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Citation: Chin. Phys. B, 2021, 30 (**7**): 070307. DOI: 10.1088/1674-1056/abff2a
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SPECIAL TOPIC — Quantum computation and quantum simulation

Nonlocal advantage of quantum coherence and entanglement of two spins under intrinsic decoherence*

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(Received 18 March 2021; revised manuscript received 14 April 2021; accepted manuscript online 8 May 2021)

We investigate the nonlocal advantage of quantum coherence (NAQC) and entanglement for two spins coupled via the Heisenberg interaction and under the intrinsic decoherence. Solutions of this decoherence model for the initial spin-1/2 and spin-1 maximally entangled states are obtained, based on which we calculate the NAQC and entanglement. In the weak region of magnetic field, the NAQC behaves as a damped oscillation with the time evolves, while the entanglement decays exponentially (behaves as a damped oscillation) for the spin-1/2 (spin-1) case. Moreover, the decay of both the NAQC and entanglement can be suppressed significantly by tuning the magnetic field and anisotropy of the spin interaction to some decoherence-rate-determined optimal values.

Keywords: quantum coherence, quantum entanglement, intrinsic decoherence

PACS: 03.65.Yz, 03.65.Ud, 75.10.Pq, 42.50.Lc **DOI:** 10.1088/1674-1056/abff2a

1. Introduction

Quantum coherence originates from the superposition principle of the basis states, and it is different from that of the interference phenomenon in classical physics. Among the various characterizations of quantumness (*e.g.*, entanglement,[1] quantum discord,[2] *etc.*), quantum coherence is the the most fundamental one, and in some sense, the essence of quantum correlations,[3,4] although it characterizes quantumness of the whole system S, whereas quantum correlations are related to the interrelation between subsystems of S. Moreover, quantum coherence is an indispensable resource for achieving the quantum advantage of quantum computation, quantum communication, and quantum metrology tasks.[5]

Due to its fundamental role in the basic theory of quantum mechanics and applications in new quantum technologies, it is necessary to quantify coherence. In 2014, Baumgratz and his coauthors[6] constructed a resource theoretic framework of coherence, and proposed to quantify the amount of coherence in a state ρ by its "shortest distance" to the set of incoherent states. Some well-defined measures within such a framework include the l_1 norm and relative entropy of coherence,[6] the entanglement-based coherence measures,[7] the robustness of coherence,[8] the intrinsic randomness of coherence,[9] the coherence of formation,[10] the maximum relative entropy of coherence,[11] and the skew information measure of coherence.[12] There are also several coherence measures defined within slightly different frameworks, see Ref. [4].

Based on the above measures, researchers further analyzed quantitatively the role of quantum coherence in specific quantum computational tasks. Some notable progresses include the advantage of quantum state merging,[13] deterministic quantum computation with one qubit,[14] the Deutsch–Jozsa algorithm,[15] the Grover search algorithm,[16] and the phase discrimination tasks.[8,11,12] Quantum coherence is also a resource for enhancing efficiency of the quantum heat engine.[17] As a fundamental concept in quantum theory, it has also been used to interpret the wave-particle duality[18,19] and various form of quantum correlations such as quantum entanglement[7,20] and quantum discord.[20–23]

From a practical point of view, decoherence remains a main obstacle for carrying out quantum computation tasks, and different systems may face different sources of decoherence.[5] Hence it is significant to give a quantitative description of the decoherence process. The various coherence measures facilitate the development of such a task. In recent years, some studies, including the quantitative analyses of the decoherence process of different systems,[24–27] the evolution equation of coherence under completely positive and trace preserving operations,[28] and the conditions for freezing coherence,[29–32] have been performed. Effects of active operations on coherence, such as the coherence-preserving

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11774406 and 11934018), the National Key Research and Develepment Program of China (Grant Nos. 2016YFA0302104 and 2016YFA0300600), the Strategic Priority Research Program of Chinese Academy of Sciences (Grant No. XDB28000000), and the Research Program of Beijing Academy of Quantum Information Sciences (Grant No. Y18G07).

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operations,[33] the cohering power of a channel,[34–36] and the energy cost for creating coherence,[37] have also been discussed.

In this work, we explore the nonlocal advantage of quantum coherence (NAQC) and entanglement under intrinsic decoherence.[38] The NAQC was defined based on steered coherence under local operations and classical communication.[39,40] It reveals a kind of quantum correlation which is stronger than entanglement (it is also stronger than Bell nonlocality for the two-qubit states[41]). The shareability of NAQC by sequential observers,[42] its role in studying quantum criticality of the spin systems,[43] and its behavior under noisy channels,[44–46] have been explored. For two spins under intrinsic decoherence, we will show that the decay of both the NAQC and entanglement can be noticeably suppressed by tuning the system parameters to appropriate values.

2. Measures of NAQC and entanglement

As a preliminary, we recall how to quantify NAQC and entanglement in a $(d\times d)$-dimensional state ρ_{AB}. First, the NAQC was defined based on the resource theory of coherence,[6] and one can obtain different criteria for capturing NAQC in ρ_{AB} by using different coherence measures.[39] We will use the relative entropy of coherence which has a clear physical interpretation.[10] For a d-dimensional state ρ, it was defined to be the relative entropy $S(\rho\|\delta)$ minimized over all d-dimensional diagonal density operator δ in the reference basis $\{|i\rangle\}$, and can be solved analytically as[6]

$$C_{\rm re}^{\{|i\rangle\}}(\rho)=S(\rho_d)-S(\rho), \tag{1}$$

where $\rho_d=\sum_i\langle i|\rho|i\rangle|i\rangle\langle i|$, $S(\rho_d)=-\operatorname{tr}(\rho_d\log_2\rho_d)$ is the von Neumann entropy of ρ_d, and likewise for $S(\rho)$.

Based on Eq. (1), one can derive the criterion for capturing NAQC. There are two different frameworks related to such a problem, both of which are formulated by first measuring one of the mutually unbiased observables $\{A_k\}$ (*e.g.*, A_i) on party A and then calculating the average coherence of the ensemble $\{\rho_{B|A_i^a},p_{a|A_i}\}$, with $p_{a|A_i}$ being the probability of obtaining the outcome a and $\rho_{B|A_i^a}$ the corresponding postmeasurement state of B. But for the first framework, the coherence of $\rho_{B|A_i^a}$ is calculated with respect to the basis spanned by the eigenbasis of $A_j\neq A_i$ and then being averaged over all $A_j\neq A_i$,[39] while for the second framework, it is calculated only with respect to the optimal basis spanned by the eigenbasis of $A_{\tilde{\alpha}_i}$, with $\{A_{\tilde{\alpha}_i}\}$ being a permutation of the set $\{A_k\}$ which gives the maximum average coherence of $\{\rho_{B|A_i^a},p_{a|A_i}\}$, *i.e.*, one should maximize the average coherence of $\{\rho_{B|A_i^a},p_{a|A_i}\}$ over all possible permutations of the set $\{A_k\}$.[40] As the criterion formulated within the second framework captures a wider region of NAQC states than that formulated within the first framework,[40] we will make use of it in this paper. Then the criterion for capturing the NAQC in ρ_{AB} can be obtained as

$$\tilde{C}_{\rm re}^{na}(\rho_{AB})=\sum_{i,a}p_{a|A_i}C_{\rm re}^{A_{\tilde{\alpha}_i}}(\rho_{B|A_i^a})>C_{\rm re}^{m}, \tag{2}$$

where the bound $C_{\rm re}^{m}$ was given in Refs. [39,40], *e.g.*, $C_{\rm re}^{m}\simeq 2.2320$ for $d=2$ and $C_{\rm re}^{m}\simeq 5.0065$ for $d=3$. For the two-qubit case, the NAQC has been observed experimentally in an optics-based platform.[47]

Next, we recall the entanglement measure negativity which was introduced by Vidal and Werner[48] and was defined as

$$N(\rho_{AB})=\frac{\left\|\rho_{AB}^{\rm T_A}\right\|_1-1}{2}, \tag{3}$$

where $\|X\|_1=\operatorname{tr}(X^\dagger X)^{1/2}$ denotes the trace norm of X, and the superscript $\rm T_A$ in $\rho_{AB}^{\rm T_A}$ denotes the partial transpose of ρ_{AB} with respect to the subsystem A. Such a measure characterizes the degree of violation of the positive partial transpose (PPT) criterion which is a necessary separability condition [it is also sufficient for the (2×2)- and (2×3)-dimensional states].

3. The intrinsic decoherence model

We consider the intrinsic decoherence model, for which the equation of motion for a system described by the Hamiltonian $\hat{H}$ is given by[38]

$$\frac{{\rm d}\rho}{{\rm d}t}=\frac{1}{\gamma}\left({\rm e}^{-{\rm i}\gamma\hat{H}}\rho\,{\rm e}^{{\rm i}\gamma\hat{H}}-\rho\right), \tag{4}$$

and it is formulated based on the hypothesis that on sufficiently short time steps, the system will evolves in a stochastic sequence of identical unitary transformation instead of evolving continuously and unitary in the whole evolution process, and the decoherence rate γ is proportional to this minimum time step.[38]

3.1. Solution of the model

The decoherence model of Eq. (4) is usually solved by expanding its right-hand side (RHS) to the first order in γ, which yields

$$\frac{{\rm d}\rho}{{\rm d}t}=-{\rm i}[\hat{H},\rho]-\frac{\gamma}{2}\{\hat{H}^2,\rho\}+\gamma\hat{H}\rho\hat{H}, \tag{5}$$

where $[\,]$ and $\{\,\}$ denote, the commutator and anticommutator, respectively. Then by denoting $\{\epsilon_k\}$ and $\{|\psi_k\rangle\}$ the eigenvalues and eigenstates of $\hat{H}$, respectively, and $a_{kl}=\langle\psi_k|\rho(0)|\psi_l\rangle$, with $\rho(0)$ being the initial state, equation (5) can be solved as[49]

$$\rho^{(1)}(t)=\sum_{kl}a_{kl}\,{\rm e}^{-{\rm i}t(\epsilon_k-\epsilon_l)-\frac{1}{2}\gamma t(\epsilon_k-\epsilon_l)^2}|\psi_k\rangle\langle\psi_l|, \tag{6}$$

where $\rho^{(1)}(t)$ is introduced for distinguishing the solutions of Eq. (4) with its RHS being expanded to different orders

in γ. Based on this solution, decay of Bell nonlocality,[50] entanglement,[51–53] and entropic uncertainty,[54,55] have been extensively investigated.

By further expanding the RHS of Eq. (4) to the order of γ^2, one has

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\mathrm{i}\left[\hat{H},\rho\right] - \frac{\gamma}{2}\left\{\hat{H}^2,\rho\right\} + \gamma\hat{H}\rho\hat{H} + \frac{1}{6}\mathrm{i}\gamma^2\left\{\hat{H}^3,\rho\right\} + \frac{1}{2}\mathrm{i}\gamma^2\left[\hat{H}\rho\hat{H},\hat{H}\right], \quad (7)$$

and similarly to Eq. (5), this equation can be solved as

$$\rho^{(2)}(t) = \sum_{kl} a_{kl}\,\mathrm{e}^{-\mathrm{i}t(\epsilon_k-\epsilon_l)-\frac{1}{2}\gamma t(\epsilon_k-\epsilon_l)^2+\frac{1}{6}\mathrm{i}\gamma^2 t(\epsilon_k-\epsilon_l)^3}|\psi_k\rangle\langle\psi_l|. \quad (8)$$

Apart from expanding the RHS of Eq. (4) to different orders in γ, one can also transform it into the following form:[56]

$$\frac{\mathrm{d}\tilde{\rho}}{\mathrm{d}t} = \tilde{\Lambda}\tilde{\rho}, \quad (9)$$

where $\tilde{\rho}$ is a column vector with the elements $\tilde{\rho}_{(i-1)d+j} = \rho_{ij}$ ($i,j = 1,2,\ldots,d$ and $d = \dim\rho$). Moreover, $\tilde{\Lambda}$ is a $(d^2\times d^2)$-dimensional matrix constructed from the RHS of Eq. (4). To be explicit, by defining

$$\Lambda^{(ij)} = \frac{1}{\gamma}\left(\mathrm{e}^{-\mathrm{i}\gamma\hat{H}}\rho^{(ij)}\,\mathrm{e}^{\mathrm{i}\gamma\hat{H}} - \rho^{(ij)}\right), \quad (10)$$

with $\rho^{(ij)}$ being a $(d\times d)$-dimensional matrix with one element of 1 in the i-th row and j-th column and all the other elements are zero, then the elements of $\tilde{\Lambda}$ can be obtained as

$$\tilde{\Lambda}_{d(k-1)+l,d(i-1)+j} = \Lambda^{(ij)}_{kl} \quad (i,j,k,l = 1,2,\ldots,d). \quad (11)$$

The solution of Eq. (9) can be written formally as

$$\tilde{\rho}(t) = \mathrm{e}^{\tilde{\Lambda}t}\tilde{\rho}(0), \quad (12)$$

and the elements of $\rho(t)$ are given by $\rho_{ij}(t) = \tilde{\rho}_{d(i-1)+j}(t)$. Different from $\rho^{(1)}(t)$ and $\rho^{(2)}(t)$, the accuracy of the solution $\rho(t)$ depends on the accuracy for diagonalizing $\hat{H}$ and $\tilde{\Lambda}$.

The NAQC of the thermal states of various spin systems has been studied.[57–59] In this paper, we focus on the intrinsic decoherence effects on NAQC of the spin system. We consider the following Hamiltonian (in units of $\hbar$):

$$\hat{H} = J(s_1^x s_2^x + s_1^y s_2^y + \Delta s_1^z s_2^z) + B(s_1^z + s_2^z), \quad (13)$$

where $s_n^{x,y,z}$ are the spin-s operators at site n, J is the coupling strength of two spins, Δ characterizes anisotropy of the coupling, and B is the transverse magnetic field.

We consider the cases of $s = 1/2$ and 1, for which $\hat{H}$ can be diagonalized exactly, thus the accuracy of the solution $\rho(t)$ depends solely on the diagonalization of $\tilde{\Lambda}$. For $s = 1/2$, the eigenvalues and eigenvectors of $\hat{H}$ can be derived as

$$\epsilon_{1,2} = -\frac{1}{4}J\Delta \pm \frac{1}{2}J,\ \ \epsilon_{3,4} = \frac{1}{4}J\Delta \pm B,$$
$$|\psi_{1,2}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle),\ |\psi_3\rangle = |00\rangle,\ |\psi_4\rangle = |11\rangle, \quad (14)$$

and for $s = 1$, by denoting $\eta = \sqrt{\Delta^2+8}$, the eigenvalues and eigenvectors of $\hat{H}$ can be obtained as

$$\epsilon_{1,2} = -\frac{1}{2}J\Delta \pm \frac{1}{2}J\eta,\ \epsilon_{3,4} = B \pm J,\ \epsilon_5 = -J\Delta,$$
$$\epsilon_{6,7} = -B \pm J,\ \epsilon_{8,9} = J\Delta \pm 2B,$$
$$|\psi_{1,2}\rangle = \sqrt{\frac{2}{\eta(\eta\pm\Delta)}}\left(|02\rangle + \frac{\Delta\pm\eta}{2}|11\rangle + |20\rangle\right),$$
$$|\psi_{3,4}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle),\ |\psi_5\rangle = \frac{1}{\sqrt{2}}(|02\rangle - |20\rangle),$$
$$|\psi_{6,7}\rangle = \frac{1}{\sqrt{2}}(|12\rangle \pm |21\rangle),\ |\psi_8\rangle = |00\rangle,\ |\psi_9\rangle = |22\rangle. \quad (15)$$

3.2. Comparison of the different solutions

To compare the accuracy of the solutions $\rho^{(1)}(t)$, $\rho^{(2)}(t)$, and $\rho(t)$, we consider the following initial states:

$$|\Psi\rangle_{1/2} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle),\ |\Psi\rangle_1 = \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle), \quad (16)$$

where $|\Psi\rangle_{1/2}$ ($|\Psi\rangle_1$) is for the spin-1/2 (spin-1) case. We consider the two states for they are useful in quantum information processing tasks such as quantum teleportation.[60,61]

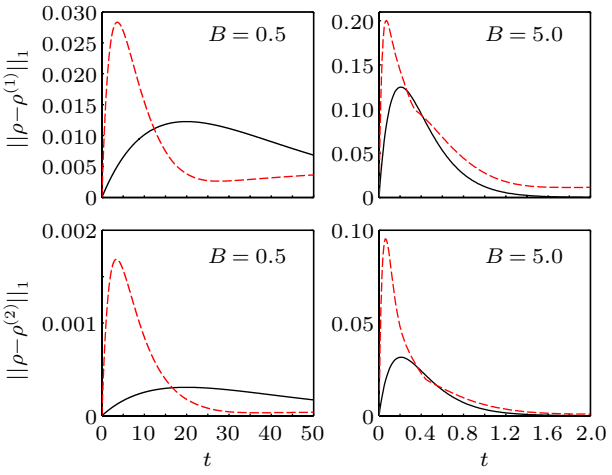


Fig. 1. $\|\rho-\rho^{(1)}\|_1$ and $\|\rho-\rho^{(2)}\|_1$ *versus* t for the initial states $|\Psi\rangle_{1/2}$ (solid black) and $|\Psi\rangle_1$ (dashed red) with different B. The other parameters are given by $J = 1$, $\Delta = 0$, and $\gamma = 0.1$.

First, for the spin-1/2 case, the four nonzero a_{kl} are given by $a_{33,34,43,44} = 1/2$. As a consequence, one can obtain from Eqs. (6), (8), and (14) that both $\rho^{(1)}(t)$ and $\rho^{(2)}(t)$ are independent of Δ. As for the solution $\rho(t)$, although $\tilde{\Lambda}$ is a function of Δ, $\tilde{\rho}(t)$ is determined only by the elements of $\tilde{\Lambda}$ lie in the ith column with $i \in \{1,4,13,16\}$ due to the specific form of $\tilde{\rho}(0)$, and by using Eqs. (10) and (11), one can obtain that the elements lie in the first and 16th columns of $\tilde{\Lambda}$ are all zero, while the only nonzero element lies in its fourth column is $\tilde{\Lambda}_{4,4} = (\mathrm{e}^{-2\mathrm{i}B\gamma}-1)/\gamma$ and that lies in its 13th column is $\tilde{\Lambda}_{13,13} = \tilde{\Lambda}^*_{4,4}$, then one can obtain analytically that

$$\rho(t) = \frac{1}{2}\Big(|00\rangle\langle 00| + |11\rangle\langle 11|$$

$$+ \mathrm{e}^{\tilde{\Lambda}_{4,4}t}|00\rangle\langle 11| + \mathrm{e}^{\tilde{\Lambda}_{13,13}t}|11\rangle\langle 00|\Big), \quad (17)$$

so $\rho(t)$ is also independent of Δ. One can also note that $\rho^{(1)}(t)$ and $\rho^{(2)}(t)$ are in fact the first and second order approximations of $\rho(t)$, as one has $\tilde{\Lambda}_{4,4} \simeq -2\mathrm{i}B - 2B^2\gamma + 4B^3\gamma^2/3$ by expanding it to the second order in γ. By using the trace norm to evaluate the accuracy of different solutions, one can obtain

$$\left\|\rho - \rho^{(n)}\right\|_1 = \left|\exp\left(\tilde{\Lambda}_{4,4}t\right) - \exp\left(\sum_{k=1}^{n}\frac{(-2\mathrm{i}B)^k\gamma^{k-1}t}{k!}\right)\right|, \quad (18)$$

from which one can see that $\rho^{(1)}(t)$ and $\rho^{(2)}(t)$ may yield inaccurate results for certain B, see, *e.g.*, the solid lines showed in Fig. 1. Moreover, when $B\gamma = k\pi$ ($k \in \mathbb{Z}$), one has $\tilde{\Lambda}_{4,4} = 0$, so $\rho(t)$ will remain unchanged. But it should note that such an observation does not hold for a general initial state.

For the spin-1 case, as the dimension of $\tilde{\Lambda}$ is still relatively small, it can be diagonalized numerically. In Fig. 1, we show the time dependence of the trace norm $\|\rho - \rho^{(1)}\|_1$ and $\|\rho - \rho^{(2)}\|_1$ with $\Delta = 0$ and different B. In the long-time region, one can see that the three solutions are approximately the same. In the short-time region, however, the accuracy of $\rho^{(1)}(t)$ and $\rho^{(2)}(t)$ are relatively low. So when using them as solutions of Eq. (4), one should pay attention to their accuracy.

4. Dynamics of NAQC and negativity

In this section, we use $\rho(t)$ obtained from Eq. (12) to calculate the time-evolved NAQC and negativity, and explore their decay behaviors for the initial states of Eq. (16). Occasionally, we also give some intuitive analysis by using the approximate solutions of Eqs. (6) and (8).

4.1. The NAQC

For the spin-1/2 case, we show in Fig. 2 the t dependence of $\tilde{C}_{\mathrm{re}}^{na}(\rho)$ with different strengths of B and the B dependence of $\tilde{C}_{\mathrm{re}}^{na}(\rho)$ at different times t. One can see that $\tilde{C}_{\mathrm{re}}^{na}(\rho)$ behaves as a damped oscillation when the time evolves and approaches to the steady-state value 1 in the limit of $t \to \infty$. Such a behavior can be understood from Eq. (17), which gives

$$\mathrm{e}^{\tilde{\Lambda}_{4,4}t} = \mathrm{e}^{-[2\sin^2(B\gamma) + \mathrm{i}\sin(2B\gamma)]t/\gamma}, \quad (19)$$

that is, the decay of $\tilde{C}_{\mathrm{re}}^{na}(\rho)$ is due to the term $\mathrm{e}^{-2\sin^2(B\gamma)t/\gamma}$ and its oscillation is due to the term $\mathrm{e}^{-\mathrm{i}\sin(2B\gamma)t/\gamma}$. Moreover, as the NAQC in $\rho(t)$ is achieved when $\tilde{C}_{\mathrm{re}}^{na}(\rho) > C_{\mathrm{re}}^{m}$, then as is shown in the top panel of Fig. 2, the NAQC disappears after several rounds of damped oscillations. From the bottom panel of Fig. 2, one can see that the extent to which $\tilde{C}_{\mathrm{re}}^{na}(\rho)$ can be enhanced by tuning B in the weak field region is finite. But when B is tuned to $B_{\mathrm{c}} = l\pi/\gamma$ ($l = 0, 1, \ldots$), one can obtain from Eq. (17) that the state $\rho(t)$ will be the same to the initial state $|\Psi\rangle_{1/2}$, so $\tilde{C}_{\mathrm{re}}^{na}(\rho) \equiv 3$, *i.e.*, $\rho(t)$ is always NAQC correlated.

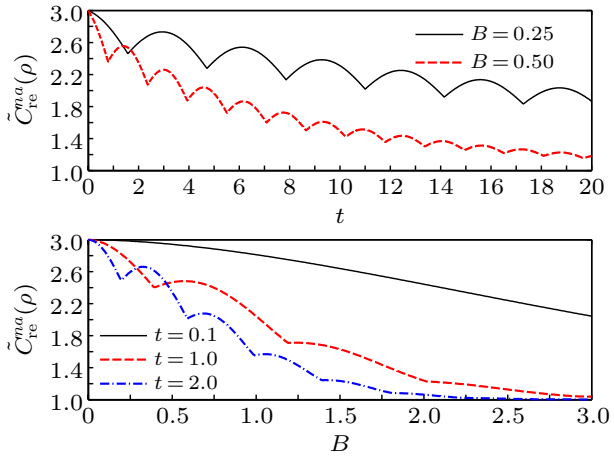


Fig. 2. $\tilde{C}_{\mathrm{re}}^{na}(\rho)$ *versus* t with different B and $\tilde{C}_{\mathrm{re}}^{na}(\rho)$ *versus* B at different times t for the initial state $|\Psi\rangle_{1/2}$. The other parameters are given by $J = 1$ and $\gamma = 0.1$.

Next, we consider the spin-1 Hamiltonian $\hat{H}$ and the initial state $|\Psi\rangle_1$. Different from that of the spin-1/2 case, the nonzero coefficients $\{a_{kl}\}$ for the present case correspond to those $k, l \in \{1, 2, 8, 9\}$, and the evolved density operator depends on all the involved parameters. In Fig. 3, we display the dependence of $\tilde{C}_{\mathrm{re}}^{na}(\rho)$ on the evolution time t, the transverse magnetic field B, and the anisotropic parameter Δ. From these plots, one can see that $\tilde{C}_{\mathrm{re}}^{na}(\rho)$ also behaves as a damped oscillation as the time t evolves, and qualitatively, this could be understood from the approximate solutions of Eqs. (6) and (8). But for this case, $\tilde{C}_{\mathrm{re}}^{na}(\rho)$ becomes smaller than the bound $C_{\mathrm{re}}^{m} \simeq 5.0065$ after a very short evolution time t_{c}, *e.g.*, $t_{\mathrm{c}} \simeq 0.6128$ when $B = 0$, $\Delta = 0$, and $\gamma = 0.1$. Moreover, in the infinite time limit, as the three solutions are the same, then one can obtain from Eq. (6) that the steady state $\rho(\infty)$ is given by

$$\rho(\infty) = \sum_{m,n\in\{1,2,8,9\}} a_{mn}\delta_{\epsilon_m,\epsilon_n}|\psi_m\rangle\langle\psi_n|, \quad (20)$$

where $\delta_{\epsilon_m,\epsilon_n}$ is the Kronecker delta, *i.e.*, $\delta_{\epsilon_m,\epsilon_n} = 1$ when $\epsilon_m = \epsilon_n$ and 0 otherwise. Then one can see that $\tilde{C}_{\mathrm{re}}^{na}(\rho(\infty))$ depends on J, Δ, and B. Its maximum is about 4.4686, which occurs at $B = 0$ and $\Delta = 1$, irrespective of J.

Moreover, as can be seen from the bottom panels of Fig. 3, $\tilde{C}_{\mathrm{re}}^{na}(\rho)$ decreases with the increase of B and Δ in the relative short-time region (*e.g.*, $t = 0.1$), and apart from this short-time region, $\tilde{C}_{\mathrm{re}}^{na}(\rho)$ may be enhanced to some extents by tuning the strengths of B or Δ to certain appropriate values in the weak B and Δ regions, but it cannot exceed the bound $C_{\mathrm{re}}^{m} \simeq 5.0065$. Furthermore, as for the spin-1/2 case, it has been shown that when B is tuned to the critical value $l\pi/\gamma$ ($l = 0, 1, \ldots$), $\tilde{C}_{\mathrm{re}}^{na}(\rho)$ will remains unchanged during the evolution process, then it is natural to ask whether such a phenomenon also occurs for the spin-1 case. Our numerical calculation shows that for $B = l\pi/\gamma$ and Δ is relatively small, the

dependence of $\tilde{C}_{\rm re}^{na}(\rho)$ on time t is similar to that showed in the top panel of Fig. 3, thus the extent to which it can be enhanced is still finite. But if one can further tune the strength of the anisotropy to $\Delta_{\rm c} = 2m\pi/\gamma$ ($m = 1,2,\ldots$), as is shown in the top panel of Fig. 4, $\tilde{C}_{\rm re}^{na}(\rho)$ can also be enhanced significantly. In particular, the larger the positive integer m, the larger the enhancement to $\tilde{C}_{\rm re}^{na}(\rho)$. But such an effect is very sensitive to the deviation of Δ from $\Delta_{\rm c}$. As can be seen from the bottom panel of Fig. 4, even a very small deviation of Δ from $\Delta_{\rm c}$ can induce noticeably decrease of $\tilde{C}_{\rm re}^{na}(\rho)$, especially in the long-time region.

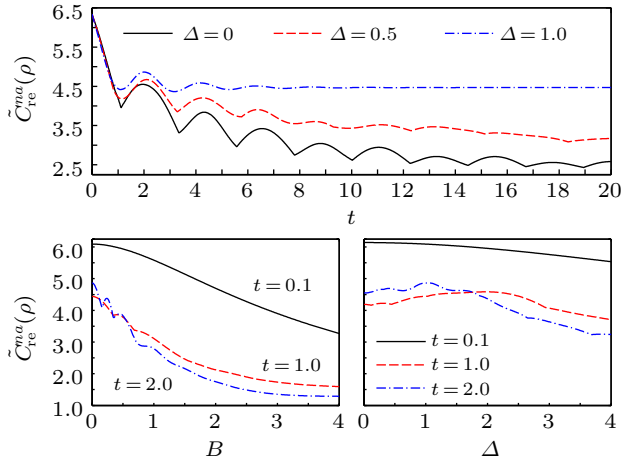


Fig. 3. $\tilde{C}_{\rm re}^{na}(\rho)$ *versus* t with fixed $B = 0$ and different Δ (top), $\tilde{C}_{\rm re}^{na}(\rho)$ *versus* B with $\Delta = 1$ and different t (bottom left), and $\tilde{C}_{\rm re}^{na}(\rho)$ *versus* Δ with $B = 0$ and different t (bottom right), all for the initial state $|\Psi\rangle_1$. The other parameters are given by $J = 1$ and $\gamma = 0.1$.

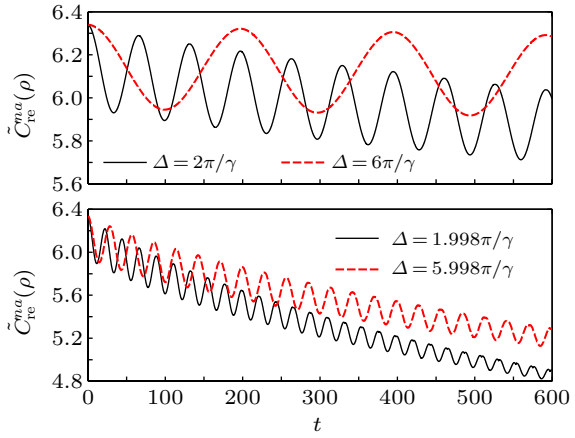


Fig. 4. $\tilde{C}_{\rm re}^{na}(\rho)$ *versus* t for the initial state $|\Psi\rangle_1$ with $B = 0$ and Δ locating at (top) or in the vicinity of $\Delta = 2m\pi/\gamma$ (bottom). The other parameters are given by $J = 1$ and $\gamma = 0.1$.

Before ending this section, we would like to remark that for the initial state of the form $|\Phi\rangle_1 = \alpha|00\rangle + \beta|22\rangle$ ($|\alpha|^2 + |\beta|^2 = 1$) and the spin-1 system Hamiltonian with $B = 0$, one can obtain from Section 3 that $\rho(t)$ will remains unchanged. In particular, $\tilde{C}_{\rm re}^{na}(\rho) > C_{\rm re}^{m}$ when $\alpha \in (0.3577, 0.9339)$ and $\beta = (1-\alpha^2)^{1/2}$. But its strength is weak, and its maximum is of about 5.0882, which occurs at $\alpha = \beta = 1/\sqrt{2}$.

4.2. The negativity

It has been shown that the NAQC captures a kind of quantum correlation stronger than entanglement.[39,40] Then it is natural to compare decay behaviors of the NAQC with that of the entanglement of the time-evolved state $\rho(t)$ obtained within the framework of intrinsic decoherence. In this subsection, we will consider such a problem, aimed at revealing the similarities and differences between decay behaviors of these two different forms of quantum correlations.

We first consider the spin-1/2 case, for which the negativity of $\rho(t)$ can be obtained analytically as

$$N(\rho) = \frac{1}{2}\,{\rm e}^{-2\sin^2(B\gamma)t/\gamma}, \tag{21}$$

from which one can obtain that when $B = k\pi/\gamma$ ($k = 0,1,\ldots$), the negativity $N(\rho)$ of the time-evolved state will remain the constant value 0.5, irrespective of the evolution time t of the two spins. For general values of B, however, $N(\rho)$ will decay exponentially with time and approach to the asymptotic value 0 in the infinite time limit. This indicates that the state $\rho(t)$ is always entangled for the initial state $|\Psi\rangle_{1/2}$. As the NAQC of $\rho(t)$ disappears after several rounds of damped oscillations (see Fig. 2), this also confirms the finding that what the NAQC captures is a type of quantum correlation which is stronger than quantum entanglement.[39,40]

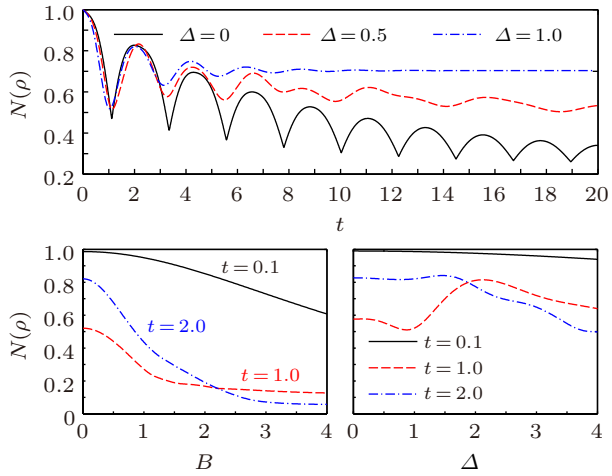


Fig. 5. $N(\rho)$ *versus* t with $B = 0$ and different Δ (top), $N(\rho)$ *versus* B with $\Delta = 1$ and different t (bottom left), and $N(\rho)$ *versus* Δ with $B = 0$ and different t (bottom right), all for the initial state $|\Psi\rangle_1$. The other parameters are given by $J = 1$ and $\gamma = 0.1$.

Next, we consider the spin-1 case for the same initial state $|\Psi\rangle_1$ when discussing the NAQC. We calculated numerically the negativity $N(\rho)$ of $\rho(t)$, and the corresponding exemplified plots are shown in Fig. 5. Similar to $\tilde{C}_{\rm re}^{na}(\rho)$, $N(\rho)$ also behaves as a damped oscillation with the time evolves and approaches to its steady-state value when $t \to \infty$. From Eq. (20), one can further obtain that the maximum steady-state value is about 0.7037, which occurs at $B = 0$ and $\Delta = 1$. This behavior is different from that of the spin-1/2 case for which $N(\rho(\infty)) = 0$. It is also different from that of $\tilde{C}_{\rm re}^{na}(\rho(\infty))$, as the

entanglement of $\rho(\infty)$ maintains finite value and the NAQC of $\rho(\infty)$ disappears completely.

Moreover, in the region of weak (B,Δ), as can be seen from the bottom two panels of Fig. 5, $N(\rho)$ can be increased to some extent by tuning Δ and B. When $B = l\pi/\gamma\,(l = 0,1,\ldots)$ and $\Delta = 2m\pi/\gamma\,(m = 1,2,\ldots)$, similar to the case of $\tilde{C}^{na}_{\rm re}(\rho)$, $N(\rho)$ can also be enhanced noticeably (we do not give the plots here for conciseness of the paper).

5. Summary

In summary, we have investigated the decay process of both NAQC and entanglement for two spins within the framework of intrinsic decoherence, and the two spins are coupled via the Heisenberg *XXZ* model. We first presented solutions $\rho^{(n)}(t)$ of the intrinsic decoherence model by expanding its decoherence term to the n-th order in γ, and then compared its accuracy with the solution $\rho(t)$ obtained by introducing a generalized superoperator. By choosing the initial maximally entangled states, we obtained analytical result of $\rho(t)$ for the spin-1/2 case and numerical result of $\rho(t)$ for the spin-1 case, and showed explicitly that $\rho^{(n)}(t)$ may yield very inaccurate results under certain circumstances. So we used the solution $\rho(t)$ in the subsequent investigation of NAQC and entanglement.

For two spins interact via the Heisenberg *XXZ* model with weak transverse magnetic field, the NAQC of the initial maximally entangled states behave as a damped oscillation with the time t evolves, while the negativity decays exponentially (behaves as a damped oscillation) for the spin-1/2 (spin-1) case. Moreover, we have also shown that for the spin-1/2 case, the time-evolved state is independent of Δ and it will be immune of the intrinsic decoherence if $B = l\pi/\gamma\,(l = 0,1,\ldots)$. For the spin-1 case, the rapid decay of both NAQC and entanglement can be noticeably suppressed when $B = l\pi/\gamma\,(l = 0,1,\ldots)$ and $\Delta = 2m\pi/\gamma\,(m = 1,2,\ldots)$. Such a suppression effect can be strengthened by increasing m. This shows that by tuning the system parameters to appropriate strengths, the detrimental effect of intrinsic decoherence can be noticeably suppressed and the quantum correlations of two spins may be preserved for a long time.

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