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Wave–particle duality relation with a quantum N-path beamsplitter*

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The wave–particle duality relation derived by Englert sets an upper bound of the extractable information from wave and particle properties in a two-path interferometer. Surprisingly, previous studies demonstrated that the introduction of a quantum beamsplitter in the interferometer could break the limitation of this upper bound, due to interference between wave and particle states. Along the other line, a lot of efforts have been made to generalize this relation from the two-path setup to the N-path case. Thus, it is an interesting question that whether a quantum N-path beamsplitter can break the limitation as well. This paper systemically studies the model of a quantum N-path beamsplitter, and finds that the generalized wave–particle duality relation between interference visibility and path distinguishability is also broken in certain situations. We further study the maximal extractable information's reliance on the interference between wave and particle properties, and derive a quantitative description. We then propose an experimental methodology to verify the break of the limitation. Our work reflects the effect of quantum superposition on wave–particle duality, and exhibits a new aspect of the relation between visibility and path distinguishability in N-path interference. Moreover, it implies the observer's influence on wave–particle duality.

Keywords: wave–particle duality, interference visibility, path distinguishability, quantum N-path beamsplitter

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1. Introduction

Bohr's principle of complementarity[1,2] indicates that a quanton may possess various properties, and depending on the exclusive detecting devices, we will observe one of the corresponding behaviours. Wave–particle duality,[3] which considers a quanton's behaviour in a two-path interferometer, is the most famous instance. If we choose to detect along which path the quanton propagates, in order to obtain information from its particle property, no interference visibility as information from its wave property is extractable, and vice versa. This can be understood more easily by a Mach–Zehnder interferometer (MZI), with presence or absence of the second beamsplitter (BS) to reveal the photon's wave or particle property respectively.[4] Because these two cases are incompatible, appearance of one behaviour suppresses that of the other, and complete wave and particle properties can never be observed simultaneously. The general relation between them is described by the inequality derived by Englert,[5] expressed as

$$V^2 + D^2 \leq 1, \tag{1}$$

where V is the interference visibility and D is the path distinguishability. It considers the intermediate situation where incomplete wavelike and particlelike behaviours are revealed, and sets a limitation on the amount of the total extractable information from wave and particle properties. Experiments with different kinds of quanton have verified this inequality.[4,6–9]

Surprisingly, previous studies demonstrated that the introduction of a quantum BS (QBS)[10–13] broke the limitation set by Eq. (1).[14] This occurs because as the QBS is in the superposition between presence and absence, it is entangled with wave and particle states of the quanton. When it collapses on a certain state, wave and particle states are enabled to be superimposed,[15] and interference between them causes the break.

On the other side, it is not natural to generalize the inequality of Eq. (1) to N-path case,[16,17] because dispute exists on finding an operational measure of how much wave property is observed. According to studies on N-path interference, visibility and path distinguishability are not complementary in some situations, and V is argued not to be a good measure for wave property.[18–20] Efforts have been made to handle with this issue.[21–29] In Ref. [30], the averaged visibility and path distinguishability are considered in all $N(N-1)/2$ two-path interferences, and are derived

$$\frac{2}{N(N-1)}\sum_{\text{pairs}} V_{mn} + \frac{2}{N(N-1)}\sum_{\text{pairs}} D_{Q_{mn}} \leq 1, \tag{2}$$

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where V_{mn} is the visibility of interference between paths m and n, and $D_{Q_{mn}}$ is the success probability in unambiguous quantum state discrimination (UQSD)[31–34] between the corresponding which-path detector (WPD) states.

However, it is unclear whether a generalized QBS is able to break the limitation set by Eq. (2) in N-path interference. In this paper, we use quantum Fourier transform[35] as a N-path BS (BS_N), and propose a model of N-path interferometer with a quantum BS_N (QBS_N). We study relation between the averaged visibility and path distinguishability, and find similar phenomena that the limitation in Eq. (2) is broken, but not for all detection ports. We further analyze the case where wave property partially interferes with particle property, and provide a quantitative description of the relation between wave property's interference with particle property and the maximal extractable information from these two properties. We then propose an implementation of a N-path interferometer with a QBS_N, and an experiment methodology to verify the break of the limitation. Our work exhibits the effect of quantum superposition on wave and particle duality, which provides a new aspect of the relation between visibility and path distinguishability in N-path interference.

2. Theoretical analysis

2.1. Duality relation with a QBS

In this part, we recap the break of the duality relation in Eq. (1), with a QBS introduced.[11] The whole system consists of the photon system (labeled as s) and the QBS system (labeled as c). When the photon enters the two-path MZI shown in Fig. 1(a), absence of the QBS makes the combined system in the state $|\mathrm{absence}\rangle_\mathrm{c}|p\rangle_\mathrm{s}$, where

$$|p\rangle_\mathrm{s} = \frac{1}{\sqrt{2}}\left(|0\rangle_\mathrm{s} + \mathrm{e}^{\mathrm{i}\phi}|1\rangle_\mathrm{s}\right), \tag{3}$$

and ϕ is the relative phase between path 0 and path 1. Since for the photon in the state $|p\rangle_\mathrm{s}$, the probability that we detect it at port 0 (or 1) in Fig. 1(a) stays invariant with ϕ, $|p\rangle_\mathrm{s}$ describes the photon's particle behaviour.

When the QBS is present in the interferometer, the state of the combined system is $|\mathrm{presence}\rangle_\mathrm{c}|w\rangle_\mathrm{s}$, where

$$|w\rangle_\mathrm{s} = \mathrm{e}^{\mathrm{i}\frac{\phi}{2}}\left(\cos\frac{\phi}{2}|0\rangle_\mathrm{s} - \mathrm{i}\sin\frac{\phi}{2}|1\rangle_\mathrm{s}\right), \tag{4}$$

which causes the probability to oscillate with ϕ and describes the photon's wave behaviour. When the QBS system is initially in $(1/\sqrt{2})|\mathrm{absence}\rangle_\mathrm{c} + (1/\sqrt{2})|\mathrm{presence}\rangle_\mathrm{c}$, the photon system is entangled with it as

$$\frac{1}{\sqrt{2}}\left(|\mathrm{absence}\rangle_\mathrm{c}|p\rangle_\mathrm{s} + |\mathrm{presence}\rangle_\mathrm{c}|w\rangle_\mathrm{s}\right). \tag{5}$$

When the QBS system finally collapses on $\cos\alpha|\mathrm{absence}\rangle_\mathrm{c} + \sin\alpha|\mathrm{presence}\rangle_\mathrm{c}$, the photon system becomes in a superposition between wave and particle states

$$|\psi\rangle_\mathrm{s} = F(\alpha,\phi)\left(\cos\alpha|p\rangle_\mathrm{s} + \sin\alpha|w\rangle_\mathrm{s}\right), \tag{6}$$

with the normalization factor

$$F(\alpha,\phi) = \left(1 + \sqrt{2}\cos\alpha\sin\alpha\cos\phi\right)^{-\frac{1}{2}}. \tag{7}$$

It is notable that, $|p\rangle_\mathrm{s}$ and $|w\rangle_\mathrm{s}$ generally are not orthogonal, and the normalization factor depends on α and phase ϕ.

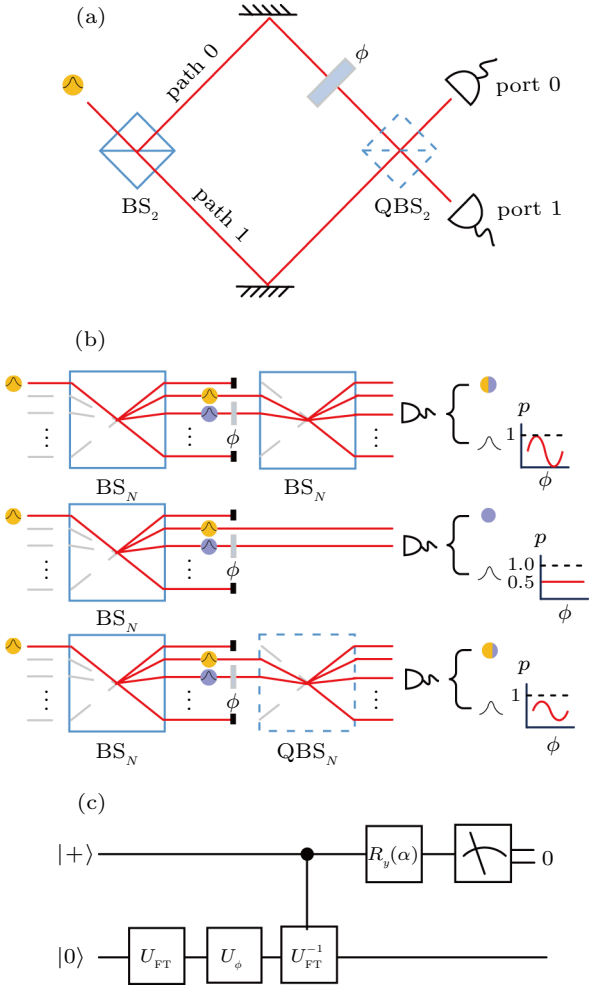


Fig. 1. Theoretical models. (a) A two-path interferometer with a QBS. The subscript of BS and QBS is used to denote number of paths, in contrast with the N-path case. A photon enters the MZI, with QBS in the superposition between absence and presence. Relative phase ϕ between these two paths is adjustable. (b) Our generalization of (a) to the N-path form. The above and the middle part correspond to presence and absence of the second BS_N respectively, and the bottom part denotes a QBS_N in the superposition between presence and absence. The dashed gray paths represent that no photon is input to them. For a photon detected at a certain port, we describe the extractable information from its particle and wave properties. p is the probability for the photon to be detected at this port. Information from wave property is obtained by visibility of p. To distinguish these two paths, we color the photon yellow in one path and blue in the other, and information from particle property is expressed with difference between areas of each color. (c) A circuit model of implantation of (b). We use quantum Fourier transform U_{FT} and its reverse U_{FT}^{-1} as BS_N. An ancillary system initially prepared as $|+\rangle_\mathrm{c} = (1/\sqrt{2})(|0\rangle_\mathrm{c} + |1\rangle_\mathrm{c})$ controls the presence and absence of U_{FT}^{-1}. With rotation $R_y(\alpha)$ and selecting the result of 0 after measurement on computational basis of the controlling system, there is a quantum superposition between wave and particle states of the photon system.

For a photon in a two-path interferometer, $|0\rangle_s$ and $|1\rangle_s$ are two possible states for it to propagate along the two paths. When the photon is in the quantum superposition between $|0\rangle_s$ and $|1\rangle_s$, the interference between the two paths causes the constructive or destructive output intensity. For a photon in the state $|\psi\rangle_s$, $|p\rangle_s$ and $|w\rangle_s$ are also two possible states that it may stay in, and can also be regarded as two possible "paths" where the photon may "propagate" in a fictitious interferometer. The interference between these two "paths" (i.e. $|p\rangle_s$ and $|w\rangle_s$) causes the constructive or destructive output "intensity" (i.e. V^2 and D^2) similarly, and for the constructive cases Eq. (1) may be broken.

To explain this in detail, we consider extractable information of $|\psi\rangle_s$ from wave and particle properties in Eq. (1). Since the QBS is asymmetric for most cases, different detection ports should be treated separately.[36] For port 0, the output intensity is proportional to

$$\begin{aligned} p_0(\alpha,\phi) &= F^2(\alpha,\phi)\big(\cos^2\alpha\,|_s\langle 0|p\rangle_s|^2 \\ &\quad +\sin^2\alpha\,|_s\langle 0|w\rangle_s|^2 + I_V\big) \\ &= \frac{1+\sin^2\alpha\cos\phi+2I_V(\alpha,\phi)}{2+4I_V(\alpha,\phi)-\sqrt{2}\sin 2\alpha}. \end{aligned} \tag{8}$$

To illustrate the effect of the interference between $|p\rangle_s$ and $|w\rangle_s$, we have divided $p_0(\alpha,\phi)$ into three terms, the particle term (i.e., $\cos^2\alpha\,|_s\langle 0|p\rangle_s|^2$), the wave term (i.e. $\sin^2\alpha\,|_s\langle 0|w\rangle_s|^2$), and the interference term $I_V(\alpha,\phi) = (\sqrt{2}/4)\sin 2\alpha(\cos\phi+1)$. $I_V(\alpha,\phi)$ causes $p_0(\alpha,\phi)$ no longer in a cosine relation with ϕ, and for some values of α it may enhance the visibility. By varying ϕ, we get the maximum and minimum $p_{0\max}(\alpha)$ and $p_{0\min}(\alpha)$. Information from wave property is related with interference visibility[37]

$$V_0(\alpha) = \frac{p_{0\max}(\alpha)-p_{0\min}(\alpha)}{p_{0\max}(\alpha)+p_{0\min}(\alpha)}. \tag{9}$$

Information from particle property is obtained by the path distinguishability $D_0(\alpha)$. For a photon detected at port 0, we have (unnormalized) probabilities of it coming from path 0 and 1 as

$$\begin{aligned} p_{00}(\alpha) &= \frac{1}{4}\left(\cos^2\alpha+\frac{1}{2}\sin^2\alpha+I_D(\alpha)\right), \\ p_{01}(\alpha) &= \frac{1}{8}\sin^2\alpha, \end{aligned} \tag{10}$$

where $I_D(\alpha) = \sqrt{2}\sin\alpha\cos\alpha$ is the interference term in $p_{00}(\alpha)$. $D_0(\alpha)$ is derived by difference between them (in next subsection, we will discuss this in details),

$$D_0(\alpha) = \frac{|p_{00}(\alpha)-p_{01}(\alpha)|}{p_{00}(\alpha)+p_{01}(\alpha)}. \tag{11}$$

Similar to $I_V(\alpha,\phi)$, $I_D(\alpha)$ affects $p_{00}(\alpha)$ and $D_0(\alpha)$, and for some values of α it may enhance the path distinguishability. Amount of total extractable information from both properties is expressed as $V_0^2(\alpha)+D_0^2(\alpha)$. As shown in Ref. [11], limitation of 1 in Eq. (1) is broken for some values of α. Similar results will be found if we detect port 1.

2.2. Different ways to obtain the photon's path information

In the original derivation of the duality relation of Eq. (1), the authors considered entanglement between the photon and the WPD system, and the interferometer consists of two symmetric BSs.[5] The photon's path information is revealed by discrimination between the corresponding states of WPD. Later in Refs. [4,36], the authors pointed out that, the path distinguishability D in Eq. (1) could also be introduced by an asymmetric output BS, which differs from the case where the input BS is asymmetric, and path predictability rather than path distinguishability is offered.[4] In this subsection, we will show that, though path distinguishability is considered in different setups and appears to be in different forms, it has the same meaning — "the measure of how much better one can do by using prior information and detectors than by just random guessing".[25]

In the first symmetric interferometer with WPD system in pure quantum states,[5] the photon's output state is $(1/2)\big(|0\rangle_s\big(|d_0\rangle_d+e^{i\phi}|d_1\rangle_d\big)+|1\rangle_s\big(|d_0\rangle_d-e^{i\phi}|d_1\rangle_d\big)\big)$, where $|d_0\rangle_d$ and $|d_1\rangle_d$ are states of WPD corresponding to path 0 and 1. The output intensity of port 0 is proportional to $\big||d_0\rangle_d+e^{i\phi}|d_1\rangle_d\big|^2$, and the visibility V defined by Eq. (9) is calculated to be $|_d\langle d_0|d_1\rangle_d|$. To obtain the photon's path information, one needs to distinguish between $|d_0\rangle_d$ and $|d_1\rangle_d$ to conclude which path the photon comes from. According to the theory of minimum-error state discrimination (MESD),[38] when $|d_0\rangle_d$ and $|d_1\rangle_d$ appear in equal probability, the maximal probability $P_{\max}$ of a successful distinction between them is $(1/2)+(1/2)\sqrt{1-|_d\langle d_0|d_1\rangle_d|^2}$. On the other side, the success probability is $1/2$ by just random guessing, and the difference between them is $(1/2)\sqrt{1-|_d\langle d_0|d_1\rangle_d|^2}$. For purpose of normalization, $D=\sqrt{1-|_d\langle d_0|d_1\rangle_d|^2}=\sqrt{1-V^2}$. When mixed states of WPD are related with the paths, V^2+D^2 is smaller than 1.

In the second asymmetric interferometer without a WPD system,[4,36] the photon's output state is $(1/\sqrt{2})(|0\rangle_s(\sqrt{R}+e^{i\phi}\sqrt{1-R})+|1\rangle_s(\sqrt{1-R}-e^{i\phi}\sqrt{R}))$, where R is the reflectivity of the output BS. Following the same definition, the visibility V is calculated to be $2\sqrt{R(1-R)}$. For a photon detected at port 0, the probabilities of it taking path $|0\rangle_s$ and $|1\rangle_s$ are $p_{00}=R$ and $p_{01}=1-R$. The optimal strategy to guess the path is to opt for path 0 if $p_{00}>p_{01}$ and opt for path 1 otherwise, and $P_{\max}=\max\{p_{00},p_{01}\}$. The difference is $P_{\max}-(1/2)=(1/2)\,|p_{00}-p_{11}|$. With normalization,

$D=|1-2R|=\sqrt{1-V^2}$. Though the definition of D changes, it has the same meaning with that in the first symmetric interferometer. The benefit of this model is the ellipsis of WPD, and it provides an operational definition of D. In practice, p_{00} (p_{01}) can be obtained by blocking path 1 (0) and recording the output intensity at port 0. In Ref. [11], the analysis of wave–particle duality with a QBS uses this definition of D, and the introduce of a QBS enables V^2+D^2 to break the limitation of 1.

In Ref. [30], instead of MESD, the authors define $D_{Q_{mn}}$ as the success probability in UQSD between WPD states $|d_m\rangle_d$ and $|d_n\rangle_d$. According to the relevant theory,[31–33] $D_{Q_{mn}}=1-|_d\langle d_m|d_n\rangle_d|=1-V_{mn}$. With $D_{Q_{mn}}=1-\sqrt{1-D_{mn}^2}$, where $D_{mn}=\sqrt{1-|_d\langle d_m|d_n\rangle_d|^2}$ is the definition of distinguishability in original duality relation of Eq. (1), $V_{mn}+D_{Q_{mn}}=1$ is equivalent with $V_{mn}^2+D_{mn}^2=1$.[22,23] Considering all (m,n) pairs, Eq. (2) is derived. Similarly, $D_{Q_{mn}}+V_{mn}$ is smaller than 1 for the mixed state case.

In our following analysis, as a generalization of Ref. [11], we consider a N-path interferometer with a $\mathrm{QBS_N}$ and study Eq. (2) in such a configuration. For each two-path interference by path m and n, we adopt the operational definition of D_{mn} in the asymmetric interferometer, and convert it to $D_{Q_{mn}}$ by the relation between them. This is sensible according to the above explanation.

2.3. An interferometer with quantum N-path beamsplitter

In Fig. 1(b), we propose our N-path interferometer with a QBS_N. The first BS_N generates a symmetric superposition among N paths. To investigate the limitation set by Eq. (2), each time we select two of the N paths and block the others. With presence of the second BS_N, path character of the detected photon is indistinguishable, and no information from its particle property is extractable. In contrast, full information from wave property is obtained by interference visibility of the output intensity, which equals to 1. While in the absence case, full information from particle property is obtained, but the output intensity stays at a constant with varying ϕ. When a QBS_N is in the intermediate situation, wave and particle states are superimposed, and part of the information from each property is extractable.

In Fig. 1(c), we provide an implementation of a QBS_N with a combined 2-level and N-level system. We embody the BS_N as quantum Fourier transform U_{FT} with the elements $(U_{\mathrm{FT}})_{mn}=\frac{1}{\sqrt{N}}\mathrm{e}^{2mn\pi\mathrm{i}/N}$ and its reverse U_{FT}^{-1}. Since all of the elements in U_{FT} and U_{FT}^{-1} have an equal amplitude, they make a maximal overlap of the N paths, and works as a BS_N.[39–42] When $N=2$, our scheme reduces to the MZI. Quantum feature of the second QBS_N is realized by quantum control from the ancillary 2-level system.

We analyze extractable information from wave and particle properties defined as averaged visibility $V_k(\alpha)$ and path distinguishability $D_{Q_k}(\alpha)$ in Eq. (2) for detection port k. Supposing we select path m and n, denoted as (m,n), and block all the other paths before the controlled-U_{FT}^{-1}, combined state of these two systems is expressed as

$$\begin{aligned}|\psi\rangle_{cs} &= \frac{1}{\sqrt{2}}|0\rangle_{\mathrm{c}}\left(\cos\alpha|p_{mn}\rangle_{\mathrm{s}}+\sin\alpha|w_{mn}\rangle_{\mathrm{s}}\right)\\ &\quad+\frac{1}{\sqrt{2}}|1\rangle_{\mathrm{c}}\left(-\sin\alpha|p_{mn}\rangle_{\mathrm{s}}+\cos\alpha|w_{mn}\rangle_{\mathrm{s}}\right),\end{aligned}\tag{12}$$

where $|p_{mn}\rangle_{\mathrm{s}}=(1/\sqrt{2})\left(|m\rangle_{\mathrm{s}}+\mathrm{e}^{\mathrm{i}\phi_{mn}}|n\rangle_{\mathrm{s}}\right)$, and $|w_{mn}\rangle_{\mathrm{s}}=(1/\sqrt{2})\left(U_{\mathrm{FT}}^{-1}|m\rangle_{\mathrm{s}}+\mathrm{e}^{\mathrm{i}\phi_{mn}}U_{\mathrm{FT}}^{-1}|n\rangle_{\mathrm{s}}\right)$. ϕ_{mn} is the relative phase between path m and n, and we assume $n>m$. $|w_{mn}\rangle_{\mathrm{s}}$ and $|p_{mn}\rangle_{\mathrm{s}}$ correspond to wave state and particle state in the interference between path m and n, respectively.

When the controlling system collapses on $|0\rangle_{\mathrm{c}}$ after rotation $R_y(\alpha)$, we have the superposition between $|p_{mn}\rangle_{\mathrm{s}}$ and $|w_{mn}\rangle_{\mathrm{s}}$ in the photon system,

$$|\psi\rangle_s=F_{mn}(\alpha,\phi_{mn})\left(\cos\alpha|p_{mn}\rangle_{\mathrm{s}}+\sin\alpha|w_{mn}\rangle_{\mathrm{s}}\right).\tag{13}$$

Likewise, the normalization factor

$$\begin{aligned}F_{mn}(\alpha,\phi_{mn}) &= \Big(1+\frac{\sin\alpha\cos\alpha}{\sqrt{N}}\Big(\cos\frac{2m^2\pi}{N}+\cos\frac{2n^2\pi}{N}\\ &\quad+2\cos\frac{2mn\pi}{N}\cos\phi_{mn}\Big)\Big)^{-\frac{1}{2}}.\end{aligned}\tag{14}$$

To derive visibility $V_{k,mn}(\alpha)$ of interference between path m and n, we calculate probability of the output intensity to be

$$\begin{aligned}p_{k,mn}(\alpha,\phi_{mn}) &= F_{mn}^2(\alpha,\phi_{mn})(\cos^2\alpha\,|_{\mathrm{s}}\langle k|p_{mn}\rangle_{\mathrm{s}}|^2\\ &\quad+\sin^2\alpha\,|_{\mathrm{s}}\langle k|w_{mn}\rangle_{\mathrm{s}}|^2\\ &\quad+\sin 2\alpha\mathrm{Re}(_{\mathrm{s}}\langle k|p_{mn}\rangle_{ss}\langle w_{mn}|k\rangle_{\mathrm{s}})),\end{aligned}\tag{15}$$

where Re represents the real part of a complex number. The three terms summed up in Eq. (15) correspond to the contribution of particle state, wave state, and the interference between them to the output respectively. The visibility for path m and n is derived from Eq. (9). An average over all $N(N-1)/2$ pairs gives $V_k(\alpha)$ as the extractable information from wave property of the input photon.

$D_{Q_k}(\alpha)$ can be obtained similarly with the two-path case. For the photon detected at port k, the probability of it coming from path $l=m$ or n is

$$p_{kl}(\alpha)=\frac{\sin^2\alpha}{4N^2}+\left(\frac{\cos^2\alpha}{4N}+\frac{\sin 2\alpha}{4N^{\frac{3}{2}}}\cos\frac{2l^2\pi}{N}\right)\delta_{kl},\tag{16}$$

where δ_{kl} equals to 1 if $k=l$ and 0 otherwise. Extractable information from particle property is expressed as $D_{Q_{k,mn}}(\alpha)=$

$1-\sqrt{1-D^2_{k,mn}(\alpha)}$, with

$$D_{k,mn}(\alpha)=\frac{|p_{km}-p_{kn}|}{p_{km}+p_{kn}}. \tag{17}$$

2.4. Deriving break of the limitation

In this subsection, we fix the detection port at $k=0$ for convenience. A more general case will be discussed later. We divide all the pairs into two parts. The first part corresponds to $N-1$ pairs containing path 0, namely $(0,n)$. We derive visibility

$$V_{0,0n}(\alpha)=\frac{|ad-bc|}{bd-ac}, \tag{18}$$

with four parameters

$$\begin{aligned} a&=\frac{\sin^2\alpha}{N}+\frac{\sin 2\alpha}{2\sqrt{N}},\quad b=a+\frac{\cos^2\alpha}{2},\\ c&=\frac{\sin 2\alpha}{\sqrt{N}},\quad d=\frac{c}{2}\left(1+\cos\frac{2n^2\pi}{N}\right)+1, \end{aligned} \tag{19}$$

and path distinguishability

$$\begin{aligned} D_{Q_{0,0n}}(\alpha)&=1-\sqrt{1-D^2_{0,0n}(\alpha)},\\ D_{0,0n}(\alpha)&=\frac{\left|\cos^2\alpha+\dfrac{\sin 2\alpha}{\sqrt{N}}\right|}{\cos^2\alpha+\dfrac{2\sin^2\alpha}{N}+\dfrac{\sin 2\alpha}{\sqrt{N}}}. \end{aligned} \tag{20}$$

For the remaining second part, namely $(m\neq 0,n)$, we have

$$p_{0,mn}(\alpha,\phi_{mn})=\frac{\sin^2\alpha}{N^2}(1+\cos\phi_{mn}). \tag{21}$$

For $\sin\alpha\neq 0$, the minimum is 0 at $\cos\phi_{mn}=-1$, and we have $V_{0,mn}(\alpha)=1$. Otherwise, $V_{0,mn}(\alpha)=0$ since $p_{0,mn}(\alpha,\phi_{mn})\equiv 0$. Meanwhile, $D_{0,mn}(\alpha)=0$ under this circumstance. Then we focus on extractable information from average of the first part

$$\begin{aligned} V'_0(\alpha)&=\frac{1}{N-1}\sum_{n=1}^{N-1}V_{0,0n}(\alpha),\\ D'_{Q_0}(\alpha)&=\frac{1}{N-1}\sum_{n=1}^{N-1}D_{Q_{0,0n}}(\alpha). \end{aligned} \tag{22}$$

Since $D_{Q_{0,0n}}(\alpha)$ is independent of n, it equals to $D'_{Q_0}(\alpha)$. When $\tan\alpha_0=-\sqrt{N}$, $D'_{Q_0}(\alpha_0)$ reaches the maximum of 1, and at this time

$$V'_0(\alpha_0)=\frac{1}{N-1}\sum_{n=1}^{N-1}\frac{2}{N-\cos\frac{2n^2\pi}{N}}, \tag{23}$$

which is always positive. On the other side, summation of the two averaged quantities over pairs in second part equals to 1 at this time. Thus, according to Eq. (2), amount of total extractable information expressed as $V_0(\alpha_0)+D_{Q_0}(\alpha_0)$ exceeds 1. In Figs. 2(a) and 2(b), we calculated $V_0(\alpha)$, $D_{Q_0}(\alpha)$, and their summation, when $N=2$ and $N=4$ with α varied from 0 to π, and maximum of $V_0+D_{Q_0}$ reaches 1.667 and 1.278 when $\tan\alpha=-\sqrt{2}$ and $\tan\alpha=-\sqrt{4}$. As N gets larger, ratio of the first $N-1$ pairs over all the $N(N-1)/2$ pairs gets smaller. Consequently, $D_{Q_0}(\alpha_0)$ and $V_0(\alpha_0)+D_{Q_0}(\alpha_0)$ decreases obviously, but the summation still exceeds 1. Figure 2(c) shows the comparison among cases with N from 2 to 7, and $\alpha_0=\pi-\arctan\sqrt{N}$ gets closer to $\pi/2$ as N gets larger.

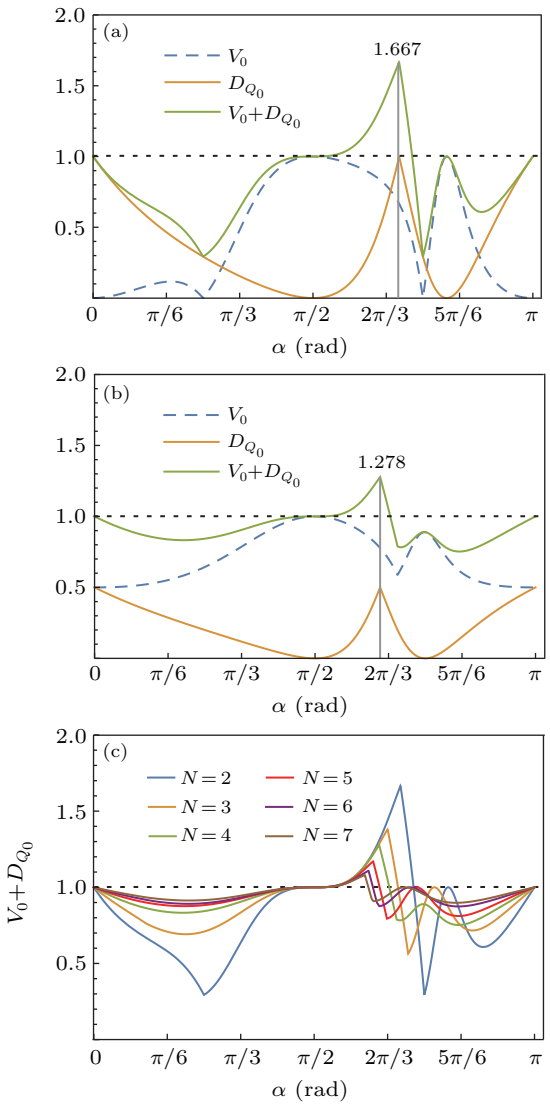


Fig. 2. Theoretical results with $k=0$. (a) Extractable information V_0 from wave property, information D_{Q_0} from particle property and their summation when $N=2$. The maximums of D_{Q_0} and $V_0+D_{Q_0}$ occur when $\tan\alpha=-\sqrt{2}$. It is in accordance with the result of V^2+D^2 in Ref. [11]. (b) The same calculation when $N=4$. The maximums of D_{Q_0} and $V_0+D_{Q_0}$ occur when $\tan\alpha=-\sqrt{4}$ and drop dramatically compared with (a). $V_0(\alpha)$ and $V_0(\alpha)+D_{Q_0}(\alpha)$ are discontinuous at $\sin\alpha=0$, where $V_0=0$ and $D_{Q_0}=0.5$. (c) Amount of total extractable information with N from 2 to 7. $V_0+D_{Q_0}$ decreases as N gets larger, and tends to become less variant with α, but still exceeds the limitation of 1. More details are analyzed in the main text.

2.5. A more general case when $k\neq 0$

For a more general case where port $k\neq 0$, we calculated $V'_k(\alpha)$ and $D'_{Q_k}(\alpha)$ for the first part, i.e. pairs containing path k, as quantities for the second part are the same with those when $k=0$. We exhibit derivation of visibility and path dis-

tinguishability for the cases $(k, n > k)$, and cases of the counterpart $(m, k > m)$ are handled similarly.

Starting from Eqs. (14) and (15), we have

$$p_{k,kn}(\alpha, \phi_{kn}) = \frac{a\cos\phi_{kn} + e\sin\phi_{kn} + b}{c\cos\phi_{kn} + d}, \tag{24}$$

with

$$\begin{aligned} a &= \frac{\sin^2\alpha}{N}\cos\frac{2k(k-n)\pi}{N} + \frac{\sin 2\alpha}{2\sqrt{N}}\cos\frac{2kn\pi}{N}, \\ b &= \frac{\cos^2\alpha}{2} + \frac{\sin^2\alpha}{N} + \frac{\sin 2\alpha}{2\sqrt{N}}\cos\frac{2k^2\pi}{N}, \\ c &= \frac{\sin 2\alpha}{2\sqrt{N}}\cos\frac{2kn\pi}{N}, \\ d &= 1 + \frac{\sin 2\alpha}{2\sqrt{N}}\left(\cos\frac{2k^2\pi}{N} + \cos\frac{2n^2\pi}{N}\right), \\ e &= -\frac{\sin^2\alpha}{N}\sin\frac{2k(k-n)\pi}{N} + \frac{\sin 2\alpha}{2\sqrt{N}}\sin\frac{2kn\pi}{N}. \end{aligned} \tag{25}$$

By derivation over ϕ_{kn}, we get the maximum and minimum of $p_{k,kn}$. The visibility

$$V_{k,kn}(\alpha) = \frac{\sqrt{(ad-bc)^2 + e^2(d^2-c^2)}}{|bd-ac|}, \tag{26}$$

$$V'_k(\alpha) = \frac{1}{N-1}\sum_{n=1}^{N-1} V_{k,kn}(\alpha). \tag{27}$$

When $\sin 2\alpha \neq 0$, path distinguishability is expressed as

$$D'_k(\alpha) = -\frac{1}{1 + r_k(\alpha)}, \tag{28}$$

where

$$r_k(\alpha) = \frac{\dfrac{2}{N}\tan\alpha}{\cot\alpha + \dfrac{2}{\sqrt{N}}\cos\dfrac{2k^2\pi}{N}}. \tag{29}$$

The interval peak value of $D'_k(\alpha)$ occurs at the minimum of $r_k(\alpha)$, with $\tan\alpha_0 = -\sqrt{N}\sec(2k^2\pi/N)$ and $r_k(\alpha_0) = -2\sec^2(2k^2\pi/N)$. When $k^2/N = z/2$ with $z \in \mathbb{N}$, $D'_{Q_k}(\alpha_0)$ reaches the maximum 1 with non-zero $V'_k(\alpha_0)$, and the amount of extractable information from both properties exceeds 1. In Fig. 3(a), we exhibit $V'_k(\alpha) + D'_{Q_k}(\alpha)$ when $N = 6$ with different k. The exceeding occurs only when $k = 0$ and 3, with $k^2/N = 0$ and $3/2$, which is in accordance with our analysis. Contrast with the previous study on the two-path case with a QBS,[11] where the violation can be observed for both detection ports, in the N-path case, the violation is not available for some values of N and the detection port k, which is exhibited for photons detected at port 1 and 2. This reflects that, even though Eq. (2) is derived from pairs of paths, a QBS$_N$ embodied by quantum Fourier transform can not be regarded as a simple construction of several two-path QBSs, and it makes the state of superposition between wave and particle properties for path m and n differ from that derived with a two-path QBS. There are exceptions where $k^2/N \neq z/2$, but $r_k(\alpha)$ is also small enough to break the limitation. The case where $N = 7$ and $k = 2$ is an instance.

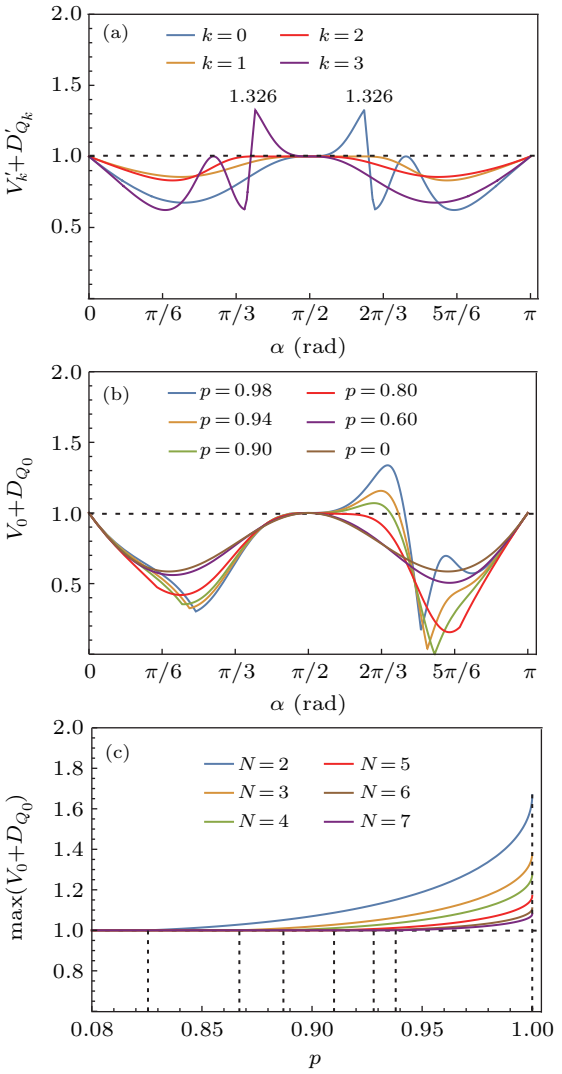


Fig. 3. Results of further calculations. (a) $V'_k + D'_{Q_k}$ with different k when $N = 6$. The case for detection port k is identical to that for $N-k$, due to the periodicity of each element in quantum Fourier transform. Break of the limitation corresponds to the situation when $k^2/N = 0$ or $3/2$. (b) $V_0 + D_{Q_0}$ with different p when $N = 2$. (c) A quantitative relation between the maximum of $V_0 + D_{Q_0}$ and p, with N varied from 2 to 7. As p decreases, the maximum drops dramatically. The minimal value of p that makes the violation for each N is 0.825, 0.867, 0.887, 0.910, 0.928 and 0.938 respectively.

2.6. Quantitative analysis of wave–particle interference's effect

We further analyze the interference between wave and particle properties in this model, and provide a quantitative relation between the maximum of $V_k + D_{Q_k}$ and the interference. In Eq. (15), not only output intensity of the wave part, but also that of the interference between the wave part and particle part, oscillate with ϕ_{mn}. The difference between their phases is $2k^2\pi/N$, for $k = m$ or n. Both wave state and interference between wave and particle states give information from wave property. The break occurs when $2k^2\pi/N = z\pi$, which indicates the enhancement on the oscillation of $|w_{mn}\rangle_s$ by that of the interference between $|p_{mn}\rangle_s$ and $|w_{mn}\rangle_s$.

To quantitatively analyze the wave–particle superposition's effect on the violation of Eq. (2), we consider a QBS_N in partial quantum superposition of $|\text{absence}\rangle_\text{c}$ and $|\text{presence}\rangle_\text{c}$, which can be realized by initializing the controlling system in Fig. 1(c) as

$$\rho_\text{c} = p|+\rangle_\text{c}\langle+|_\text{c} + (1-p)\left(\frac{1}{2}|0\rangle_\text{c}\langle 0|_\text{c} + \frac{1}{2}|1\rangle_\text{c}\langle 1|_\text{c}\right), \quad (30)$$

where parameter p represents the coherence of ρ_c. Consequently, in the interference between path m and n, the photon is in the partial quantum superposition between wave and particle states,

$$\begin{aligned}\rho_\text{s} \;=\; & p|\psi\rangle_\text{s}\langle\psi|_\text{s} + (1-p)\Big(\cos^2\alpha|p_{mn}\rangle_\text{s}\langle p_{mn}|_\text{s} \\ & + \sin^2\alpha|w_{mn}\rangle_\text{s}\langle w_{mn}|_\text{s}\Big). \end{aligned} \quad (31)$$

The parameter p quantifies the quantum superposition between wave and particle properties. A straightforward deduction indicates the result is to replace all the $\sin 2\alpha$ terms in the previous representations by $p\sin 2\alpha$, and it makes no difference for pairs that does not contain path k. For convenience, we fix the detection port k at 0. The result for $N=2$ is shown in Fig. 3(b), and α_0 that makes maximum of $V_0 + D_{Q_0}$ decreases as p gets smaller. In Fig. 3(c), we exhibit the maximum of $V_0 + D_{Q_0}$'s reliance on p with N varied from 2 to 7, and the maximum decreases dramatically as p gets smaller and finally stays at a constant of 1. As N gets larger, the maximum decreases faster with p. Theoretically, cases for all values of N and k can be calculated, and a general quantitative description of wave–particle interference's effect on the break of the limitation is available.

3. An optical scheme of the N-path interferometer with a QBS_N

Based on the circuit model in Fig. 1(c), we propose an optical scheme to demonstrate the N-path interference with a QBS_N using a single photon in $2N$ path modes, and an experiment methodology to verify the break of Eq. (2) in this setup.

The whole scheme is shown in Fig. 4, and can be divided into two parts, each containing N paths. Both parts are placed on a U_FT with the dimension of N to generate a superposition of N paths with an equal amplitude, and a phase adjuster U_ϕ to vary the phase of each path. Then U_FT^{-1} with the dimension of N is placed only on paths from N to $2N-1$. Finally, U_α, which serves as $R_y(\alpha)$, is composed of N variable BSs that combine paths i and $i+N$ for i from 0 to $N-1$, with a reflectivity of $\cos^2\alpha$. The photon is initially prepared as $(1/\sqrt{2})\big(|0\rangle_\text{s} + |n\rangle_\text{s}\big)$, and finally evolves into $(1/\sqrt{2})|0\rangle_\text{c}\big(\cos\alpha|p_N\rangle_\text{s} + \sin\alpha|w_N\rangle_\text{s}\big) + (1/\sqrt{2})|1\rangle_\text{c}\big(\sin\alpha|p_N\rangle_\text{s} - \cos\alpha|w_N\rangle_\text{s}\big)$, where $|p_N\rangle_\text{s} = \sum_{j=0}^{N-1}(1/\sqrt{N})|j\rangle_\text{s}\mathrm{e}^{\mathrm{i}\phi_j}$ and $|w_N\rangle_\text{s} = \sum_{j=0}^{N-1}U_\text{FT}^{-1}(1/\sqrt{N})|j\rangle_\text{s}\mathrm{e}^{\mathrm{i}\phi_j}$ are particle and wave states for N paths respectively. For convenience, we express the $2N$ path modes with a product of a qubit and a quNit. When only the first N paths are detected, and the rest are ignored, the first qubit collapses onto $|0\rangle$, and the photon becomes in a quantum superposition between wave and particle states for N paths. To measure the visibility V_{mn} and path distinguishability D_{mn} or $D_{Q_{mn}}$ for interference between path m and n at detection port k, we can block all the other paths, leaving two pairs of paths (m,n) and $(m+N, n+N)$, vary the relative phase in each pair with the same amount, and follow the operations in Ref. [11].

The whole setup is feasible with near-term devices. Since the input is fixed as a simple superposition between path 0 and N, the first two U_FTs can be simplified with several BSs. U_ϕ can be implemented by a phase adjuster on each path. U_α can be implemented with variable BSs. U_FT^{-1} as a BS_N can be implemented using the methods in Refs. [43,44], which propose a scheme to construct an arbitrary unitary transformation on finite dimensional path modes. Besides, since the parameter α which decides the state of the QBS_N, can be determined even after the detection of the photon when entangled with an ancillary system, a delayed-choice verification of break of the limitation is available.

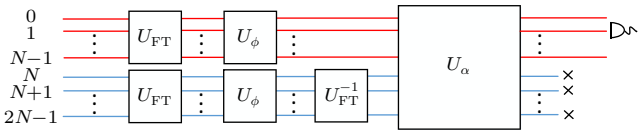


Fig. 4. An optical scheme to experimentally demonstrate the N-path interference with a QBS_N. To realize the combined system of a N-level photon system and a 2-level controlling system, $2N$ path modes are needed totally. The U_FT^{-1} as the output BS_N is present only when the photon propagates on path from N to $2N-1$, which makes the output BS_N quantum controlled.

4. Discussion

4.1. Observer's influence on visibility and path distinguishability

As a further explanation, we point out here that the break of the limitation and the interference between $|p_{mn}\rangle_\text{s}$ and $|w_{mn}\rangle_\text{s}$ are the consequence of loss of detection, which is caused by postselection from the view of observers. We explain this by a two-path MZI, and cases for N-path interference can be generalized similarly.

The general description of a BS is a unitary transformation

$$B(R) = \begin{pmatrix} \sqrt{R} & \sqrt{1-R} \\ \sqrt{1-R} & -\sqrt{R} \end{pmatrix}. \quad (32)$$

Presence and absence of a symmetric BS give $R=1/2$ and $R=1$ respectively, which corresponds to minimal and maximal path distinguishability. When we consider a QBS and postselect the controlling system with $|\alpha\rangle_\text{c} = \cos\alpha|0\rangle_\text{c} + \sin\alpha|1\rangle_\text{c}$, we actually make a transformation as

$$B_Q(\alpha) = \cos\alpha B(0) + \sin\alpha B(1/2), \quad (33)$$

which is non-unitary and causes losses.

As we make postselection, cases where the controlling system collapses on the orthogonal state $|\alpha^{\perp}\rangle_{\rm c}$ are discarded. The observed phenomenon orients from an incomplete detection. If we supplement the discarded information from $|\alpha^{\perp}\rangle_{\rm c}$, there will be no loss of information, and the inequality still holds. This corresponds to the case where we detect the photon regardless of state of the controlling system. Thus, the break occurs as if we subjectively pick part of the observation and make an artificial interference between wave and particle properties. A similar point of view is indicated in Ref. [10].

4.2. Comparison with previous similar studies

In Refs. [45,46], it was pointed out that the loss in MZI could cause the break of the limitation, and loss indeed exists in the interferometer with a QBS (or QBS$_N$) when postselection is conducted. However, as discussed below, although the breaks of Eq. (1) in both models are related with the loss in the interferometer, they are not equivalent, and the model proposed in those studies can not generate the quantum superposition between wave and particle states.

Similar to Subsection 4.1, the loss in Refs. [45,46] can be represented by the non-unitary matrix as

$$\begin{aligned} B'(R_1,R_2) &= \begin{pmatrix} \sqrt{1-R_1} & 0 \\ 0 & \sqrt{1-R_2} \end{pmatrix} B\left(\frac{1}{2}\right) \\ &= \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{1-R_1} & \sqrt{1-R_1} \\ \sqrt{1-R_2} & -\sqrt{1-R_2} \end{pmatrix}, \end{aligned} \tag{34}$$

where R_1 and R_2 are reflectivities of the two beamsplitters after the standard MZI and represent the losses. Since $B'(R_1,R_2)$ and $B_Q(\alpha)$ are different except for the case where $\cos\alpha = 0$ and $R_1 = R_2 = 0$ which corresponds to a lossless MZI, these two models are generally not equivalent. The former model with two additional beamsplitters can not be used to study the effect of the interference between wave and particle states on the duality relation. Besides, as mentioned in Ref. [45], the loss in their model has no effect on the visibility. However, in the model of a QBS (or QBS$_N$), both the visibility and path distinguishability are affected, which is exhibited in Fig. 2.

4.3. Wave–particle duality with a quantum WPD[47]

As a supplementary analysis, we consider wave–particle duality in a symmetric N-path interferometer, with a quantum WPD. Similar to a QBS$_N$, when a quantum WPD's position degree of freedom is initially in $(1/\sqrt{2})|\text{absence}\rangle_{\rm c} + (1/\sqrt{2})|\text{presence}\rangle_{\rm c}$, and finally collapses on $\sin\alpha|\text{presence}\rangle_{\rm c} + \cos\alpha|\text{absence}\rangle_{\rm c}$, the combined system is in the state of wave–particle superposition $\cos\alpha|p\rangle_{sd} + \sin\alpha|w\rangle_{sd}$, where

$$|w\rangle_{sd} = \frac{1}{\sqrt{N}}\left(|0\rangle_{\rm s} + \cdots + |N-1\rangle_{\rm s}\right)|d_{\rm ini}\rangle_d,$$

$$|p\rangle_{sd} = \frac{1}{\sqrt{N}}\left(|0\rangle_{\rm s}|d_0\rangle_d + \cdots + |N-1\rangle_{\rm s}|d_{N-1}\rangle_d\right), \tag{35}$$

and $|d_{\rm ini}\rangle_d$ is the initial state of WPD. We can rewrite it as

$$\frac{1}{\sqrt{N}}\left(|0\rangle_{\rm s}|\tilde{d}_0\rangle_d + \cdots + |N-1\rangle_{\rm s}|\tilde{d}_{N-1}\rangle_d\right), \tag{36}$$

where $|\tilde{d}_i\rangle_d = \cos\alpha|d_i\rangle_d + \sin\alpha|d_{\rm ini}\rangle_d$. In this case, the quantum superposition between wave and particle properties changes state of WPD corresponding to each path, but the combined state is still in the standard form of entanglement between the photon and the WPD, and relations of wave–particle duality derived in previous studies[23,25,26] based such a configuration still hold, where quantum coherence instead of visibility is considered as a measure for wave property. In this sense, coherence tends to be a more suitable than visibility, which is in agreement with the relevant studies.

5. Conclusion

In conclusion, we study the extractable information from wave and particle properties in N-path interference, with a QBS$_N$ in the quantum superposition between presence and absence. The consequent interference between wave and particle states of the quanton causes summation of averaged visibility and path distinguishability to break the limitation of 1. Through a quantitative analysis, we find that the maximum of $V + D_Q$ decreases dramatically, when wave property interferes with particle property partially. As N getters larger, the maximum decreases faster. We then propose a scheme to experimentally verify our analysis. Our work exhibits the effect of quantum superposition on wave–particle duality, and provides a new aspect of relation between visibility and path distinguishability in N-path interference.

References

[1] Bohr N 1928 *Naturwissenschaften* **16** 245
[2] Bohr N 1928 *Nature* **121** 580
[3] Feynman R P, Leighton R B and Sands M 1965 *The Feynman Lectures on Physics* (American Association of Physics Teachers)
[4] Jacques V, Wu E, Grosshans F, Treussart F, Grangier P, Aspect A, and Roch J F 2008 *Phys. Rev. Lett.* **100** 220402
[5] Englert B G 1996 *Phys. Rev. Lett.* **77** 2154
[6] Dürr S, Nonn T and Rempe G 1998 *Phys. Rev. Lett.* **81** 5705
[7] Peng X, Zhu X, Fang X, Feng M, Liu M and Gao K 2003 *J. Phys. A: Math. Gen.* **36** 2555
[8] Peng X, Zhu X, Suter D, Du J, Liu M and Gao K 2005 *Phys. Rev. A* **72** 052109
[9] Schwindt P D D, Kwiat P G and Englert B G 1999 *Phys. Rev. A* **60** 4285
[10] Ionicioiu R and Terno D R 2011 *Phys. Rev. Lett.* **107** 230406
[11] Tang J S, Li Y Y, Xu X Y, Xiang G Y, Li C F and Guo G C 2012 *Nat. Photon.* **6** 600
[12] Peruzzo A, Shadbolt P, Brunner N, Popescu S and O'Brien J L 2012 *Science* **2012** 634
[13] Kaiser F, Coudreau T, Milman P, Ostrowsky D B and Tanzilli S 2012 *Science* **2012** 637
[14] Tang J S, Li Y L, Li C F and Guo G C 2013 *Phys. Rev. A* **88** 014103
[15] Wang K, Xu Q, Zhu S and Ma X S 2019 *Nat. Photon.* **13** 872

[16] Dürr S 2001 *Phys. Rev. A* **64** 042113
[17] Paul T and Qureshi T 2017 *Phys. Rev. A* **95** 042110
[18] Mei M and Weitz M 2001 *Phys. Rev. Lett.* **86** 559
[19] Luis A 2001 *J. Phys. A: Math. Gen.* **34** 8597
[20] Bimonte G and Musto R 2003 *J. Phys. A: Math. Gen.* **36** 11481
[21] Baumgratz T, Cramer M and Plenio M B 2014 *Phys. Rev. Lett.* **113** 140401
[22] Siddiqui M A and Qureshi T 2015 *Theor. Exp. Phys.* **2015** 083A02
[23] Bera M N, Qureshi T, Siddiqui M A and Pati A K 2015 *Phys. Rev. A* **92** 012118
[24] Jia A A, Yang J, Yan S H, Hu Q Q, Luo Y K and Zhu S Y 2015 *Chin. Phys. B* **24** 080302
[25] Bagan E, Bergou J A, Cottrell S S and Hillery M 2016 *Phys. Rev. Lett.* **116** 160406
[26] Bagan E, Calsamiglia J, Bergou J A and Hillery M 2018 *Phys. Rev. Lett.* **120** 050402
[27] Roy P and Qureshi T 2019 *Phys. Scr.* **94** 095004
[28] Mishra S, Venugopalan A and Qureshi T 2019 *Phys. Rev. A* **100** 042122
[29] Bagan E, Bergou J A and Hillery M 2020 *Phys. Rev. A* **102** 022224
[30] Qureshi T 2019 *Phys. Rev. A* **100** 042105
[31] Ivanovic I D 1987 *Phys. Lett. A* **123** 257
[32] Dieks D 1988 *Phys. Lett. A* **126** 303
[33] Peres A 1988 *Phys. Lett. A* **128** 19
[34] Jaeger G and Shimony A 1995 *Phys. Lett. A* **197** 83
[35] Shor P W 1997 *SIAM J. Comput.* **26** 1484
[36] Li L, Liu N L and Yu S X 2012 *Phys. Rev. A* **85** 054101
[37] Born M and Wolf E 1965 *Principles of Optic,* 3rd ed (New York: Pergamon)
[38] Barnett S M and Croke S 2009 *Adv. Opt. Photon.* **1** 238
[39] Englert B G, Kaszlikowski D, Kwek L C and Chee W H 2008 *Int. J. Quantum Inform.* **6** 129
[40] Żukowski M, Zeilinger A and Horne M A 1997 *Phys. Rev. A* **55** 2564
[41] Vourdas A and Dunningham J A 2005 *Phys. Rev. A* **71** 013809
[42] Su Z E, Li Y, Rohde P P, Huang H L, Wang X L, Li L, Liu N L, Dowling J P, Lu C Y and Pan J W 2017 *Phys. Rev. Lett.* **119** 080502
[43] Reck M, Zeilinger A, Bernstein H J and Bertani P 1994 *Phys. Rev. Lett.* **73** 58
[44] Clements W R, Humphreys P C, Metcalf B J, Kolthammer W S and Walmsley I A 2016 *Optica* **3** 1460
[45] Jia A A, Huang J H, Zhang T C and Zhu S Y 2014 *Phys. Rev. A* **89** 042103
[46] Wang Z H, Tian Y L, Yang C, Zhang P F, Li G and Zhang T C 2016 *Phys. Rev. A* **94** 062124
[47] Wang D Y, Wu J J, Ding J F, Liu Y W, Huang A Q and Yang X J 2021 *Entropy* **23** 122