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Adaptive synchronization of a class of fractional-order complex-valued chaotic neural network with time-delay*

Mei Li(李梅)[1,2], Ruo-Xun Zhang(张若洵)[3], and Shi-Ping Yang(杨世平)[1,†]

[1] College of Physics, Hebei Normal University, Shijiazhuang 050024, China
[2] Department of Computer Science, North China Electric Power University, Baoding 071003, China
[3] College of Primary Education, Xingtai University, Xingtai 054001, China

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This paper is concerned with the adaptive synchronization of fractional-order complex-valued chaotic neural networks (FOCVCNNs) with time-delay. The chaotic behaviors of a class of fractional-order complex-valued neural network are investigated. Meanwhile, based on the complex-valued inequalities of fractional-order derivatives and the stability theory of fractional-order complex-valued systems, a new adaptive controller and new complex-valued update laws are proposed to construct a synchronization control model for fractional-order complex-valued chaotic neural networks. Finally, the numerical simulation results are presented to illustrate the effectiveness of the developed synchronization scheme.

Keywords: adaptive synchronization, fractional calculus, complex-valued chaotic neural networks, time-delay

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1. Introduction

In recent years, the theory of fractional calculus and its application have become a most intense research topics. Compared with traditional integer calculus, fractional calculus and related models can better describe certain actual physical systems and capture their essential properties and behaviors. So far, it has generally become an efficient tool in physics, artificial intelligence, mathematical science, chemical engineering science, control engineering science, optimization combination, material science, electronic information, digital signal processing, cognitive science, and other fields.[1–6] Some interesting results about the dynamic behaviors of fractional-order systems have been derived.[7–16] Thus, the study of the theory and applications of nonlinear fractional-order differential systems are of high theoretical and significant practical importance.

Complex-valued neural networks (CNNs) are the progressive forms of real-valued neural networks (RNNs). Compared with RNNs, the CNNs have the advantages of simple network structure, concise training process, and strong ability to handle complex signals, which can be used to solve problems that cannot be solved by real-valued neural networks. Hence, it is widely used in physical systems dealing with electromagnetic, optical, ultrasonic, and quantum waves as well as in fields such as filtering, speech synthesis, and remote sensing. Owing to the memorability and heritability of fractionalorder models, fractional calculus is combined into complex-valued neural networks to form fractional-order complex-valued neural network models, which further expands the basic theory and application capability of neural networks.

Over the last few years, the dynamical behaviors, stability analysis and synchronous control of neural networks have become an area of particular interest and active research, and some excellent outcomes have been achieved.[17–28] For instance in Refs. [17,18], the projection synchronization and adaptive synchronization of fractional-order memristor neural networks were discussed. In Ref. [19], the synchronization of fractional-order complex-valued neural networks (FOCVNNs) was studied with linear delay feedback control. In Refs. [20–22] the global Mittag–Leffler synchronization of fractional-order neural networks was investigated. In Refs. [23–28], the the stability, finite-time stability, and global Mittag–Leffler stability of fractional-order time-delayed complex-valued neural networks were analyzed, respectively. Most of the literature in this area, however, was devoted to the study of stability analysis and periodic oscillations of various neural networks, only a few researches was involved with the study of fractional-order complex-valued chaotic neural networks (FOCVCNNs). However, the engineering applications of chaotic systems have been studied widely and intensively in recent years.[29–31] Furthermore, fractional-order complex-valued chaos can generate complex and unpredictable dynamic behaviors, which will enhance resistance to attacks, and achieve higher transmission efficiency in secure communications. This property extensively broadens the application of fractional-order neural networks. Owing to the limited speed of signal transmission between neurons, delays usually occur in neural networks. The time delays will

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†Corresponding author. E-mail: yangship@hebtu.edu.cn
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affect or even destroy the dynamic properties of neural networks. Nevertheless, to the best of our knowledge, there are very few studies on the chaotic dynamics of fractional-order complex-valued neural networks with time delays. Based on the above discussion, this paper is to investigate the chaos in FOCVCNNs and the synchronization problems of FOCVCNNs. Fractionalorder complex variable differential inequalities, stability theory and adaptive control methods for fractionalorder complex-valued systems are used in order to achieve the synchronization of FOCVCNNs.

2. Problem formulation and preliminaries

In this section, we first address some definitions and theorems about fractional calculus.

Definition 1[1] The fractional-order integral of a function f is defined as

$$I^{\alpha}f(t) = {}_{t_0}D_t^{-\alpha}f(t) = \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-\tau)^{\alpha-1}f(\tau)\,\mathrm{d}\tau, \quad (1)$$

where $t \geq t_0$, $\alpha > 0$.

Definition 2[1] The Caputo fractional-order differentiation of a real-variable function f is described by the following equation:

$${}_{t_0}^{\mathrm{C}}D_t^{\alpha}f(t) = \frac{1}{\Gamma(n-\alpha)}\int_{t_0}^{t}\frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}}\,\mathrm{d}\tau, \quad (2)$$

where $t \geq t_0$, n is a positive integer and $n-1 < \alpha < 1$.

Lemma 1[32] Assume $z \in \mathbb{C}$ to be a differentiable complex-valued function. If there are $t \geq t_0$ and $\alpha \in (0,1]$, the following inequality holds:

$${}_{t_0}^{\mathrm{C}}D_t^{\alpha}\overline{z(t)}z(t) \leq \overline{z(t)}\,{}_{t_0}^{\mathrm{C}}D_t^{\alpha}z(t) + \left({}_{t_0}^{\mathrm{C}}D_t^{\alpha}\overline{z(t)}\right)z(t). \quad (3)$$

Lemma 2[33,34] (stability theory for fractional-order systems) Assume $V_1(t)$ to be a uniformly continuous and derivable Lyapunov function and $V_2(t)$ is a derivable non-negative function.

If

$$V(t) = V_1(t) + V_2(t), \quad (4)$$

$${}_0^{\mathrm{C}}D_t^{\alpha}V(t) \leq -\theta V_1(t), \quad (5)$$

where θ is a positive constant, then $\lim_{t\to\infty} V_1(t) = 0$.

Lemma 3[35] Assume $V(t)$ and $U(t)$ are two nonnegative continuous functions, and satisfy ${}_0^{\mathrm{C}}D_t^{\alpha}(V(t)+U(t)) \leq -\sigma V(t) + \rho$ where $0 < \alpha < 1$, $\sigma > 0$ and $\rho > 0$, then $V(t) \leq (V(0)+U(0)-(\rho/\sigma))E_{\alpha,1}(-\sigma t^{\alpha})(\rho/\sigma)$.

Remark 1 Since $E_{\alpha,1}(-\sigma t^{\alpha}) \to 0 (\sigma > 0)$ and $t \to \infty$, if $\rho = 0$, we can derive Lemma 2 from this lemma.

Lemma 4[36] For any $\alpha, \beta \in \mathbb{C}$, and any $\delta > 0$, the following inequality will hold: $\alpha\bar{\beta} + \bar{\alpha}\beta \leq \delta\alpha\bar{\alpha} + (1/\delta)\beta\bar{\beta}$.

3. Main results

3.1. Model description

We consider a class of FOCVCNN described by the following equations:

$$\begin{aligned} {}_0^{\mathrm{C}}D_t^{\alpha}z_i(t) &= -c_i z_i(t) + \sum_{j=1}^{n} a_{ij}f_j(z_j(t)) \\ &\quad + \sum_{j=1}^{n} b_{ij}g_j(z_j(t-\tau)) + I_i(t), \\ z_i(s) &= \phi_i(s), \end{aligned} \quad (6)$$

where $t \geq 0, i = 1,2,\ldots,n$, $s \in [-\tau, 0]$, $0 < \alpha < 1$, $z_i(t) \in \mathbb{C}$ is the complex variable of the i-th neuron at time t, $f_j(\cdot), g_j(\cdot) \in \mathbb{C}$ denote the activation functions without and with timedelay, $a_{ij}, b_{ij} \in \mathbb{C}$ represent the connection weights and delayed connection weights respectively, $c_i \in \mathbb{R}, \tau > 0$ are the constant delay times, I_i denotes the corresponding external input. The complex-valued functions $f_j(\cdot)$, $g_j(\cdot)$ and I_i satisfy the following assumptions.

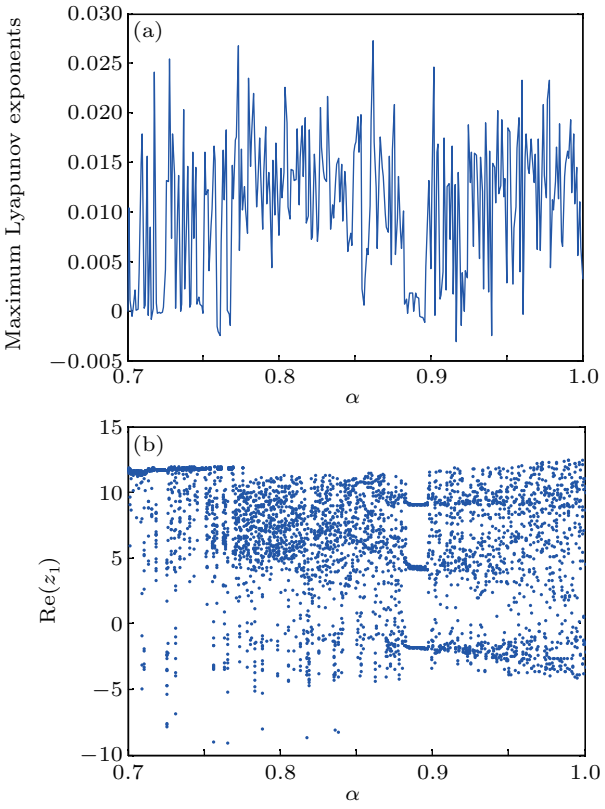


Fig. 1. Dynamic behaviors of FOCVNN (7), showing (a) maximal Lyapunov exponents and (b) bifurcation diagram.

Assumption 1 For any $\mu, \nu \in \mathbb{C}$, there exist real numbers $l_j, h_j > 0$ that satisfy the following inequalities:

$$|f_j(\mu) - f_j(\nu)| \leq l_j|\mu - \nu|,$$
$$|g_j(\mu) - g_j(\nu)| \leq h_j|\mu - \nu|.$$

Assumption 2 For any $v \in \mathbb{C}$, there exist real numbers $\gamma_i > 0$ and $\varepsilon_i > 0$ that satisfy the following inequalities $|f_j(v)| \leq \gamma_i$ and$|I_i(v)| \leq \varepsilon_i$.

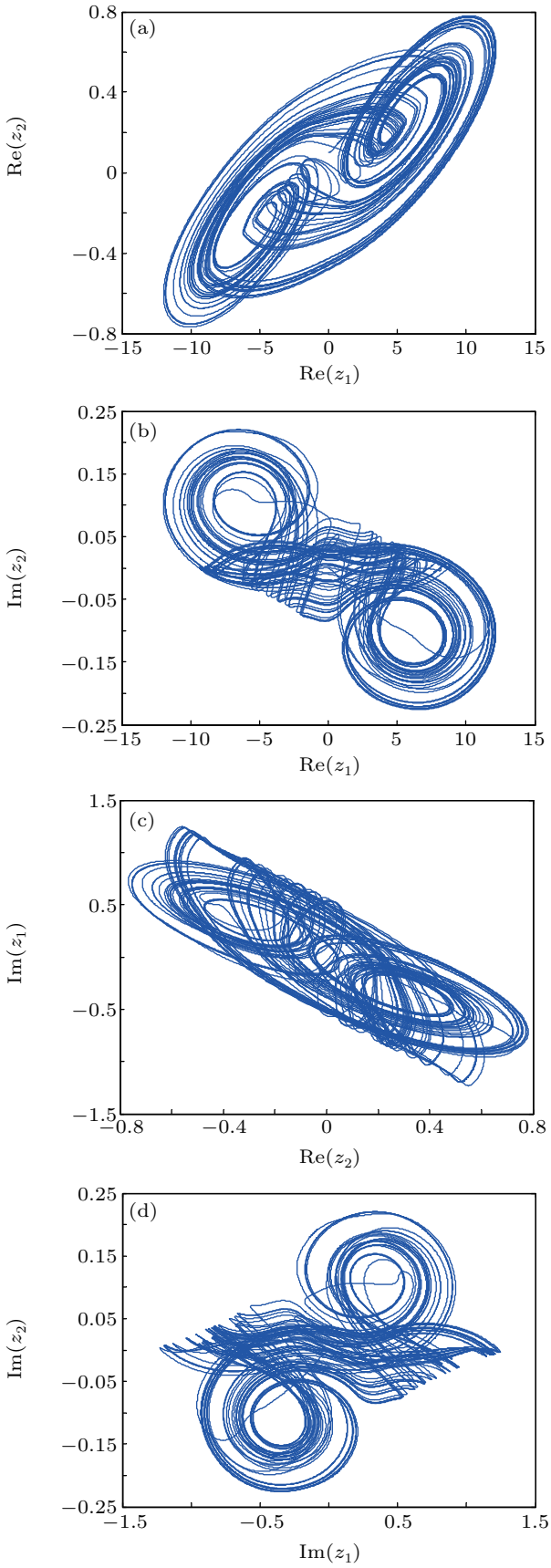


Fig. 2. Chaotic attractors of FOCVCNN (7), showing (a) Re(z_1)–Re(z_2), (b) Re(z_1)–Im(z_2), (c) Re(z_2)–Im(z_1), and (d) Im(z_1)–Im(z_2).

Equation system (6) can exhibit chaotic behaviors if appropriate network parameters and delay times are chosen. We consider the following FOCVCNN:

$$\begin{aligned}
{}_0^{\mathrm{C}}D_t^{\alpha} z_1(t) &= -c_1 z_1(t) + a_{11} f(z_1(t)) + a_{12} f(z_2(t)) \\
&\quad + b_{11} f(z_1(t-\tau)) + b_{12} f(z_2(t-\tau)), \\
{}_0^{\mathrm{C}}D_t^{\alpha} z_2(t) &= -c_2 z_2(t) + a_{21} f(z_1(t)) + a_{22} f(z_2(t)) \\
&\quad + b_{21} f(z_1(t-\tau)) + b_{22} f(z_2(t-\tau)). \qquad (7)
\end{aligned}$$

We choose $\tau = 1$,

$$\begin{aligned}
&a_{11} = 1 + \frac{\pi}{4} - 0.2\mathrm{i}, \quad a_{12} = 20 - 2\mathrm{i}, \\
&a_{21} = 0.1 - 0.1\mathrm{i}, \quad a_{22} = 1 + \frac{\pi}{4} - 0.2\mathrm{i}, \\
&b_{11} = -1.5 + 0.5\mathrm{i}, \quad b_{12} = 0.1 - 0.1\mathrm{i}, \\
&b_{21} = 0.1 - 0.1\mathrm{i}, \quad b_{22} = -1.5 + 0.5\mathrm{i}, \\
&f(z) = \frac{1}{2}(|z+1| - |z-1|).
\end{aligned}$$

The maximum Lyapunov exponent (MLE) spectrum is shown in Fig. 1(a), and the bifurcation diagram is presented in Fig. 1(b). From Figs. 1(a) and 1(b), it can be seen that equation system (7) is chaotic when the fractionalorder is at some specific values. Figure 2 shows the chaotic attractor trajectory of each complex state quantity of system (7) when $\alpha = 0.96$, which also verifies the correctness of the above conclusion.

3.2. Adaptive synchronization of fractional-order complex-valued chaotic neural networks

System (6) is chosen as a master system and the controlled response system is described by

$$\begin{aligned}
{}_0^{\mathrm{C}}D_t^{\alpha} \boldsymbol{w}_i(t) &= -c_i \boldsymbol{w}_i(t) + \sum_{j=1}^{n} \alpha_{ij} f_j(\boldsymbol{w}_j(t)) \\
&\quad + \sum_{j=1}^{n} \beta_{ij} g_j(\boldsymbol{w}_j(t-\tau)) + I_i(t) + u_i(t), \\
w_i(s) &= \phi_i(s), \qquad (8)
\end{aligned}$$

where $t \geq 0, i = 1, 2, \ldots, n$, $s \in [-\tau, 0]$, $w_i(t) \in \mathbb{C}$ is the complex variable of the i-th neuron of the response system, and $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^{\mathrm{T}}$ is the controller to be determined.

The synchronization error between master system (6) and slave system (8) can be written as $e_i(t) = w_i(t) - z_i(t)$. The fractional-order error dynamic system is determined from the following equation:

$$\begin{aligned}
{}_0^{\mathrm{C}}D_t^{\alpha} \boldsymbol{e}_i(t) &= -c_i \boldsymbol{e}_i(t) + \sum_{j=1}^{n} [a_{ij} f_j(\boldsymbol{w}_j(t)) - a_{ij} f_j(\boldsymbol{z}_j(t))] \\
&\quad + \sum_{j=1}^{n} [b_{ij} g_j(\boldsymbol{w}_j(t-\tau)) - b_{ij} g_j(\boldsymbol{z}_j(t-\tau))] \\
&\quad + u_i(t). \qquad (9)
\end{aligned}$$

Theorem 1 If Assumptions 1 and 2 hold, the asymptotic synchronization of systems (6) and (8) can be implemented under the following adaptive controller:

$$u_i(t) = -k_i(t) e_i(t) - m_i \frac{e_i(t)}{\overline{e_i(t)}\, e_i(t)} \overline{e_i(t-\tau)} e_i(t-\tau), \quad (10)$$

where $i = 1, 2, \ldots, n$.

The adaptive laws are as follows:

$$ {}_0^{\mathrm{C}}D_t^{\alpha} k_i = \sigma_i \overline{e_i(t)} e_i(t), \tag{11} $$

$$ {}_0^{\mathrm{C}}D_t^{\alpha} m_i = \varepsilon_i \overline{e_i(t-\tau)} e_i(t-\tau), \tag{12} $$

where σ_i and ε_i are the positive constants.

Proof The Lyapunov candidate functions can be presented by

$$ V_1(t) = \sum_{i=1}^{n} \bar{e}_i(t) e_i(t) = \mathrm{e}^{\mathrm{H}}(t) e(t), $$

$$ V_2(t) = \sum_{i=1}^{n} \left[\frac{1}{\sigma_i} (k_i - k_i^*)^2 + \frac{1}{\varepsilon_i} (m_i - m_i^*)^2 \right], $$

where k_i^* and m_i^* are two positive constants to be determined.

According to Lemma 1, we have

$$ \begin{aligned} {}_0^{\mathrm{C}}D_t^{\alpha}(V_1(t)+V_2(t)) &= {}_0^{\mathrm{C}}D_t^{\alpha} \sum_{i=1}^{n} \bar{e}_i(t) e_i(t) + {}_0^{\mathrm{C}}D_t^{\alpha} \sum_{i=1}^{n} \left[\frac{1}{\sigma_i} (k_i - k_i^*)^2 + \frac{1}{\varepsilon_i} (m_i - m_i^*)^2 \right] \\ &\le \sum_{i=1}^{n} \bar{e}_i(t) {}_0^{\mathrm{C}}D_t^{\alpha} e_i(t) + \sum_{i=1}^{n} e_i(t) \overline{{}_0^{\mathrm{C}}D_t^{\alpha} e_i(t)} + \sum_{i=1}^{n} \left[\frac{2}{\sigma_i} (k_i - k_i^*) {}_0^{\mathrm{C}}D_t^{\alpha} k_i + \frac{2}{\varepsilon_i} (m_i - m_i^*) {}_0^{\mathrm{C}}D_t^{\alpha} m_i \right]. \end{aligned} $$

From Lemma 4 and Assumption 1 it follows that

$$ \begin{aligned} {}_0^{\mathrm{C}}D_t^{\alpha}(V_1(t)+V_2(t)) &\le -\sum_{i=1}^{n} 2c_i \bar{e}_i(t) e_i(t) + \sum_{i=1}^{n} \sum_{j=1}^{n} \left[(a_{ij}\bar{a}_{ij} + l_j^2) \bar{e}_i(t) e_i(t) + b_{ij}\bar{b}_{ij} \bar{e}_i(t) e_i(t) + h_j^2 \overline{e_i(t-\tau)} e_i(t-\tau) \right] \\ &\quad - \sum_{i=1}^{n} 2 \left[k_i^* \bar{e}(t)_i e_i(t) - m_i^* \overline{e_i(t-\tau)} e_i(t-\tau) \right] \\ &= \sum_{i=1}^{n} \left[-2c_i - 2k_i^* + \sum_{j=1}^{n} (a_{ij}\bar{a}_{ij} + l_j^2 + b_{ij}\bar{b}_{ij}) \right] \bar{e}_i(t) e_i(t) + \sum_{i=1}^{n} \left(\sum_{j=1}^{n} h_j^2 - 2m_i^* \right) \overline{e_i(t-\tau)} e_i(t-\tau). \end{aligned} $$

Let

$$ k_i^* = \frac{1}{2} \sum_{j=1}^{n} \left(a_{ij}\bar{a}_{ij} + l_j^2 + b_{ij}\bar{b}_{ij} \right) - c_i + 1, $$

$$ m_i^* = \frac{1}{2} \sum_{j=1}^{n} h_j^2, $$

we have the following inequality ${}_0^{\mathrm{C}}D_t^{\alpha}(V_1(t)+V_2(t)) \le -V_1(t)$.

According to Lemma 2 or Lemma 3, we can obtain cc $\lim_{t\to\infty} V_1(t) = \lim_{t\to\infty} e^{\mathrm{H}}(t) e(t) = 0$, which indicates $\lim_{t\to\infty} e(t) = 0$ that shows that the synchronization between systems (6) and (8) can be achieved.

Remark 2 According to previous researches in Refs. [27, 37,38], a common approach to analyzing the fractional-order complex-valued neural networks is to decompose the complex-valued neural network into two fractional-order real-valued neural networks according to their real part and imaginary part, and then obtain a criterion about synchronization by investigating these real-valued neural networks. Unfortunately, there are two disadvantages in this approach. First, the dimensionality of the real-valued neural network is twice that of the complex-valued neural network, thereby increasing the complexity of computation and analysis. Moreover, not all complex functions can be divided into real part and imaginary part in practical applications. Unlike previous researches, the whole analysis process is performed in the complex-valued domain in our proposed method. The synchronization conditions are derived by using the complex function theory, without dividing the original complex-valued neural network into two real-valued networks, which effectively reduces the complexity of analysis and computation. In addition, the proposed method can be applied to other complex-valued systems, such as fractional-order complex-valued chaotic systems.

Remark 3 Compared with feedback control, adaptive control in this paper can automatically adjust the certain parameters of system to accomplish synchronization, thereby greatly reducing the control cost.

4. Numerical simulations

In this section, numerical simulations of fractional-order complex-valued chaotic neural network (FOCVCNNs) synchronization is provided in order to demonstrate the validity of the previously proposed scheme. We use the prediction-correction method to numerically solve these problems.[38] The FOCVCNN (7) is selected as the driving system, and the controlled response system is described as follows:

$$ \begin{aligned} {}_0^{\mathrm{C}}D_t^{\alpha} \boldsymbol{w}_1(t) &= -c_1 \boldsymbol{w}_1(t) + a_{11} f(\boldsymbol{w}_1(t)) + a_{12} f(\boldsymbol{w}_2(t)) \\ &\quad + b_{11} f(\boldsymbol{w}_1(t-\tau)) + b_{12} f(\boldsymbol{w}_2(t-\tau)) + u_1(t), \\ {}_0^{\mathrm{C}}D_t^{\alpha} \boldsymbol{w}_2(t) &= -c_2 \boldsymbol{w}_2(t) + a_{21} f(\boldsymbol{w}_1(t)) + a_{22} f(\boldsymbol{w}_2(t)) \\ &\quad + b_{21} f(\boldsymbol{w}_1(t-\tau)) + b_{22} f(\boldsymbol{w}_2(t-\tau)) \\ &\quad + u_2(t). \end{aligned} \tag{13} $$

Let the synchronization error be $e_i(t) = w_i(t) - z_i(t)$, $(i = 1, 2)$ and according to Theorem 1, the controller is se-

lected as

$$u_i(t) = -k_i(t)e_i(t) - m_i\frac{e_i(t)}{\overline{e_i(t)}e_i(t)}\overline{e_i(t-\tau)}e_i(t-\tau), \quad i = 1,2.$$

The adaptive laws are shown below:

$$\begin{aligned} {}_0^{\mathrm{C}}D_t^{\alpha}k_i &= \sigma_i\overline{e_i(t)}e_i(t), \\ {}_0^{\mathrm{C}}D_t^{\alpha}m_i &= \varepsilon_i\overline{e_i(t-\tau)}e_i(t-\tau). \end{aligned}$$

Set the delay $\tau = 1$, and the connection weights will be as follows:

$$\begin{aligned} a_{11} &= 1+\frac{\pi}{4}-0.2\mathrm{i}, \quad a_{12} = 20-2\mathrm{i}, \\ a_{21} &= 0.1-0.1\mathrm{i}, \quad a_{22} = 1+\frac{\pi}{4}-0.2\mathrm{i}. \end{aligned}$$

Also, set the delayed connection weights to be below:

$$\begin{aligned} b_{11} &= -1.5+0.5\mathrm{i}, \quad b_{12} = 0.1-0.1\mathrm{i}, \\ b_{21} &= 0.1-0.1\mathrm{i}, \quad b_{22} = -1.5+0.5\mathrm{i}. \end{aligned}$$

The activation function is

$$f(z) = \frac{|z+1|-|z-1|}{2},$$

and the fractional-order is $\alpha = 0.9$. The initial conditions are set to be as follows:

$$\begin{aligned} &[z_1(s), z_2(s)]^{\mathrm{T}} = [0.01+0.01i, 0.1-0.1i]^{\mathrm{T}}, \\ &[w_1(s), w_2(s)]^{\mathrm{T}} = [0.1i, 0.1]^{\mathrm{T}}, \\ &\forall s \in [-1,0], \quad k_i(0) = m_i(0) = 0.1, \quad \sigma_i = \varepsilon_i = 1, \quad i = 1,2. \end{aligned}$$

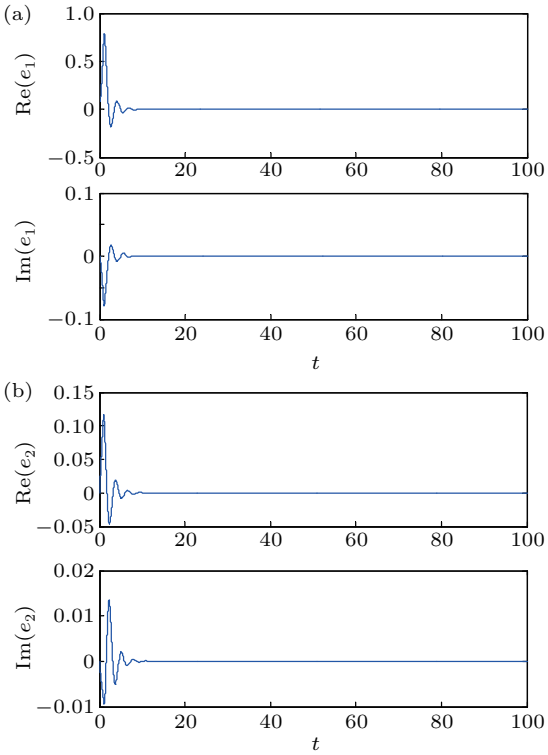


Fig. 3. Synchronization errors of fractional-order complex-valued chaotic neural networks with control parameters $\sigma_i = \varepsilon_i = 6$ $(i = 1,2)$ for (a) Re(e_1) and Im(e_1), and (b) Re(e_2) and Im(e_2).

The numerical results are depicted in Fig. 3, indicating that error dynamic system (9) converges to zero. The drive system (7) and response system (13) can be asymptotically synchronized under the action of the adaptive controller (10). The simulation results confirm the effectiveness of the proposed scheme.

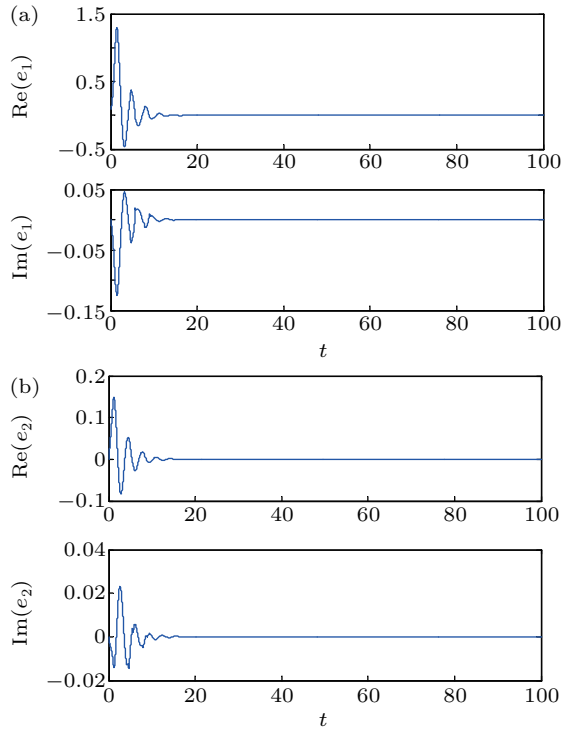


Fig. 4. Synchronization errors of fractional-order complex-valued chaotic neural networks with control parameters $\sigma_i = \varepsilon_i = 1$ $(i = 1,2)$ for (a) Re(e_1) and Im(e_1), and (b) Re(e_2) and Im(e_2).

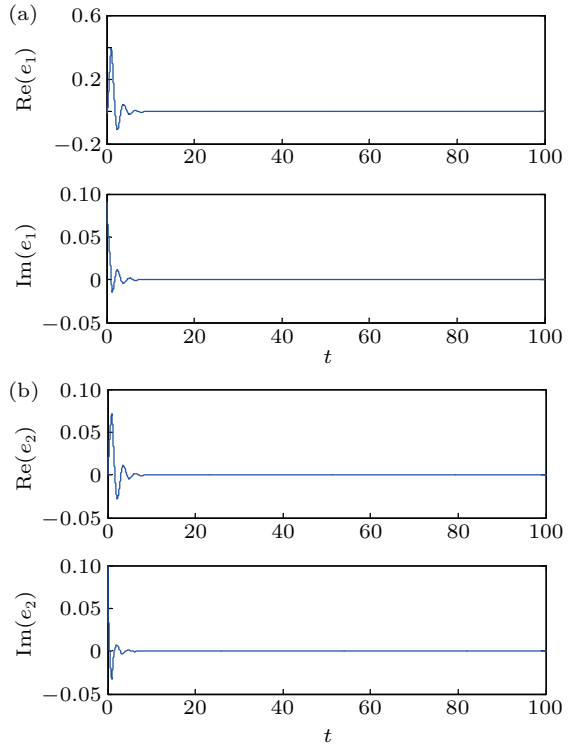


Fig. 5. Synchronization errors of fractional-order complex-valued chaotic neural networks with control parameters $\sigma_i = \varepsilon_i = 20$ $(i = 1,2)$ $\sigma_i = \varepsilon_i = 6$ $(i = 1,2)$ for (a) Re(e_1) and Im(e_1), and (b) Re(e_2) and Im(e_2).

In order to investigate the synchronization performance of the proposed method, numerical simulations are conducted by setting the control parameters to beof $\sigma_i = \varepsilon_i = 1$ $(i = 1,2)$ and $\sigma_i = \varepsilon_i = 10$ $(i = 1,2)$, respectively. The synchronization results are shown in Figs. 4 and 5, indicating that the increasing of the control parameters can speed up the synchronization of system, but the synchronization parameters should not be too large or too small. In addition, through the simulation, we find that when the control parameters are set to be $\varepsilon_i = 0$, the two systems can still achieve synchronization. We will further attests to this in our future research work.

5. Conclusion

In this paper, the synchronization problem of fractional-order complex-valued chaotic neural networks is discussed. The chaotic behaviors of a class of fractional-order complex-valued neural network are investigated, and the synchronization of FOCVCNNs is accomplished in the complex domain by an adaptive controller. The effectiveness of the proposed scheme is further confirmed by numerical simulations. In the future research, we will study the synchronization problem of fractional-order complex-valued chaotic neural networks with external disturbances and unknown parameters. We will also further investigate how to implement this synchronization scheme with actual circuits and how to apply it to the secure communication.

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