spreading on social harmony.

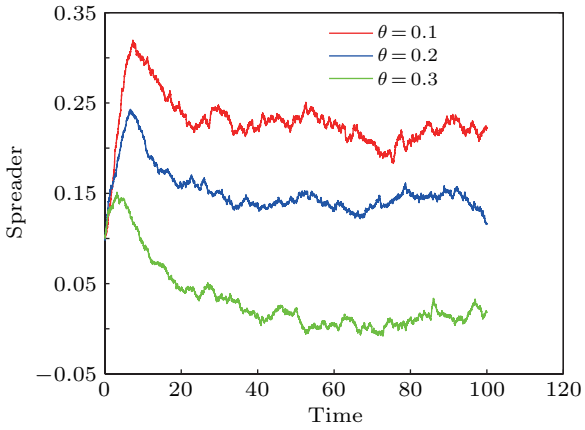


Fig. 5. Trajectories of the solutions around the endemic equilibrium with different values of θ.

7. Conclusion

This paper studied the *ISR* stochastic rumor propagation model with Lévy jump, which can more accurately reflect the rumor propagation law in a region with a relatively small range, and can be used to study the propagation process of fast and destructive rumors. We used Lyapunov method to study the stochastic model, and obtained the relationship between the solution process of *ISR* system with stochastic jump and its deterministic system. When $R_0 < 1$, the solution process of *ISR* system with Lévy jump tends to the rumor-free equilibrium point of the deterministic system, then the rumor tends to extinction. When $R_0 > 1$, the solution process of the system with Lévy jump makes stochastic oscillations around the rumor equilibrium point of the deterministic system, and the amplitude of the oscillations is related to the magnitude of the disturbance to the system. At this point, the rumor continues to spread. In addition, through numerical simulation, it can be concluded that the role of media is crucial to reduce the rumor propagation rate, and increasing media coverage will reduce the basic reproduction number, thus reduce the number of individual spreaders.

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