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Large-amplitude dust acoustic solitons in an opposite polarity dusty plasma with generalized polarization force

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Linear and nonlinear dust acoustic (DA) waves have been investigated in an opposite polarity dusty plasma comprising negatively and positively charged dust grains, Maxwellian electrons and ions, including the generalized polarization force effect. The properties of linear DA waves have been significantly altered by the dual dust polarity and polarization force. Large amplitude DA solitons have been discussed in the framework of the Sagdeev potential technique. Our results show that both rarefactive and compressive solitons can exist in such a dusty plasma. The basic features of the Sagdeev potential have been examined under the effect of the polarization force parameter R, the ratio of the charge number of the positive dust to that of the negative dust Z, and the Mach number M. The results show that these parameters play a significant role in determining the region of existence of large amplitude DA solitons.

Keywords: large amplitude, opposite polarity dusty plasma, polarization force

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1. Introduction

After the theoretical prediction of the dust acoustic (DA) wave in a dusty plasma in the early 1990s by Rao *et al.*,[1] Barkan *et al.*[2] confirmed the existence of such a wave in laboratory plasmas. The DA wave is one of the fundamental collective excitations in dusty plasma composed of electrons, ions, and negatively charged dust particles. It is a very low frequency mode that arises due to the longitudinal dust oscillation resulting from the interplay of dust inertia, the screened electric field, and the background pressures of hot ions and electrons. Over the past three decades, a large number of works have been carried out to discuss various aspects of the linear and nonlinear propagation of DA waves in different dusty plasma systems.[3–8] Most of these studies focused on a three-component dusty plasma system comprised of electrons, ions and negatively charged dust particles. Although there are many observations in both space[9–12] and laboratory[13–15] plasmas that indicate the existence of both positively and negatively charged dust populations, it is well known that the presence of positively charged dust grains depends upon various charging mechanisms, such as secondary electron emission from dust grain surfaces,[16] ultraviolet photo-ionization,[17] and thermionic emission by radiative heating.[18]

It is noted that the coexistence of both positively and negatively charged dust in plasmas introduces a new dusty plasma model which is widely known as opposite polarity dusty plasma (OPDP).[19,20] This new field of dusty plasmas has attracted a lot of attention in recent years. However, most of the studies on DA waves in OPDP have focused on small-amplitude solitary waves in the framework of the reductive perturbation technique.[21–26] Actually, the perturbation technique is valid only for small-amplitude waves, but it is not suitable for large-amplitude solitary waves. In fact, large amplitude solitary waves can be studied analytically only by using non-perturbative methods; namely, the pseudopotential method of Sagdeev[27] (which is also called the Sagdeev potential method). Chatterjee and Kundu[28] reported that a dusty plasma with an opposite polarity dust fluid and Boltzmann distributed ions admits both the rarefractive and compressive solitary waves associated with nonlinear large-amplitude DA waves. Verheest *et al.*[29] used the Sagdeev potential approach to investigate large amplitude DA solitary waves in a dusty plasma containing both cold and adiabatic negative/positive dust. They found that solitary waves can only occur for nonlinear structure velocities smaller than the adiabatic dust thermal velocity, leading to a novel dust-acoustic-like mode based on the interplay between the two dust species. Maharaj *et al.*[30] studied large amplitude solitary waves in the framework of the Sagdeev potential approach in an opposite polarity dusty plasma comprised of a cold negative dust fluid, adiabatic positive dust fluid, Boltzmann electrons and nonthermal ions. They found that both rarefactive and compressive solitary waves existed. Ahmad *et al.*[31] discussed the essential features of arbitrary amplitude DA solitary waves by employing the Sagdeev potential approach in an adiabatic opposite polarity dusty plasma with vortex-like distributed electrons and ions. They found that the basic features of dust-acoustic solitary structures are significantly modified due to

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the polarity of dust, the trapping of plasma particles (electrons and ions), and the temperatures of dust fluids. Abulwafa *et al.*[32] showed that both types of compressive and rarefactive DA solitons can coexist and propagate in a four-component dusty plasma system containing non-extensive electrons and ions as well as positively and negatively charged dust grains. They employed the Sagdeev potential and phase-portrait approaches. They found that the Mach number, non-extensive parameter and dust temperature ratio have constructive effects on the profile of dust acoustic solitons.

Actually, there are many forces acting on a massive charged dust grain embedded in a plasma. One of them is the polarization force, which arises due to any deformation in the Debye sheath around the dust grain in a background of nonuniform plasmas. Hamaguchi and Farouki[33,34] illustrated the concept of polarization force on the dust grain and its importance in a dusty plasma. The first introduction of the polarization force into the dust momentum equation was by Khrapak *et al.*[35] They investigated the propagation characteristic of a linear DA wave and mentioned that the polarization force has an important effect on the massive dust particles. It is noted that the effect of polarization force on small-amplitude DA solitary waves has attracted a lot of attention,[26,36–39] while few studies have focused on large-amplitude solitary waves in the framework of the Sagdeev potential approach. For example, Bandyopadhyay *et al.*[40] used the Sagdeev potential approach to investigate the effect of polarization force on large-amplitude DA solitary waves in a dusty plasma composed of negatively charged dust grains and Maxwellian distributed electrons and ions. They found that the presence of a polarization force leads to a decrease in the Mach number regime where solitary structures can exist. El-Taibany *et al.*[41] investigated the linear and nonlinear features of gravitoelectrostatic opposite polarity dusty plasma in the framework of the Sagdeev potential approach, taking into account the effect of polarization force on the negatively charged dust grains. They found that the real wave frequency is reduced by the presence of polarization force. Most of the above investigations focused only on the the limiting case $T_e \gg T_i$, in which the Debye length approximated by $\lambda_D \approx \lambda_{Di}$ and the polarization force arises mainly due to the polarization of plasma ions around the negatively charged dust grain. Here, T_e (T_i) is the electron (ion) temperature and λ_{Di} is the Debye length of ions. But this approximation is not congruous in general in an opposite polarity dusty plasma. Therefore, the major goal of the current paper is to examine the effect of generalized polarization force (which is valid to either $T_e \gg T_i$ or $T_e \ll T_i$) on the propagation regimes of the large-amplitude DA solitary waves in an opposite polarity dusty plasma system consisting of negatively and positively charged dust grains, electrons and ions obeying Maxwellian distribution, where the generalized polarization force is assumed to act on both negative and positive dust grains. In particular, we employ the Sagdeev potential approach to identify the dusty plasma parameters where the large-amplitude DA solitons occur and examine domains for which compressive and rarefactive solitary waves may exist.

The manuscript is organized as follows: The theoretical model containing the basic governing equations of plasma dynamics is described in Section 2. Section 3 is devoted to investigating linear DA waves and their related dispersion relation. The nonlinear large amplitude DA solitons are studied by using the Sagdeev potential approach in Section 4. Finally, conclusions are given in Section 5.

2. The theoretical model

We consider a collisionless, unmagnetized four-component dusty plasma consisting of negatively and positively charged cold dust fluids in addition to Boltzmann distributed electrons and ions in the presence of the generalized polarization force acting on the massive charged dust grains (whether positive or negative). The number densities of electrons n_e and ions n_i can be written as

$$n_e = n_{e0} \exp\left(\frac{e\varphi}{k_B T_e}\right), \tag{1}$$

$$n_i = n_{i0} \exp\left(-\frac{e\varphi}{k_B T_i}\right), \tag{2}$$

where n_{i0} (n_{e0}) and T_i (T_e) are the unperturbed number density and temperature of the electrons (ions), respectively. φ is the electrostatic potential, e is the electronic charge and k_B is the Boltzmann constant. In the current investigation, we consider that the net force on the dust grains is a combination of the electrostatic and polarization forces. According to Hamaguchi and Farouki,[33,34] the polarization force is given by

$$\boldsymbol{F}_p = -\frac{Q_d^2}{8\pi\varepsilon_0\lambda_D^2}\nabla\lambda_D, \tag{3}$$

where Q_d is the charge of the dust grain and λ_D is the linearized effective Debye length corresponding to dusty plasma having thermal electrons and ions

$$\lambda_D = \sqrt{\frac{\varepsilon_0 k_B T_i T_e}{e^2\left(n_e T_i + n_i T_e\right)}}. \tag{4}$$

Now, to re-derive the generalized expression of polarization force, we rewrite Eq. (4) in the following form:

$$\lambda_D = \lambda_{D0}\sqrt{\frac{n_{i0}T_e + n_{e0}T_i}{\left(n_e T_i + n_i T_e\right)}}, \tag{5}$$

where $\lambda_{D0} = \left[\varepsilon_0 k_B T_i T_e / e^2 (n_{i0}T_e + n_{e0}T_i)\right]^{1/2}$. Substituting Eq. (5) into Eq. (3), and after some algebraic calculations, we obtain

$$\boldsymbol{F}_p = \frac{Q_d^2}{16\pi\varepsilon_0\lambda_{D0}}\frac{\nabla(n_i + \sigma_i n_e)}{\sqrt{n_{i0}(1+\sigma_i\rho)(n_i+\sigma_i n_e)}}, \tag{6}$$

where $\rho = n_{e0}/n_{i0}$ represents the number density ratio of electron-to-ion and $\sigma_i = T_i/T_e$ is the temperature ratio of the ion-to-electron. Using $k_B T_e \nabla n_e = e n_e \nabla\varphi$, and $k_B T_i \nabla n_i = -e n_i \nabla\varphi$ into Eq. (6), the expression of the generalized polarization force can be written as

$$\boldsymbol{F}_{pd} = -|Q_d| R_d \frac{n_i - \sigma_i^2 n_e}{\sqrt{n_{i0}(1+\sigma_i\rho)(n_i+\sigma_i n_e)}} \nabla\varphi, \tag{7}$$

where $R_d = Z_d e^2/(16\pi\varepsilon_0 k_B T_i \lambda_{D0})$ is a parameter that determines the effect of the polarization force. Here, the subscript d denotes the negatively charged dust grains ($d = n$) and the positively charged dust grains ($d = p$). It is clear from Eqs. (3) and (7) that the generalized polarization force is independent of a sign of dust charge. It is always in the direction of decreasing Debye length whether the dust charge is positive or negative.

For the limiting case $T_e n_e \gg T_i n_i$, Eq. (7) tends to $\boldsymbol{F}_{pn} \approx -eZ_n R_i (n_i/n_{i0})^{1/2} \nabla\varphi$, which arises mainly due to the polarization of plasma ions around negatively charged dust where $R_i = Z_n e^2/(16\pi\varepsilon_0 k_B T_i \lambda_{Di0})$. On the other hand, if $T_e n_e \ll T_i n_i$, Eq. (7) reduces to $\boldsymbol{F}_{pp} \approx eZ_p R_e (n_e/n_{e0})^{1/2} \nabla\varphi$, which arises mainly due to the polarization of plasma electrons around positively charged dust where $R_e = Z_p e^2/(16\pi\varepsilon_0 k_B T_e \lambda_{De0})$. Accordingly, the generalized expression of the polarization force (7) is valid for either $T_e n_e \gg T_i n_i$ or $T_e n_e \ll T_i n_i$. In other words, our findings remain suitable for opposite polarity dusty plasma.

Since the condition whereby the potential perturbation is sufficiently small (i.e., $|e\varphi| \ll k_B T_e$, $k_B T_i$), we can expand the electron and ion densities given in Eqs. (1) and (2) by Taylor expansion around $\varphi = 0$ as $n_e/n_{e0} = 1 + (e\varphi/k_B T_e) + (e\varphi/k_B T_e)^2/2 + \cdots$, $n_i/n_{i0} = 1 - (e\varphi/k_B T_i) + (e\varphi/k_B T_i)^2/2 + \cdots$, substituting these into Eq. (7), and after some calculations, the expansion of the polarization force can be expressed as

$$\boldsymbol{F}_{pd} = -Z_d e R_d \alpha_1 \left[1 + P_0 \left(\frac{e\varphi}{k_B T_i}\right) + \cdots\right] \nabla\varphi, \tag{8}$$

where $\alpha_1 = (1-\rho\sigma_i^2)/(1+\rho\sigma_i)$, and $P_0 = (\alpha_1^2 - 4\alpha_2)/2\alpha_1$ with $\alpha_2 = (1+\rho\sigma_i^3)/2(1+\rho\sigma_i)$. Here, we used $|Q_d| = Z_d e$ where Z_d is the number of charges residing on the dust grain surface. It is clear that for negatively charged dust grains, $R_d = R_n = R = Z_n e^2/16\pi\varepsilon_0 k_B T_i \lambda_{D0}$, where R is a parameter determining the effects of the polarization force due to the interaction between plasma ions and negatively charged dust grains. On the other hand, for positively charged dust grains, $R_d = R_p = Z_p e^2/16\pi\varepsilon_0 k_B T_i \lambda_{D0} = ZR$, which is a parameter determining the effects of the polarization force due to the interaction between plasma electrons and positively charged dust grains where $Z = Z_p/Z_n$, and Z_p (Z_n) is the number of the charges on the positive (negative) dust grain surface. Accordingly, the one-dimensional equations for positively and negatively charged dust grains are as follows:[26]

$$\frac{\partial n_n}{\partial t} + \frac{\partial}{\partial x}(n_n u_n) = 0, \tag{9}$$

$$\frac{\partial u_n}{\partial t} + u_n \frac{\partial u_n}{\partial x} = \frac{Z_n e}{m_n}\frac{\partial\varphi}{\partial x} - \frac{Z_n e}{m_n} P \left(1 + P_0 \frac{e\varphi}{k_B T_i}\right) \frac{\partial\varphi}{\partial x}, \tag{10}$$

$$\frac{\partial n_p}{\partial t} + \frac{\partial}{\partial x}(n_p u_p) = 0, \tag{11}$$

$$\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial x} = -\frac{Z_p e}{m_p}\frac{\partial\varphi}{\partial x} - \frac{Z_p e}{m_p} ZP \left(1 + P_0 \frac{e\varphi}{k_B T_i}\right) \frac{\partial\varphi}{\partial x}, \tag{12}$$

$$\frac{\partial^2\varphi}{\partial x^2} = \frac{e}{\varepsilon_0}\left(n_e + Z_n n_n - Z_p n_p - n_i\right), \tag{13}$$

where n_n (n_p), u_n (u_p) and m_n (m_p) are the number density, the velocity, and the mass of the negatively (positively) charged dust grains, respectively, and $P = R\alpha_1$. At equilibrium, the quasi-neutrality condition requires $Z_n n_{n0} + n_{e0} = Z_p n_{p0} + n_{i0}$, where n_{p0} (n_{n0}) is the equilibrium number density of positive (negative) dust.

3. Linear analysis

In order to study the dispersion properties of the linear DA waves from the basic Eqs. (9)–(13), we carry out the usual normal mode analysis. Accordingly, we expand the dependent variables (n_n, n_p, u_n, u_p and φ) in terms of their equilibrium and perturbed parts as $n_n = n_{n0} + n_{n1}$, $n_p = n_{p0} + n_{p1}$, $u_n = 0 + u_{n1}$, $u_p = 0 + u_{p1}$ and $\varphi = 0 + \varphi_1$. We assume that all perturbed quantities ($n_{n1}, n_{p1}, u_{n1}, u_{p1}$, φ_1) are proportional to $e^{i(kx-\omega t)}$, where ω is the angular wave frequency and k is the wave number. Thus, substituting $\partial/\partial t = -i\omega$ and $\partial/\partial x = ik$ into the linearized form of Eqs. (9)–(13), we obtain the following linear equations:

$$-\omega n_{n1} + k n_{n0} u_{n1} = 0, \tag{14}$$

$$-\omega u_{n1} - k\frac{Z_n e}{m_n}(1-P)\varphi_1 = 0, \tag{15}$$

$$-\omega n_{p1} + k n_{p0} u_{p1} = 0, \tag{16}$$

$$-\omega u_{p1} + k\frac{Z_p e}{m_p}(1+ZP)\varphi_1 = 0, \tag{17}$$

$$k^2\varphi_1 + \left(\frac{e^2 n_{e0}}{\varepsilon_0 k_B T_e} + \frac{e^2 n_{i0}}{\varepsilon_0 k_B T_i}\right)\varphi_1 + \frac{eZ_n}{\varepsilon_0} n_{n1} - \frac{eZ_p}{\varepsilon_0} n_{p0} = 0. \tag{18}$$

By solving Eqs. (14) and (15), we obtain the perturbed number density for negative dust as

$$n_{n1} = -\frac{k^2}{\omega^2}\frac{eZ_n n_{n0}}{m_n}(1-P)\varphi_1. \tag{19}$$

Similarly, solving Eqs. (16) and (17), the perturbed number density for positive dust is

$$n_{p1} = \frac{k^2}{\omega^2}\frac{eZ_p n_{p0}}{m_p}(1+ZP)\varphi_1. \tag{20}$$

Substituting Eqs. (19) and (20) into the linearized Poisson's equation (18), and after simplification, we obtain the following linear dispersion relation:

$$\omega^2 = \frac{k^2\lambda_{\rm D0}^2\omega_{\rm pn}^2\left[1-P+\mu\beta Z\left(1+ZP\right)\right]}{1+k^2\lambda_{\rm D0}^2}, \tag{21}$$

where $\omega_{\rm pn} = \sqrt{n_{\rm n0}Z_{\rm n}^2e^2/\varepsilon_0 m_{\rm n}}$ is the usual dusty plasma frequency and $\lambda_{\rm D0} = \lambda_{\rm Di0}\lambda_{\rm De0}/\sqrt{\lambda_{\rm Di0}^2+\lambda_{\rm De0}^2}$ with $\lambda_{\rm Di0} = \sqrt{\varepsilon_0 k_{\rm B}T_{\rm i}/n_{\rm i0}e^2}$ and $\lambda_{\rm De0} = \sqrt{\varepsilon_0 k_{\rm B}T_{\rm e}/n_{\rm e0}e^2}$ are the ions and electron Debye lengths. Here, we define $\mu = m_{\rm n}/m_{\rm p}$, and $\beta = Z_{\rm p}n_{\rm p0}/Z_{\rm n}n_{\rm n0} = 1+\mu_{\rm e}-\mu_{\rm i}$, with $\mu_{\rm e} = n_{\rm e0}/Z_{\rm n}n_{\rm n0}$, $\mu_{\rm i} = n_{\rm i0}/Z_{\rm n}n_{\rm n0}$. In the long wavelength limit, (i.e., for $k^2\lambda_{\rm D0}^2 \ll 1$), Eq. (21) leads to

$$\frac{\omega}{k} \approx C_{\rm Dn}\sqrt{1-P(1-\mu\beta Z^2)+\mu\beta Z}. \tag{22}$$

Equation (22) is the phase velocity of the linear DA wave in the long-wavelength limit where $C_{\rm Dn} = \lambda_{\rm D0}\omega_{\rm pn}$ is the usual DA wave phase velocity, given by Rao *et al.*[1] It is noted that the difference from the conventional dispersion relation of Rao *et al.*[1] is the extra term $\sqrt{1-P(1-\mu\beta Z^2)+\mu\beta Z}$, which is due to taking into account both the polarization force and positive dust component in our model. The contribution term due to the presence of polarization force is $P(1-\mu\beta Z^2)$, while the contribution term due to the presence of the positive dust component is $\mu\beta Z$. Clearly, in the absence of the positive dust component, Eq. (22) reduces to the relation $C_{\rm Dn}\sqrt{1-P}$, which is in good agreement with the work of Khrapak *et al.*[35]

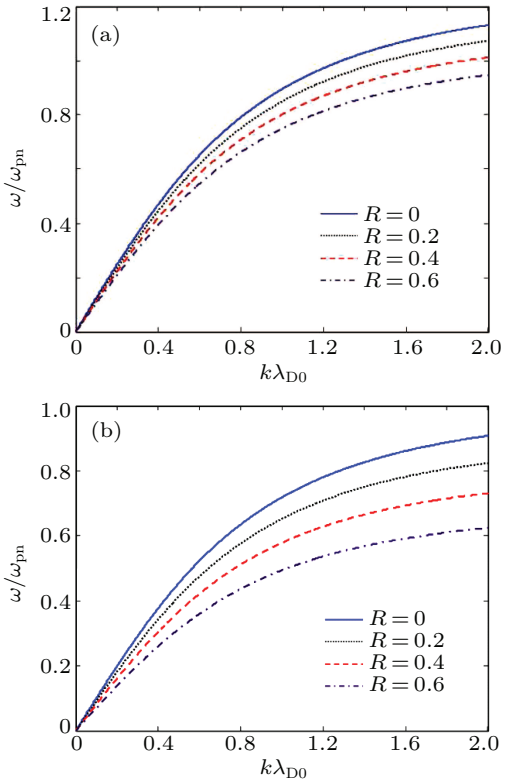


Fig. 1. The linear dispersion relation for dust-acoustic waves for different values of polarization force parameter R. (a) is plotted for $Z = 0.2$ and (b) is plotted for $Z = 0.01$ where $\mu = 5$, $\mu_{\rm i} = 0.7$, $\mu_{\rm e} = 0.3$, $\sigma_{\rm i} = 0.2$.

It is clear from Eq. (21) that the dispersion properties of the wave mode are modified by the polarization force parameter (R), the positive-to-negative dust charge number ratio Z ($= Z_{\rm p}/Z_{\rm n}$), and the negative-to-positive dust mass ratio μ ($= m_{\rm n}/m_{\rm p}$). These modified properties are displayed in Figs. 1 and 2. As depicted in Fig. 1, the dispersion relation is affected by the presence of polarization force. It is observed that the normalized wave frequency ($\omega/\omega_{\rm pn}$) increases with the increase of normalized wave number ($k\lambda_{\rm D0}$), but it decreases with the polarization force parameter (R). The comparison between Figs. 1(a) and 1(b) shows that the normalized wave frequency increases as Z increases. The influence of the polarization force becomes weaker for larger values of Z [see Fig. 1(a)]. Figure 2 illustrates the normalized wave frequency ($\omega/\omega_{\rm pn}$) against the normalized wave number ($k\lambda_{\rm D0}$) for different values of μ. It is obvious that the normalized wave frequency increases with the increase of both μ and $k\lambda_{\rm D0}$. This means that the large value of μ leads to the existence of DA waves with larger phase velocity.

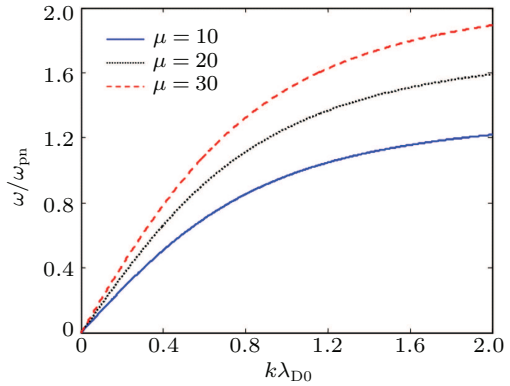


Fig. 2. The linear dispersion relation for dust-acoustic waves for different values of μ where $R = 0.5$, $Z = 0.2$, $\mu_{\rm i} = 0.7$, $\mu_{\rm e} = 0.3$, and $\sigma_{\rm i} = 0.2$.

4. Nonlinear analysis

4.1. Derivation of the Sagdeev potential

To study the nonlinear propagation of large amplitude DA solitary waves in our dusty plasma model in the framework of the Sagdeev potential approach, it is more appropriate to normalize the various quantities in Eqs. (9)–(13) as follows: $N_{\rm n} = n_{\rm n}/n_{\rm n0}$, $N_{\rm p} = n_{\rm p}/n_{\rm p0}$, $U_{\rm n} = u_{\rm n}/C_{\rm Dn}$, $U_{\rm p} = u_{\rm p}/C_{\rm Dn}$, $\phi = e\varphi/k_{\rm B}T_{\rm i}$, $T = t\omega_{\rm pn}$, and $X = x/\lambda_{\rm Dn}$, where $C_{\rm Dn} = (Z_{\rm n}k_{\rm B}T_{\rm i}/m_{\rm n})^{1/2}$ and $\lambda_{\rm Dn} = (\varepsilon_0 k_{\rm B}T_{\rm i}/n_{\rm n0}Z_{\rm n}e^2)^{1/2}$ are the DA velocity and Debye length, respectively. After the normalization, we obtain a dimensionless system of nonlinear equations. For their solutions, we assume that the dependent variables X, T depend on a single independent variable $\eta = X - MT$, where M is the Mach number (DA wave velocity/$C_{\rm Dn}$). Then, we can express Eqs. (9)–(13) in terms of the normalized variables as

$$-M\frac{{\rm d}N_{\rm n}}{{\rm d}\eta} + \frac{\rm d}{{\rm d}\eta}\left(N_{\rm n}U_{\rm n}\right) = 0, \tag{23}$$

$$-M\frac{\mathrm{d}U_\mathrm{n}}{\mathrm{d}\eta}+U_\mathrm{n}\frac{\mathrm{d}U_\mathrm{n}}{\mathrm{d}\eta}=\frac{\mathrm{d}\phi}{\mathrm{d}\eta}-P(1+P_0\phi)\frac{\mathrm{d}\phi}{\mathrm{d}\eta}, \tag{24}$$

$$-M\frac{\mathrm{d}N_\mathrm{p}}{\mathrm{d}\eta}+\frac{\mathrm{d}}{\mathrm{d}\eta}\left(N_\mathrm{p}U_\mathrm{p}\right)=0, \tag{25}$$

$$-M\frac{\mathrm{d}U_\mathrm{p}}{\mathrm{d}\eta}+U_\mathrm{p}\frac{\mathrm{d}U_\mathrm{p}}{\mathrm{d}\eta}=-\mu Z\frac{\mathrm{d}\phi}{\mathrm{d}\eta}-\mu Z^2P(1+P_0\phi)\frac{\mathrm{d}\phi}{\mathrm{d}\eta}, \tag{26}$$

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2}=\mu_\mathrm{e}\exp(\sigma_\mathrm{i}\phi)-\mu_\mathrm{i}\exp(-\phi)+N_\mathrm{n}-\beta N_\mathrm{p}. \tag{27}$$

Applying the boundary conditions, namely, $N_\mathrm{n}\to 1$, $N_\mathrm{p}\to 1$; $U_\mathrm{n}\to 0$; $U_\mathrm{p}\longrightarrow 0$; $\mathrm{d}\phi/\mathrm{d}\eta\to 0$ at $\eta\to\pm\infty$, the normalized number densities of negatively and positively charged dust grains are given, respectively, as

$$N_\mathrm{n}=\frac{M}{\sqrt{M^2+2(1-P)\phi-PP_0\phi^2}}, \tag{28}$$

$$N_\mathrm{p}=\frac{M}{\sqrt{M^2-2\mu Z(1+ZP)\phi-\mu Z^2PP_0\phi^2}}. \tag{29}$$

Substituting Eqs. (28) and (29) into Eq. (27), we obtain an equation for the electrostatic potential ϕ in the form

$$\begin{aligned}\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2}&=\mu_\mathrm{e}\exp(\sigma_\mathrm{i}\phi)-\mu_\mathrm{i}\exp(-\phi)\\&+\frac{M}{\sqrt{M^2+2(1-P)\phi-PP_0\phi^2}}\\&-\frac{\beta M}{\sqrt{M^2-2\mu Z(1+ZP)\phi-\mu Z^2PP_0\phi^2}}.\end{aligned} \tag{30}$$

Multiplying both sides of Eq. (30) by $\mathrm{d}\phi/\mathrm{d}\eta$ and integrating once with respect to η, and taking into account the boundary conditions $\mathrm{d}\phi/\mathrm{d}\eta\to 0$ and $\phi\to 0$ as $\eta\to\pm\infty$, we get the following equation:

$$\frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}\eta}\right)^2+V(\phi)=0. \tag{31}$$

Equation (31) represents the form of an energy integral for an oscillating particle of unit mass, with pseudo-position ϕ, pseudo-time η, and pseudo-potential well $V(\phi)$. Here, the mathematical expression for a pseudo-potential (or Sagdeev's potential) $V(\phi)$ is given by

$$\begin{aligned}V(\phi)&=\mu_\mathrm{i}[1-\exp(-\phi)]+\frac{\mu_\mathrm{e}}{\sigma_\mathrm{i}}[1-\exp(\sigma_\mathrm{i}\phi)]\\&-\frac{M}{\sqrt{-PP_0}}\ln\left|\frac{\sqrt{\Lambda_\mathrm{n}}+1-P-PP_0\phi}{M\sqrt{-PP_0}+1-P}\right|\\&+\frac{\beta M}{Z\sqrt{-\mu PP_0}}\ln\left|\frac{\sqrt{\Lambda_\mathrm{p}}-\mu(1+ZP)-\mu ZPP_0\phi}{M\sqrt{-\mu PP_0}-\mu(1+ZP)}\right|,\end{aligned} \tag{32}$$

where $\Lambda_\mathrm{n}=-PP_0[M^2+2(1-P)\phi-PP_0\phi^2]$ and $\Lambda_\mathrm{p}=-\mu PP_0[M^2-2\mu Z(1+ZP)\phi-\mu Z^2PP_0\phi^2]$. It can be noticed that the Sagdeev's potential $V(\phi)$ is significantly affected by the polarization force as well as other plasma parameters. In the absence of polarization force, i.e., in the limiting case ($R\to 0$), the Sagdeev potential $V(\phi)$ is reduced to

$$\begin{aligned}V(\phi)&=\mu_\mathrm{i}[1-\exp(-\phi)]+\frac{\mu_\mathrm{e}}{\sigma_\mathrm{i}}[1-\exp(\sigma_\mathrm{i}\phi)]+M^2\\&\times\left[1-\left(1+\frac{2\phi}{M^2}\right)^{1/2}\right]\\&+\frac{\beta M^2}{\mu Z}\left[1-\left(1-\frac{2\mu Z\phi}{M^2}\right)^{1/2}\right],\end{aligned} \tag{33}$$

which completely agrees with the result obtained by Chatterjee and Kundu.[27]

4.2. Existence conditions for DA solitons

The solitonic solutions of Eq. (31) exist only if the Sagdeev potential (32) satisfies the conditions: (i) $V(\phi)=\mathrm{d}V(\phi)/\mathrm{d}\phi=0$, for $\phi=0$; (ii) $\mathrm{d}^2V(\phi)/\mathrm{d}\phi^2<0$ at $\phi=0$, so that a maximum occurs at the origin (the fixed point at the origin is unstable); (iii) $V(\phi)<0$ for ϕ lying between 0 and ϕ_m. In the interval $0<\phi<\phi_\mathrm{m}$, compressive solitons (i.e., positive potential solitons) exist. In the interval $\phi_\mathrm{m}<\phi<0$, rarefactive solitons (i.e., negative potential solitons) exist. Here, ϕ_m is the amplitude of the solitary wave or soliton.

Equation (32) indicates that condition (i) is always satisfied. For the second condition (ii), we can obtain

$$\begin{aligned}S(M)&=\frac{\mathrm{d}^2V(\phi)}{\mathrm{d}\phi^2}\,|_{\phi=0}\\&=\frac{\beta\mu Z(1+ZP)+1-P}{M^2}-(\mu_e\sigma_\mathrm{i}+\mu_\mathrm{i})<0.\end{aligned} \tag{34}$$

In order for the local maximum condition to be satisfied at the origin, viz. $[\mathrm{d}^2V(\phi)/\mathrm{d}\phi^2]_{\phi=0}<0$, one requires $M>M_\mathrm{cr}$, where M_cr is the critical Mach number (the minimum value of M at which the soliton amplitude goes to zero or the value of M at which the second derivative changes its sign). The critical Mach number M_cr can be calculated from Eq. (34) as

$$M_\mathrm{cr}=\left[\frac{\beta\mu Z(1+ZP)+1-P}{\mu_\mathrm{e}\sigma_\mathrm{i}+\mu_\mathrm{i}}\right]^{1/2}. \tag{35}$$

This gives the lower limit on the range of Mach numbers that allows DA solitons to exist.

Figure 3 shows how the critical Mach number M_cr is modified by the polarization force parameter (R) and the charge number ratio (Z). From this figure, we determine that M_cr decreases (increases) with the increase in R (Z). For $Z=0.01$, the critical Mach number $M_\mathrm{cr}<1$ for all values of $R>0.35$, while for $Z=0.2$, the critical Mach number $M_\mathrm{cr}>1$ for all values of R. Physically, $M_\mathrm{cr}<1$ means that the existence of subsonic ($M<1$) DA solitons is possible in our model, whereas supersonic ($M>1$) DA solitons are admissible for $M_\mathrm{cr}>1$. Furthermore, we found that in the absence of polarization force (as $R=0$), which for Z is taken as $Z=0.01$ or $Z=0.2$, DA solitons can only exist with higher values of

($M > 1$) (i.e., supersonic DA solitons exist). Now, if we consider the limiting case for $Z = 0$ or $\beta = 0$ (i.e., in the absence of a positively charged dust component), Eq. (35) leads to

$$M_{\rm cr} = \left(\frac{1-P}{\mu_{\rm e}\sigma_{\rm i}+\mu_{\rm i}}\right)^{1/2}, \tag{36}$$

which agrees with Bandyopadhyay *et al.*[40]

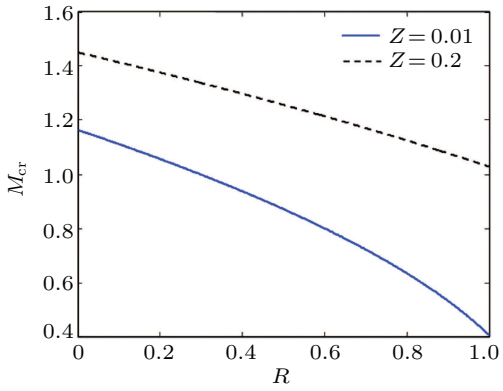


Fig. 3. Variation of the critical Mach number $M_{\rm cr}$ with polarization force parameter R for two different values of Z where $\mu = 5$, $\mu_{\rm i} = 0.7$, $\mu_{\rm e} = 0.3$, $\sigma_{\rm i} = 0.2$.

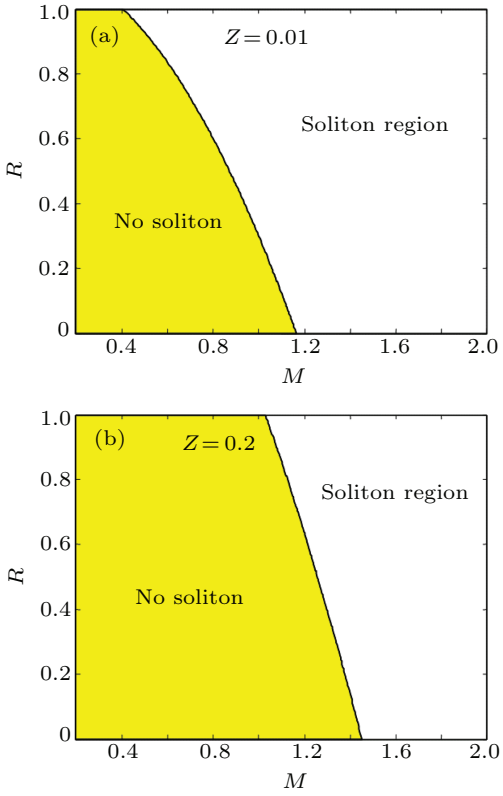


Fig. 4. Contour plots showing the existence regions of DA solitons in space (M, R). (a) is plotted for $Z = 0.01$ and (b) is plotted for $Z = 0.2$ where $\mu = 5$, $\mu_{\rm i} = 0.7$, $\mu_{\rm e} = 0.3$, and $\sigma_{\rm i} = 0.2$.

The contour of the condition (34) in the space (M, R) is plotted in Fig. 4. This figure shows the regions of existence of DA solitons in the space (M, R) for two different values of Z. Figure 4(a) is plotted for $Z = 0.01$, while Fig. 4(b) is plotted for $Z = 0.2$. It is noted that the curve $S(M) = 0$ (in which $M = M_{\rm cr}$) divides the space (M, R) into two areas, one on the left of the curve, in which $S(M) > 0$, and the other on the right of the curve where $S(M) < 0$. It shows that the soliton solutions of Eq. (31) exist at the right region of the curves in which $S(M) < 0$. But in the left region, in which $S(M) > 0$, the soliton solutions of Eq. (31) do not exist. Furthermore, Figs. 4(a) and 4(b) show that the value of Z has a significant effect on the admissible range of the Mach number for which DA solitons occur. It is clear from Fig. 4(a) that, for $Z = 0.01$ (i.e., for lower value of the charge number of positive dust), localized soliton structures can propagate for $M > 1.16$ when $R = 0$. It can also be seen from Fig. 4(a) that the region of the existence domain of DA solitons increases with the increased values of R, and for $R \geq 0.3$, the region of the existence domain of subsonic DA solitons is possible. On the other hand, when $Z = 0.2$, the region of the existence domain of DA solitons becomes narrower, as shown in Fig. 4(b). This means that when the contribution of the positive dust component (via Z) is enhanced, only supersonic solitons can exist.

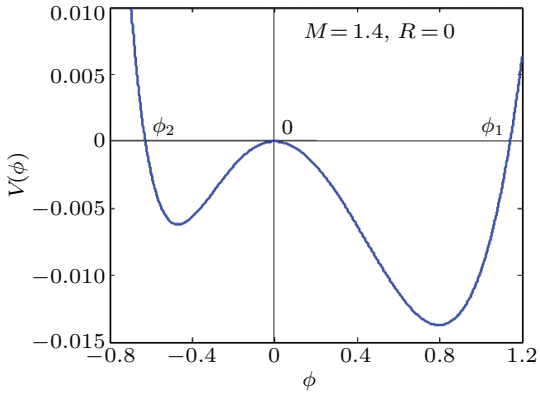


Fig. 5. A profile of the Sagdeev potential $V(\phi)$ versus ϕ in the absence of polarization force (via $R = 0$) where $M = 1.4$, $Z = 0.1$, $\mu = 5$, $\mu_{\rm i} = 0.7$, $\mu_{\rm e} = 0.3$, and $\sigma_{\rm i} = 0.2$.

Figure 5 depicts a profile of the Sagdeev potential curve $V(\phi)$ versus the electrostatic potential ϕ in the absence of the polarization force effect [i.e., when $R = 0$, and $V(\phi)$ given in Eq. (33)] with the consideration of the typical dusty plasma parameters, namely, $M = 1.4$, $Z = 0.1$, $\mu_{\rm i} = 0.7$, $\mu_{\rm e} = 0.3$, $\mu = 5$, and $\sigma_{\rm i} = 0.2$. It is obvious from this figure that the Sagdeev potential $V(\phi)$ has a negative value in the interval of $\phi = 0$ to $\phi = \phi_1$ with one side ending on a maximum and the other side just crossing the zero value of $V(\phi)$. This will produce a compressive-type soliton. In the other region, for $\phi = \phi_2$ from ϕ_2 to $\phi = 0$, a rarefactive soliton is produced. The potential well created in the positive ϕ-axis is deeper than those created in the negative ϕ-axis. This means that the amplitude (width) of the rarefactive soliton is lower (higher) than that of the compressive soliton.

The influence of polarization force (via $R = 0.2$) on a Sagdeev potential profile $V(\phi)$ (which is determined by Eq. (32)) is shown in Fig. 6 with the same values of dusty

plasma parameters as Fig. 5. From this figure we observe that Sagdeev potential wells are formed in both the positive and negative ϕ-axes, i.e., the compressive and rarefactive DA solitons coexist. Due to the presence of polarization force, the potential well formed in the negative ϕ-axis is very deep in comparison with that formed in the positive ϕ-axis. From a comparison between Figs. 5 and 6, one can also see that, for a fixed value of $M = 1.4$, the presence of polarization force leads to an increase (a decrease) of the amplitude (width) for both compressive and rarefactive DA solitons, as shown in Fig. 6. The presence of polarization force has a profound effect on the rarefactive solitons compared with compressive solitons.

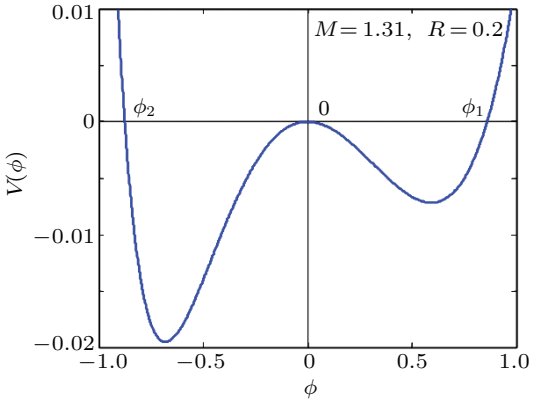


Fig. 6. A profile of the Sagdeev potential $V(\phi)$ versus ϕ in the presence of polarization force (via $R = 0.2$) where $M = 1.4$, $Z = 0.1$, $\mu = 5$, $\mu_i = 0.7$, $\mu_e = 0.3$, and $\sigma_i = 0.2$.

In addition, there is also another condition determining the criterion for the existence of compressive and rarefactive solitons used in a number of investigations.[20,24] This condition requires that the dusty plasmas would support rarefactive (compressive) DA solitons if $\mathrm{d}^3V(\phi)/\mathrm{d}\phi^3 < 0 (> 0)$ for $\phi = 0$, and $M = M_{cr}$.

At the critical value of M, the condition, $[\mathrm{d}^3V(\phi)/\mathrm{d}\phi^3]_{\phi=0,M=M_{cr}} < 0\ (> 0)$, can be expressed as

$$F(M_{cr}) = \mu_i - \mu_e\sigma_i^2 + \frac{PP_0\left(\beta\mu Z^2 - 1\right)}{M_{cr}^2} + \frac{3}{M_{cr}^4}\left[\beta\mu^2 Z^2(1+ZP)^2 - (1-P)^2\right], \quad (37)$$

where $F(M_{cr})$ determines the criterion for the existence of compressive or rarefactive solitons, i.e., if $F(M_{cr}) < 0$, the soliton solution of Eq. (31) corresponds to rarefactive solitons. On the other hand, when $F(M_{cr}) > 0$, Eq. (31) is solved only for compressive solitons. Figure 7 shows the contour plot of the existence domain region of compressive or rarefactive DA solitons. Figure 7(a) is plotted without a polarization force (via $R = 0$), and Fig. 7(b) is plotted with a polarization force (via $R = 0.5$) for $\mu_e = 0.3$, $\mu = 5$, and $\sigma_i = 0.2$. From these figures, we can see that both positive and negative potential solitons can create in our dusty plasma model. The potential profile is negative (i.e., the appearance of rarefactive solitons) on the left of the curve of $F(M_{cr}) = 0$, in which $F(M_{cr}) < 0$, and positive (i.e., the appearance of compressive solitons) on the right of the curve where $F(M_{cr}) > 0$. Due to the presence of the polarization force effect, the compressive soliton region becomes wider, while the rarefactive soliton region shrinks (as shown in Fig. 7(b)).

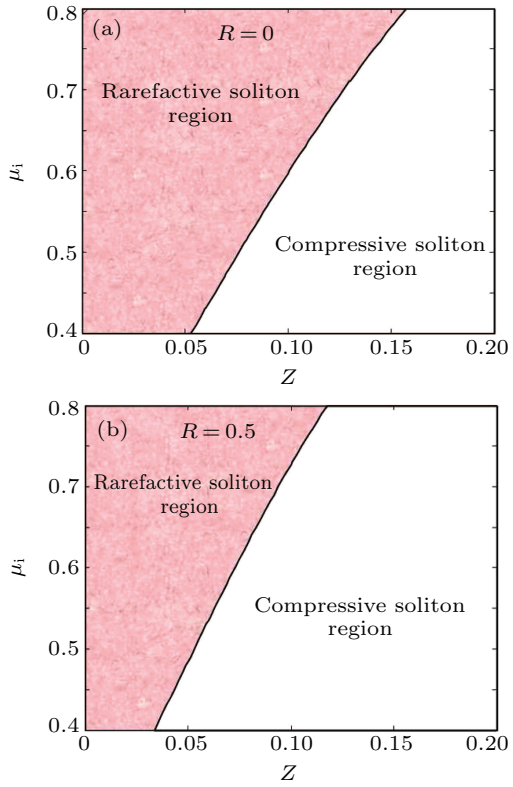


Fig. 7. Contour plot showing the region of the existence domain of compressive or rarefactive DA solitons in space (Z, μ_i). (a) is plotted in the absence of polarization force (via $R = 0$); and (b) is plotted in the presence of polarization force (via $R = 0.5$) where $\mu = 5$, $\mu_e = 0.3$, and $\sigma_i = 0.2$.

Figure 8 illustrates the variation of the Sagdeev potential curves against the negative ϕ-axis, without polarization force [i.e., when $R = 0$, and $V(\phi)$ is given by Eq. (33)], for different values of Mach number M. It is clear from this figure that for $Z = 0.01$, and when M exceeds the value of M_{cr} which is $M_{cr} = 1.1464$, only rarefactive solitons are formed in the supersonic region with a Mach number range $M_{cr} < M \leq 1.3$. Note that the enlargement of M leads to an increase in both the width and depth of the Sagdeev potential wells (i.e., an increase of the soliton amplitude and a decrease of the soliton width). In contrast, when the effect of polarization force is taken into account [i.e., when $R \neq 0$, and $V(\phi)$ is given by Eq. (32)], the critical Mach number value M_{cr} drops to $M_{cr} = 0.8717$. This means that there is a possibility of negative potential solitons forming in the subsonic region (where the wave speed is less than sound speed) in the range $0.87 < M \leq 0.96$, as shown in Fig. 9. It is also obvious from Fig. 9 that the amplitude of DA solitons increases with the increased values of M, but their

width becomes narrower. Moreover, the consideration of the effect of the polarization force leads to a decrease in both the lower and higher values of the Mach number for which DA solitons exist.

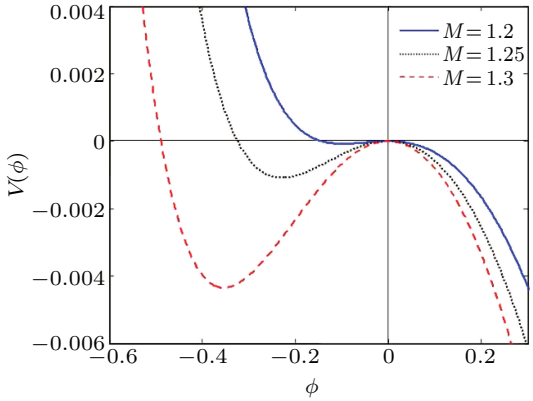


Fig. 8. Behavior of the Sagdeev potential $V(\phi)$ versus ϕ for different values of M when M exceeds $M_{\rm cr} = 1.1464$ where $Z = 0.01$, $R = 0$, $\mu_{\rm i} = 0.7$, $\mu_{\rm e} = 0.3$, $\mu = 5$, and $\sigma_{\rm i} = 0.2$.

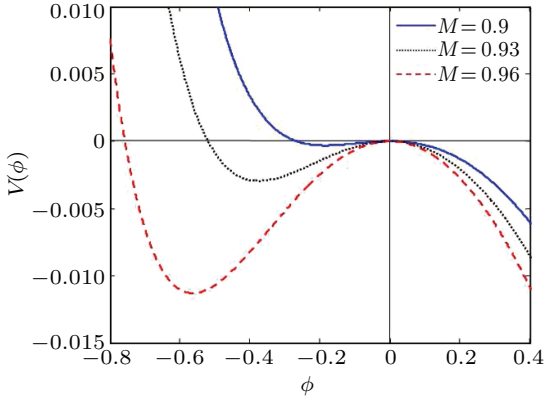


Fig. 9. Behavior of the Sagdeev potential $V(\phi)$ versus ϕ for different values of M when M exceeds $M_{\rm cr} = 0.8717$ where $Z = 0.01$, $R = 0.5$, $\mu_{\rm i} = 0.7$, $\mu_{\rm e} = 0.3$, $\mu = 5$, and $\sigma_{\rm i} = 0.2$.

The formation of potential wells in the positive ϕ-axis, which corresponds to the formation of compressive DA solitons for different values of M and for large values of $Z = 0.2$, is displayed in Fig. 10. Here, the Sagdeev's potential $V(\phi)$ corresponds to Eq. (32) with the dusty plasma parameters: $Z = 0.2$, $R = 0.2$, $\mu_i = 0.7$, $\mu_{\rm e} = 0.3$, $\mu = 5$, $\sigma_{\rm i} = 0.2$, and M exceeds $M_{\rm cr} = 1.3678$. It is evident that compressive DA solitons are formed in the Mach number range $1.5 \leq M \leq 1.76$. Physically, this means that only supersonic DA solitons can exist in such plasmas. This figure also indicates that the amplitude of compressive DA solitons increases with the increased values of M, while their width reduces.

On the other hand, the effect of the polarization force parameter (via R) on the formation of Sagdeev potential wells in the positive ϕ-axis is illustrated in Fig. 11 when the Mach-number is taken as $M = 1.6$ and other plasma parameters are $Z = 0.2$, $\mu_{\rm i} = 0.7$, $\mu_{\rm e} = 0.3$, $\mu = 5$, and $\sigma_{\rm i} = 0.2$. It can be noted that the depth and width of the Sagdeev potential wells increase with an increase in the polarization force (as R increases). This means that, by increasing the polarization force, the amplitude of the DA soliton increases but its width decreases slightly. This reflects the fact that the polarization force decreases the critical value of the Mach number.

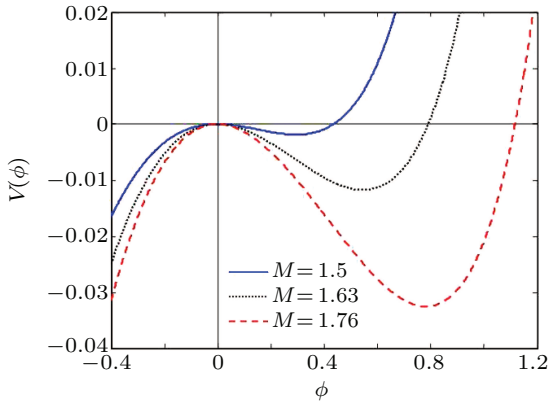


Fig. 10. Behavior of the Sagdeev potential $V(\phi)$ versus ϕ for different values of M when M exceeds $M_{\rm cr} = 1.3678$ where $Z = 0.2$, $R = 0.5$, $\mu_{\rm i} = 0.7$, $\mu_{\rm e} = 0.3$, $\mu = 5$, and $\sigma_{\rm i} = 0.2$.

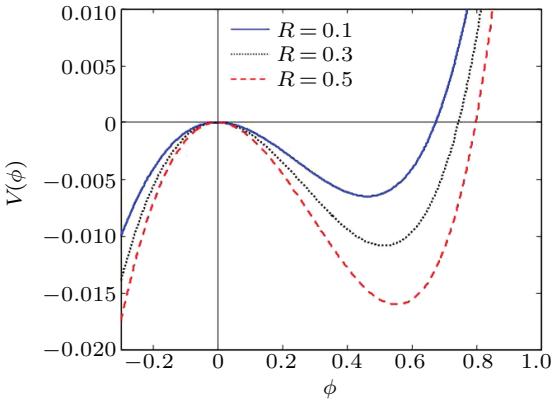


Fig. 11. Behavior of the Sagdeev potential $V(\phi)$ versus ϕ for different values of R where $M = 1.6$, $\mu_{\rm i} = 0.7$, $\mu_{\rm e} = 0.3$, $\mu = 5$, and $\sigma_{\rm i} = 0.2$, and $Z = 0.2$.

5. Conclusion

By including the effect of generalized polarization force, the linear and nonlinear properties of the DA waves in an opposite polarity dusty plasma system consisting of positively charged dust grains, negatively charged dust grains and Maxwellian electrons and ions are studied. The linear dispersion relation of a DA wave has been derived. An energy integral equation involving the Sagdeev potential, which describes large amplitude DA solitons, has been obtained by using the Sagdeev potential approach. The analysis of the Sagdeev potential indicates that both the rarefractive and compressive DA solitons can form in our dusty plasmas model. The effects of the dusty plasma parameters, such as the polarization force parameter R, the ratio of the charge number of positive dust to that of negative dust Z and the Mach number M, on the Sagdeev potential profile have been examined. For large polarization force with a lower value of $Z = 0.01$, large amplitude DA solitons can exist in the subsonic regime ($M < 1$) or

supersonic regime ($M > 1$), whilst in the absence of the polarization force effect, our results demonstrate the possibility of the propagation of DA solitons in the supersonic regime. We have also found that the effect of the polarization force makes the electrostatic solitary structures spikier. Moreover, it has been found that the presence of the positive dust component does not only significantly modify the basic properties of the electrostatic solitary potential structures, but also causes the coexistence of the positive and negative solitary potential structures.

Finally, the parameters that we have chosen in our numerical analysis are relevant to different regions of space, viz., cometary tails, mesosphere, and Jupiter magnetosphere.[11,12] Thus, the results of this paper would be useful in understanding the basic nonlinear feature of DA wave propagation in space dusty plasmas, especially those including polarization force.

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