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A sign-function receiving scheme for sine signals enhanced by stochastic resonance*

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To address the problem that it is difficult to detect an intermediate frequency (IF) signal at the receiving end of a communication system under extremely low signal-to-noise ratio (SNR) conditions, we propose a stochastic resonance (SR)-enhanced sine-signal detection method based on the sign function. By analyzing the SR mechanism of the sine signal and combining it with the characteristics of a dual-sequence frequency-hopping (DSFH) receiver, a periodic stationary solution of the Fokker–Planck equation (FPE) with a time parameter is obtained. The extreme point of the sine signal is selected as the decision time, and the force law of the electromagnetic particles is analyzed. A receiving structure based on the sign function is proposed to maximize the output difference of the system, and the value condition of the sign function is determined. In order to further improve the detection performance, in combination with the central-limit theorem, the sampling points are averaged N times, and the signal-detection problem is transformed into a hypothesis-testing problem under a Gaussian distribution. The theoretical analysis and simulation experiment results confirm that when N is 100 and the SNR is greater than 20 dB, the bit-error ratio (BER) is less than 1.5×10^{-2} under conditions in which the signal conforms to the optimal SR parameters.

Keywords: stochastic resonance, Fokker–Planck equation, sine signal detection, sign-function receiving structure

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1. Introduction

Frequency-hopping communications are widely used in military communications because of their good anti-jamming and anti-interception performance. However, with the development of communications interference technologies and the widespread popularity of civil wireless communications, the background noise and interference experienced by communications systems have increased significantly, resulting in the reception of signals that are often swamped by strong background noise, and which show characteristically low SNRs. Research into how to realize emergency military communications under low SNR conditions is of great significance for taking the initiative on the battlefield. The dual-sequence frequency-hopping (DSFH) communication mode selects different transmit channels based on transmission code elements 0 and 1, and by modulating the information in the radio-frequency (RF) part, it can obtain an innate anti-jamming ability.[1,2] As a means of emergency military communication, DSFH is mostly used for emergency communications, i.e., when high-speed military communication methods are disrupted and unusable, so it can accept a lower communication rate. However, in harsh electromagnetic environments with extremely low SNRs, the performance of DSFH communications is also affected. A way of detecting weak useful signals at the receiver is the key to improving the performance of DSFH communications.

The core idea of the traditional weak signal linear detection method is to improve the SNR by suppressing noise, but when noise suppression is carried out, the signal to be detected is also suppressed or lost, which is not conducive to the detection of the weak signal.[3–7] Stochastic resonance (SR), as a signal processing method developed in recent years, describes a kind of nonlinear physical phenomenon: under specific conditions, the nonlinear system, weak signal, and noise produce synergy; the noise energy is transferred to the signal via a nonlinear system, so as to enlarge the useful signal and suppress the noise.[8–10] McNamara *et al.*[11] studied the bistable system model and simplified the system to a two-state model under adiabatic approximation conditions. First, the output power spectrum of the system was calculated, and then the output SNR was calculated from the power spectrum. The results showed that the output SNR of the system reached a maximum under conditions of non-zero noise intensity. Xu[12] pointed out that when the noise intensity of the system exceeds the intensity range that makes the SR phenomenon occur, it is possi-

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ble to optimize the system parameters to obtain the best system response speed, and finally to make the system produce output in the resonant region. Mitaim *et al.*[13] proposed an adaptive SR theory and studied the problem of optimal matching between the noise and the signal type. Asdi[14] further studied adaptive SR theory and applied it to weak signal detection. Under Gaussian white-noise conditions, Galdi[15] analyzed the problem of the optimal reception of an SR-processed signal and gave three test statistics for average value, zero-crossing detection, and non-coherent detection according to the maximum likelihood criterion, which provided the basic concept of applying SR to signal detection. Chen[16] pointed out that there are some limitations in measuring the signal-processing gain of SR with the SNR as the index, proposed a generalized SR receiving structure, and derived the detection probability and false-alarm probability under different decision criteria. Wang[17] improved the traditional energy detection structure using SR theory, simulated the particle trajectory using a Langevin equation, and obtained numerical simulation results for the SR output. Liu[18] studied the energy-detection algorithm based on stochastic resonance under extremely low SNR conditions. By adding SR noise to the signal, the received signal was guided to resonate with the SR noise, but the method required that the mean value of the received signal was non-zero. Liu[19] studied the detection performance of a DSFH signal processed by stochastic resonance under colored noise conditions and obtained the relationship curve between the detection performance of DSFH under strong colored noise conditions and the correlation time of the colored noise. In Ref. [20], based on the energy reception and matched filtering method, a reception method based on a quadratic polynomial was proposed, but the performance of this method was similar to those of traditional receiving methods under low SNR conditions.

In this paper, a detection method for the SR output signal based on the sign function is proposed. By analyzing the SR mechanism of a DSFH IF signal and introducing an optimal decision time, a periodic stationary solution of the Fokker–Planck equation (FPE) with a time parameter is obtained. According to the characteristics of the IF sine signal, the extreme point of the sine signal is selected as the decision time. A sign-function receiving structure is designed, which improves the signal detection performance of the DSFH receiver under low-SNR conditions, and analyzes the feasibility of applying the sign-function receiving method in an engineered implementation.

This paper is organized as follows: in the second section, we introduce the transmitting and receiving model for DSFH signals, and analyze the SR process of the received signal. In the third part, we introduce the proposed method based on the sign function, and analyze the receiving performance. The fourth section verifies the correctness of the theory through numerical simulation, and compares the sign-function reception method with the traditional reception method under low-SNR conditions. The last section draws some conclusions.

2. SR detection theory of DSFH signals

2.1. DSFH communication system model

The DSFH communication mode selects the corresponding occupied-channel and dual-channel according to the transmitted symbol. As shown in Fig. 1, FS_0 and FS_1 are two independent pseudo-random sequences, used to control frequency-hopping carrier generation. At a time t, if the transmitted symbol is 1, then channel 1 is the occupied channel, and the channel selector selects the working frequency $S_1(t)$ of channel 1 for transmission; if the transmitted symbol is 0, then channel 0 is the occupied channel, and the channel selector selects the working frequency $S_0(t)$ of channel 0 for transmission.

Supposing that the transmitted symbol sequence $b = (\ldots 1,0,1,0,\ldots)$, then the corresponding frequency sequence is $(\ldots, f_3, f_5, f_1, f_4, \ldots)$, and the output signal of DSFH can be expressed as

$$\begin{aligned} s(t) &= \cos\left[2\pi f_{i,n}(t - nT_{\mathrm{s}}) + \varphi\right] \\ &\quad \times \left[\varepsilon(t - nT_{\mathrm{s}}) - \varepsilon(t - (n+1)T_{\mathrm{s}})\right], \end{aligned} \tag{1}$$

where T_{s} is the symbol period, $\varepsilon(t)$ is the step signal, and $f_{i,n}$ is the transmitted signal frequency at the n-th hop. According to the different transmitted code elements, $f_{i,n} = \begin{cases} f_{0,n}, & \text{code 0}, \\ f_{1,n}, & \text{code 1}, \end{cases}$ and $f_{0,n} \neq f_{1,n}$.

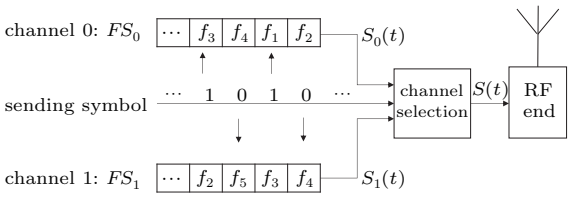


Fig. 1. DSFH transmitter.

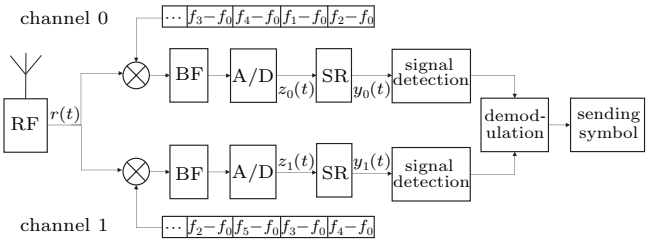


Fig. 2. DSFH receiver.

The DSFH receiver is shown in Fig. 2. The RF front end mixes the received signal $r(t)$ with the local carrier and obtains the IF signal $z_i(t)$ after band-pass filtering (BF) and A/D conversion. According to whether the channel is an occupied channel, $z_i(t)$ can be expressed as

$$z_i(t)=\begin{cases}\cos\left[2\pi f_0(t-nT_s)+\varphi\right]\left[\varepsilon(t-nT_s)-\varepsilon(t-(n+1)T_s)\right]+n(t), & \text{occupied channel,}\\ n(t), & \text{dual channel,}\end{cases} \tag{2}$$

where f_0 is the center frequency of the BF and $n(t)$ is Gaussian white noise, satisfying $E[n(t)]=0$, $E[n(t)n(t+s)]=2D\delta(s)$.

The IF received signal $z_i(t)$ is enhanced by the SR system, obtaining the signal $y_i(t)$ to be detected. The signal detection module judges the channel state by detecting whether there is a sine signal with the frequency f_0 in $y_i(t)$, and finally demodulates the transmitted symbol.

2.2. SR principle of the sine signal

To better illustrate the mechanism of SR, it is assumed that there are electromagnetic moving particles in the system, and that the sine signal and noise will drive the particles to move in the system according to a certain law. In order to analyze this kind of motion, we describe the process by which SR processes the DSFH IF signal from two perspectives: the orbits of the particles and the probability density function (PDF) of particles in orbit.

The trajectory of a particle in a bistable SR system can be described by the Langevin equation

$$\frac{\mathrm{d}x}{\mathrm{d}t}=ax-bx^3+A\cos(\omega_0 t+\varphi)+n(t), \tag{3}$$

where a and b are non-negative coefficients which determine the structure of the bistable system. The sine signal frequency $\omega_0=2\pi f_0$.

According to adiabatic approximation theory, SR can only process small, low-frequency signals, but the IF of DSFH is generally 1 kHz for SR systems, which is a large parameter signal. It is necessary to scale the IF signal to convert it into a small parameter signal that can be processed by SR. Therefore, the linear transformation formula $x=x\sqrt{b/a}$, $t=at$ is introduced. As a result, the Langevin equation can be expressed as

$$\frac{\mathrm{d}x}{\mathrm{d}t}=x-x^3+\sqrt{\frac{b}{a^3}}A\cos\left(\frac{\omega_0}{a}t+\varphi\right)+\sqrt{\frac{2Db}{a^2}}n(t). \tag{4}$$

The Langevin equation describes the orbit of a particle in the process of SR motion, reflects the position of the particle in orbit at a certain time, and reflects the SR behavior of the signal at the micro level. However, due to the presence of noise, the position of the particle at a certain time is uncertain at the macro level, so we pay more attention to the statistical properties of particle orbits in the SR process. The FPE corresponding to Eq. (4) is

$$\frac{\partial\rho(x,t)}{\partial t}=-\frac{\partial}{\partial x}\left\{\left[x-x^3+\sqrt{\frac{b}{a^3}}A\cos\left(\frac{\omega_0}{a}t+\varphi\right)\right]\cdot\rho(x,t)\right\}+\frac{\partial^2}{\partial x^2}\left[\frac{Db}{a^2}\rho(x,t)\right], \tag{5}$$

where $\rho(x,t)$ is the PDF of the SR particle in position x at time t.

In Eq. (5), the concrete expression of $\rho(x,t)$ cannot be obtained because of the existence of the non-autonomous item $\cos\left(\frac{\omega_0}{a}t+\varphi\right)\rho(x,t)$. But at the DSFH receiver, the signal is converted into a digital signal after A/D sampling; therefore, for each sampling time t_0, $\rho(x,t_0)$ reflects the probability distribution of particles at time t_0, and due to the introduction of t_0, the FPE can be transformed into an autonomous quadratic differential equation with $\frac{\partial\rho(x,t_0)}{\partial t}=0$, (integral Eq. (5)) to get

$$\rho(x,t_0)=G(t_0)^{-1}\exp\left(\frac{a^2}{Db}\left(\frac{1}{2}x^2-\frac{1}{4}x^4\right)+\sqrt{\frac{b}{a^3}}xA\cos\left(\frac{\omega_0}{a}t_0+\varphi\right)\right), \tag{6}$$

where $G(t_0)$ is the probability normalization coefficient related to t_0.

It can be seen from Eq. (6) that the addition of a sine signal is equivalent to the application of a tractive force to electromagnetic particles, which increases the probability of particles gathering at a potential well, and the size and direction of the tractive forces acting on particles are different at different sampling times: when the sine signal of the SR system satisfies $A\cos\left(\frac{\omega_0}{a}t_0+\varphi\right)>0$ at a certain time, the particles are driven by a positive force and tend to gather at the positive potential well; when the sine signal of the SR system satisfies $A\cos\left(\frac{\omega_0}{a}t_0+\varphi\right)<0$ at a certain time, the particles are driven by a negative force and tend to gather at the negative potential well. Moreover, when the sampling time is $A\cos\left(\frac{\omega_0}{a}t_0+\varphi\right)=\pm A$, the tractive force received by the particles is at a maximum, and the probability of the particles appearing in the potential well is also at a maximum. Although the direction and size of the electromagnetic particles are different at different times in a sine period, due to the semi periodicity of sine signals, when the system tends to be stable, the probability density of the system output over the whole sine period is symmetrically distributed.

3. Receiving structure based on sign-function

Because the two channels of DSFH are completely symmetric, for any channel, the problem of signal detection can correspond to binary hypothesis testing:

$$\left.\begin{array}{l}H_0: y(t)=n(t)\\ H_1: y(t)=x(t)+n(t)\end{array}\right\}, \tag{7}$$

where $x(t)$ is the sine signal enhanced by SR.

In the second part, by analyzing the SR mechanism of the sine signal, we come to the conclusion that the magnitude and direction of the force of the sine signal on electromagnetic particles are different at different times. In order to ensure the maximum probability of electromagnetic particles appearing in the potential well, we select the "wave peak and trough" point of the sine signal as the decision time. At this time, the driving effect of the sine signal on the electromagnetic particles is at its most obvious, and the output drift of the SR system is the largest. To detect the existence of the sine signal, although the tractive effects of peaks and troughs on electromagnetic particles are different, the traditional signal detection method cannot distinguish the peak versus trough states of the sine signal at the decision time. Since the probability of peaks and troughs is equal, the phase difference between the wave peak and the wave trough in a same period is π, so the output probability density of the system is obtained by averaging the driving effects of peaks and troughs, which is

$$\rho(x,t_0) = \left(2\int_{x_1}^{x_2}\exp\left\{\frac{a^2}{Db}\left(\frac{1}{2}x^2-\frac{1}{4}x^4\right)+\sqrt{\frac{b}{a^3}}xA\cos\left(\frac{\omega}{a}t_0+\varphi\right)\right\}\mathrm{d}x\right)^{-1} \times\left\{\exp\left[\frac{a^2}{Db}\left(\frac{1}{2}x^2-\frac{1}{4}x^4\right)+\sqrt{\frac{b}{a^3}}xA\cos\left(\frac{\omega}{a}t_0+\varphi\right)\right]+\exp\left[\frac{a^2}{Db}\left(\frac{1}{2}x^2-\frac{1}{4}x^4\right)+\sqrt{\frac{b}{a^3}}xA\cos\left(\frac{\omega}{a}t_0+\varphi+\pi\right)\right]\right\}. \quad (8)$$

It can be seen that when the traditional detection method is used to detect the signal, the output probability density of the system under the H_0 and H_1 assumptions is a symmetrical distribution, and the difference between them is small, which is not conducive to distinguishing between the two different assumptions.

To solve the above problem, an enhanced SR signal detection method based on the sign function is proposed. The receiving structure is shown in Fig. 3.

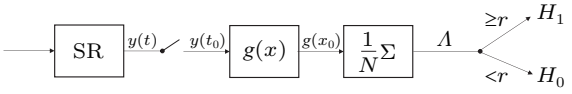


Fig. 3. Sign-function receiving structure of the SR enhanced signal.

The signal with noise is enhanced by SR to get the signal $y(t)$ to be detected, and the signal value $y(t_0)$ at the decision time is obtained by sampling the decision switch. The sign function $g(x)$ is used to process the signal value $y(t_0)$ to get the system's output value $g(x_0)$ at the decision time. The system test statistic Λ is obtained by averaging N independent output values. When Λ is greater than a threshold r, the system is judged to be in the state H_1; when Λ is less than threshold r, the system is judged to be in the state H_0.

For the convenience of theoretical analysis, we assume that the SR behavior of electromagnetic particles is instantaneous, and that the state of electromagnetic particles is only related to the current driving signal value $A\cos\left(\frac{\omega_0}{a}t_0+\varphi\right)$. Under the condition that the transmitter and the receiver are synchronized, the phase information of the IF sine signal at the receiver is known, and the positions of the peaks and troughs of the sine signal can be determined accordingly. The time corresponding to the peak point of the sine signal is recorded as t_u, and the time corresponding to the trough point of the sine signal is recorded as t_d.

We define the sign function $g(x)$ as follows:

$$g(x) = \begin{cases} x, & t_0 = t_\mathrm{u}, \\ -x, & t_0 = t_\mathrm{d}. \end{cases} \quad (9)$$

When the decision time satisfies the condition t_u, the output value of $g(x)$ is equal to the input value; when the decision time satisfies the condition t_d, the $g(x)$ output is the opposite of the input value. After the sign-function processing, the output value $g(x_0)$ of the system can be regarded as only being driven by the peak point of the sine signal, and its probability density is

$$\rho[g(x_0)|H_1] = \left\{\int\exp\left(\frac{a^2}{Db}\left(\frac{1}{2}x^2-\frac{1}{4}x^4\right)+\sqrt{\frac{b}{a^3}}xA\right)\mathrm{d}x\right\}^{-1} \times\exp\left[\frac{a^2}{Db}\left(\frac{1}{2}x^2-\frac{1}{4}x^4\right)+\sqrt{\frac{b}{a^3}}xA\right], \quad (10)$$

$$\rho[g(x_0)|H_0] = \left\{\int\exp\left(\frac{a^2}{Db}\left(\frac{1}{2}x^2-\frac{1}{4}x^4\right)\right)\mathrm{d}x\right\}^{-1} \times\exp\left[\frac{a^2}{Db}\left(\frac{1}{2}x^2-\frac{1}{4}x^4\right)\right]. \quad (11)$$

In order to further improve the detection performance, in each decision, we continuously record the output value $g(x_0)$ of the sign function N times; we sum and average it to obtain the system test statistic, Λ. When N is sufficiently large, according to the central limit theorem, Λ obeys the following distribution:

$$\Lambda = \frac{1}{N}\sum g(x_0) \sim \begin{cases} N(\mu_1,\sigma_1^2/N), & H_1, \\ N(\mu_0,\sigma_0^2/N), & H_0, \end{cases} \quad (12)$$

where

$$\mu_1 = \int g(x_0)\rho[g(x_0)|H_1]\,\mathrm{d}x,$$

$$\begin{aligned}\mu_0 &= \int g(x_0)\,\rho\left[g(x_0)\,|H_0\right]\mathrm{d}x,\\ \sigma_1^2 &= \int \left(g(x_0)-\mu_1\right)^2\rho\left[g(x_0)\,|H_1\right]\mathrm{d}x,\\ \sigma_0^2 &= \int \left(g(x_0)-\mu_0\right)^2\rho\left[g(x_0)\,|H_0\right]\mathrm{d}x.\end{aligned} \tag{13}$$

So far, the problem of SR enhanced signal detection after sign-function processing has been transformed into a hypothesis testing problem under a Gaussian distribution, and the system's bit-error ratio (BER) can be expressed as

$$\begin{aligned}P_{\mathrm{e}} &= P(H_1)\int_{-\infty}^{r} f\left[\Lambda(H_1)\right]\mathrm{d}\Lambda + P(H_0)\int_{r}^{+\infty} f\left[\Lambda(H_0)\right]\mathrm{d}\Lambda\\ &= P(H_1)\,\Phi\left(\frac{r-(\mu_1-\mu_0)}{\sqrt{\sigma_1^2/N+\sigma_0^2/N}}\right)\\ &\quad + P(H_0)\left[1-\Phi\left(\frac{r-(\mu_0-\mu_1)}{\sqrt{\sigma_1^2/N+\sigma_0^2/N}}\right)\right],\end{aligned} \tag{14}$$

where r is the decision threshold.

According to the minimum error probability criterion, the optimal decision threshold satisfies

$$\begin{aligned}&P(H_1)\,\Phi\left(\frac{r-(\mu_1-\mu_0)}{\sqrt{\sigma_1^2/N+\sigma_0^2/N}}\right)\\ &= P(H_0)\left[1-\Phi\left(\frac{r-(\mu_0-\mu_1)}{\sqrt{\sigma_1^2/N+\sigma_0^2/N}}\right)\right].\end{aligned} \tag{15}$$

Usually, the DSFH system makes 0 and 1 symbols appear with equal probability through source coding to maximize information entropy. So, given $p(H_1)=p(H_0)$, a simplified version of formula (15) has

$$\frac{(r-\mu_1)^2}{\sigma_1^2}-\frac{(r-\mu_0)^2}{\sigma_0^2}=\frac{2}{N}\mathrm{In}\frac{\sigma_1}{\sigma_0}; \tag{16}$$

the optimal decision threshold r can be calculated by Eq. (16).

At this time, the communication rate R of the DSFH communication system is

$$R=\frac{2f_0}{N}\left[1+P_{\mathrm{e}}\log_2(P_{\mathrm{e}})+(1-P_{\mathrm{e}})\log_2(1-P_{\mathrm{e}})\right]. \tag{17}$$

4. Simulation results and discussion

A Simulink simulation model based on a sign-function receiving structure is built, and the simulation parameters are set as follows: The frequency-hopping band is 30–50 MHz, the RF sampling rate is 192 MHz, the IF frequency is 1 kHz, and the IF sampling rate is 192 kHz.

4.1. Time and frequency domain waveform of sine signal enhanced by SR

The time-domain and time–frequency diagrams of a sine signal mixed with noise are shown in Figs. 4(a) and 4(b), respectively. The time-domain waveform after SR system enhancement is shown in Fig. 4(c), and the corresponding time–frequency diagram is shown in Fig. 4(d).

When the input SNR is −14 dB, the useful signal cannot be distinguished in the time-domain diagram (Fig. 4(a)), and in the time–frequency diagram (Fig. 4(b)), the signal energy is randomly distributed at each frequency point, which therefore does not reflect any characteristics of a 1 kHz sine signal. Following SR system enhancement, the signal in the time-domain diagram (Fig. 4(c)) appears to have an obvious 1kHz periodic component, and from the time–frequency diagram (Fig. 4(d)), it can be seen that the signal energy is concentrated in the low-frequency area and reaches a maximum at 1 kHz, which verifies the SR system's enhancement effect using a weak sine signal.

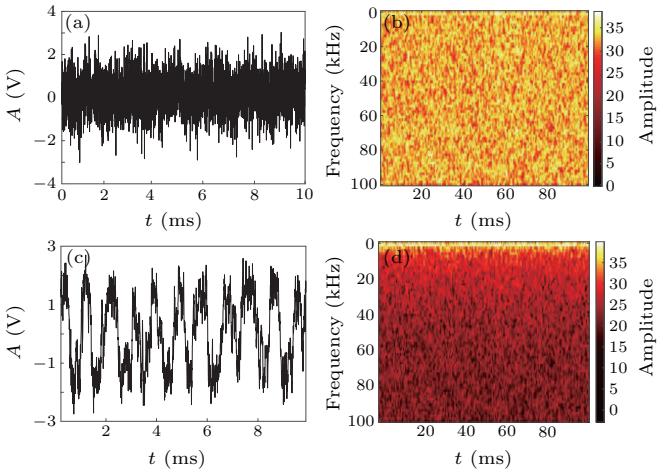


Fig. 4. Time and frequency-domain waveforms of the IF signal before and after SR enhancement (SNR = −14 dB, $a=1\times10^4$, $b=2.6\times10^{12}$). (a) Time-domain waveform of the input signal. (b) Time–frequency diagram of the input signal. (c) Time-domain waveform of the SR output signal. (d) Time–frequency diagram of the SR output signal.

4.2. The probability density of the SR output before and after processing by the sign function

The probability density of the SR output value is shown in Fig. 5(a). Both the theoretical analysis and simulation curves show that electromagnetic particles always tend to gather at the potential wells on both sides of the bistable system, and the addition of an IF signal increases the probability of particles gathering at the potential wells, resulting in the difference in signal probability density under the two assumptions. However, due to the periodicity of the sine signal and the randomness of the Gaussian noise, the probability density of the output signal under the two assumptions is symmetrical, and the difference is small, which is not conducive to distinguishing between the two assumptions.

After the sign-function processing, the probability distribution of the system output value $g(x_0)$ is shown in Fig. 5(b). At this time, the system's output probability density is no longer symmetrical under the H_1 assumption, while the system's output probability density is still symmetrical under the

H_0 assumption. This is because the driving force of the sine signal on the electromagnetic particles at the time of the wave peaks and troughs are positive and negative driving forces, respectively. After the sign-function processing of the SR output, the output value corresponding to the peak time remains unchanged, and the output value corresponding to the trough time is reversed. At this time, the overall output value of the system can be considered equivalent to one driven only by the peak of the sine signal, forcing the particles to gather at the positive-side potential well, with the result that the output probability density of the system is no longer symmetrical, thus increasing the difference of $\rho(x,t)$ under the two different assumptions and improving the detect ability of the sine signal.

In Fig. 5, there is a certain difference between the theoretical value and the simulation value; this is because the theoretical value is obtained under the assumption of the adiabatic approximation theory, and the transition time between the states of the system is not considered. But in fact, the bistable system is a dynamic system, so there may be a phase difference between the input signal and the signal enhanced by SR, which means that the sampling signal at the decision time may not be driven by the sine signal's extremum, which means that the theoretical and simulated values have a certain difference. The dynamic performance of a bistable system is related to the system parameters, the input's signal frequency, and the input's SNR. In practical engineering applications, we can follow certain adjustment strategies according to the actual situation to weaken or eliminate the impact of phase delay on detection performance.

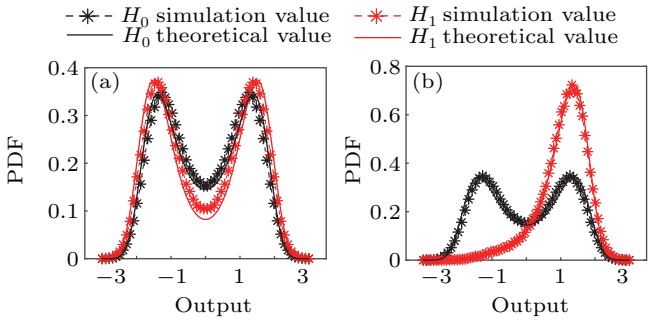


Fig. 5. The probability density of the SR output before and after processing by the sign function (SNR $= -14$ dB, $a = 1 \times 10^4$, $b = 2.6 \times 10^{12}$). (a) Probability density of the SR output value. (b) Probability density after sign-function processing.

4.3. System output with different decision points

The theoretical and simulated values of the probability density of the system output statistics Λ are shown in Fig. 6 for varying numbers of decision points N. The theoretical values are obtained according to Eq. (12), and the simulated values are the probability density of the system output statistics. It can be seen from the figure that when N is small, the probability density of the system output statistic Λ is quite different from that of the Gaussian distribution, because it does not meet the conditions of the central limit theorem; when $N \geq 50$, the probability density of the system output statistic Λ is in good agreement with the Gaussian distribution. Both the theoretical and simulated curves show that with an increase of N, the expected value of the system output remains unchanged and the variance decreases, which causes the system output more different under different assumptions; this is conducive to distinguishing between the two different assumptions.

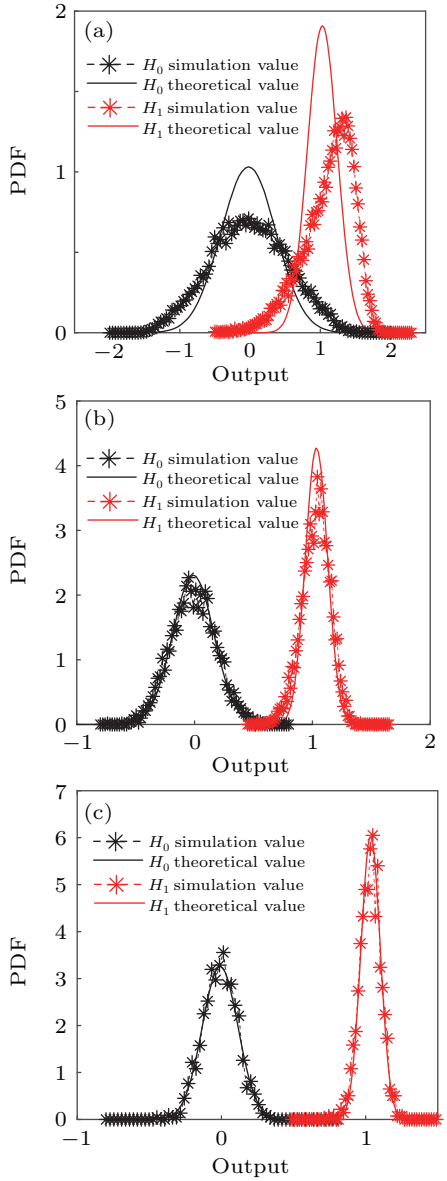


Fig. 6. Probability density of system output statistics at different N. (a) Probability distribution of statistics when $N = 10$. (b) Probability distribution of statistics when $N = 50$. (c) Probability distribution of statistics when $N = 100$.

4.4. System output BER under different conditions

The BERs of different receiving structures are shown in Fig. 7, in which the differences in different receiving structures is only the change in the method used to get the output value of the system, and the final test statistics for comparison with the threshold are obtained by averaging the output val-

ues of the system, which are recorded N times. It can be seen that the sign-function-based receiving structure has the lowest BER, followed by the quadratic-polynomial receiving structure, and the energy-detection structure has the highest BER, which indicates that the sign-function receiving structure has better performance than the other receiving structures.

This is because the traditional signal-detection method does not process the SR output values differently at different sampling times, and because of the half periodicity of the sine signal, the probabilities of the positive and negative forces acting on the electromagnetic particles are equal. In this case, if the output probability density of the system is directly counted, it is equivalent to averaging the effects of the forces on the particles at different times, resulting in a symmetrical distribution of the output probability density of the system and low signal-detection performance. The sign-function-based receiving structure not only considers the influence of the existence of a sine signal on the system's output, but also distinguishes the SR outputs at different times through the sign function, and makes equivalent constraints on the force directions of the electromagnetic particles, which increases the output difference of the system under different assumptions and is more conducive to signal detection.

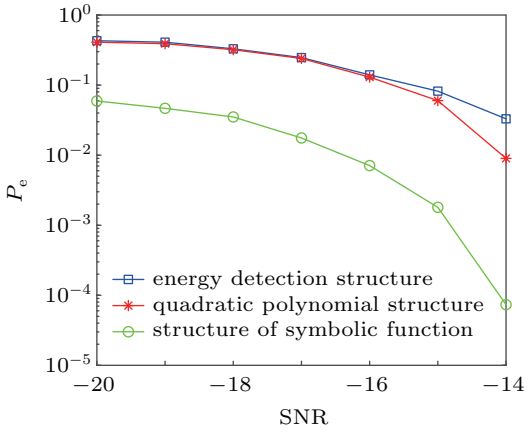


Fig. 7. System output BERs for different receiving structures.

System output BERs are shown for different numbers of sampling points in Fig. 8. It can be seen that with an increase in the sampling points N, the system BER becomes smaller and smaller. The reason for this is that when the SNR is determined, the expectation of the system's output value is also determined; however the number of decision points N determines the system's output variance. According to the central limit theorem, the output of the system obeys a Gaussian distribution, and its variance is σ^2/N. With an increase in N, the output variance of the system decreases. Under the different assumptions, the output separation of the system increases, and the detection performance also improves. When the sign-function receiving structure is adopted, because the decision result is determined by N mutually independent peak-and-trough sampling points, it requires that there are $N/2$ complete sine periodic signals in each symbol period. When the communication frequency and the sampling rate of the DSFH system are fixed, if we want to improve the detection performance, we need to increase the number of decision points N, which exchanges the system's effectiveness for reliability.

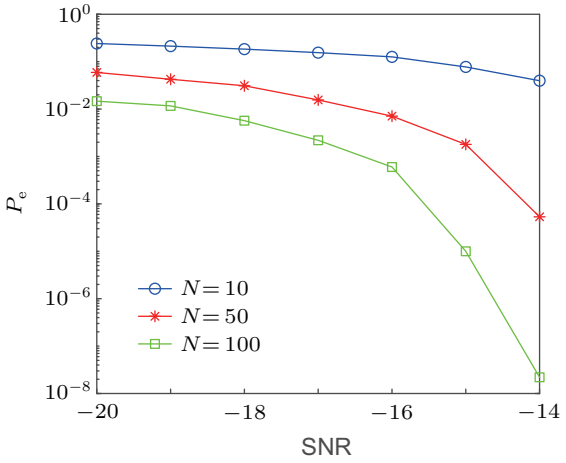


Fig. 8. BER of the sign-function receiving structure for different values of N.

5. Conclusion

In this paper, a SR signal-detection method based on the sign function is proposed, which improves the communication performance of DSFH at very low SNRs. By analyzing the kinetic response of the SR system to the action of a sine signal, the optimal decision time is determined, and a periodic stationary solution of the FPE with a time parameter is obtained. According to the kinetic characteristics of SR particles at different times, a receiving structure is designed based on the sign function, which transforms the sign of the SR output at decision time, so as to maximize the system output difference with or without a signal. The simulation results show that the receiving method based on the sign-function is more suitable for SR output signal detection than the traditional signal-detection method. When it is applied to the DSFH receiver, it can realize emergency communications using the DSFH system under very low SNR conditions.

References

[1] Fitzek and Frank H P 2006 *IEEE International Conference on Communications* Istanbul, Turkey, June 11–15, 2006 p. 5016
[2] Quan H D, Zhao H and Cui P Z 2015 *Wirel. Pers. Commun.* **81** 1159
[3] Xiang G X 2000 *IEEE Trans. Signal. Process.* **48** 3122
[4] Jang G J and Lee T W 2003 *J. Mach. Learn. Res.* **4** 1365
[5] Patel A and Kosko B 2011 *IEEE Trans. Signal. Process.* **59** 488
[6] Azar Y and Campisi M 2014 *International Conference on Advances in Computing, Communications and Informatics* Delhi, India, September 24–27, 2014 p. 1021
[7] Wang S and Wang F Z 2018 *Acta Phys. Sin.* **67** 160502 (in Chinese)
[8] Benzi R, Sutera A and Vulpiani A 1981 *J. Phys. A* **14** L453
[9] He D, Lin Y P, He C and Jiang L G 2010 *IEEE Trans. Veh. Technol.* **59** 1680

[10] Fu Y X, Kang Y M and Liu R N 2020 *IEEE Trans. Instrum. Meas.* **1** 99
[11] Mcnamara B and Wiesenfeld K 1989 *Phys. Rev. A* **39** 4854
[12] Xu B H, Duan F B, Bao R H and Li J L 2002 *Chaos. Soliton. Fract.* **13** 633
[13] Mitaim S and Kosko B 1998 *Proc. IEEE* **86** 2152
[14] Asdi A S and Tewfik A H 1995 *International Conference on Acoustics, Speech, and Signal Processing* Detroit, USA, May 9–12, 1995 p. 1332
[15] Galdi V, Pierro V and Pinto I M 1998 *Phys. Rev. E* **57** 6470
[16] Chen H and Varshney P K 2008 *IEEE Trans. Signal. Process.* **56** 5031
[17] Wang J, Ren X, Zhang S W, Zhang D M, Li H S and Li S Q 2014 *IEEE Trans. Wirel. Commun.* **13** 4014
[18] Liu J and Li Z 2015 *Iet. Commun.* **9** 101
[19] Liu G K, Kang Y M, Quan H D, Sun H X, Cui P Z and Guo C 2019 *Radioengineering.* **27** 618
[20] Liu G K, Quan H D, Kang Y M, Sun H X, Cui P Z and Han Y M 2019 *Acta Phys. Sin.* **68** 210501 (in Chinese)