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Fano interference and transparency in a waveguide-nanocavity hybrid system with an auxiliary cavity*

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We investigate theoretically single photon transport in one-dimensional waveguide coupled to a pair of cavities, which are denoted by the first cavity and the auxiliary cavity. Two cases with no atom and one atom embedded in the first cavity are discussed. The Fano dips in the transmission spectrum and locations of transparency window are calculated. When no atom is embedded in the first cavity, there exists a transparency window under the condition that the first cavity and the auxiliary cavity are not resonant. The locations of the transparency window and Fano line type depend strongly on the eigen frequency of the auxiliary cavity and the coupling strength between the auxiliary cavity and the waveguide. When one atom is embedded in the first cavity, we show that the transparency window exists even though the first cavity, the atom and the auxiliary cavity are resonant. The Fano line type is strongly dependent on the eigen frequency of the auxiliary cavity and the coupling strength. Our results have potential applications in design of quantum devices at the level of single photon, such as single photon switch and single photon routers.

Keywords: Fano interference, waveguide quantum electrodynamics, auxiliary cavity

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1. Introduction

Realization of strong coupling between atoms and photons plays important roles in quantum information processing. Many groups have reported the realization of strong couplings between atoms and photons in one-dimensional waveguide in recent years.[1–9] Typical waveguides include metallic nanowires,[1] fibers,[2] supercondcuting transmission lines[6,7] and photonic crystal waveguides.[3,4,9] Based on the strong coupling, photon scattering properties in the one-dimensional waveguide have also been investigated widely.[10–38] By controlling single photon reflection and transmission spectra, different kinds of quantum devices and quantum logic gates have been proposed and realized, which includes single photon switching,[10–12] transistors,[13–15] diodes,[16,17] routers,[22–24,27] circulators,[28,29] and frequency-combs.[25,26] An atom-waveguide system with strong coupling is also a good platform to investigative many quantum effects, such as collective emission,[39] collective Lamb shift,[40] and photon bound states,[41] due to the phase (or time delay) induced by photons propagating in waveguides. Many of these developments can be found in the recent excellent reviews.[7,42–45]

It is well-known that photonic crystal (PhC) cavity can enhance the interaction between photons due to its ultrasmall mode volume and low dissipation rate. Thus, there are many reports on single photons transporting in waveguides interacting with a quantum emitter embedded in a PhC cavity. The photon scattering properties can be manipulated by the frequency of the PhC cavity, the interaction strength between the atom and the PhC cavity, etc.

On the other side, several investigations show that many quantum optical phenomena induced by the interaction between atom and photon in a PhC cavity can be manipulated by an auxiliary cavity (AC). Recently, Liu *et al.* investigated the vacuum Rabi oscillations for a single quantum emitter insider a cavity, where the cavity couples to an AC.[46] After that, Chen predicted that AC can induce vacuum Rabi splitting in a semiconductor quantum dot-first cavity system.[47] Furthermore, the ACs enhancing the dipole induced transparency[48] and manipulating the optomechanically induced transparency[49] have also been reported recently. In most of these models, the AC and the PhC cavity are coupled directly. In this paper, we investigate AC-assisted controlling Fano interference effects and transparency in the transmission spectrum of single photon scattering. In our model, the AC and PhC cavity are coupled through the waveguide, thus they interact indirectly. For a more general case, the PhC cavity is denoted as the first cavity (FC). The calculations show that the Fano dips and locations of transparency window depend strongly on the eigen frequency, coupling strength and location of the AC. The scheme may pave the way towards the realization of solid integrable quantum

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devices at single-photon levels.

2. Theoretical models for AC-assisted controlling single photon transport

The model we consider is shown in Fig. 1. The FC and AC are coupled by a waveguide. The separation between the FC cavity and AC is L. J_c (J_a) denotes the coupling strength between the FC (AC) and the waveguide. An atom with ground state $|g\rangle$ and excited state $|e\rangle$ is embedded in the FC. The configuration can be realized in the solid system using a usual photonic crystal technique.[30,50] A parallel line-defect photonic waveguide sustains the photon propagation, which couples to two separated point-defect cavities. A quantum dot is embedded in one of the point-defect cavity. The other point-defect cavity plays the role of AC. A similar possible physical configuration was given in Ref. [51].

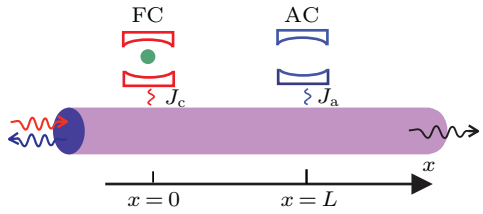


Fig. 1. The configurations of AC-assisted controlling single photon propagation. The FC and AC are coupled indirectly by a waveguide. The green dot denotes the two-level system.

After linearization of the photonic dispersion of the waveguide mode near the eigenfrequency of the FC, the system can be described by the following Hamiltonian (with $\hbar = 1$):

$$\begin{aligned} H = & \int \mathrm{d}x[C_{\mathrm{R}}^{\dagger}(x)\left(-\mathrm{i}v_{\mathrm{g}}\frac{\partial}{\partial x}\right)C_{\mathrm{R}}(x)+C_{\mathrm{L}}^{\dagger}(x)\left(\mathrm{i}v_{\mathrm{g}}\frac{\partial}{\partial x}\right)C_{\mathrm{L}}(x)] \\ & +\omega_{\mathrm{e}}\sigma_{\mathrm{ee}}+\omega_{\mathrm{c}}c^{\dagger}c+\omega_{\mathrm{a}}a^{\dagger}a+g(c\sigma_{\mathrm{eg}}+c^{\dagger}\sigma_{\mathrm{ge}}) \\ & +J_{\mathrm{c}}\int \mathrm{d}x\delta(x)[C_{\mathrm{R}}^{\dagger}(x)c+C_{\mathrm{L}}^{\dagger}(x)c+\mathrm{H.c.}] \\ & +J_{\mathrm{a}}\int \mathrm{d}x\delta(x-L)[C_{\mathrm{R}}^{\dagger}(x)a+C_{\mathrm{L}}^{\dagger}(x)a+\mathrm{H.c.}], \end{aligned} \tag{1}$$

where $C_{\mathrm{R}}^{\dagger}(x)$ ($C_{\mathrm{L}}^{\dagger}(x)$) denotes creating a right (left) propagation at x; v_{g} is the group velocity of the single photon; ω_{e} is the eigenfrequency of the excited state $|e\rangle$; ω_{c} and ω_{a} are the eigenfrequencies of the modes of the FC and AC, respectively. We have set the energy scale such that the eigenfrequency of the ground state of the two-level system is zero. Here, σ_{mn} $(m,n=\mathrm{e},\mathrm{g})$ is the dipole operator; $c^{\dagger}$ ($a^{\dagger}$) represents the bosonic creation operator of the FC (AC); g is the coupling strength between the FC and the atom, which can be manipulated by the location of the atom in the FC; $\delta(x)$ denotes the delta function.

The general eigenstate of the Hamiltonian H can be given in the following form since we concentrate on single photon transport properties:

$$\begin{aligned} |\Psi\rangle = & \int \mathrm{d}x[\phi_{\mathrm{R}}(x)C_{\mathrm{R}}^{\dagger}(x)|0\rangle+\phi_{\mathrm{L}}(x)C_{\mathrm{L}}^{\dagger}(x)|0\rangle]+u_{\mathrm{c}}c^{\dagger}|0\rangle \\ & +u_{\mathrm{a}}a^{\dagger}|0\rangle+u_{\mathrm{e}}\sigma_{\mathrm{eg}}|0\rangle, \end{aligned} \tag{2}$$

where $\phi_{\mathrm{R}}(x)$ and $\phi_{\mathrm{L}}(x)$ are the probability amplitudes of the right and left propagating photon, respectively; $|0\rangle$ denotes the vacuum state with the atom in the ground state and zero photon among the waveguide, the FC and the AC; u_{c} and u_{a} are the excitation amplitudes of the FC and AC modes, respectively; and u_{e} is is the excitation amplitude of atomic state $|e\rangle$.

For the single photon incident from the left of the waveguide, $\phi_{\mathrm{R}}(x)$ and $\phi_{\mathrm{L}}(x)$ take the form

$$\phi_{\mathrm{R}}(x) = \exp(\mathrm{i}kx)[\theta(-x)+a\theta(x)\theta(L-x)+t\theta(x-L)], \tag{3}$$

$$\phi_{\mathrm{L}}(x) = \exp(-\mathrm{i}kx)[r\theta(-x)+b\theta(x)\theta(L-x)], \tag{4}$$

where $\theta(x)$ is the unit step function with $\theta(0)=1/2$; t and r are the transmission and reflection amplitudes, respectively; $a\theta(x)\theta(L-x)$ and $b\theta(x)\theta(L-x)$ represent the wave functions of the single photon located between 0 and L.

From the eigenvalue equation $H|\Psi\rangle=\omega|\Psi\rangle$, finally, one can obtain

$$t = \frac{k_{\mathrm{a}}(g^2-k_{\mathrm{c}}k_{\mathrm{e}})}{k_{\mathrm{e}}\Gamma_{\mathrm{a}}\Gamma_{\mathrm{c}}-\mathrm{e}^{2\mathrm{i}kL}k_{\mathrm{e}}\Gamma_{\mathrm{a}}\Gamma_{\mathrm{c}}+\mathrm{i}g^2\Gamma_{\mathrm{a}}-\mathrm{i}k_{\mathrm{c}}k_{\mathrm{e}}\Gamma_{\mathrm{a}}-\mathrm{i}k_{\mathrm{a}}k_{\mathrm{e}}\Gamma_{\mathrm{c}}+g^2k_{\mathrm{a}}-k_{\mathrm{a}}k_{\mathrm{c}}k_{\mathrm{e}}}, \tag{5}$$

$$r = \frac{-k_{\mathrm{e}}\Gamma_{\mathrm{a}}\Gamma_{\mathrm{c}}+\mathrm{e}^{2\mathrm{i}kL}k_{\mathrm{e}}\Gamma_{\mathrm{a}}\Gamma_{\mathrm{c}}-\mathrm{i}\,\mathrm{e}^{2\mathrm{i}kL}g^2\Gamma_{\mathrm{a}}+\mathrm{i}\,\mathrm{e}^{2\mathrm{i}kL}k_{\mathrm{c}}k_{\mathrm{e}}\Gamma_{\mathrm{a}}+\mathrm{i}k_{\mathrm{a}}k_{\mathrm{e}}\Gamma_{\mathrm{c}}}{k_{\mathrm{e}}\Gamma_{\mathrm{a}}\Gamma_{\mathrm{c}}-\mathrm{e}^{2\mathrm{i}kL}k_{\mathrm{e}}\Gamma_{\mathrm{a}}\Gamma_{\mathrm{c}}+\mathrm{i}g^2\Gamma_{\mathrm{a}}-\mathrm{i}k_{\mathrm{c}}k_{\mathrm{e}}\Gamma_{\mathrm{a}}-\mathrm{i}k_{\mathrm{a}}k_{\mathrm{e}}\Gamma_{\mathrm{c}}+g^2k_{\mathrm{a}}-k_{\mathrm{a}}k_{\mathrm{c}}k_{\mathrm{e}}}, \tag{6}$$

where $k_m=\omega-\omega_m$ $(m=\mathrm{e},\mathrm{a},\mathrm{c})$, $\Gamma_{\mathrm{a}}=J_{\mathrm{a}}^2/v_{\mathrm{g}}$ and $\Gamma_{\mathrm{c}}=J_{\mathrm{c}}^2/v_{\mathrm{g}}$.

2.1. Without atom

Let us consider the simpler scenario of no atom embedded in the FC first. Then Eqs. (5) and (6) degenerate into

$$t = \frac{-k_{\mathrm{a}}k_{\mathrm{c}}}{\Gamma_{\mathrm{a}}\Gamma_{\mathrm{c}}-\mathrm{e}^{2\mathrm{i}kL}\Gamma_{\mathrm{a}}\Gamma_{\mathrm{c}}-\mathrm{i}k_{\mathrm{c}}\Gamma_{\mathrm{a}}-\mathrm{i}k_{\mathrm{a}}\Gamma_{\mathrm{c}}-k_{\mathrm{a}}k_{\mathrm{c}}}, \tag{7}$$

$$r = \frac{-\Gamma_{\mathrm{a}}\Gamma_{\mathrm{c}}+\mathrm{e}^{2\mathrm{i}kL}\Gamma_{\mathrm{a}}\Gamma_{\mathrm{c}}+\mathrm{i}\,\mathrm{e}^{2\mathrm{i}kL}k_{\mathrm{c}}\Gamma_{\mathrm{a}}+\mathrm{i}k_{\mathrm{a}}\Gamma_{\mathrm{c}}}{\Gamma_{\mathrm{a}}\Gamma_{\mathrm{c}}-\mathrm{e}^{2\mathrm{i}kL}\Gamma_{\mathrm{a}}\Gamma_{\mathrm{c}}-\mathrm{i}k_{\mathrm{c}}k_{\mathrm{e}}\Gamma_{\mathrm{a}}-\mathrm{i}k_{\mathrm{a}}\Gamma_{\mathrm{c}}-k_{\mathrm{a}}k_{\mathrm{c}}}. \tag{8}$$

The results are the same as those of two emitters coupled in a one-dimensional waveguide. The transmission amplitude reaches zero at $k_{\mathrm{a}}=k_{\mathrm{c}}=0$, i.e., $\omega=\omega_{r1}=\omega_{\mathrm{a}}$ and $\omega=\omega_{r2}=\omega_{\mathrm{c}}$. The locations of peaks (LP) in the transmission spectrum can be found when the following equations are

satisfied,

$$2\Gamma_a\Gamma_c\sin^2(kL)+k_c\Gamma_a\sin(2kL)=0, \tag{9}$$

$$\Gamma_a\Gamma_c\sin(2kL)+k_c\Gamma_a\cos(2kL)+k_a\Gamma_c=0. \tag{10}$$

The LPs are strongly dependent on kL due to waveguide-mediated dipole-dipole interaction between the two cavities. A symmetric transmission spectrum appears when $kL=n\pi/2$ with $\omega_a=\omega_c$ and $\Gamma_a=\Gamma_c$, where n is an integer. When $kL\neq n\pi/2$, the transmission spectrum shows Fano-type transmission with strong asymmetry. We have analyzed the physical mechanism for the asymmetric Fano-type transmission spectrum.[19] When $\omega_a\neq\omega_c$ with $kL=n\pi$, there is a transparency window in the transmission spectrum which is located at $\omega=(\omega_a+\omega_c)/2$. In this paper, we further discuss the effect of AC on the LP of the transparency.

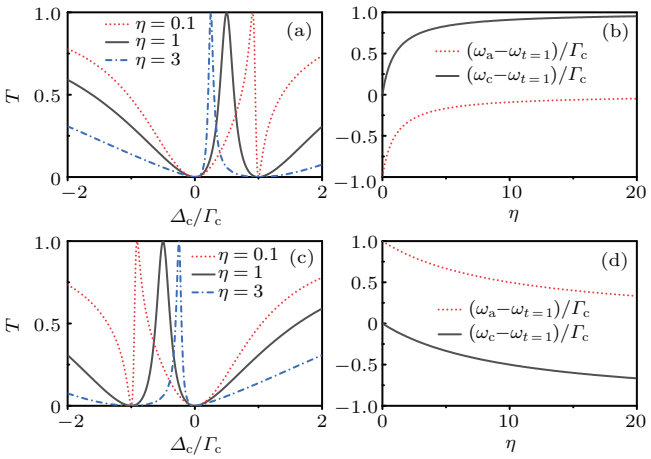


Fig. 2. Single photon transmission spectra (a) and (c) with different η. (b) and (d) The distance between the Fano dips and the LP. In (a) and (b), $\omega_a-\omega_c=\Gamma_c$, while in (c) and (d), $\omega_a-\omega_c=-\Gamma_c$. The other parameters are $\Gamma_c=10^{-5}\omega_c$, $kL=\pi$.

It is not difficult to find that t reaches unit when $\omega=\omega_{t=1}\equiv(\omega_c\Gamma_a+\omega_a\Gamma_c)/(\Gamma_a+\Gamma_c)$ with $\omega_a\neq\omega_c$ and $kL=n\pi$. Here $\omega_{t=1}$ depends strongly on Γ_a and ω_a. Thus, by adjusting the AC, one can obtain the scattering transparency window. Figure 2(a) shows $T\equiv|t|^2$ as a function of $\Delta_c=\omega-\omega_c$ with different η, where $\eta\equiv\Gamma_a/\Gamma_c$. It shows that the peak shifts from a value close to ω_a to a value close to ω_c with increasing of Γ_a. As we discussed above, there are two dips located at ω_a and ω_c in the transmission spectrum, respectively. The transmission spectrum is symmetric with respect to $(\omega_a+\omega_c)/2$ when $\Gamma_a=\Gamma_c$. However, when $\Gamma_a\neq\Gamma_c$, the transmission spectrum exhibits strong Fano-type, as shown by the dotted lines and dashed dotted lines in Fig. 2(a). The LP is also shifted. Figure 2(b) shows the distance between the LP and $\omega_a(\omega_c)$. It exhibits clearly that the peak gets close to ω_c with increasing η. The Fano line type near ω_c shows strong asymmetry when η is large. Figures 2(c) and 2(d) show T and $\omega_a(\omega_c)-\omega_{t=1}$, respectively, for the case of $\omega_a-\omega_c=-\Gamma_c$. One can find that one of the dips shifts to $-\Gamma_c$ due to changing of ω_a. It is noted that $\omega_a-\omega_{t=1}$ becomes to be negative when $\omega_c-\omega_{t=1}$ is positive. However, with increasing η, the peak still gets close to ω_c.

By carefully analyzing Eqs. (9) and (10), one can find a peculiar solution with $\omega=\omega_{t=1}=\omega_c+\Gamma_c$ under the condition that $kL=n\pi-\pi/4$ and $\Gamma_a+\omega_a=\Gamma_c+\omega_c(\omega_a\neq\omega_c)$. Figure 3 exhibits T as a function of Δ_c with different ω_a. It reaches unit at $\omega_c+\Gamma_c$, which is consistent with the analytical analysis. There are still two dips in the transmission spectrum. One of the dip shifts due to different ω_a. However, the other dip keeps at zero since ω_c is fixed. Different from the case of $kL=n\pi$, the Fano line-type between ω_c and $\omega_c+\Gamma_c$ is almost the same even though Γ_a is different.

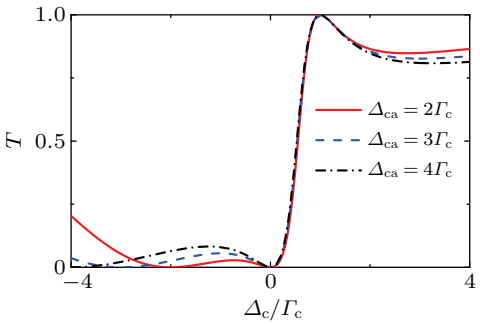


Fig. 3. Single photon transmission spectrum with different ω_a. In the calculations, $\Gamma_a+\omega_a=\Gamma_c+\omega_c$, $\Gamma_c=10^{-5}\omega_c$, $kL=3\pi/4$, $\Delta_{ca}\equiv\omega_c-\omega_a$.

2.2. With atom

Now, let us consider the case in which an atom is embedded in the FC. By carefully analyzing Eq. (5), one can find that there are three dips in the transmission spectrum, which locate at

$$\omega_{r1}=\omega_a, \tag{11}$$

$$\omega_{r2}=\frac{(\omega_e+\omega_c)+\sqrt{(\omega_c-\omega_e)^2+4g^2}}{2}, \tag{12}$$

$$\omega_{r2}=\frac{(\omega_e+\omega_c)-\sqrt{(\omega_c-\omega_e)^2+4g^2}}{2}. \tag{13}$$

To explain this, we review the single photon scattering properties in the one-dimensional waveguide coupling to a two-level system firstly. In such a system, the incident photon is reflected back with probability of 100% when it is resonant with the two-level system, and the two-level system is called "quantum mirror". In our model, the AC can be equivalent to a two-level system. Thus, when the incident photon is resonant with the AC, i.e., $\omega=\omega_a$, it will be reflected back. The other two peaks of reflection spectrum result from the interaction between the FC and the atom. The system composed of FC and a two-level atom can be treated by using the dressed state picture. In the dressed state, one can get two eigenfrequencies, which are given by ω_{r2} and ω_{r3}, respectively. Thus,

the incident photon with frequency ω_{r2} (ω_{r3}) will be reflected back with probability of 100%.

The LPs in the transmission spectrum can be determined by the following two equations:

$$2k_e\Gamma_a\Gamma_c\sin^2(kL)-g^2\Gamma_a\sin(2kL)+k_ck_e\Gamma_a\sin(2kL)=0, \quad (14)$$

$$k_e\Gamma_a\Gamma_c\sin(2kL)-g^2\Gamma_a\cos(2kL)+k_ck_e\Gamma_a\cos(2kL)$$
$$+k_ak_e\Gamma_c=0. \quad (15)$$

It is easy to find that there are two transparency windows, which separately locate at $\omega=\omega_{t=1}=\omega_c\pm g\sqrt{\eta/(1+\eta)}$, with $kL=\pi$ when FC, AC and the atom are resonant. Note that there will be no transparency window for the case of no atom embedded in the FC if the FC and AC are resonant with $kL=\pi$. Figure 4(a) shows T as a function of Δ_c. It exhibits that the spectrum keeps symmetric with respect to ω_c with different η. However, the transparency window shifts with η, which is consistent with the analytical results. Furthermore, the width of the transparency window is dependent on Γ_a. With increasing Γ_a, the transparency window around $\omega_{t=1}$ becomes narrower. When the atom and the FC are detuned ($\omega_a=\omega_c\neq\omega_e$), the transmission spectrum becomes asymmetrical and the transparency windows shift to $\omega=\omega_{t1}=(\omega_c+\omega_e\pm\sqrt{(\omega_c-\omega_e)^2+4g^2\eta/(1+\eta)})/2$. Since the dips in the transmission spectrum are independent of η, one can adjust Γ_a to obtain sharp nonlinear Fano line-type, as shown by the red dashed line in Fig. 4(b).

When $kL=n\pi\mp\pi/4$, for the case that the FC and AC are resonant, one can also find a transparency window locating at $\omega_{t1}=\omega_c\pm\eta\Gamma_c$ under the condition

$$\Delta_{ce}\equiv\omega_c-\omega_e=\mp(\eta\Gamma_c-g^2/[(\eta-1)\Gamma_c]). \quad (16)$$

Figure 5 shows T as a function of Δ_c with different η. It exhibits that one can adjust the locations of transparency window by carefully choosing the relationship between g and Γ_c to satisfy Eq. (16). There are three dips in the transmission spectrum. One of the dips is fixed since $\omega_a=\omega_c$ keeps unchanged. The locations of the other two dips change with different g, as we discussed above. It is notable that there is no transparency window when $kL=n\pi+\pi/4$ for the case of no atom embedded in the FC.

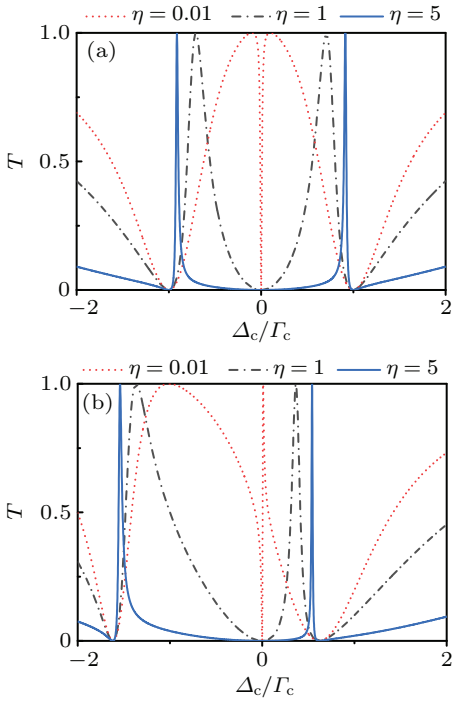


Fig. 4. Single photon transmission spectrum with different η for the FC embedded within an atom. (a) The FC, AC and the atom are resonant with $\omega_a=\omega_c=\omega_c$. (b) The atom and the FC are non-resonant with $\omega_e=\omega_c-\Gamma_c$ and $\omega_a=\omega_c$. In the calculations, $\Gamma_c=g=10^{-5}\omega_c$, $kL=\pi$.

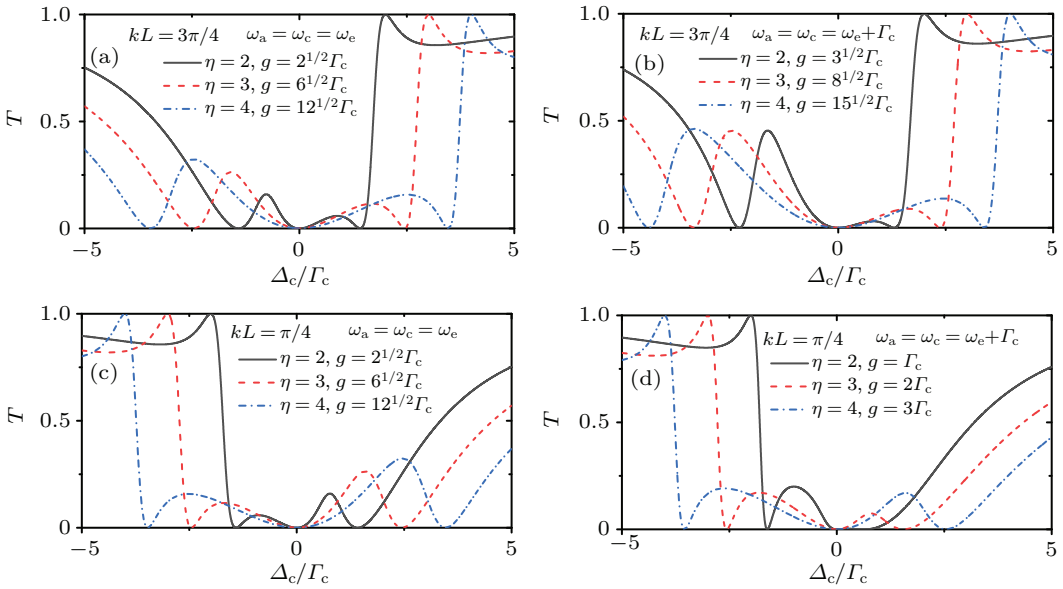


Fig. 5. Single photon transmission spectrum for the FC embedded within an atom. In (a) and (c), the FC, AC and the atom are resonant. In (b) and (d), the FC and the AC are resonant but the atom is detuned with $\omega_c-\omega_e=\Gamma_c$. The other parameter in the calculations is given by $\Gamma_c=10^{-5}\omega_c$.

3. Discussion

The Fano interference effects due to absorption and emission from the two or more emitters coupled to one waveguide have been discussed before.[19,37] The case of $kL = n\pi$ is studied widely in these reports. In the present article, we discuss analytically the dips and positions of peak in the transmission spectrum in detail and find that when no atom is embedded in the FC, there exists a transparency window with $kL = (n - 1/4)\pi$. When an atom is embedded in the FC, there exists two transparency windows with $kL = \pi$ and one transparency window with $kL = (n \pm 1/4)\pi$ in the case that FC and AC are resonant. The intrinsic physical mechanism results from energy structure modifications of the FC by the embedded atom. The locations of Fano dips and Fano-types can be modulated by the AC.

4. Summary

In summary, we have theoretically investigated Fano interference and transparency in a waveguide-nanocavity hybrid system with an auxiliary cavity. When no atom is embedded in the FC, there is a transparency window locating at $(\omega_a + \omega_c)/2$ with $kL = \pi$ under the condition of $\omega_a \neq \omega_c$. Furthermore, we also find that when $kL = (n - 1/4)\pi$, a transparency window will appear under the condition $\Gamma_a + \omega_a = \Gamma_c + \omega_c$. The locations of Fano dips are determined by ω_a and ω_c. When one atom is embedded in the FC, the locations of transparency windows depend on the coupling strength between the AC and the waveguide for the case of $kL = \pi$. Comparing with the case of no atom embedded in the FC, there are transparency windows in the transmission spectrum when $kL = (n \pm 1/4)\pi$. The Fano dips depend strongly on ω_a, ω_c and g. With increasing Γ_c, the Fano line-type near the transparency window becomes more sharp. Our results may find applications in designing quantum devices at single-photon level such as sensing, and single photon switching.

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