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Fine-grained uncertainty relation for open quantum system*

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The fine-grained uncertainty relation (FUR) is investigated for accelerating open quantum system, which manifests the celebrated Unruh effect, a crucial piece of the jigsaw for combining relativity and quantum physics. For a single detector, we show that the inevitable Unruh decoherence can induce a smaller FUR uncertainty bound, which indicates an additional measurement uncertainty may exist. For an open system combined with two detectors, via a nonlocal retrieval game, the related FUR uncertainty bound is determined by the non-classical correlation of the system. By estimating the maximal violation of Bell inequality for an accelerating system, we show that the FUR uncertainty bound can be protected from Unruh decoherence, due to quantum correlation generated through Markovian dynamics.

Keywords: open quantum system, fine-grained uncertainty relation, Unruh effect

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1. Introduction

Since it was originally proposed by Heisenberg in 1927,[1] the uncertainty principle has been a subject of recurring interest for over ninety years. The principle bounds our ability to simultaneously predict two incompatible observables of a quantum particle, thus distinguishing the quantum world from its classical complement. Nowadays, it has been recast, in an information-theoretical framework, with the uncertainty quantified by entropic measures,[2–4] and has many important applications in entanglement witnesses,[5–9] quantum metrology,[10] quantum cryptographic protocols[11,12] (see Ref. [13] for a recent review).

However, even entropic formula of uncertainty principle is fairly useful in various cases, it is still a rather coarse way of measuring the uncertainty of a set of measurements. In particular, one cannot distinguish the uncertainty inherent in obtaining any combination of outcomes for different measurements, since an entropic function can only encode the probability distribution of outcomes as a whole. To overcome this defect, Oppenheim and Wehner proposed a new form of uncertainty relation, *i.e.*, a fine-grained uncertainty relation (FUR).[14] For a set of measurements $T=\{t\}$, associating with every combination of possible outcomes $\boldsymbol{x}=(x^{(1)},\ldots,x^{(n)})$, there exist a set of inequalities

$$\left\{\mathcal{P}(\rho;\boldsymbol{x})\equiv\sum_{t=1}^{n}p(t)p(x^{(t)}|\rho)\leq\zeta_{\boldsymbol{x}}\middle|\boldsymbol{x}\in\boldsymbol{B}^{\times n}\right\},\qquad(1)$$

where $\boldsymbol{B}^{\times n}$ is the set involving all possible combinations of outcomes, $p(t)$ is the probability of choosing a particular measurement, and $p(x^{(t)}|\rho)$ is the probability that one obtains the outcome $x^{(t)}$ after performing measurement t on the state ρ. To measure the uncertainty, the maximum in function $\zeta_{\boldsymbol{x}}=\max_{\rho}\sum_{t=1}^{n}p(t)p(x^{(t)}|\rho)$ should be evaluated over all states allowed on a particular system. It is easy to tell that one cannot obtain outcomes with certainty for all measurements simultaneously when $\zeta_{\boldsymbol{x}}<1$. Like entropic formula, the FUR can also be used as an entanglement witness in practice.[15]

On the other hand, FUR becomes an important link part between various disparate subfields. For instance, in their original work,[14] Oppenheim and Wehner showed that FUR can determine bipartite quantum nonlocality through a nonlocal retrieval game, and further discriminate among classical, quantum, and superquantum correlations involved in a multipartite system.[16,17] Based on FUR, a steering inequality has been given to capture bipartite Einstein–Podolsky–Rosen steering from both theoretical and experimental views.[18,19] Similar to entropic uncertainty, a quantum-memory-assisted FUR is proposed to improve the lower bound of inequality, which may benefit quantum cryptography.[20] Moreover, a profound link between the FUR and thermodynamics has been found,[21,22] which claims that a violation of uncertainty relation implies a violation of second thermodynamical law.

While most studies focus on closed systems, a complete account of FURs requires understanding them for open quantum systems, due to inevitable decoherence or dissipation from the interaction between systems and environment in any realistic regime. In particular, such environment decoherence limits the ability and task quality in various quantum computation protocols. In recent years, particular interests have been attracted for an accelerating quantum system through an external

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field.[23–25] Even when the environment in its vacuum state, it is known[26] that quantum system can be spontaneously excited as if it was in a thermal bath with temperature $T = a/2\pi$, proportional to its proper acceleration. This celebrated Unruh effect is a key piece of the jigsaw for combining both relativity and quantum physics,[27] which provides a strong motivation to measure it and related phenomena in the laboratory. However, given the difficulty of direct observation (one needs an acceleration of about 10^{23} m/s^2 to reach room temperature), alternative approaches using cutting-edge quantum technology have been developed. For instance, with significant influence of Unruh decoherence on practical quantum information tasks, it may be possible to measure indirectly the Unruh effect on localized quantum systems, which in principle can be measured, transformed and exploited for quantum information tasks.[28,29] In particular, for the localized detector as an open system, its Berry phase[30] and entanglement[31–33] depend sensitively on Unruh temperature, thus have been suggested to be good probes on Unruh effect. For qubits restricted in moving cavity, entropic uncertainty bound[34] and FUR[35] as the entanglement witnesses were also investigated, showed a significant modification attributed to the (relativistic) motion status of system may happen. On the other hand, it was suggested to employ a quantum simulator to recreate a synthetic Unruh effect in the laboratory by Bose–Einstein condensate (BEC),[36] NMR,[37] and superconducting circuits,[38] *etc*.

In this paper, we explore FUR for open quantum system under relativistic motion, and find the uncertainty bound does depend on the motion state of the system. We work in the detector-field picture and study a localized system composed of multi-Unruh–De Witt detectors (UDD) in Minkowski space, each modeled by an accelerating two-level atom. Assuming a weakly interaction with a bath of fluctuating scalar field, the detectors behave like an open quantum system, suffering from an environment decoherence attributed to thermality of Rindler vacua, a manifestation of the Unruh effect. Without loss of generality, we particularly focus on two typical scenarios with single-UDD and two-UDD respectively. For a single detector, the Unruh effect is manifested by purely environment decoherence, which forces the state of detector into approaching an equilibrium thermal state. Once estimating the FUR uncertainty bound ζ for the single detector, we show an increased uncertainty in quantum measurement would be induced, which is a direct result of Unruh decoherence. On the other hand, for a multi-detectors system, it was known[23] that the quantum correlation could be generated through its Markovian evolution. With this novel feature of the multipartite open system, the Unruh decoherence may be overcome in certain conditions. To demonstrate it, we particularly study the FUR for a two-UDD system and find that the uncertainty bound can be protected from the Unruh effect, due to the intrinsic correlation generated under the relativistic motion. For a general case with n-UDD ($n > 2$), one may expect that a similar reduced Unruh decoherence could also exist,[39] as an n-UDD system shares the same collective correlation dynamics like a bipartite system but can only be calculated numerically.

The paper is organized as follows. In Section 2, we solve the master equation of multi-UDD system, and give explicitly its density matrix. In Section 3, we evaluate the FUR for single-UDD and two-UDD system respectively, and show that how the uncertainty bound of FUR can be modified by Unruh effect. In Section 4, the summary and discussion are given. Throughout the paper, we use natural units with $G = c = \hbar = k_{\rm B} = 1$.

2. Master equation of accelerating qubits

To proceed, we first recall the full dynamics of multi-UDD state in flat spacetime, which effectively is governed by a Lindblad master equation. Without loss of generality, the total Hamiltonian of the combined system (detectors + environment) is

$$H_{\rm tot} = H_{n\text{-atoms}} + H_\Phi + H'_n. \tag{2}$$

Here $H_{n\text{-atoms}}$ is the Hamiltonian of n-UDD in the common comoving frame, each modeled by a two-level atom with energy level spacing ω. H_Φ is the standard Hamiltonian of free massless scalar fields $\Phi(x)$, whose details do not need to be specified in our approach.[23] For later convenience, we particularly consider $n = 1, 2$ cases. Since each atom internal dynamics is driven by a 2×2 matrix, we choose Hamiltonian of UDDs in a concise form as

$$H_{1\text{-atoms}} = \frac{\omega}{2}\sigma_3, \quad H_{2\text{-atoms}} = \frac{\omega}{2}\Sigma_3, \tag{3}$$

where the symmetrized bipartite operators $\Sigma_3 \equiv \sigma_3^{(A)}\otimes \mathbf{1}^{(B)} + \mathbf{1}^{(A)}\otimes\sigma_3^{(B)}$ is defined by Pauli matrices $\sigma_3^{(\alpha)}$ and identical matrix $\mathbf{1}^{(\alpha)}$, with superscript $\alpha = \{A, B\}$ labeling distinct atoms. We assume the UDD couples to the massless scalar field, through an 1-atom interaction $H'_1 = \eta\sigma_3\Phi$ or 2-atoms interaction with dipole form[40] $H'_2 = \eta\Big[(\sigma_2^{(A)}\otimes\mathbf{1}^{(B)})\Phi(t, \boldsymbol{x}^{(A)}) + (\mathbf{1}^{(A)}\otimes\sigma_2^{(B)})\Phi(t, \boldsymbol{x}^{(B)})\Big]$, where η is a small dimensionless coupling constant.

To derive the dynamics of detectors' state, a separable initial state of total system is assumed, *i.e.*, $\rho_{\rm tot} = \rho_{n\text{-atoms}}(0)\otimes|0\rangle\langle 0|$, where $|0\rangle$ is the vacuum state of field $\Phi(x)$. Then, in a weak coupling limit, the Markovian dynamics of detectors' density matrix can be induced from $\rho_{\rm tot}(t)$, by tracing over all field degrees of freedom, *i.e.*, $\rho_{n\text{-atoms}}(t) = {\rm Tr}_\Phi\rho_{\rm tot}(t)$. This just means that in the limit of weak couplings, the changes in the evolution of the system occur on very large time-scales, that the details of the internal environment dynamics of the scalar field Φ become irrelevant. The detectors' density matrix now satisfies a master equation in Kossakowski–Lindblad

form[41,42]

$$\frac{\partial \rho_{n\text{-atoms}}(t)}{\partial t} = -\mathrm{i}[H_{\text{eff}}, \rho_{n\text{-atoms}}(t)] + \mathcal{L}[\rho_{n\text{-atoms}}(t)]. \quad (4)$$

Here $\mathcal{L}[\rho_{n\text{-atoms}}]$ is the decoherence term produced by the coupling with external fields. For $n = 1,2$, it reads respectively as

$$\begin{aligned}\mathcal{L}[\rho_{1\text{-atoms}}] &= \frac{1}{2}\sum_{i,j=1,2,3} C_{ij}\left[2\sigma_j\rho_{1\text{-atoms}}\sigma_i - \{\sigma_i\sigma_j, \rho_{1\text{-atoms}}\}\right],\\ \mathcal{L}[\rho_{2\text{-atoms}}] &= \frac{1}{2}\sum_{\substack{i,j=1,2,3\\ \alpha,\beta=A,B}} C_{ij}^{(\alpha\beta)}\left[2\sigma_j^{(\beta)}\rho_{2\text{-atoms}}\sigma_i^{(\alpha)}\right.\\ &\quad \left. - \{\sigma_i^{(\alpha)}\sigma_j^{(\beta)}, \rho_{2\text{-atoms}}\}\right], \end{aligned} \quad (5)$$

where the Kossakowski matrices are denoted as $C_{ij}^{(\bullet)}$, with $(\bullet)$ reads as $(\alpha\beta)$ for $n = 2$. Moreover, since $\rho_{2\text{-atoms}}$ now lives in a 4-dimensional Hilbert space which is a tensor product of single atom's state space, we use the superscript $\alpha = \{A, B\}$ to specify the subspace that Pauli matrices $\sigma_i^{(\alpha)}$ live.[43] They are determined by Fourier transforms of the Wightman functions of scalar field $G^{(\bullet)}(x-y) = \langle 0|\Phi(x)\Phi(y)|0\rangle$, like $\mathcal{G}^{(\bullet)}(\omega) = \int_{-\infty}^{\infty} \mathrm{d}\Delta t\, \mathrm{e}^{\mathrm{i}\omega\Delta t} G^{(\bullet)}(\Delta t)$, and can be written explicitly as

$$C_{ij}^{(\bullet)} = \Gamma_+^{(\bullet)}\delta_{ij} - \mathrm{i}\Gamma_-^{(\bullet)}\varepsilon_{ijk}\delta_{3k} + \Gamma_0^{(\bullet)}\delta_{3i}\delta_{3j} \quad (6)$$

with

$$\Gamma_\pm^{(\bullet)} = \frac{1}{2}\left[\mathcal{G}^{(\bullet)}(\omega) \pm \mathcal{G}^{(\bullet)}(-\omega)\right], \quad \Gamma_0^{(\bullet)} = \mathcal{G}^{(\bullet)}(0) - \Gamma_+^{(\bullet)}. \quad (7)$$

Let us express the density matrix $\rho_{n\text{-atoms}}$ in Bloch form, *e.g.*, for single atom, we have

$$\rho_{1\text{-atoms}}(t) = \frac{1}{2}\left(1 + \sum_{i=1}^{3} r_i(t)\sigma_i\right). \quad (8)$$

Substituting Eq. (8) into the master equation (4) and assuming that the initial state of the atom is $|\psi(0)\rangle = \cos(\theta/2)|+\rangle + \sin(\theta/2)|-\rangle$, one can easily work out the time-dependent reduced density matrix

$$\rho_{1\text{-atoms}}(t) = \frac{1}{2}\begin{pmatrix} 1 + \mathrm{e}^{-4t\Gamma_+}(R+\sin\theta) - R & \mathrm{e}^{-2t\Gamma_+ - \mathrm{i}\Omega t}\cos\theta \\ \mathrm{e}^{-2t\Gamma_+ + \mathrm{i}\Omega t}\cos\theta & 1 - \mathrm{e}^{-4t\Gamma_+}(R+\sin\theta) + R \end{pmatrix}, \quad (9)$$

where a ratio $R \equiv \Gamma_-/\Gamma_+$ is defined, and frequency ω is renormalized to Ω by Lamb shift.[23]

For a two-atoms system, the Bloch representation of its density matrix becomes

$$\rho_{2\text{-atoms}}(t) = \frac{1}{4}\left(\mathbf{1}^{(A)}\otimes\mathbf{1}^{(B)} + \sum_{i=1}^{3}\tilde{r}_i(t)\Sigma_i + \sum_{i,j=1}^{3}\tilde{r}_{ij}(t)\sigma_i^{(A)}\otimes\sigma_j^{(B)}\right). \quad (10)$$

Just like before, one can substitute the above equation into Eq. (4), and obtain cumbersome time-dependent solutions by exactly integration. Nevertheless, what really attracts us in the two-atoms case is the possible quantum correlation generation during its Markovian evolution,[43] as well as its implication in evaluating FUR bound. Therefore, we can simply investigate the final equilibrium state of the two-atoms system. It was shown[23] that there always exists a proper interatomic separation L, below which the generated correlation can persist asymptotically in final equilibrium states under environment dissipation. In such a case, all the Kossakowski matrices become equal $\{C_{ij}^{(\bullet)}|C_{ij}^{AA} = C_{ij}^{BB} = C_{ij}^{AB} = C_{ij}^{BA} \equiv C_{ij}\}$. By resolving the master equation (4), the final reduced density matrix of two-atom at asymptotic equilibrium is[31,44]

$$\rho_{2\text{-atoms}} = \begin{pmatrix} \frac{(3+\tau)(R-1)^2}{4(3+R^2)} & 0 & 0 & 0 \\ 0 & \frac{3-\tau-(\tau+1)R^2}{4(3+R^2)} & \frac{2(\tau-R^2)}{4(3+R^2)} & 0 \\ 0 & \frac{2(\tau-R^2)}{4(3+R^2)} & \frac{3-\tau-(\tau+1)R^2}{4(3+R^2)} & 0 \\ 0 & 0 & 0 & \frac{(3+\tau)(R+1)^2}{4(3+R^2)} \end{pmatrix}. \quad (11)$$

One can observe that the final equilibrium state of 2-atoms system now depends on the ratio $R = \Gamma_-/\Gamma_+$ characterizing the scalar bath, as well as the choice of the initial state encoded in $\tau = \sum_i \tilde{r}_{ii}(0)$, a dimensionless constant of motion satisfying $-3 \leqslant \tau \leqslant 1$ to keep $\rho_{2\text{-atoms}}(0)$ positive.

3. FURs for open quantum systems

The main goal of the paper is to explore the FUR for an accelerating open quantum system, whose Unruh effect may induce a nontrivial modification on the uncertainty bound of FUR. In Minkowski spacetime, detector

with acceleration a follows a hyperbolic trajectory $x^{\mu}(t) = \left(a^{-1}\sinh(at), a^{-1}\cosh(at), 0, 0\right)$, leading to a Wightman function as[25]

$$G(t-t') = -\frac{a^2}{16\pi^2 \sinh^2\left[a\left(t-t'\right)/2 - \mathrm{i}\varepsilon\right]}.$$

Substituting the Wightman function back into Eq. (7), we obtain $\Gamma_+ = (\omega/4\pi)[(\mathrm{e}^{\beta\omega}+1)/(\mathrm{e}^{\beta\omega}-1)]$, $\Gamma_- = \omega/4\pi$, and $R = (\mathrm{e}^{\beta\omega}-1)/(\mathrm{e}^{\beta\omega}+1)$ (apparently, R is a monotonic function of acceleration a, *e.g.*, $R = 0$ for infinity acceleration, and $R = 1$ for $a = 0$) with β is the inverse of Unruh temperature $\beta \equiv 1/T = 2\pi/a$, from which the explicit form of Eqs. (9) and (11) are obviously. With all these results, we are sufficiently prepared to study the FUR for these localized quantum systems.

3.1. FUR for accelerating qubit

We now evaluate the maximum of FUR (1) with uncertainty bound read as $\zeta_{\boldsymbol{x}} = \max_{\rho} \sum_{t=1}^{n} p(t) p(x^{(t)}|\rho)$. For simplicity, we consider two projective measurements with two outcomes, *e.g.*, in eigenbases $X \equiv \sigma_x = \{0^x, 1^x\}$ and $Z \equiv \sigma_z = \{0^z, 1^z\}$, whose eigenstates in common convention are $\{|+\rangle, |-\rangle\}$ and $\{|0\rangle, |1\rangle\}$, respectively. Assuming each measurement is chosen with probability $1/2$, the FUR (1) can be rewritten as

$$\left\{\forall\rho : \frac{1}{2}\left(p\left(X|\rho\right) + p\left(Z|\rho\right)\right) \leq \zeta_{\boldsymbol{y}} \mid \boldsymbol{y} \in \{0,1\}^2\right\}. \quad (12)$$

For instance, for any pure state, it is known[21] that

$$\frac{1}{2}\left(p\left(0^x|\rho\right) + p\left(0^z|\rho\right)\right) \leq \frac{1}{2} + \frac{1}{2\sqrt{2}},$$

which is also hold for all other pairs of outcomes $(0^x, 1^z), (1^x, 0^z)$, and $(1^x, 1^z)$. In particular, we call those states that saturate a particular inequality are maximally certain states (MCS).

Back to our model, we concern the open system constituted by a two-level single atom. As its initial state is a pure state, the bound of FUR for the above measurements should be $1/2 + 1/2\sqrt{2}$. However, once the atom is accelerating, the state of the system becomes Eq. (9) due to the Unruh effect. This therefore may alter the uncertainty bound of Eq. (12) into motion-status-dependent.

From the density matrix Eq. (9), we can calculate straightforwardly the measurement probability for the outcomes $(0^x, 0^z)$,

$$\begin{aligned} p(0^z|\rho_{\text{1-atoms}}) &\equiv \mathrm{tr}\left(|0\rangle\langle 0|\rho_{\text{1-atoms}}\right) \\ &= \frac{1}{2}\left(1 + \mathrm{e}^{-4t\Gamma_+}\left(R + \sin\theta\right) - R\right), \\ p(0^x|\rho_{\text{1-atoms}}) &\equiv \mathrm{tr}\left(|+\rangle\langle +|\rho_{\text{1-atoms}}\right) \\ &= \frac{1}{2}\left(1 + \mathrm{e}^{-2t\Gamma_+}\cos\theta\cos\Omega t\right). \end{aligned} \quad (13)$$

Then, the uncertainty bound of FUR (12) is

$$\zeta_{(0^x,0^z)} \equiv \max_{\rho} \frac{1}{2}\left[p(0^z|\rho_{\text{1-atoms}}) + p(0^x|\rho_{\text{1-atoms}})\right].$$

For a uniform accelerating detector at a fixed time, its density matrix depends on different initial state preparation by parameter θ. This means that, to obtain the maximum of the left-hand side of FUR, we should evaluate it for all $\theta \in [0, 2\pi]$. We can now write the uncertainty bound explicitly

$$\begin{aligned} \zeta_{(0^x,0^z)} &= \frac{1}{4}\max_{\theta}[2 - R + \mathrm{e}^{-2t\Gamma_+}(\cos\theta\cos\Omega t \\ &\quad + \mathrm{e}^{-2t\Gamma_+}(\sin\theta + R))], \end{aligned} \quad (14)$$

where Γ_+ and R have been given before, which depend on the detector's acceleration a and evolution time t. By extending the above calculation for other possible pairs of outcomes, like $(0^x, 1^z), (1^x, 0^z)$, and $(1^x, 1^z)$, we obtain three other uncertainty bounds

$$\begin{aligned} \zeta_{(0^x,1^z)} &= \frac{1}{4}\max_{\theta}[2 + R + \mathrm{e}^{-2t\Gamma_+}(\cos\theta\cos\Omega t \\ &\quad - \mathrm{e}^{-2t\Gamma_+}(\sin\theta + R))], \\ \zeta_{(1^x,0^z)} &= \frac{1}{4}\max_{\theta}[2 - R - \mathrm{e}^{-2t\Gamma_+}(\cos\theta\cos\Omega t \\ &\quad - \mathrm{e}^{-2t\Gamma_+}(\sin\theta + R))], \\ \zeta_{(1^x,1^z)} &= \frac{1}{4}\max_{\theta}[2 + R - \mathrm{e}^{-2t\Gamma_+}(\cos\theta\cos\Omega t \\ &\quad + \mathrm{e}^{-2t\Gamma_+}(\sin\theta + R))]. \end{aligned} \quad (15)$$

After substituting the explicit representations of Γ_+ and R derived before into Eq. (15), we can depict four FUR uncertainty bounds in Fig. 1.

For the accelerating single atom, we observe a clear decrease of ζ compared with those of its initial pure state, *i.e.*, $\zeta_{\text{accel}} < 1/2 + 1/2\sqrt{2}$, which means a larger FUR uncertainty bound. This is not surprising as the thermality from inevitable Unruh decoherence just introduces classical noise in quantum measurement. In particular, as $t \to \infty$, the off-diagonal terms of density matrix (9) is vanishing, and state of single atom becomes thermal, $\rho_{\text{1-atoms}}(\infty) = \frac{1}{2}\mathrm{diag}(1-R, 1+R)$. Asymptotically, the FUR uncertainty bounds (15) become

$$\begin{aligned} \zeta_{(0^x,0^z)} &= \zeta_{(1^x,0^z)} = \frac{2-R}{4}, \\ \zeta_{(0^x,1^z)} &= \zeta_{(1^x,1^z)} = \frac{2+R}{4}, \end{aligned} \quad (16)$$

where $R \in [0, 1]$ is a monotonic function of acceleration. On the other hand, while larger acceleration means greater Unruh decoherence, we observe a drastically increase of ζ with growing acceleration for specific measurement outcomes $(0^x, 0^z)$ and $(1^x, 0^z)$, as shown in Figs. 1(a) and 1(c), that implies a degradation on the measurement uncertainty. However, from the asymptotic value of FUR bounds (16), we know that the upper bound of $\zeta_{(0^x,0^z)}$ and $\zeta_{(1^x,0^z)}$ can at most reach to $0.25 < 1/2 + 1/2\sqrt{2}$ as $a \to \infty$. Therefore, no violation of standard uncertainty principle can happen.

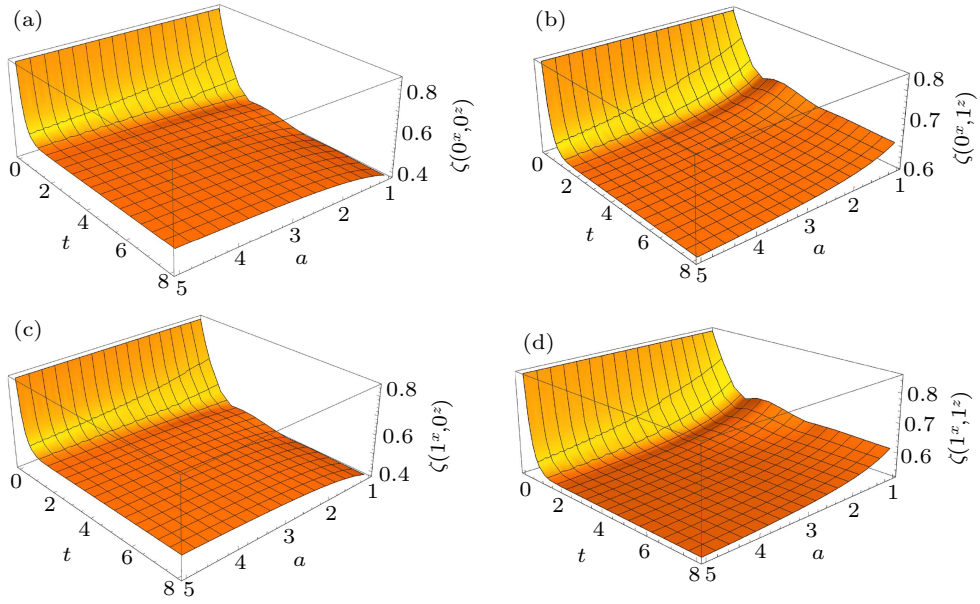


Fig. 1. The FUR uncertainty bound ζ for pair measurement outcomes (a) $(0^x,0^z)$, (b) $(1^x,1^z)$, (c) $(0^x,1^z)$, (d) $(1^x,0^z)$, respectively, and all of which are the functions of detector's acceleration a and time t. With larger acceleration a, a greater Unruh decoherence is triggered, inducing a nontrivial modification on the uncertainty bound ζ. For simplicity, we adopt the parametrization in natural units as in Ref. [29] and evaluate both uncertainty bounds for $\Omega = 1$.

3.2. FUR for accelerating two-atoms

For the case of a bipartite system of UDD, each modeled by a two-level atom, the upper bound on their FUR has a profound link with quantum nonlocality between atoms. In particular, by displaying a nonlocal retrieval game, the FUR uncertainty bound can discriminate different theories with the help of Bell-like inequalities.[14] On the other hand, it is known[23,45] that nontrivial quantum correlation can be generated through the competition between Unruh decoherence and the Markovian dynamics of the system. Therefore, we expect that a similar modification on FUR uncertainty bound may also exhibit under the relativistic acceleration.

Let us recall that how to display the nonlocal retrieval game proposed in Ref. [14] on the two-atoms system. According to the game, Alice and Bob receive questions "s" and "t" with some probability distribution, *e.g.*, $p(s,t) = p(s)p(t)$, and their answer "a" or "b" will be winning answers determined by the set of rules. In the prescribed game, Alice and Bob receive respective binary questions $s,t \in \{0,1\}$, and win the game if their respective outcomes $a,b \in \{0,1\}$ satisfy $a \oplus b = st$. Then, the probability of winning the game for a physical theory described by bipartite state $\rho_{2\text{-atoms}}$ is given by

$$\mathcal{P}^{\text{game}}(\rho;s,t) = \sum_{s,t} p(s,t) \sum_{a} p(a,b \mid s,t)_{\rho} \tag{17}$$

with

$$p(a,b \mid s,t)_{\rho} = \sum_{b} V(a,b \mid s,t) \left\langle \left(A_s^a \otimes B_t^b \right) \right\rangle_{\rho},$$

where A_s^a and B_t^b are projectors of observable s and t with outcome a and b, could be $A_s^a = [I + (-1)^a \boldsymbol{s}\cdot\boldsymbol{\sigma}]/2$ and $B_t^b = [I + (-1)^b \boldsymbol{t}\cdot\boldsymbol{\sigma}]/2$ respectively. The condition function is given as $V(a,b \mid s,t) = 1(\text{iff } a \oplus b = st)$. Given $p(s,t) = p(s)p(t) = 1/4$, it was shown that the FUR can be expressed in terms of the maximum winning probability of prescribed retrieval nonlocal games, and the degree of nonlocality of the underlying physical theory. Therefore, we have an FUR bound like[16]

$$\zeta^{\text{game}} = \max_{\rho} \mathcal{P}^{\text{game}}(\rho;s,t) = \max_{\rho} \frac{1}{2} \left(1 + \frac{\langle \mathcal{B}_{\text{CHSH}} \rangle_{\rho}}{4} \right), \tag{18}$$

where $\mathcal{B}_{\text{CHSH}}$ corresponds to the well-known Bell-CHSH operator.[46] As Bell inequality is bounded by 2 classically, we have $\zeta^{\text{game}} = 3/4$. For quantum physics, we have $\zeta^{\text{game}} = 1/2 + \sqrt{2}/2$ read from Tsirelson bound $\langle \mathcal{B}_{\text{CHSH}} \rangle = 2\sqrt{2}$.

To calculate the maximal violation of Bell inequality that determines the FUR uncertainty bound, we employ the Horodeckis' theorem of bipartite system.[47] Then the maximum of $\langle \mathcal{B}_{\text{CHSH}} \rangle$ is given as $\langle \mathcal{B}_{\max} \rangle_{\rho} = 2\sqrt{\nu_1 + \nu_2}$, where ν_1, ν_2 are two largest eigenvalues of matrix $T_{\rho}^t T_{\rho}$, and the components of correlation matrix $T_{\rho} = (t_{ij})$ are defined as $t_{ij} = \text{Tr}[\rho_{2\text{-atoms}} \sigma_i \otimes \sigma_j]$. From the final equilibrium state (11), we can straightforwardly obtain a diagonal $T^t_{\rho_{2\text{-atoms}}} T_{\rho_{2\text{-atoms}}}$ matrix with three eigenvalues

$$\lambda_1 = \lambda_2 = \left(\frac{\tau - R^2}{3 + R^2} \right)^2, \quad \lambda_3 = \left[\frac{\tau + (\tau+2)R^2}{3 + R^2} \right]^2, \tag{19}$$

which indicates that $\langle \mathcal{B}_{\max} \rangle_{\rho_{2\text{-atoms}}}$ should depend on the initial state of detectors labeling by τ and system's acceleration a.

For different values of acceleration, we need to select the two largest eigenvalues in $\{\lambda_1, \lambda_2, \lambda_3\}$, that gives the probabil-

ity of winning the retrieval game

$$\mathcal{P}^{\text{game}} = \frac{1}{2} + \frac{1}{8}\max_{\tau}\{2\lambda_1, \lambda_1 + \lambda_3\}. \tag{20}$$

In Fig. 2 we depict the winning probability $\mathcal{P}^{\text{game}}$ as a function of initial state preparation encoded in τ and two-atoms system's acceleration a. Apparently, as the evaluation runs over all possible states, the maximal value of $\mathcal{P}^{\text{game}}$ happens at $\tau = -3$. Substituting this back into $\langle \mathcal{B}_{\max} \rangle_{\rho}$, we find that the violation of Bell inequality can reach Tsirelson bound $2\sqrt{2}$. Hence, the FUR uncertainty bound ζ^{game} allowed for accelerating two-atoms open system is $1/2 + \sqrt{2}/2$. In other words, it is independent of the Unruh decoherence attributed to acceleration, the FUR uncertainty bound $1/2 + \sqrt{2}/2$ can be protected for the accelerated bipartite system. This is quite different from the single atom case, where Unruh decoherence induces a significant increase in the measurement uncertainty. As mentioned before, in two-atoms system, however the Unruh decoherence can be overcome by the Markovian evolution of bipartite system,[45] manifested by the generation of quantum correlation.

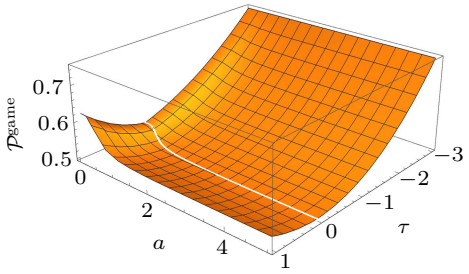


Fig. 2. The probability of winning the retrieval game $\mathcal{P}^{\text{game}}$ as a function of detectors' acceleration a and the initial state preparation characterized by the dimensionless constant $\tau = \sum_i \tilde{r}_{ii}(0)$ satisfying $-3 \leqslant \tau \leqslant 1$ to keep $\rho_{2\text{-atoms}}(0)$ positive. One can observe that the maximal value of $\mathcal{P}^{\text{game}}$ can reach for initial state satisfying $\tau = -3$. For simplicity, we adopt the parametrization in natural units as in Ref. [29] and evaluate both uncertainty bounds for $\Omega = 1$.

4. Conclusion

To summarize, in the present work we explore the FUR uncertainty bound for the open quantum system under relativistic acceleration, which is modified by the celebrated Unruh effect. We found that for the single detector system, the time-dependent uncertainty bound is a function of atom's acceleration. With nonvanishing acceleration, the inevitable Unruh decoherence induces a larger uncertainty in quantum measurements than the inertial detector case, which gives a smaller bound $\zeta_{\text{accel}} < 1/2 + \sqrt{2}/2$. For a two-detectors system, via a nonlocal retrieval game, the bipartite FUR uncertainty bound is determined by the violation of Bell inequality for the open system. Due to the quantum correlation generated during the system's Markovian evolution, we found that with a particular choice of initial state, the FUR uncertainty bound for inertial case[14] can be protected from Unruh decoherence.

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