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Quantum multicast schemes of different quantum states via non-maximally entangled channels with multiparty involvement*

Yan Yu(于妍)[1], Nan Zhao(赵楠)[1,†], Chang-Xing Pei(裴昌幸)[1], and Wei Li(李玮)[2]

[1] State Key Laboratory of Integrated Services Networks, Xidian University, Xi'an 710071, China
[2] Cloud Computing Center of Xi'an Branch of Shaanxi Telecom, China Telecom, Xi'an 710002, China

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Due to the unavoidable interaction between the quantum channel and its ambient environment, it is difficult to generate and maintain the maximally entanglement. Thus, the research on multiparty information transmission via non-maximally entangled channels is of academic value and general application. Here, we utilize the non-maximally entangled channels to implement two multiparty remote state preparation schemes for transmitting different quantum information from one sender to two receivers synchronously. The first scheme is adopted to transmit two different four-qubit cluster-type entangled states to two receivers with a certain probability. In order to improve success probabilities of such multicast remote state preparation using non-maximally entangled channels, we put forward the second scheme, which deals with the situation that is a synchronous transfer of an arbitrary single-qubit state and an arbitrary two-qubit state from one sender to two receivers. In particular, its success probability can reach 100% in principle, and independent of the entanglement degree of the shared non-maximally entangled channel. Notably, in the second scheme, the auxiliary particle is not required.

Keywords: quantum multicast communications, non-maximally entangled states, remote state preparation

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1. Introduction

Quantum entanglement as a significant quantum resource plays an essential role in the field of applications of the quantum information science, and the theory and technology have achieved some breakthroughs.[1–5] By utilized a shared entangled state, quantum teleportation (QT)[6] is firstly proposed to transmit an unknown qubit from one site to another. Subsequently, Lo[7] gave another intriguing method called remote state preparation (RSP) to transmit a pure known quantum state. Compared with teleportation, RSP requires less classical communication cost, and the sender knows the complete knowledge about the state. Owing to its advantages and features, RSP has been investigated by researchers theoretically and experimentally.[8–19] Further, RSP is extended to multiparty quantum information transmission of multiqubit entangled states, and related schemes are proposed via different quantum channels,[20–25] including maximally entangled and non-maximally entangled. Apparently, the optimal success probability and fidelity can be achieved with the help of the maximally entangled one. However, due to the unavoidable interaction between the quantum channel and its ambient environment, it is difficult to generate and maintain the maximally entanglement. So far, the research of multiparty RSP via non-maximally entangled channels has made dramatic progress.[26–32]

Recently, Chen *et al.*[28] put forward the controlled RSP of two-qubit state using an eight-qubit partially entangled channel. In 2017, a multiparty RSP scheme was introduced in Ref. [29] where the channel consists of two sets of GHZ states. In 2018, Wei *et al.*[30] gave a probabilistic controlled RSP with multiparty based on non-maximally entangled state. In 2019, a multicast scheme[31] employing the eight-qubit non-maximally entangled channel was given for transmitting different two-qubit state to two receivers synchronously. In 2020, Zhao *et al.*[32] utilized a six-qubit partially entangled channel to broadcast the arbitrary single-qubit state to three remote receivers. Moreover, cluster state is the specific entanglement and can be efficiently applied to information processing tasks. In general, a cluster state is expressed as[33]

$$|C_N\rangle = \frac{1}{2^{N/2}} \bigotimes_{a=1}^{N} (|0\rangle_a \sigma_z^{(a+1)} + |1\rangle_a), \tag{1}$$

with the convention $\sigma_z^{(N+1)} \equiv 1$. It has been shown that one-dimensional N-qubit cluster states are generated in arrays of N qubits mediated with an Ising-type interaction. Especially, the cluster state has the properties of the GHZ-class and the W-class entangled states, and robust against decoherence.[34] Thus, many authors proceeded to focus on RSP of transmitting cluster-type states in recent years.[35–41] For example, by two non-maximally entangled GHZ-type states, Wei *et al.*[37] presented an efficient RSP scheme of preparing a four-qubit entangled cluster-type state.

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†Corresponding author. E-mail: zhaonan@xidian.edu.cn

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However, for above-mentioned schemes, the requirements about the low complexity and synchronous diverse information transmission are not satisfied. In this work, we wish to study the multiparty multicast RSP via non-maximally entangled channel from one sender to multiple receivers. In the first scheme, with the probability $16\zeta_0^2\eta_0^2$, we show that each receiver can obtain different four-qubit cluster-type states synchronously. In addition, the achievement ratio of such a kind of multicast schemes using the non-maximally entangled channels largely depends upon the parameters of the shared quantum channel. In fact, the non-maximally entangled channel is inevitably encountered because of the influence from the environment. Furthermore, we are going to improve the probabilities of such multicast RSP schemes in the second scheme. It can be implemented successfully, and independent of entangled degree of the channel. We now explain the protocols in the next two sections, followed by discussions and comparison in Section 4, and the paper concludes with Section 5.

2. The multicast scheme of different four-qubit cluster-type states via twelve-qubit non-maximally entangled channel

Firstly let us name the two receivers Bob and Charlie, while the sender remains Alice. Suppose Alice intends to help Bob to prepare $|\mu_1\rangle$ and to help Charlie to prepare $|\mu_2\rangle$ synchronously. Here $|\mu_1\rangle$ and $|\mu_2\rangle$ are given by

$$\begin{aligned}|\mu_1\rangle &= x_0|0000\rangle + x_1|0011\rangle + x_2|1100\rangle - x_3|1111\rangle,\\ |\mu_2\rangle &= y_0|0000\rangle + y_1|0011\rangle + y_2|1100\rangle - y_3|1111\rangle,\end{aligned} \quad (2)$$

where the parameters x, y are real-valued, satisfy the normalized condition $\sum_{i=0}^{3} z_i = 1 (z = x, y)$, and are known to sender exactly while unknown to receivers. Assume that all the participants share previously generated two non-maximally entangled quantum resources, i.e.,

$$\begin{aligned}|\Gamma^1\rangle &= (\zeta_0|000000\rangle + \zeta_1|000111\rangle\\ &\quad + \zeta_2|111000\rangle - \zeta_3|111111\rangle)_{123456},\\ |\Gamma^2\rangle &= (\eta_0|000000\rangle + \eta_1|000111\rangle\\ &\quad + \eta_2|111000\rangle - \eta_3|111111\rangle)_{7,8,9,10,11,12},\end{aligned} \quad (3)$$

where the real numbers ζ, η satisfy $\sum_{i=0}^{3}|\omega_i|^2 = 1$ $(\omega = \zeta, \eta)$ and $|\omega_0| \leq \min\{|\omega_j|\}$ $(j = 1,2,3)$ $(\omega = \zeta, \eta)$. The particles $(2,3,5,6)$ and $(8,9,11,12)$ are sent to receivers Bob and Charlie, respectively, sender Alice is in procession of particles $(1,4,7,10)$. Then the detailed implementation procedures are shown below.

$[(S_1)]$ Concretely, an auxiliary particle with initial state $|0\rangle_{a_1}$ is firstly introduced by Alice, afterwards she performs a joint unitary transformation $\widetilde{\Omega}$,

$$\widetilde{\Omega} = \mathrm{diag}(E_2, \Xi_1, \Xi_2, \Xi_3) \quad (4)$$

on $|\Gamma^1\rangle_{A_1B_1B_2A_2B_3B_4} \otimes |0\rangle_{a_1}$ under the basis $\{|000\rangle_{A_1A_2a_1}, |001\rangle_{A_1A_2a_1}, \ldots, |111\rangle_{A_1A_2a_1}\}$, where Ξ_i are defined as

$$\Xi_i = \begin{pmatrix} \zeta_0/\zeta_i & -\sqrt{1-(\zeta_0/\zeta_i)^2} \\ \sqrt{1-(\zeta_0/\zeta_i)^2} & \zeta_0/\zeta_i \end{pmatrix} \quad (i = 1,2,3), \quad (5)$$

and E_2 is a 2×2 identity matrix. Then Alice gets

$$\begin{aligned}&|\widetilde{\Gamma^1}\rangle_{A_1B_1B_2A_2B_3B_4a_1}\\ &= \zeta_0(|000000\rangle + |000111\rangle + |111000\rangle\\ &\quad - |111111\rangle)_{A_1B_1B_2A_2B_3B_4} \otimes |0\rangle_{a_1}\\ &\quad + [\zeta_1\sqrt{1-(\zeta_0/\zeta_1)^2}|000111\rangle + \zeta_2\sqrt{1-(\zeta_0/\zeta_2)^2}|111000\rangle\\ &\quad - \zeta_3\sqrt{1-(\zeta_0/\zeta_3)^2}|111111\rangle]_{A_1B_1B_2A_2B_3B_4} \otimes |1\rangle_{a_1}.\end{aligned} \quad (6)$$

By measuring the particle $|0\rangle_{a_1}$ under the basis $\{|0\rangle, |1\rangle\}$ (donate as M_{a_1}), Eq. (6) would turn to $|\widetilde{\Gamma^1}\rangle_{A_1B_1B_2A_2B_3B_4} = \frac{1}{2}(|000000\rangle + |000111\rangle + |111000\rangle - |111111\rangle)$ with the probability $4\zeta_0^2$. Similarly, Alice may change $|\Gamma^2\rangle$ to $|\widetilde{\Gamma^2}\rangle_{A_3C_1C_2A_4C_3C_4}$ with the probability $4\eta_0^2$ by measuring the introduced auxiliary particle $|0\rangle_{a_2}$ (donated as M_{a_2}).

Therefore, with the probability $16\zeta_0^2\eta_0^2$, the initial state of the partially entangled quantum resources turns to $|\widetilde{\Psi}\rangle_{12}$ which is composed of two six-qubit cluster-type states,

$$\begin{aligned}|\widetilde{\Psi}\rangle_{12} &= |\widetilde{\Gamma^1}\rangle_{A_1B_1B_2A_2B_3B_4} \otimes |\widetilde{\Gamma^2}\rangle_{A_3C_1C_2A_4C_3C_4}\\ &= \frac{1}{4}(|000000\rangle + |000111\rangle + |111000\rangle\\ &\quad - |111111\rangle)_{A_1B_1B_2A_2B_3B_4}\\ &\quad \otimes (|000000\rangle + |000111\rangle + |111000\rangle\\ &\quad - |111111\rangle)_{A_3C_1C_2A_4C_3C_4}.\end{aligned} \quad (7)$$

$[(S_2)]$ A projective measurement (donated as M_A) is performed by Alice on particles (A_1, A_2, A_3, A_4) under a set of orthogonal bases $\{|\varepsilon^1\rangle \ldots |\varepsilon^{16}\rangle\}$, which composed of the computation basis $\{|0000\rangle, |0001\rangle, \ldots, |1111\rangle\}$,

$$(|\varepsilon^1\rangle \ldots |\varepsilon^{16}\rangle)^{\mathrm{T}} = M_{A_1} \otimes M_{A_2}(|0000\rangle, |0001\rangle, \ldots, |1111\rangle)^{\mathrm{T}}. \quad (8)$$

Here construct a 4×4 matrix $M(z)$,

$$M(z) = \begin{pmatrix} z_0 & z_1 & z_2 & -z_3 \\ -z_1 & z_0 & z_3 & z_2 \\ -z_2 & -z_3 & z_0 & -z_1 \\ z_3 & -z_2 & z_1 & z_0 \end{pmatrix}, \quad (9)$$

and

$$\begin{aligned}M_{A_1} &= M(x), \quad (z = x),\\ M_{A_2} &= M(y) \times U_A, \quad (z = y),\end{aligned} \quad (10)$$

where $U_A = \mathrm{diag}(1,1,1,-1)$. Therefore, the total systemic state $|\widetilde{\Psi}\rangle_{12}$ in Eq. (7) can also be of the form

$$\begin{aligned}|\widetilde{\Psi}\rangle_{12} &= \frac{1}{4}[|\varepsilon^1\rangle_{A_1A_2A_3A_4}(x_0|0000\rangle + x_1|0011\rangle\\ &\quad + x_2|1100\rangle - x_3|1111\rangle)_{B_1B_2B_3B_4}\end{aligned}$$

$$
\begin{aligned}
&(y_0|0000\rangle + y_1|0011\rangle + y_2|1100\rangle - y_3|1111\rangle)_{C_1C_2C_3C_4} \\
&+ |\varepsilon^2\rangle_{A_1A_2A_3A_4}(x_0|0000\rangle + x_1|0011\rangle \\
&+ x_2|1100\rangle - x_3|1111\rangle)_{B_1B_2B_3B_4} \\
&(y_0|0011\rangle - y_1|0000\rangle + y_2|1111\rangle + y_3|1100\rangle)_{C_1C_2C_3C_4} \\
&+ \cdots \\
&+ |\varepsilon^{16}\rangle_{A_1A_2A_3A_4}(x_0|1111\rangle + x_1|1100\rangle \\
&- x_2|0011\rangle + x_3|0000\rangle)_{B_1B_2B_3B_4} \\
&(y_0|1111\rangle + y_1|1100\rangle - y_2|0011\rangle \\
&+ y_3|0000\rangle)_{C_1C_2C_3C_4}]. \qquad (11)
\end{aligned}
$$

[(S_3)] In response to the different measurement outcomes of sender Alice, receiver Bob operates on his qubits (B_1, B_2, B_3, B_4) with an appropriate unitary transformation U_B, and Charlie reconstructs the state U_C as well. All the situations are shown in Table 1.

Table 1. Measurement results and unitary operations of each party in multicast scheme of arbitrary four-qubit cluster states via twelve-qubit non-maximally entangled channel.

Alice's results	States of particles (B_1,B_2,B_3,B_4)	Bob's operation	States of particles (C_1,C_2,C_3,C_4)	Charlie's operation
$\|\varepsilon^1\rangle_{A_1A_2A_3A_4}$	$x_0\|0000\rangle + x_1\|0011\rangle + x_2\|1100\rangle - x_3\|1111\rangle$	$I_1\otimes I_2\otimes I_3\otimes I_4$	$y_0\|0000\rangle + y_1\|0011\rangle + y_2\|1100\rangle - y_3\|1111\rangle$	$I_1\otimes I_2\otimes I_3\otimes I_4$
$\|\varepsilon^2\rangle_{A_1A_2A_3A_4}$	$x_0\|0000\rangle + x_1\|0011\rangle + x_2\|1100\rangle - x_3\|1111\rangle$	$I_1\otimes I_2\otimes I_3\otimes I_4$	$y_0\|0011\rangle - y_1\|0000\rangle + y_2\|1111\rangle + y_3\|1100\rangle$	$I_1\otimes I_2\otimes X_3\otimes Y_4$
$\|\varepsilon^3\rangle_{A_1A_2A_3A_4}$	$x_0\|0000\rangle + x_1\|0011\rangle + x_2\|1100\rangle - x_3\|1111\rangle$	$I_1\otimes I_2\otimes I_3\otimes I_4$	$y_0\|1100\rangle - y_1\|1111\rangle - y_2\|0000\rangle - y_3\|0011\rangle$	$X_1\otimes Y_2\otimes Z_3\otimes I_4$
$\|\varepsilon^4\rangle_{A_1A_2A_3A_4}$	$x_0\|0000\rangle + x_1\|0011\rangle + x_2\|1100\rangle - x_3\|1111\rangle$	$I_1\otimes I_2\otimes I_3\otimes I_4$	$y_0\|1111\rangle + y_1\|1100\rangle - y_2\|0011\rangle + y_3\|0000\rangle$	$Y_1\otimes X_2\otimes X_3\otimes X_4$
$\|\varepsilon^5\rangle_{A_1A_2A_3A_4}$	$x_0\|0011\rangle - x_1\|0000\rangle + x_2\|1111\rangle + x_3\|1100\rangle$	$I_1\otimes I_2\otimes X_3\otimes Y_4$	$y_0\|0000\rangle + y_1\|0011\rangle + y_2\|1100\rangle - y_3\|1111\rangle$	$I_1\otimes I_2\otimes I_3\otimes I_4$
$\|\varepsilon^6\rangle_{A_1A_2A_3A_4}$	$x_0\|0011\rangle - x_1\|0000\rangle + x_2\|1111\rangle + x_3\|1100\rangle$	$I_1\otimes I_2\otimes X_3\otimes Y_4$	$y_0\|0011\rangle - y_1\|0000\rangle + y_2\|1111\rangle + y_3\|1100\rangle$	$I_1\otimes I_2\otimes X_3\otimes Y_4$
$\|\varepsilon^7\rangle_{A_1A_2A_3A_4}$	$x_0\|0011\rangle - x_1\|0000\rangle + x_2\|1111\rangle + x_3\|1100\rangle$	$I_1\otimes I_2\otimes X_3\otimes Y_4$	$y_0\|1100\rangle - y_1\|1111\rangle - y_2\|0000\rangle - y_3\|0011\rangle$	$X_1\otimes Y_2\otimes Z_3\otimes I_4$
$\|\varepsilon^8\rangle_{A_1A_2A_3A_4}$	$x_0\|0011\rangle - x_1\|0000\rangle + x_2\|1111\rangle + x_3\|1100\rangle$	$I_1\otimes I_2\otimes X_3\otimes Y_4$	$y_0\|1111\rangle + y_1\|1100\rangle - y_2\|0011\rangle + y_3\|0000\rangle$	$Y_1\otimes X_2\otimes X_3\otimes X_4$
$\|\varepsilon^9\rangle_{A_1A_2A_3A_4}$	$x_0\|1100\rangle - x_1\|1111\rangle - x_2\|0000\rangle - x_3\|0011\rangle$	$X_1\otimes Y_2\otimes Z_3\otimes I_4$	$y_0\|0000\rangle + y_1\|0011\rangle + y_2\|1100\rangle - y_3\|1111\rangle$	$I_1\otimes I_2\otimes I_3\otimes I_4$
$\|\varepsilon^{10}\rangle_{A_1A_2A_3A_4}$	$x_0\|1100\rangle - x_1\|1111\rangle - x_2\|0000\rangle - x_3\|0011\rangle$	$X_1\otimes Y_2\otimes Z_3\otimes I_4$	$y_0\|0011\rangle - y_1\|0000\rangle + y_2\|1111\rangle + y_3\|1100\rangle$	$I_1\otimes I_2\otimes X_3\otimes Y_4$
$\|\varepsilon^{11}\rangle_{A_1A_2A_3A_4}$	$x_0\|1100\rangle - x_1\|1111\rangle - x_2\|0000\rangle - x_3\|0011\rangle$	$X_1\otimes Y_2\otimes Z_3\otimes I_4$	$y_0\|1100\rangle - y_1\|1111\rangle - y_2\|0000\rangle - y_3\|0011\rangle$	$X_1\otimes Y_2\otimes Z_3\otimes I_4$
$\|\varepsilon^{12}\rangle_{A_1A_2A_3A_4}$	$x_0\|1100\rangle - x_1\|1111\rangle - x_2\|0000\rangle - x_3\|0011\rangle$	$X_1\otimes Y_2\otimes Z_3\otimes I_4$	$y_0\|1111\rangle + y_1\|1100\rangle - y_2\|0011\rangle + y_3\|0000\rangle$	$Y_1\otimes X_2\otimes X_3\otimes X_4$
$\|\varepsilon^{13}\rangle_{A_1A_2A_3A_4}$	$x_0\|1111\rangle + x_1\|1100\rangle - x_2\|0011\rangle + x_3\|0000\rangle$	$Y_1\otimes X_2\otimes X_3\otimes X_4$	$y_0\|0000\rangle + y_1\|0011\rangle + y_2\|1100\rangle - y_3\|1111\rangle$	$I_1\otimes I_2\otimes I_3\otimes I_4$
$\|\varepsilon^{14}\rangle_{A_1A_2A_3A_4}$	$x_0\|1111\rangle + x_1\|1100\rangle - x_2\|0011\rangle + x_3\|0000\rangle$	$Y_1\otimes X_2\otimes X_3\otimes X_4$	$y_0\|0011\rangle - y_1\|0000\rangle + y_2\|1111\rangle + y_3\|1100\rangle$	$I_1\otimes I_2\otimes X_3\otimes Y_4$
$\|\varepsilon^{15}\rangle_{A_1A_2A_3A_4}$	$x_0\|1111\rangle + x_1\|1100\rangle - x_2\|0011\rangle + x_3\|0000\rangle$	$Y_1\otimes X_2\otimes X_3\otimes X_4$	$y_0\|1100\rangle - y_1\|1111\rangle - y_2\|0000\rangle - y_3\|0011\rangle$	$X_1\otimes Y_2\otimes Z_3\otimes I_4$
$\|\varepsilon^{16}\rangle_{A_1A_2A_3A_4}$	$x_0\|1111\rangle + x_1\|1100\rangle - x_2\|0011\rangle + x_3\|0000\rangle$	$Y_1\otimes X_2\otimes X_3\otimes X_4$	$y_0\|1111\rangle + y_1\|1100\rangle - y_2\|0011\rangle + y_3\|0000\rangle$	$Y_1\otimes X_2\otimes X_3\otimes X_4$

According to the above-mentioned three steps, one can see that each receiver can obtain the different cluster-type states synchronously with the probability $16\zeta_0^2\eta_0^2$, and the detailed steps are shown in Fig. 1.

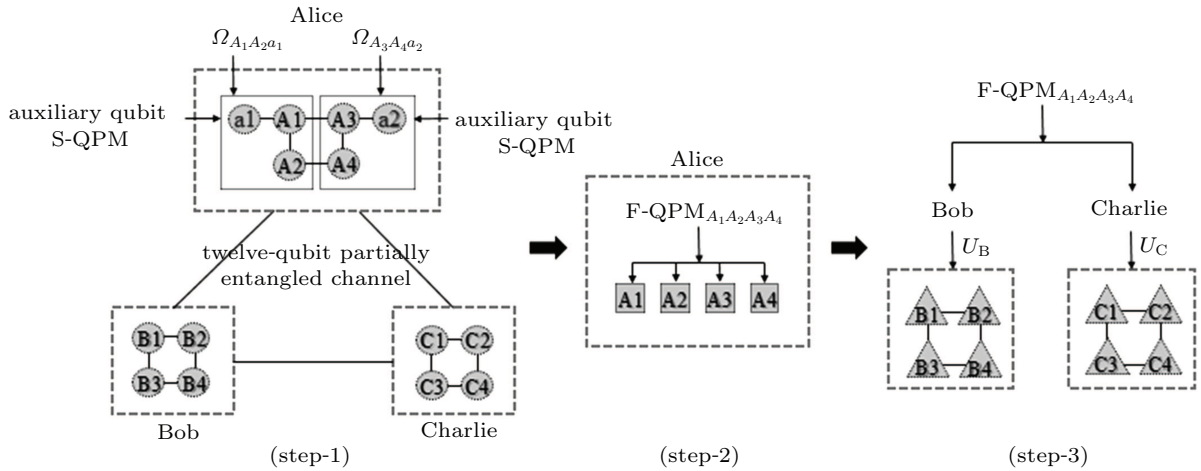


Fig. 1. Schematic diagram for the multicast scheme implementation of (S_1)–(S_3). A twelve-qubit non-maximally entangled state is used as the quantum channel. S-QPM and F-QPM stand for the single- and four-qubit projective measurements, which are performed by Alice. U_B and U_C stand for the appropriate unitary operations on particles (B_1,B_2,B_3,B_4) and (C_1,C_2,C_3,C_4), respectively, which are performed by Bob and Charlie.

3. The deterministic multicast scheme of arbitrary single- and two-qubit states via the nine-qubit non-maximally entangled channel

Supposed that there are four parties, under the control of the supervisor David, sender Alice would like to assist receiver Bob remotely in the preparation of the arbitrary single-qubit state $|v_1\rangle$ and assists receiver Charlie in the preparation of the arbitrary two-qubit states $|v_2\rangle$,

$$
\begin{aligned}
|v_1\rangle &= \alpha_1|0\rangle + \beta_1|1\rangle, \\
|v_2\rangle &= \alpha_2|00\rangle + \beta_2|01\rangle + \gamma_2|10\rangle + \delta_2|11\rangle, \qquad (12)
\end{aligned}
$$

where the parameters $\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_2, \delta_2$ are real, and satisfy $\alpha_1^2 + \beta_1^2 = 1$, $\alpha_2^2 + \beta_2^2 + \gamma_2^2 + \beta_2^2 = 1$. Note that parameters are known to sender while unknown to the receivers.

To accomplish the task of this protocol, three non-maximally entangled quantum resources shared by four participants are shown as follows:

$$
\begin{aligned}
|MS^1\rangle &= \frac{1}{\sqrt{2}}(|000\rangle + l_1|111\rangle + k_1|110\rangle)_{123},\\
|MS^2\rangle &= \frac{1}{\sqrt{2}}(|000\rangle + l_2|111\rangle + k_2|110\rangle)_{456},\\
|MS^3\rangle &= \frac{1}{\sqrt{2}}(|000\rangle + l_3|111\rangle + k_3|110\rangle)_{789}, \qquad (13)
\end{aligned}
$$

where l_i and k_i are assumed to be real and $l_i^2 + k_i^2 = 1$ $(i = 1,2,3)$. Among these nine particles, Alice possesses (1,4,7), Bob and Charlie are in possession of 2 and (5,8) respectively, and David owns the rest (3,6,9). Thus, the combined state $|\Phi\rangle_9$ turns to

$$
\begin{aligned}
|\Phi\rangle_9 \;=\; & |MS^1\rangle_{A_1BD_1} \otimes |MS^2\rangle_{A_2C_1D_2} \otimes |MS^3\rangle_{A_3C_2D_3}\\
=\; & \frac{1}{2\sqrt{2}}(|000\rangle + l_1|111\rangle + k_1|110\rangle)_{A_1BD_1}\\
& \otimes (|000\rangle + l_2|111\rangle + k_2|110\rangle)_{A_2C_1D_2}\\
& \otimes (|000\rangle + l_3|111\rangle + k_3|110\rangle)_{A_3C_2D_3}. \qquad (14)
\end{aligned}
$$

The detailed steps of the deterministic multicast scheme via the non-maximally entangled quantum channel are shown in the following.

$[(S_1)]$ A projective measurement is made by Alice on particles (A_1, A_2, A_3) in the orthogonal basis $\{|\varepsilon^i\rangle\}(i = 1,2,\ldots,8)$ composed of computational bases $\{|000\rangle, |001\rangle, \ldots, |111\rangle\}$,

$$
(|\varepsilon^1\rangle, |\varepsilon^2\rangle, \ldots, |\varepsilon^8\rangle)^{\mathrm{T}} = M_A(|000\rangle, |001\rangle, \ldots, |111\rangle)^{\mathrm{T}}, \qquad (15)
$$

where M_A in Eq. (15) is taken to be

$$
M_A = \begin{pmatrix}
\alpha_1\alpha_2 & \alpha_1\beta_2 & \alpha_1\gamma_2 & \alpha_1\delta_2 & -\beta_1\alpha_2 & -\beta_1\beta_2 & -\beta_1\gamma_2 & -\beta_1\delta_2\\
-\alpha_1\beta_2 & \alpha_1\alpha_2 & \alpha_1\delta_2 & -\alpha_1\gamma_2 & \beta_1\beta_2 & -\beta_1\alpha_2 & -\beta_1\delta_2 & \beta_1\gamma_2\\
-\alpha_1\gamma_2 & -\alpha_1\delta_2 & \alpha_1\alpha_2 & \alpha_1\beta_2 & \beta_1\gamma_2 & \beta_1\delta_2 & -\beta_1\alpha_2 & -\beta_1\beta_2\\
\alpha_1\delta_2 & -\alpha_1\gamma_2 & \alpha_1\beta_2 & -\alpha_1\alpha_2 & -\beta_1\delta_2 & \beta_1\gamma_2 & -\beta_1\beta_2 & \beta_1\alpha_2\\
\beta_1\alpha_2 & \beta_1\beta_2 & \beta_1\gamma_2 & \beta_1\delta_2 & \alpha_1\alpha_2 & \alpha_1\beta_2 & \alpha_1\gamma_2 & \alpha_1\delta_2\\
-\beta_1\beta_2 & \beta_1\alpha_2 & \beta_1\delta_2 & -\beta_1\gamma_2 & -\alpha_1\beta_2 & \alpha_1\alpha_2 & \alpha_1\delta_2 & -\alpha_1\gamma_2\\
-\beta_1\gamma_2 & -\beta_1\delta_2 & \beta_1\alpha_2 & \beta_1\beta_2 & -\alpha_1\gamma_2 & -\alpha_1\delta_2 & \alpha_1\alpha_2 & \alpha_1\beta_2\\
\beta_1\delta_2 & -\beta_1\gamma_2 & \beta_1\beta_2 & -\beta_1\alpha_2 & \alpha_1\delta_2 & -\alpha_1\gamma_2 & \alpha_1\beta_2 & -\alpha_1\alpha_2
\end{pmatrix}. \qquad (16)
$$

While Eq. (14) can also be of the form

$$
\begin{aligned}
|\Phi\rangle_9 \;=\; & \frac{1}{2\sqrt{2}}(|\varepsilon^1\rangle_{A_1A_2A_3} \otimes |\varsigma_1\rangle_{D_1D_2D_3BC_1C_2}\\
& + |\varepsilon^2\rangle_{A_1A_2A_3} \otimes |\varsigma_2\rangle_{D_1D_2D_3BC_1C_2}\\
& + \cdots + |\varepsilon^8\rangle_{A_1A_2A_3} \otimes |\varsigma_8\rangle_{D_1D_2D_3BC_1C_2}), \qquad (17)
\end{aligned}
$$

where $|\varsigma_1\rangle$ is taken as an example, i.e.,

$$
\begin{aligned}
& |\varsigma_1\rangle_{D_1D_2D_3BC_1C_2}\\
=\; & (\alpha_1\alpha_2|000000\rangle + k_3\alpha_1\beta_2|000001\rangle\\
& + l_3\alpha_1\beta_2|001001\rangle + k_2\alpha_1\gamma_2|000010\rangle\\
& + k_2k_3\alpha_1\delta_2|000011\rangle + k_2l_3\alpha_1\delta_2|001011\rangle\\
& + l_2\alpha_1\gamma_2|010010\rangle + l_2k_3\alpha_1\delta_2|010011\rangle\\
& + l_2l_3\alpha_1\delta_2|011011\rangle - k_1\beta_1\alpha_2|000100\rangle\\
& - k_1k_3\beta_1\beta_2|000101\rangle - k_1l_3\beta_1\beta_2|001101\rangle\\
& - k_1k_2\beta_1\gamma_2|000110\rangle - k_1k_2k_3\beta_1\delta_2|000111\rangle\\
& - k_1k_2l_3\beta_1\delta_2|001111\rangle - k_1l_2\beta_1\gamma_2|010110\rangle\\
& - k_1l_2k_3\beta_1\delta_2|010111\rangle - k_1l_2l_3\beta_1\delta_2|011111\rangle\\
& - l_1\beta_1\alpha_2|100100\rangle - l_1k_3\beta_1\beta_2|100101\rangle\\
& - l_1l_3\beta_1\beta_2|101101\rangle - l_1k_2\beta_1\gamma_2|100110\rangle\\
& - l_1k_2k_3\beta_1\delta_2|100111\rangle - l_1k_2l_3\beta_1\delta_2|101111\rangle\\
& - l_1l_2\beta_1\gamma_2|110110\rangle - l_1l_2k_3\beta_1\delta_2|110111\rangle\\
& - l_1l_2l_3\beta_1\delta_2|111111\rangle)_{D_1D_2D_3BC_1C_2}. \qquad (18)
\end{aligned}
$$

After sender's projective measurement, by classical communication, she broadcasts her measurement outcomes to Bob and Charlie synchronously.

$[(S_2)]$ If controller David consents to provide the assistance, he will perform three measurements on particles D_1, D_2, D_3 in the following bases synchronously,

$$
\begin{aligned}
|\tau_+\rangle_{D_j} &= \frac{1}{\sqrt{l_j^2 + (1+k_j)^2}}[(1+k_j)|0\rangle + l_j|1\rangle],\\
|\tau_-\rangle_{D_j} &= \frac{1}{\sqrt{l_j^2 + (1-k_j)^2}}[(1-k_j)|0\rangle - l_j|1\rangle],\\
& (j = 1,2,3). \qquad (19)
\end{aligned}
$$

As an example, one can see that if Alice's result is $|\varepsilon_1\rangle$, David's results are $|\tau_+\rangle_{D_1}|\tau_+\rangle_{D_2}|\tau_+\rangle_{D_3}$, the states of particles B and (C_1, C_2) evolve as

$$
\begin{aligned}
& |\varsigma_1\rangle_{D_1D_2D_3BC_1C_2} \xrightarrow{|\tau_+\rangle_{D_1}|\tau_+\rangle_{D_2}|\tau_+\rangle_{D_3}}\\
& \frac{1}{8}\rho_1(\alpha_1|0\rangle - \beta_1|1\rangle)_B \otimes (\alpha_2|00\rangle\\
& + \beta_2|01\rangle + \gamma_2|10\rangle + \delta_2|11\rangle)_{C_1C_2}, \qquad (20)
\end{aligned}
$$

where $\rho_1 = \sqrt{l_1^2 + (1+k_1)^2}\sqrt{l_2^2 + (1+k_2)^2}\sqrt{l_3^2 + (1+k_3)^2}$.

$[(S_3)]$ According to the above measurement outcomes, two receivers perform recovery operations on their particles to obtain $|\nu_1\rangle$ and $|\nu_2\rangle$, respectively. Cases under $|\varepsilon^1\rangle_{A_1A_2A_3}$ are taken as an example listed in Table 2.

Table 2. Measurement results and unitary operations of each party in deterministic multicast scheme of arbitrary single- and two-qubit states via the nine-qubit non-maximally entangled quantum channel under the orthogonal basis vectors $|\varepsilon^1\rangle_{A_1A_2A_3}$.

Alice's results	David's results of (D_1, D_2, D_3)			Bob's operation	Charlie's operation
$\|\varepsilon^1\rangle_{A_1A_2A_3}$	$\|\tau_+\rangle_{D_1}$	$\|\tau_+\rangle_{D_2}$	$\|\tau_+\rangle_{D_3}$	σ_z	$I\otimes I$
			$\|\tau_-\rangle_{D_3}$	σ_z	$I\otimes\sigma_z$
		$\|\tau_-\rangle_{D_2}$	$\|\tau_+\rangle_{D_3}$	σ_z	$\sigma_z\otimes I$
			$\|\tau_-\rangle_{D_3}$	σ_z	$\sigma_z\otimes\sigma_z$
	$\|\tau_-\rangle_{D_1}$	$\|\tau_+\rangle_{D_2}$	$\|\tau_+\rangle_{D_3}$	I	$I\otimes I$
			$\|\tau_-\rangle_{D_3}$	I	$I\otimes\sigma_z$
		$\|\tau_-\rangle_{D_2}$	$\|\tau_+\rangle_{D_3}$	I	$\sigma_z\otimes I$
			$\|\tau_-\rangle_{D_3}$	I	$\sigma_z\otimes\sigma_z$

To be simplified, the other cases of Alice's results ($|\varepsilon^2\rangle_{A_1A_2A_3}\ldots|\varepsilon^8\rangle_{A_1A_2A_3}$) are no longer depicted here as they are the similar analyses. For all the 64 measurement results, Bob can always obtain the arbitrary single-qubit state $|\nu_1\rangle$ by appropriate unitary operations U_B, and Charlie obtains arbitrary two-qubit state $|\nu_2\rangle$ by $U_{C_1C_2}$ synchronously. Thus, the success probability of this scheme is 100%. For clarity, its implementation of quantum circuit is displayed in Fig. 2.

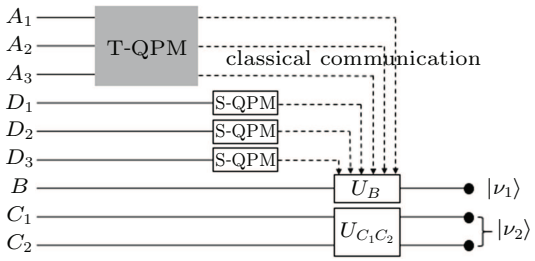


Fig. 2. Schematic quantum circuit for deterministic multicast scheme of arbitrary single- and two-qubit states via the nine-qubit non-maximally entangled quantum channel. S-QPM and T-QPM denote single- and three-qubit projective measurements under a set of complete orthogonal basis vectors $\{|\varepsilon\rangle\}_{A_1A_2A_3}$ and $\{|\tau\rangle\}_D$, respectively. U_B and $U_{C_1C_2}$ stand for the appropriate unitary operations on particles B and (C_1, C_2), respectively, which are performed by receivers.

4. Discussion and comparison

According to the theoretical analysis in Sections 2 and 3, by non-maximally entangled channel, we propose two quantum multicast schemes including one sender and multiple receivers, which enable each receiver to obtain different quantum states synchronously. Next, we will give the discussion and comparison about our schemes by the following two aspects.

4.1. Success probability

In this section, we will analyze the success probabilities of two schemes, which are mathematically expressed as follows. For the first one, the total success probability (TSP_1) reads

$$\begin{aligned}\mathrm{TSP}_1 &= \sum_{M_{a_1}}\sum_{M_{a_2}}\sum_{M_A} P_{M_{a_1}} P_{M_{a_2}|M_{a_1}} P_{M_A|M_{a_2}M_{a_1}} \\ &= 16\times\frac{1}{16}\cdot 4\zeta_0^2\cdot 4\eta_0^2 \\ &= 16\zeta_0^2\eta_0^2. \end{aligned} \tag{21}$$

The relationship between the total success probability (TSP) and parameters (ζ_0 and η_0) is shown in Fig. 3. If the parameters are chosen at the beginning, the channels will become maximally entangled.

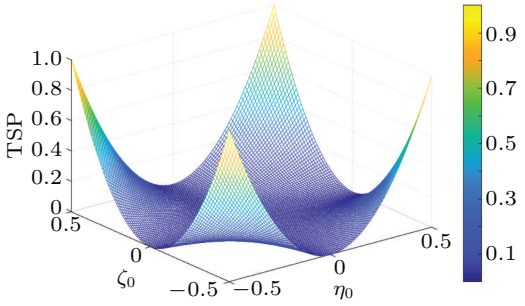


Fig. 3. The relation among TSP and the parameters ζ_0 and η_0 of channel entanglement.

For the second one, the total success probability (TSP_2) can be expressed as

$$\begin{aligned}\mathrm{TSP}_2 &= \sum_{M_{A_1}}\sum_{M_{D_1}M_{D_2}M_{D_3}} P_{M_{A_1}} P_{M_{D_1}M_{D_2}M_{D_3}|M_{A_1}} \\ &= 8\times\left(\frac{1}{2\sqrt{2}}\right)^2\times\left(\frac{1}{8}\right)^2(\rho_1^2+\rho_2^2+\cdots+\rho_8^2) \\ &= 1. \end{aligned} \tag{22}$$

It is shown that the second scheme is deterministic. As different states can be synchronously obtained by the multiple receivers, its efficiency for qubits approaches the maximal value.

4.2. Communication efficiency and comparisons

Intrinsic efficiency[42] of remote state preparation schemes is defined as

$$\mu = \frac{q_\mathrm{p}}{q_\mathrm{c}+x_t}, \tag{23}$$

where q_p denotes the number of prepared state, q_c stands for the number of quantum channel and auxiliary particles, and the number of classical bits is expressed as x_t. Applying Eq. (23), we compare the efficiencies of our proposed schemes with some other RSP schemes using non-maximally entangled channels in Table 3.

It is apparent that the intrinsic efficiencies of our schemes are highest among the others. Actually, the comparison of efficiency is only a part reflection of the actual situation. Our

proposed schemes have several other nontrivial features: (1) the non-maximally entangled channels are easy to generate compared to maximally entangled states; (2) for the second scheme, the assistance of auxiliary particle is not required, whereas in some present schemes;[26,46–49] (3) the success probability of the second scheme is independent of the channel parameters, and higher than the RSP protocols using non-maximally entangled channels.[31,32,43]

Table 3. Comparison with some other RSP schemes using non-maximally entangled channels.

Schemes	The number of prepared states (q_p)	Quantum consumption (q_c)	The number of classical bits (x_t)	Intrinsic efficiency (μ)
Ref. [43]	1	6(6+0)	5	1/11
Ref. [44]-S_1	4	11(11+0)	7	2/9
Ref. [44]-S_2	4	13(13+0)	9	2/11
Ref. [45]	2	8(8+0)	6	1/7
Scheme 1	8	14(12+2)	6	2/5
Scheme 2	3	9(9+0)	6	1/5

5. Conclusions

In addition to the conventional RSP with two parties, multiparty RSP plays a significant role in the general quantum network communication and quantum distributed computation. Different from joint remote state preparation, our concern is to investigate the multicast-based RSP with one sender to multiple receivers taking non-maximally entangled state as the shared entangled resources. In particular, each receiver can obtain different quantum information synchronously.

To this aim, we put forward two three-party multicast schemes based on the non-maximally entangled channels, where sender Alice remotely help receivers Bob and Charlie prepare different quantum states synchronously. Firstly, sender and controller construct two sets of completely orthogonal bases. After their measurements, two receivers recover the states by unitary operations. In the first scheme, each receiver can obtain the different four-qubit cluster-type states probably, and its probability depends on the channel parameters (ζ_0 and η_0). To improve the probabilities of such multicast RSP using the non-maximally entangled channel, by the nine-qubit partially entangled channel, the determined multicast CRSP scheme is given in the second scheme, so that Bob can obtain an arbitrary single-qubit state, and Charlie obtains an arbitrary two-qubit state synchronously. It is worth emphasizing that the probability of the second scheme can reach 100%, irrespective of l_i and k_i, and auxiliary resources are not required. It is shown that the non-maximally entangled channels may offer some significant advantages in quantum information transmission.

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