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Adaptive synchronization of chaotic systems with less measurement and actuation*

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We investigate the synchronization problem between identical chaotic systems only when necessary measurement (output) and actuation (input) are needed to be implemented by the adaptive controllers. A sufficient condition is derived based on the Lyapunov stability theory and Schur complementary lemma. Moreover, the theoretic result is applied to the Rikitake system and the hyperchaotic Liu system to show its effectiveness and correctness. Numerical simulations are presented to verify the results.

Keywords: chaotic systems, adaptive synchronization, linear matrix inequality

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1. Introduction

Since it was introduced by Pecora and Carroll[1] in 1990, the synchronization of chaotic systems has attracted increasing attention due to its possible applications in secure communication,[2–5] biomedical engineering,[6] information science,[7–9] and chemical reactions,[10,11] etc. A wide variety of control methods of chaos synchronization have been studied, such as observer design with or without time delay,[5,12,13] linear feedback control method,[14–16] sliding mode control,[17,18] adaptive control method,[19–27] and backstepping control method.[28–30]

In 2005, Huang[22] showed that if the vector function of a chaotic system is Lipschitz, then the synchronization problem can be achieved by an adaptive control scheme while the controllers are implemented to all the state variables. However, it has been revealed in the literature that not all the states are necessary to be controlled in order to achieve synchronization. For example, Liao *et al.*[19] showed that synchronization of a Lorenz system can be achieved only by a single adaptive feedback controller that is acted on the first or the second state equation. Similar results can be found for the Chen system,[20] the Rikitake system and the hyperchaotic Rössler system,[23] etc. Guo[23] proposed a method to reduce the number of controlled states and also observed that selecting only a part of states to be controlled does not always lead to synchronization. Thus an interesting question arises: how to verify whether the synchronization between two identical chaotic systems can be achieved if certain number of state variables are chosen to be controlled, and how to choose those state variables? To the best of our knowledge, there does not exist such a result in the literature.

Motivated by the aforementioned analysis, we investigate the above question in this article. A sufficient condition is derived based on the Lyapunov stability theory and it generalizes the results in Ref. [22], and also in Ref. [23] (in a similar way). In most cases, this condition can be formulated as an LMI feasibility problem which can be efficiently solved by numerical tool, such as LMI solver of Matlab.

The rest of this article is organized as follows. Section 2 presents the main result and its proof. Section 3 gives the applications of the proposed results to the adaptive synchronization for the Rikitake system and the hyperchaotic Liu system. Numerical simulations are given in Section 4. Finally, Section 5 contains the conclusion.

2. Sufficient conditions for identical chaotic synchronization

Consider the following nonlinear system

$$\Sigma: \quad \dot{x} = Ax + f(x), \tag{1}$$

where $x = (x_1, \ldots, x_n)^{\mathrm{T}} \in \Omega \subset \mathbb{R}^n$ is the state, $f(x) = (f_1(x), \ldots, f_n(x))^{\mathrm{T}} : \Omega \mapsto \mathbb{R}^n$, denotes the nonlinear part of the system which is assumed to be smooth and $A = (a_{ij})_{n\times n}$ is a real matrix, i.e., $a_{ij} \in \mathbb{R}$, for $1 \le i, j \le n$. Most chaotic systems can be described in the form (1), and thus without loss of generality, Ω will be regarded as an n-dimensional bounded closed set of $\mathbb{R}^n$. For the nonlinear part $f(x)$, the following assumption has been imposed.

Assumption 1 For any $x = (x_1, \ldots, x_n)^{\mathrm{T}} \in \Omega$ and $\hat{x} = (\hat{x}_1, \ldots, \hat{x}_n)^{\mathrm{T}} \in \Omega$, there exists a positive semi-definite matrix M such that

$$||f(\hat{x}) - f(x)||_2^2 \le (\hat{x} - x)^{\mathrm{T}} M (\hat{x} - x), \tag{2}$$

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where $(\hat{x}-x)=(\hat{x}_1-x_1,\ldots,\hat{x}_n-x_n)^{\mathrm{T}}$.

Remark 1 According to the fact that all smooth functions are locally Lipschitz, there exists a Lipschitz constant $l>0$ such that $||f(\hat{x})-f(x)||_2 \leq l||\hat{x}-x||_2$ when $x\in\Omega$ and $\hat{x}\in\Omega$. Then, Assumption 1 is not restrictive at all, and can be seen as a less restrictive condition than the classical Lipschitz one, since $(\hat{x}-x)^{\mathrm{T}}M(\hat{x}-x)\leq l^2||\hat{x}-x||_2^2$, where the Lipschitz constant l is equal to the square root of the maximal eigenvalue of the positive semi-definite matrix M.

Remark 2 Although inequality (2) in Assumption 1 is imposed to be satisfied locally for f, this property can be stated in a global sense by introducing the conventional saturation extension of f.[31,32] Denote $\sigma:\mathbb{R}^n\mapsto\Omega, x\mapsto\sigma(x)$ as any smooth bounded saturation function that coincides with x in Ω, i.e., $\sigma(x)=x$ for all $x\in\Omega$, and is saturated outside Ω. Then the local inequality property of nonlinear function $f(x)$ can be, without loss of generality, extended to the global one as $||f(\hat{x})-f(x)||_2^2\leq(\hat{x}-x)^{\mathrm{T}}M(\hat{x}-x)$ for $x\in\mathbb{R}^n$ and $\hat{x}\in\mathbb{R}^n$. In the rest of this article, for the sake of simplicity, we will not distinguish the local and global properties of $f(x)$ if there is no ambiguity.

For a given nonlinear system of the form (1), let us construct the following nonlinear dynamics to achieve the synchronization objective

$$\tilde{\Sigma}: \quad \dot{z}=Az+f(z)+Bu, \tag{3}$$

where $z=(z_1,\ldots,z_n)^{\mathrm{T}}$ is the state, $u=(u_1,\ldots,u_m)^{\mathrm{T}}$ denotes the designed control, and $B\in\mathbb{R}^{n\times m}$, named as the 'active matrix' before u, is to determine the controller for which lines of system (3) will be activated. For simplicity, when the i-th row of B is not zero, then we can say that the relative controller will be activated for the i-th row.

It is evident that the synchronization between systems (1) and (3) can be easily achieved if we have all states x of system (1) (i.e., $C=I_{n\times n}$, which means that all states x can be measured for system (1), and can be used to design controller u in system (3)) and if we active relative controllers for each line in system (3) (i.e., $B=I_{n\times n}$, which means each line has an independent controller u_i). However, an interesting question, as stated in the introduction, is: Can we use less measurement of system (1) to active fewer controllers u in system (3)? The positive answer of this question will be more applausive since fewer sensors are needed for system (1) to measure only the necessary states, and less energy will be consumed to actuate some necessary u in system (3).

To answer the above question, let us note

$$y=Cx$$

as the necessary states for the purpose of achieving synchronization between systems (1) and (3). In practice, those necessary states are respectively measured by different sensors. Thus, without loss of generality, we can assume that $C\in\mathbb{R}^{m\times n}$ is of full row rank. Moreover, it is assumed that each row of C contains only one non-zero element which provides the measurement of one necessary state. We would like to emphasize that such an assumption is not restrictive at all, since if C is not of such a form, then there will exist two invertible matrices Q_1 and Q_2 such that C can be transformed into Q_1CQ_2, which is exactly of the assumed form.

In order to achieve the synchronization between systems (1) and (3), we investigate the following output feedback control law:

$$u=K(Cz-y)=KCe, \tag{4}$$

where $e=z-x$ represents the synchronization error, and $K=\mathrm{diag}\{k_1,\ldots,k_m\}$ denotes the feedback gain vector.

Based on the above necessary state measurement y and the active control law u, we can state the following result.

Theorem 1 If there exist two positive definite diagonal matrices $L=\mathrm{diag}\{l_1,\ldots,l_m\}$ and $P=\mathrm{diag}\{\rho_1,\ldots,\rho_n\}$ such that

$$PBLC-\frac{1}{2}(A^{\mathrm{T}}P+PA)-\varepsilon M-\frac{\varepsilon}{4}P^2>0, \tag{5}$$

where $B=C^{\mathrm{T}}$ holds for a small positive ε, then systems (1) and (3) can be globally synchronized by the control scheme (4), where the feedback gains k_j, $1\leq j\leq m$, are updated by the following adaptive laws:

$$\dot{k}_j=-\gamma_j e_{\tau_j}^2, \tag{6}$$

where $\gamma_j>0$, $1\leq j\leq m$, are arbitrary positive constants and $1\leq\tau_1<\cdots<\tau_m\leq n$ such that the columns $\tau_1,\ldots,\tau_m$ of the matrix C are linearly independent.

Proof According to the structures of systems (1) and (3), one obtains the error dynamics

$$\dot{e}=Ae+f(z)-f(x)+Bu. \tag{7}$$

Denote $\Gamma^{-1}=\mathrm{diag}\{\frac{1}{\gamma_1},\ldots,\frac{1}{\gamma_m}\}$ and $\mathbf{1}_n=(1,1,\ldots,1)^{\mathrm{T}}$, and consider the following positive definite quadratic Lyapunov function

$$V=\frac{1}{2}e^{\mathrm{T}}Pe+\frac{1}{2}\mathbf{1}_n^{\mathrm{T}}PB\Gamma^{-1}(K+L)^2C\mathbf{1}_n.$$

Since $B=C^{\mathrm{T}}$, it is easy to see that both BKC and $B(K+L)C$ are diagonal matrices and consequently, we have $(BKC)^{\mathrm{T}}P=(BKC)P=P(BKC)$. With the above knowledge, calculating the differentiation of V with respect to time t, we obtain

$$\begin{aligned}\dot{V} &= \frac{1}{2}\dot{e}^{\mathrm{T}}Pe+\frac{1}{2}e^{\mathrm{T}}P\dot{e}-e^{\mathrm{T}}[PB(K+L)C]e\\ &= e^{\mathrm{T}}[\frac{1}{2}(A+BKC)^{\mathrm{T}}P+\frac{1}{2}P(A+BKC)]e\\ &\quad -e^{\mathrm{T}}[PB(K+L)C]e+e^{\mathrm{T}}P(f(z)-f(x))\\ &= e^{\mathrm{T}}(-PBLC+\frac{1}{2}(A^{\mathrm{T}}P+PA))e+e^{\mathrm{T}}P(f(z)-f(x))\end{aligned}$$

$$= \left[\, e^{\mathrm{T}}, \Delta f^{\mathrm{T}} \,\right] \begin{bmatrix} -PBLC + \frac{1}{2}A^{\mathrm{T}}P + \frac{1}{2}PA, & \frac{1}{2}P \\ \frac{1}{2}P, & 0 \end{bmatrix} \begin{bmatrix} e \\ \Delta f \end{bmatrix}, \quad (8)$$

where $\Delta f = f(z) - f(x)$.

Note that $f(x)$ satisfies the inequality (2) in Assumption 1, which is in fact equivalent to the following inequality:

$$\left[\, e^{\mathrm{T}}, \Delta f^{\mathrm{T}} \,\right] \begin{bmatrix} -M, & 0 \\ 0, & I \end{bmatrix} \begin{bmatrix} e \\ \Delta f \end{bmatrix} \leq 0.$$

By using the S-procedure, if the following condition

$$\begin{bmatrix} -PBLC + \frac{1}{2}A^{\mathrm{T}}P + \frac{1}{2}PA, & \frac{1}{2}P \\ \frac{1}{2}P, & 0 \end{bmatrix} < \varepsilon \begin{bmatrix} -M, & 0 \\ 0, & I \end{bmatrix} \quad (9)$$

holds for a small positive ε, then we can obtain

$$\dot{V} < \varepsilon \left[\, e^{\mathrm{T}}, \Delta f^{\mathrm{T}} \,\right] \begin{bmatrix} -M, & 0 \\ 0, & I \end{bmatrix} \begin{bmatrix} e \\ \Delta f \end{bmatrix} \leq 0,$$

which implies that V is positive and decrescent. It follows that the equilibrium point $(e_1 = \cdots = e_n = 0, k_j = k_j^*)$, for $1 \leq j \leq m$, of the error systems is uniformly stable which implies from Eqs. (4), (6) and (7) that $\dot{e}_1(t), \ldots \dot{e}_n(t)$ are also uniformly stable. Moreover, it is easy to conclude from Eq. (8) that the error states $e_1(t), \ldots, e_n(t)$ are quadratically integrable function, i.e. $e_1(t), \ldots, e_n(t) \in L_2$. Therefore, according to Barbalat's lemma, given any initial conditions, we always have $e_i(t) \to 0$, for $1 \leq i \leq n$, and $k_j \to k_j^*$, for $1 \leq j \leq m$, when t tends to infinity, which implies that the synchronization is achieved.

It remains to prove that the inequality (9) is equivalent to Eq. (5). Clearly, Eq. (9) can be written as

$$\begin{bmatrix} -PBLC + \frac{1}{2}A^{\mathrm{T}}P + \frac{1}{2}PA + \varepsilon M, & \frac{1}{2}P \\ \frac{1}{2}P, & -\varepsilon I \end{bmatrix} < 0.$$

According to Schur complementary lemma,[33] the above condition is equivalent to the following condition:

$$-PBLC + \frac{1}{2}(A^{\mathrm{T}}P + PA) + \varepsilon M + \frac{\varepsilon}{4}P^2 < 0,$$

which is equivalent to Eq. (5).

Remark 3 Note that in Eq. (5), both L and P are variables to be solved and therefore, in general, Eq. (5) is a convex quadratic matrix inequality. However, if the diagonal matrix P satisfies the condition $CP = C$, then we have $PBLC = (CP)^{\mathrm{T}}LC = C^{\mathrm{T}}LC = BLC$. Finally Eq. (5) can be simplified to a linear matrix inequality (LMI)

$$BLC - \frac{1}{2}(A^{\mathrm{T}}P + PA) - \varepsilon M - \frac{\varepsilon}{4}P^2 > 0, \quad (10)$$

which can be numerically solved via LMI solver.

Remark 4 For the case $m = n$, if we choose, in particular, $C = I_{n\times n}$ and $P = I_{n\times n}$, then the condition (10) will become

$$L - \frac{1}{2}(A^{\mathrm{T}} + A) - \varepsilon M - \frac{\varepsilon}{4}I_{n\times n} > 0, \quad (11)$$

where $L = \mathrm{diag}\{l_1, \ldots, l_n\}$. Obviously, inequality (11) can be always true if L is sufficient large, which implies that the result in Ref. [22] is just a special case of Theorem 1.

Remark 5 In a similar way, we can also obtain an extended version of the result in Ref. [23] where $k_1 = \cdots = k_m = k \in \mathbb{R}$, which is updated by the adaptive law $\dot{k} = -\gamma\Sigma_{j=1}^{m} e_{\tau_j}^2$ with $\gamma > 0$ being an arbitrary positive constant.

3. Applications

In this section, two examples are presented in order to verify the effectiveness and correctness of the proposed methodologies.

Example 1 The Rikitake system.[23,34]

The dynamics of the Rikitake system is of the form (1) with

$$A = \begin{bmatrix} -\mu & 0 & 0 \\ -a & -\mu & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad f(x) = \begin{bmatrix} x_2x_3 \\ -x_1x_3 \\ -x_1x_2 \end{bmatrix},$$

where $a = 5$, $\mu = 2$. The state trajectories $x_1(t), x_2(t), x_3(t)$ are globally bounded and hence, there exist three positive real constants $D_i, 1 \leq i \leq 3$, such that $|x_i(t)| \leq D_i, 1 \leq i \leq 3$ hold for all $t \geq 0$.

A straightforward calculation shows that for the Rikitake system, we have $||\Delta f||_2^2 \leq e^{\mathrm{T}}Me$, where

$$M = \begin{bmatrix} D_2^2 + D_3^2, & D_1D_2, & D_1D_3 \\ D_1D_2, & D_1^2 + D_3^2, & D_2D_3 \\ D_1D_3, & D_2D_3, & D_1^2 + D_2^2 \end{bmatrix}. \quad (12)$$

Select $C = (0, 0, 1)$. Put $B = C^{\mathrm{T}}$ and $K = \mathrm{diag}\{k\}$. Select $L = \mathrm{diag}\{l\}, P = \mathrm{diag}\{\rho_1, \rho_2, 1\}$, where l and ρ_1, ρ_2 are positive constants which should be determined later. Clearly, we have $CP = C$. A direct calculation gives

$$Bu = BKCe = \begin{bmatrix} 0 \\ 0 \\ ke_3 \end{bmatrix}, \quad (13)$$

where the feedback gains k is updated by $\dot{k} = -\gamma e_3^2$.

Selecting the upper bounds by $D_1 = 6$, $D_2 = 3$, $D_3 = 9$, and using Matlab's LMI toolbox, we can obtain that the LMI feasibility problem (10) is solvable for $\varepsilon = 10^{-4}$, $\rho_1 = 10$, $\rho_2 = 5$, $l = 18.6821$ and thus the synchronization problem for the Rikitake system can be achieved by the control law (13).

Example 2 The hyperchaotic Liu system.[35]

Dynamics of the hyperchaotic Liu system is of the form (1) with

$$A = \begin{bmatrix} -a & a & 0 & 0 \\ b & 0 & 0 & -1 \\ 0 & 0 & -c & 1 \\ d & 1 & 0 & 0 \end{bmatrix}, \quad f(x) = \begin{bmatrix} 0 \\ x_1x_3 \\ -x_1x_2 \\ 0 \end{bmatrix},$$

where $a = 10$, $b = 35$, $c = 1.4$, $d = 5$. The state trajectories $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t)$ are globally bounded and hence,

there exist four positive real constants D_i, $1 \le i \le 4$, such that $|x_i(t)| \le D_i$, $1 \le i \le 4$ hold for all $t \ge 0$.

By a straightforward calculation, we obtain that the hyperchaotic Liu system satisfies Assumption (1), i.e., $||\Delta f||_2^2 \le e^{\mathrm{T}} M e$, where

$$M = \begin{bmatrix} D_2^2 + D_3^2, & D_1 D_2, & D_1 D_3, & 0 \\ D_1 D_2, & D_1^2, & 0, & 0 \\ D_1 D_3, & 0, & D_1^2, & 0 \\ 0, & 0, & 0, & 0 \end{bmatrix}. \tag{14}$$

Select $C = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \end{bmatrix}$. Put $B = C^{\mathrm{T}}$, $K = \mathrm{diag}\{k_1, k_2\}$, and select $L = \mathrm{diag}\{l_1, l_2\}$, $P = \mathrm{diag}\{\rho, 1, 1, 1\}$, where l_1, l_2 and ρ are positive constants which should be determined later. Clearly, we have $CP = C$ and a direct calculation gives

$$Bu = BKCe = \begin{bmatrix} 0 \\ 4k_2 e_2 \\ 0 \\ k_1 e_4 \end{bmatrix}, \tag{15}$$

where the feedback gains k_1, k_2 are updated by $\dot{k}_1 = -\gamma_1 e_2^2$, $\dot{k}_2 = -\gamma_2 e_4^2$, since the columns 2, 4 of the matrix C are linearly independent.

Selecting the upper bounds by $D_1 = 30$, $D_2 = 52$, $D_3 = 82$, $D_4 = 100$ and using Matlab's LMI toolbox, we can obtain that the LMI feasibility problem (10) is solvable for $\varepsilon = 10^{-4}$, $\rho = 20$, $l_1 = 163.3964$, $l_2 = 54.0881$ and thus the synchronization problem for the hyperchaotic Liu system can be achieved by the control (15).

4. Numerical simulations

In this section, numerical simulations by MATLAB will be given and the fourth-order Runge–Kutta method is applied to approximate the solution of differential equations with a small chosen fixed time step size.

For the Rikitake system, the initial conditions of the master system and the salve system are chosen as $x(0) = (2, -1, 4)$ and $z(0) = (-3, 1, 6)$, respectively. Select $\gamma = 5$ and the initial conditions of the feedback gain by $k(0) = 1$. Figure 1 shows that the error states are asymptotically stable to zero by using the control law (13), while the control gains k tends to a negative constant, as t tends to infinity.

For the hyperchaotic Liu system, the initial conditions of the master system and the salve system are chosen as $x(0) = (15, 22, -46, -21)$ and $z(0) = (18, -13, -1, 37)$, respectively. Select $\gamma_1 = 5$, $\gamma_2 = 10$ and the initial conditions of the feedback gain by $k_1(0) = 50$, $k_2(0) = 100$. Figure 2 shows that the error states are asymptotically stable to zero by using the control law (15) while the control gains k_1 and k_2 tend to two negative constants, respectively, as t tends to infinity.

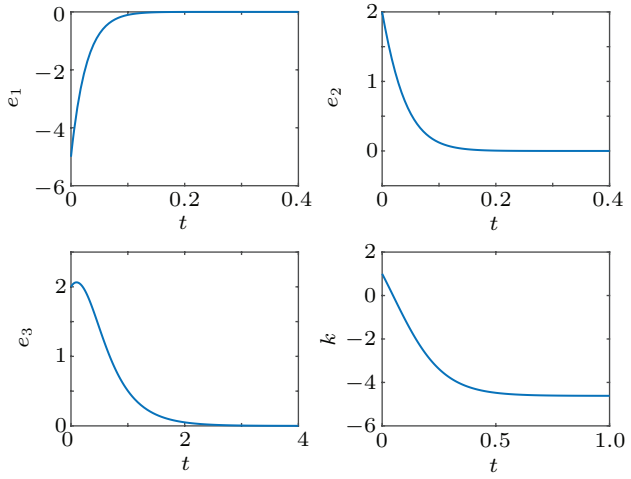


Fig. 1. Adaptive synchronization for the Rikitake system.

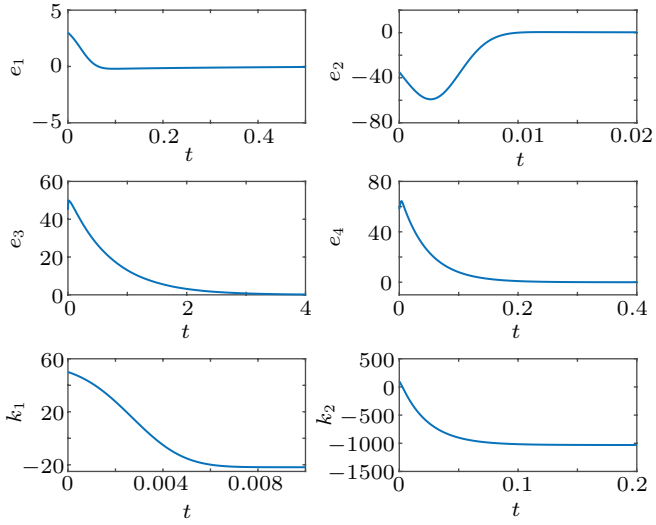


Fig. 2. Adaptive synchronization for the hyperchaotic Liu system.

5. Conclusion

In summary, we have studied the synchronization problem between two identical chaotic systems with less measurement and actuation. Under a weaker Lipschitz condition, a sufficient algebraic condition was proposed to guarantee the synchronization and meanwhile, an adaptive control scheme was designed to achieve the chaotic synchronization. The main result was proved by using the Lyapunov stability theory and Schur complementary lemma. Moreover, in most cases, this condition can be formulated as an LMI feasibility problem which can be efficiently solved by numerical tool. Our result generalized the results of Huang,[22] and also Guo[23] (in a similar way). Finally, the result has been applied to the Rikitake system and the hyperhaotic Liu system and numerical simulations are also presented to verify the result.

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