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Dynamic modeling and aperiodically intermittent strategy for adaptive finite-time synchronization control of the multi-weighted complex transportation networks with multiple delays*

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The idea of network splitting according to time delay and weight is introduced. Based on the cyber physical systems (CPS), a class of multi-weighted complex transportation networks with multiple delays is modeled. The finite-time synchronization of the proposed complex transportation networks model is studied systematically. On the basis of the theory of stability, the technique of adaptive control, aperiodically intermittent control and finite-time control, the aperiodically intermittent adaptive finite-time synchronization controller is designed. The controller designed in this paper is beneficial for understanding the synchronization in multi-weighted complex transportation networks with multiple delays. In addition, the conditions for the existence of finite time synchronization have been discussed in detail. And the specific value of the settling finite time for synchronization is obtained. Moreover, the outer coupling configuration matrices are not required to be irreducible or symmetric. Finally, simulation results of the finite-time synchronization problem are given to illustrate the correctness of the results obtained.

Keywords: complex transportation networks, adaptive finite-time synchronization, multiple delays and multi-weighted, aperiodically intermittent control

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1. Introduction

Cyber physical systems (CPS), as the unity of computing process and physical process, are the next generation of intelligent system integrating computing, communication and control. CPS realizes the interaction with the physical process through the man–machine interaction interface, and uses the networked space to manipulate a physical entity in a remote, reliable, real-time, safe and cooperative way.[1] Compared with the Internet of things, CPS puts more emphasis on control. CPS has been concerned by many research institutions and scholars since it was first proposed in 2005.[2] In recent years, CPS has become a hot research direction and a lot of achievements have been made in priority industrial field for enterprises.[3] CPS has been widely used in intelligent transportation, aerospace, smart power grid, disaster early warning, military exercise, logistics and supply chain optimization and so on. The research and application of CPS are of great strategic significance for accelerating the integration of industrialization and informatization.[4–6]

Complex dynamical network is an important method to describe and study the complex systems. According to different research aspects, many complex systems can be abstracted into a complex network of interacting individuals from the practical background. Over the past 20 years, the complex dynamical networks have been the focus of attention, and have gained more and more attention in various fields. Many scientific and technical systems can be modeled as the complex dynamical networks.[7–10]

The emergence of CPS brings about new opportunities for the researchers in the field of network, which gives rise to cooperative control of multiple robots, attitude control of multiple satellites, formation control of unmanned aerial vehicle, smart grid control, intelligent transportation system, and so on.[11,12] The outstanding characteristic of CPS is an intelligent network composed of many devices with communication, computing and decision control functions. It can make the whole system in the best state through interaction. The operation of CPS is actually a collection of information transmission, synchronization, optimization, control and other dynamic processes in the complex dynamic networks.[13]

There are rich dynamic behaviors in the analysis of complex network. Synchronization is one of the most important dynamic behaviors of the complex network.[14] Common phenomena in life, such as the resonance phenomenon, the cricket chorus, the flock of birds, and the rhythmic applause broke forth the audience in the theatre are widely known as synchronous phenomenon. Many practical problems are closely

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related to synchronization, such as the synchronous phenomena on the communications networks, the transportation networks, the human connection networks and WWW.[15–19] The synchronous study of complex networks is around the relationship and the network structure among the network behaviors. The research on synchronization control of complex networks is helpful to understand the influence of network structure on synchronization of complex systems, which can supplement the synchronization analysis of complex networks.[20–22] In the study of synchronous control of complex networks, many control methods have been introduced, such as adaptive control,[22,23] pinning control,[24,25] intermittent control,[26,27] finite-time control,[27–31] and sampled-data control.[32]

The collaborative scheduling control of the transportation network can be taken as an example of the CPS system.[11,33] In the control of multiple means of transportation, different means of transportation are required to arrive at the same place at the same time as possible. In the complex transportation networks, when the air transportation, rail transportation, road transportation and water transportation are synchronized over a period of time, the time of goods staying in the logistics center can be reduced, the empty load rate of the transportation tools can be reduced, the average time of inventory can be shortened, and sometimes even the time of inventory in the logistics warehouse center can be eliminated. Therefore, it is necessary to conduct in-depth research on the synchronization of complex transportation networks, which will provide important theoretical support for the construction planning of macro-regional logistics and the auxiliary decision-making of transport line distribution and dispatch in the future.

On the other hand, in the complex transportation networks, delay is inevitable due to the limited speed of transmission and traffic congestion. After considerable development, many complex network models that can reflect the actual characteristics have been established. In actual transportation network, sometimes the physical distance among nodes is far. The speed of vehicles, carrying capacity of transportation routes, transit and storage effect of logistics are limited. References [34,35] introduce the idea of network splitting, and establish a multi-delay complex network model. This network can be well applied to complex transportation networks with multiple delays. However, the weights of different nodes in each subnetwork are not reflected. Wang *et al.*[36] have made some attempts at splitting the weights. Wang put forward a complex network model based on weight by using the idea of network splitting and introducing the concept of weight. The model is analyzed synchronously and the pinning controller is designed. But the delays of different nodes in each subnetwork are not considered. In Ref. [37], the model of complex networks with multi-delay and multi-weight is preliminarily established. A simple linear feedback controller is designed using the technique of linear matrix inequalities. There is some conservatism in the controller design. Therefore, how to build a complex network model that can comprehensively reflect the delay and weight should be further studied.

The occurrence of delay often affects the performance of the system, resulting in the vibration and even instability of the complex transportation network. There have been many research results on synchronization in delays complex networks.[38,39] Especially, how to solve the finite-time synchronization problem of the complex transportation networks with multiple delays and multi-weighted is still challenging.

Based on the results of the above discussion and the idea of CPS, we proposed the model of multi-weighted complex transportation networks with multiple delays. On the basis of the theory of Lyapunov stability, the technique of adaptive control, aperiodically intermittent strategy and finite-time control, the adaptive aperiodically intermittent finite-time synchronization controllers have been designed. The external coupled configuration matrices do not require to be irreducibility and symmetry. Finally, the correctness and effectiveness of the proposed controllers are verified by the numerical simulations.

The remainder of this paper is organized as follows. In Section 2, based on the ideas of CPS and network splitting, the multi-weighted complex transportation network model with multiple coupling delays is established, and some necessary assumptions and lemmas are given. In Section 3, the adaptive aperiodically intermittent finite-time synchronization of the multi-weighted complex transportation networks with multiple delays is researched via the theory of Lyapunov stability, the technique of adaptive control and finite-time. The adaptive aperiodically intermittent finite-time synchronization controllers are designed. The numerical simulation results are given to illustrate the theorems in Section 4. Finally, the conclusion is drawn in Section 5.

2. Model description and preliminaries

Transportation systems can be well described by complex network models. Each region involved in transport can be used as a node in the complex transport network system. And each mode of transportation (such as air, rail, road, water, and pipeline) connecting any two regions can be abstracted to represent each side of a complex network.[37] In order to make the model better reflect the structural properties of the actual system, it is considered that the same network can be split many times, that is, the subnetwork obtained by the first split is regarded as a complete network, and then the subnetwork is split to a more detailed split result. We fully consider delay and weight in modeling and further improve the

model in this paper. First of all, different types of transportation models have different transport velocity. As the transportation velocity of the highway network, railway network, navigation network and aviation network are different, under the premise of simultaneous departure, the velocity of aviation network is the fastest, and the railway network, highway network and navigation network have a time lag of arrival time for the aviation network, respectively. Therefore, the idea of network splitting can describe the multi-delay complex transportation network system. Second, for the same traffic mode, each subnetwork is divided again according to the weight attribute of the path and station in the road network. And the weight division is to divide nodes and edges according to their weights in the network. The practical significance of its division is to divide the stations and lines in the traffic network according to their busy degree in the network. Therefore, based on the idea of network splitting and CPS, the N coupling nonlinear and diffusion of the multi-weighted complex transportation networks with multiple delays model, with each node as an n-dimensional nonlinear system is described as follows:

$$\begin{aligned}\dot{x}_i(t) &= f(x_i(t),t)+\sum_{r_0=1}^{M_0} c_{r_0}\sum_{j=1}^{N} a_{(r_0)ij}\Gamma_0 x_j(t)\\ &+\sum_{r_1=1}^{M_1} c_{r_1}\sum_{j=1}^{N} a_{(r_1)ij}\Gamma_1 x_j(t-\tau_1)+\cdots\\ &+\sum_{r_m=1}^{M_m} c_{r_m}\sum_{j=1}^{N} a_{(r_m)ij}\Gamma_m x_j(t-\tau_m)+u_i(t), \qquad (1)\end{aligned}$$

where $i=1,2,\ldots,N$. $x_i(t)=(x_{i1}(t),x_{i2}(t),\ldots,x_{\mathrm{in}}(t))^{\mathrm{T}}\in R^n$ shows the state vector of node i at time t. The continuous differentiable nonlinear function $f(x_i(t),t):R^n\times R\to R^n$ is the dynamic system of node i. $\tau_1,\tau_2,\ldots\tau_m$ are bounded arbitrary time delay. c_{r_l} means the external coupling strength of the rth subnet in the lth sub-subnet, $A_{r_\kappa}=(a_{(r_\nu)ij})\in R^{N\times N}$ $(\nu=0,1,\ldots,m)$ is the external coupling matrix. A_{r_κ} means the topology of transportation networks at time t and satisfies the zero-sum row condition. $a_{(r_\kappa)ij}$ can be 0 or 1 for $i\neq j$, the means of $a_{(r_\kappa)ij}$ is as follows: if there is not a connection from node i to node j, $a_{(r_\kappa)ij}=0$, otherwise $a_{(r_\kappa)ij}=1$ for $i,j=1,2,\ldots,N$. Positive diagonal matrix $\Gamma_\nu\in R^{n\times n}$ is the individual couplings matrix between nodes i and j at time t. $u_i(t)\in R^n$ $(i=1,2,\ldots,N)$ is the control variables of node i.

Remark 1 The model (1) is very useful for understanding the multi-weighted complex transportation networkmodel with multiple coupling delays. In the real world, many practical systems can be abstracted by the model (1), such as biological networks, and communication networks. The model (1) is a synthesis and generalization of many other models. For example, when $M_0=M_1=\cdots=M_m=1$, the model (1) is degenerated to the complex network model with multiple delays in Refs. [34,35]. When $\tau_1=\tau_2=\cdots=\tau_m=0$, the model (1) is degenerated to the multi-weighted complex networks in Ref. [36]. Moreover, the following results do not have to satisfy the conditions in the past of outer coupling configuration matrices to be symmetric or irreducible.

When the states $x_1(t)\to\cdots\to x_N(t)\to s(t)$, as $t\to T^*$, the multi-weighted complex transportation networks model with multiple delays (1) realizes finite-time synchronization. Where $s(t)\in R^n$ is the solution of an isolated node, i.e.,

$$\dot{s}(t)=f(s(t),t). \qquad (2)$$

Let $\tau=\max\{\tau_1,\ldots,\tau_m\}$, $C([-\tau,0],R^n)$ be the Banach space of continuous functions mapping the interval $[-\tau,0]$ into R^n with the norm $||\phi||=\sup\limits_{-\tau\le\theta\le 0}||\phi(\theta)||$, where $||\cdot||$ is the Euclidean norm. The rigorous mathematical definition of finite-time synchronization for the multi-weighted complex transportation networks with multiple delays (1) is given below.

Definition 1 Let $x_i(t;t_0;\phi)$, $i=1,2,\ldots,N$ be a solution of the multi-weighted complex transportation networks with multiple delays, where $\phi=(\phi_1^{\mathrm{T}},\phi_2^{\mathrm{T}},\ldots\phi_N^{\mathrm{T}})^{\mathrm{T}}$, $\phi_i=\phi_i(\theta)\in C([-\tau,0],R^n)$ are initial conditions. If for an appropriate designed the finite-time controller $u_i(t)$, there exists a positive constant T^* and a nonempty subset $\Lambda\subseteq R^n$, such that ϕ_i take values in Λ and $x_i(t;t_0;\phi)\in R^n$ for all $t\ge t_0$ and

$$\lim_{t\to T^*}||x_i(t;t_0;\phi)-s(t;t_0;s_0)||=0, \quad i=1,2,\ldots,N, \qquad (3)$$

where $s(t;t_0;s_0)$ is a solution of the system (2) with $s_0\in R^n$, then the multi-weighted complex transportation networks model with multiple delays (1) is said to realize finite-time synchronization, and $\Lambda\times\Lambda\times\cdots\times\Lambda$ is called the region of finite-time synchrony of the multi-weighted complex transportation networks model with multiple delays (1).

In order to reach the following conclusions, we give the following necessary assumptions and lemmas.

Assumption 1 There exists positive constant L_i $(i=1,2,\ldots,N)$. $f:R^n\times R\to R^n$ is a nonlinear vector-valued continuous function, which satisfies the Lipschiz condition

$$\|f(x_i(t),t)-f(s(t),t)\|\le L_i\|x_i(t)-s(t)\|. \qquad (4)$$

Lemma 1[27] Suppose that function $V(t)$ is continuous and non-negative when $t\in[-\tau,+\infty)$ and satisfies the following conditions:

$$\begin{cases}\dot{V}(t)\le-\alpha V^{\gamma}(t), & t_l\le t\le s_l,\\ \dot{V}(t)\le\beta V(t), & s_l\le t\le t_{l+1},\end{cases} \qquad (5)$$

where $l=0,1,2,\ldots$, $\alpha,\beta>0$, $0<\gamma<1$. If there exists a constant $\psi\in(0,1)$, where ψ is defined as follows:

$$\psi=\lim_{l\to\infty}\sup\frac{t_{l+1}-s_l}{t_{l+1}-t_l},$$

then the following inequality holds:

$$V^{1-\gamma}(t)\le\left(\sup_{-\tau\le s\le 0}V(s)\right)^{1-\gamma}\exp\{\beta\psi(1-\gamma)t\}$$

$$-\alpha(1-\psi)(1-\gamma)t, \quad 0 \le t \le T. \tag{6}$$

where the constant T is the settling time.

Remark 2 Lemma 1 plays an important role in the intermittent finite-time synchronization analysis and control of complex networks via periodically intermittent control. In the Lemma 1, $t_{l+1}-t_l$ is the length of the lth aperiodical control period. $t_{l+1}-s_l$ and s_l-t_l represent the non-control time and control time in the l-th control period respectively.

Lemma 2[40] For any n dimensional column vectors x, y, positive definite $n\times n$ dimensional matrix Q, the following matrix inequality holds:

$$2x^{\mathrm{T}}y \le x^{\mathrm{T}}Qx + y^{\mathrm{T}}Q^{-1}y.$$

If not specified, inequality $Q>0$ ($Q<0$, $Q\ge 0$, $Q\le 0$) means Q is a positive (or negative, or semi-positive, or semi-negative) definite matrix.

Lemma 3[41] Let $\xi_1, \xi_2, \ldots, \xi_n$ are positive numbers and $0 \le \mu_1 < \mu_2$, then

$$\left(\sum_{i=1}^{n}\xi_i^{\mu_2}\right)^{\frac{1}{\mu_2}} \le \left(\sum_{i=1}^{n}\xi_i^{\mu_1}\right)^{\frac{1}{\mu_1}}.$$

Especially, if let $\mu_2 = 1$ and $\mu_1 = \frac{1+\nu}{2}$ $(0 \le \nu < 1)$, then the following inequality holds:

$$(\xi_1+\xi_2+\cdots+\xi_n)^{\frac{1+\nu}{2}} \le \xi_1^{\frac{1+\nu}{2}} + \xi_2^{\frac{1+\nu}{2}} + \cdots + \xi_n^{\frac{1+\nu}{2}}.$$

3. Aperiodically intermittent strategy for adaptive finite-time synchronization control of the multi-weighted complex transportation networks with multiple delays

For the convenience of proof, we define the error vector as follows:

$$e_i(t) = x_i(t) - s(t), \quad i = 1,2,\ldots,N. \tag{7}$$

Then the error system corresponding to model (1) is can be described by

$$\begin{aligned}\dot{e}_i(t) \;=\; & f(x_i(t),t) - f(s(t),t) + \sum_{r_0=1}^{M_0} c_{r_0} \sum_{j=1}^{N} a_{(r_0)ij}\Gamma_0 e_j(t) \\ & + \sum_{r_1=1}^{M_1} c_{r_1} \sum_{j=1}^{N} a_{(r_1)ij}\Gamma_1 e_j(t-\tau_1) \\ & + \cdots + \sum_{r_m=1}^{M_m} c_{r_m} \sum_{j=1}^{N} a_{(r_m)ij}\Gamma_m e_j(t-\tau_m) \\ & + u_i(t), \quad (i = 1,2,\ldots,N).\end{aligned} \tag{8}$$

When the error dynamical system (8) achieves finite-time stabilization, the multi-weighted complex transportation networks with multiple delays (1) realize finite-time synchronization.

In order to enable the multi-weighted complex transportation networks with multiple delays (1) finite-time synchronization, we design the following aperiodically intermittent adaptive finite-time controllers:

$$u_i(t) = \begin{cases} -k_i(t)e_i(t) - \theta_i \operatorname{sign}(e_i(t))\,|e_i(t)|^{\sigma} \\ -\theta_i \sum\limits_{k=1}^{m}\sum\limits_{r_k=1}^{M_k} \left(C_{r_k}\int_{t-\tau_k}^{t} e_i^{\mathrm{T}}(s)e_i(s)\mathrm{d}s\right)^{\frac{1+\sigma}{2}} \dfrac{e_i(t)}{\|e_i(t)\|^2}, & t_l \le t \le s_l, \\ 0, & s_l < t < t_{l+1}. \end{cases} \tag{9}$$

and updating laws

$$\dot{k}_i(t) = \begin{cases} e_i^{\mathrm{T}}(t)e_i(t) - \theta_i \operatorname{sign}(k_i(t)-k_i^*)\,|k_i(t)-k_i^*|^{\sigma}, & t_l \le t \le s_l, \\ 0, & s_l < t < t_{l+1}. \end{cases} \tag{10}$$

where $l = 0,1,2,\ldots$, $k_i^* > 0$ $(i = 1,2,\ldots,N)$.

Using the theory of stability theory, the technique of adaptive control, aperiodically intermittent control and finite-time control theory, the synchronization controller of the multi-weighted complex transportation networks with multiple delays (1) under the aperiodically intermittent adaptive finite-time synchronization controllers (9) and (10) can be got as following.

Theorem 1 If the Assumption 1 holds, there exist positive constants $\psi \in (0,1)$ and positive definite matrix Φ defined as follows, such that the following inequality holds:

$$e\lambda_{\Phi}\psi\left(\sup_{-\tau\le s\le 0} V(s)\right)^{\frac{1-\sigma}{2}} \le 2\theta(1-\psi). \tag{11}$$

Then the error system of multi-weighted complex transportation networks with multiple delays (8) will stabilize at zero with the aperiodically intermittent adaptive finite-time controllers and updating laws (9) and (10), i.e., the multi-weighted complex transportation networks with multiple delays (1) realize finite-time synchronization. The settling time T^* can be given by $T^* = \frac{2}{\lambda_{\Phi}\psi(1-\sigma)}$ for the condi-

tion $e\lambda_{\Phi}\psi\left(\sup\limits_{-\tau\le s\le 0}V(s)\right)^{\frac{1-\sigma}{2}}=2\theta(1-\psi)$, and $T^*=T_1^*$ for the condition $e\lambda_{\Phi}\psi\left(\sup\limits_{-\tau\le s\le 0}V(s)\right)^{\frac{1-\sigma}{2}}<2\theta(1-\psi)$, respectively. Where λ_{Φ} is the biggest eigenvalue of Φ.$T_1^*$is smaller solution of the equation as follows:

$$\left(\sup_{-\tau\le s\le 0}V(s)\right)^{\frac{1-\sigma}{2}}\exp\left\{\lambda_{\Phi}\psi\left(\frac{1-\sigma}{2}\right)t\right\}=2\theta(1-\psi)\left(\frac{1-\sigma}{2}\right)t.$$

$$\Phi = 2LI_{nN}+2\sum_{r_0=1}^{M_0}c_{r_0}A_{(r_0)}\otimes\Gamma_0+\sum_{k=1}^{m}\sum_{r_k=1}^{M_k}c_{r_k}(\Gamma_{(k)}^{\mathrm{T}}\Gamma_{(k)})\otimes I_N$$

$$+\sum_{k=1}^{m}\sum_{r_k=1}^{M_k}c_{r_k}(A_{(r_k)}^{\mathrm{T}}A_{(r_k)})\otimes I_n.$$

Proof Select the following form of a Lyapunov-Krasovskii function:

$$V(t) = \sum_{i=1}^{N}\Big[e_i^{\mathrm{T}}(t)e_i(t)+(k_i(t)-k_i^*)^2 + \sum_{k=1}^{m}\sum_{r_k=1}^{M_k}c_{r_k}\int_{t-\tau_k}^{t}e_i^{\mathrm{T}}(s)\Gamma_k^{\mathrm{T}}\Gamma_k e_i(s)\mathrm{d}s\Big], \quad (12)$$

where $k_i^*>0$ is to be determined.

When $t_l\le t\le s_l$, the derivative of $V(t)$ along with the synchronous evolution $s(t)$ is satisfied

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{N}\Big[2e_i^{\mathrm{T}}(t)\dot{e}_i(t)+2(k_i(t)-k_i^*)\dot{k}_i(t)+\sum_{k=1}^{m}\sum_{r_k=1}^{M_k}c_{r_k}\left(e_i^{\mathrm{T}}(t)\Gamma_k^{\mathrm{T}}\Gamma_k e_i(t)-e_i^{\mathrm{T}}(t-\tau_k)\Gamma_k^{\mathrm{T}}\Gamma_k e_i(t-\tau_k)\right)\Big]\\
&= \sum_{i=1}^{N}\Big[2e_i^{\mathrm{T}}(t)\left(f(x_i(t),t)-f(s(t),t)+\sum_{r_0=1}^{M_0}c_{r_0}\sum_{j=1}^{N}a_{(r_0)ij}\Gamma_0 e_j(t)+\sum_{k=1}^{m}\sum_{r_k=1}^{M_k}c_{r_k}\sum_{j=1}^{N}a_{(r_k)ij}\Gamma_k e_j(t-\tau_k)\right.\\
&\quad -k_i(t)e_i(t)-\theta_i\,\mathrm{sign}(e_i(t))\,|e_i(t)|^{\sigma}-\theta_i\sum_{k=1}^{m}\sum_{r_k=1}^{M_k}\left(C_{r_k}\int_{t-\tau_k}^{t}e_i^{\mathrm{T}}(s)e_i(s)\mathrm{d}s\right)^{\frac{1+\sigma}{2}}\frac{e_i(t)}{\|e_i(t)\|^2}\Big)\\
&\quad +2(k_i(t)-k_i^*)\left(e_i^{\mathrm{T}}(t)e_i(t)-\theta_i\,\mathrm{sign}(k_i(t)-k_i^*)\,|k_i(t)-k_i^*|^{\sigma}\right)\\
&\quad +\sum_{k=1}^{m}\sum_{r_k=1}^{M_k}c_{r_k}\left(e_i^{\mathrm{T}}(t)\Gamma_k^{\mathrm{T}}\Gamma_k e_i(t)-e_i^{\mathrm{T}}(t-\tau_k)\Gamma_k^{\mathrm{T}}\Gamma_k e_i(t-\tau_k)\right)\Big]. \quad (13)\end{aligned}$$

By Lemma 2, the subsequent inequation holds:

$$\begin{aligned}&2\sum_{k=1}^{m}\sum_{r_k=1}^{M_k}c_{r_k}\sum_{j=1}^{N}a_{(r_k)ij}\Gamma_k e_i^{\mathrm{T}}(t)e_j(t-\tau_k)\\
&\le \sum_{k=1}^{m}\sum_{r_k=1}^{M_k}c_{r_k}e_i^{\mathrm{T}}(t)\left(A_{(r_k)}\right)^{\mathrm{T}}A_{(r_k)}e_i(t)\\
&\quad +\sum_{k=1}^{m}\sum_{r_k=1}^{M_k}c_{r_k}e_i^{\mathrm{T}}(t-\tau_k)\Gamma_k^{\mathrm{T}}\Gamma_k e_i(t-\tau_k). \quad (14)\end{aligned}$$

Combining with the Assumption 1, and substituting inequality (14) into Eq. (13), we can get

$$\begin{aligned}\dot{V}(t) &\le e^{\mathrm{T}}(t)\Big[2LI_{nN}+\sum_{k=1}^{m}\sum_{r_k=1}^{M_k}c_{r_k}(A_{(r_k)}^{\mathrm{T}}A_{(r_k)})\otimes I_n\\
&\quad +2\sum_{r_0=1}^{M_0}c_{r_0}A_{(r_0)}\otimes\Gamma_0\\
&\quad +\sum_{k=1}^{m}\sum_{r_k=1}^{M_k}c_{r_k}(\Gamma_{(k)}^{\mathrm{T}}\Gamma_{(k)})\otimes I_N-2I_N\otimes K^*\Big]e(t)\\
&\quad -\sum_{i=1}^{N}2\theta_i|k_i(t)-k_i^*|^{\sigma+1}-\sum_{i=1}^{N}2\theta_i|e_i(t)|^{\sigma+1}\\
&\quad -\sum_{i=1}^{N}2\theta_i\sum_{k=1}^{m}\sum_{r_k=1}^{M_k}\left(C_{r_k}\int_{t-\tau_k}^{t}e_i^{\mathrm{T}}(s)e_i(s)\mathrm{d}s\right)^{\frac{1+\sigma}{2}}. \quad (15)\end{aligned}$$

In order to guarantee $\dot{V}(t)<0$, k^* can be selected as

$$k^*>\frac{1}{2}\lambda_{\Phi}. \quad (16)$$

Because$0\le\sigma\le 1$,based on Lemma 3, it is easy to obtain the following inequality:

$$\left(\sum_{i=1}^{N}\theta_i|k_i(t)-k_i^*|^{\sigma+1}\right)^{\frac{1}{\sigma+1}}\ge\left(\sum_{i=1}^{N}\theta_i|k_i(t)-k_i^*|^2\right)^{\frac{1}{2}}, \quad (17)$$

$$\left(\sum_{i=1}^{N}\theta_i|e_i(t)|^{\sigma+1}\right)^{\frac{1}{\sigma+1}}\ge\left(\sum_{i=1}^{N}|e_i(t)|^2\right)^{\frac{1}{2}}, \quad (18)$$

and we have

$$\begin{aligned}\dot{V}(t) &\le -2\theta\left[\left(\sum_{i=1}^{N}|k_i(t)-k_i^*|^2\right)^{\frac{\sigma+1}{2}}+\left(\sum_{i=1}^{N}|e_i(t)|^2\right)^{\frac{\sigma+1}{2}}\right.\\
&\quad \left.+\sum_{i=1}^{N}\sum_{k=1}^{m}\sum_{r_k=1}^{M_k}\left(C_{r_k}\int_{t-\tau_k}^{t}e_i^{\mathrm{T}}(s)e_i(s)\mathrm{d}s\right)^{\frac{1+\sigma}{2}}\right]\\
&\le -2\theta\left[\left(\sum_{i=1}^{N}|k_i(t)-k_i^*|^2\right)+\left(\sum_{i=1}^{N}|e_i(t)|^2\right)\right.\\
&\quad \left.+\sum_{i=1}^{N}\sum_{k=1}^{m}\sum_{r_k=1}^{M_k}\left(C_{r_k}\int_{t-\tau_k}^{t}e_i^{\mathrm{T}}(s)e_i(s)\mathrm{d}s\right)\right]^{\frac{1+\sigma}{2}}\end{aligned}$$

$$\leq -2\theta V^{\frac{1+\sigma}{2}}(t). \tag{19}$$

Similarly, when $s_l < t < t_{l+1}$, using the similar method above,we can get

$$\begin{aligned}\dot{V}(t) =& \sum_{i=1}^{N}\Big[2e_i^{\mathrm{T}}(t)\dot{e}_i(t)+2(k_i(t)-k_i^*)\dot{k}_i(t)\\ &+\sum_{k=1}^{m}\sum_{r_k=1}^{M_k}c_{r_k}e_i^{\mathrm{T}}(t)\Gamma_k^{\mathrm{T}}\Gamma_k e_i(t)\\ &-\sum_{k=1}^{m}\sum_{r_k=1}^{M_k}c_{r_k}e_i^{\mathrm{T}}(t-\tau_k)\Gamma_k^{\mathrm{T}}\Gamma_k e_i(t-\tau_k)\Big]\\ \leq& e^{\mathrm{T}}(t)\,[2LI_{nN}+\sum_{k=1}^{m}\sum_{r_k=1}^{M_k}c_{r_k}(\Gamma_{(k)}^{\mathrm{T}}\Gamma_{(k)})\otimes I_N\\ &+2\sum_{r_0=1}^{M_0}c_{r_0}A_{(r_0)}\otimes\Gamma_0\\ &+\sum_{k=1}^{m}\sum_{r_k=1}^{M_k}c_{r_k}(A_{(r_k)}^{\mathrm{T}}A_{(r_k)})\otimes I_n\Big]e(t)\\ \leq& \lambda_\Phi V(t).\end{aligned} \tag{20}$$

So we can derive the following result:

$$\begin{aligned}V^{1-\frac{1+\sigma}{2}}(t)\leq&\left(\sup_{-\tau\leq s\leq 0}V(s)\right)^{1-\frac{1+\sigma}{2}}\exp\left\{\lambda_\Phi\psi(1-\frac{1+\sigma}{2})t\right\}\\ &-2\theta(1-\psi)\left(1-\frac{1+\sigma}{2}\right)t,\\ &0\leq t\leq T^*,\end{aligned} \tag{21}$$

i.e.,

$$\begin{aligned}V^{\frac{1-\sigma}{2}}(t)\leq&\left(\sup_{-\tau\leq s\leq 0}V(s)\right)^{\frac{1-\sigma}{2}}\exp\left\{\lambda_\Phi\psi(\frac{1-\sigma}{2})t\right\}\\ &-2\theta(1-\psi)\left(\frac{1-\sigma}{2}\right)t,\\ &0\leq t\leq T^*.\end{aligned} \tag{22}$$

Next, we give the value of finite-time T^*. Apparently, T^* satisfies the following equation:

$$\begin{aligned}&\left(\sup_{-\tau\leq s\leq 0}V(s)\right)^{\frac{1-\sigma}{2}}\exp\left\{\lambda_\Phi\psi\left(\frac{1-\sigma}{2}\right)t\right\}\\ =&\ 2\theta(1-\psi)\left(\frac{1-\sigma}{2}\right)t.\end{aligned} \tag{23}$$

Take the derivative of both sides of the Eq. (23) with respect to t, and properly arrange it to obtain

$$t=\frac{2}{\lambda_\Phi\psi(1-\sigma)}\ln\frac{2\theta(1-\psi)}{\lambda_\Phi\psi\left(\sup\limits_{-\tau\leq s\leq 0}V(s)\right)^{\frac{1-\sigma}{2}}}. \tag{24}$$

Substituting Eq. (24) back into Eq. (23), we can get

$$2\theta(1-\psi)=e\lambda_\Phi\psi\left(\sup_{-\tau\leq s\leq 0}V(s)\right)^{\frac{1-\sigma}{2}}. \tag{25}$$

Then we apply condition (25) to discuss the value of finite-time T^*.

If $e\lambda_\Phi\psi\big(\sup\limits_{-\tau\leq s\leq 0}V(s)\big)^{\frac{1-\sigma}{2}}>2\theta(1-\psi)$, Eq. (23) has no solution. There is no guarantee that T^* exists, so that when $t\to T^*$, $V(t)\leq 0$ holds.

If $e\lambda_\Phi\psi\big(\sup\limits_{-\tau\leq s\leq 0}V(s)\big)^{\frac{1-\sigma}{2}}=2\theta(1-\psi)$, there is a unique solution to Eq. (23) that is $T^*=\frac{2}{\lambda_\Phi\psi(1-\sigma)}$. When $t\to T^*$, $V(t)\to 0$ is established. When $t\geq T^*$, $T^*\in[t_l,s_l]$ or $T^*\in[s_l,t_{l+1}]$. When $T^*\in[t_l,s_l]$, $t\in[T^*,s_l]$ or (s_l,t_{l+1}). When $t\in[T^*,s_l]$, it can be obtained from Eq. (19) that $V(t)$ decreases monotonically. So $V(t)\leq V(T^*)=0$ is true. When $t\in(s_l,t_{l+1})$, using the inequality (20), we can get $V(t)\leq V(s_l)\exp\{\lambda_\Phi(t-s_l\}=0$. When $T^*\in[s_l,t_{l+1}]$, for all $t\in(T,t_{l+1})$ using the same method as above, we can get $V(t)\leq 0$.

If $e\lambda_\Phi\psi\big(\sup\limits_{-\tau\leq s\leq 0}V(s)\big)^{\frac{1-\sigma}{2}}>2\theta(1-\psi)$, Eq. (23) has two solutions, letting them be T_1^* and T_2^*. Might as well set $T_1^*<T_2^*$. Let $T^*=T_1^*$, similar to the discussion of T^*in the case (b), we can derive when $t\to T^*$, $V(t)\to 0$. When $t\geq T^*$, $V(t)=0$.

By analyzing, only the condition of (b) and (c) can the multi-weighted complex transportation networks with multiple delays (1) be fifinite-time synchronization. So the settling time T^* can be given by $T^*=\frac{2}{\lambda_\Phi\psi(1-\sigma)}$ for the condition (b) and $T^*=T_1^*$ for the condition (c), respectively.

The proof is completed.

Remark 3 If $t_{l+1}-t_l=T$ and $t_{l+1}-s_l=\delta$ $(l=0,1,2,\ldots)$, where T and δ are positive constants, the aperiodically adaptive intermittent finite-time control strategy becomes the adaptive periodic intermittent finite-time control strategy. Therefore, the control strategy we designed is more general and more flexible to use. And we also give the condition for finite-time synchronization and the specific time T^* of finite-time synchronization.

Remark 4 In the complex networks (1), when $M_0=M_1=\cdots=M_m=1$, the multi-weighted complex transportation networks with multiple delays (1) is degenerated to the following form:

$$\begin{aligned}\dot{x}_i(t)=&\ f(x_i(t),t)+c_0\sum_{j=1}^{N}a_{(0)ij}\Gamma_0x_j(t)\\ &+c_1\sum_{j=1}^{N}a_{(1)ij}\Gamma_1x_j(t-\tau_1)+\cdots\\ &+c_m\sum_{j=1}^{N}a_{(m)ij}\Gamma_mx_j(t-\tau_m)+u_i(t),\\ &(i=1,2,\ldots,N).\end{aligned} \tag{26}$$

Equation (26) is the complex networks with multiple delays of Ref. [34] We take the same aperiodically intermittent

adaptive finite-time controllers (9) and updating laws (10) to get the following corollary.

Corollary 1 If the Assumption 1 holds, there exist positive constants $\psi \in (0,1)$ and positive definite matrix Ω defined as follows, such that the following inequality holds:

$$e\lambda_\Omega \psi \left(\sup_{-\tau \le s \le 0} V(s) \right)^{\frac{1-\sigma}{2}} \le 2\theta(1-\psi). \tag{27}$$

Then the complex networks with multiple delays (26) will realize finite-time synchronization with the aperiodically intermittent adaptive finite-time controllers and updating laws (9)–(10). The settling time T^* can be given by $T^* = \frac{2}{\lambda_\Omega \psi(1-\sigma)}$ for the condition $e\lambda_\Omega \psi \big(\sup_{-\tau \le s \le 0} V(s) \big)^{\frac{1-\sigma}{2}} = 2\theta(1-\psi)$, and $T^* = T_1^*$ for the condition $e\lambda_\Omega \psi \big(\sup_{-\tau \le s \le 0} V(s) \big)^{\frac{1-\sigma}{2}} < 2\theta(1-\psi)$, respectively. λ_Ω is the biggest eigenvalue of Ω.$T_1^*$is smaller solution of the equation as follows:

$$\begin{aligned} &\left(\sup_{-\tau \le s \le 0} V(s) \right)^{\frac{1-\sigma}{2}} \exp\left\{ \lambda_\Omega \psi \left(\frac{1-\sigma}{2} \right) t \right\} \\ &= 2\theta(1-\psi)\left(\frac{1-\sigma}{2} \right) t. \end{aligned}$$

$$\begin{aligned} \Omega \;=\; & 2LI_{nN} + 2c_0 A_{(0)} \otimes \Gamma_0 + \sum_{k=1}^{m} c_k (\Gamma_{(k)}^{\mathrm{T}} \Gamma_{(k)}) \otimes I_N \\ & + \sum_{k=1}^{m} c_k (A_{(k)}^{\mathrm{T}} A_{(k)}) \otimes I_n. \end{aligned}$$

Remark 5 In the complex networks (1), when $\tau_1 = \tau_2 = \cdots = \tau_m = 0$, the multi-weighted complex transportation networks with multiple delays (1) is degenerated to the following form:

$$\begin{aligned} \dot{x}_i(t) \;=\; & f(x_i(t),t) + c_1 \sum_{j=1}^{N} a_{(1)ij} \Gamma_1 x_j(t) \\ & + c_2 \sum_{j=1}^{N} a_{(2)ij} \Gamma_2 x_j(t) + \cdots \\ & + c_m \sum_{j=1}^{N} a_{(m)ij} \Gamma_m x_j(t) + u_i(t), \\ & (i = 1,2,\ldots,N). \end{aligned} \tag{28}$$

Equation (28) is the complex networks with multi-weighted of Ref. [36]. Accordingly,we design the aperiodically intermittent adaptive finite-time controllers as follows:

$$\begin{aligned} & u_i(t) \\ &= \begin{cases} -k_i(t) e_i(t) - \theta_i \operatorname{sign}(e_i(t)) |e_i(t)|^{\sigma}, & t_l \le t \le s_l, \\ 0, & s_l \le t \le t_{l+1}; \end{cases} \end{aligned} \tag{29}$$

and the updating laws

$$\dot{k}_i(t) = \begin{cases} e_i^{\mathrm{T}}(t) e_i(t) \\ -\theta_i \operatorname{sign}(k_i(t) - k_i^*) |k_i(t) - k_i^*|^{\sigma}, & t_l \le t \le s_l, \\ 0, & s_l \le t \le t_{l+1}, \end{cases} \tag{30}$$

where $l = 0,1,2,\ldots$, $k_i^* > 0$ $(i = 1,2,\ldots,N)$.

Then we select the following Lyapunov–Krasovskii function:

$$V(t) = \sum_{i=1}^{N} \left[e_i^{\mathrm{T}}(t) e_i(t) + (k_i(t) - k_i^*)^2 \right], \tag{31}$$

where $k_i^* > 0$ is to be determined. We can get the following corollary.

Corollary 2 If the Assumption 1 holds, there exist positive constants $\psi \in (0,1)$ and positive definite matrix Δ defined as follows, such that the following inequality holds:

$$e\lambda_\Delta \psi V^{\frac{1-\sigma}{2}}(0) \le 2\theta(1-\psi), \tag{32}$$

Then the complex networks with multiple delays (28) will realize finite-time synchronization with the aperiodically intermittent adaptive finite-time controllers and updating laws (29)–(30). The settling time T^*can be given by $T^* = \frac{2}{\lambda_\Delta \psi(1-\sigma)}$ for the condition $e\lambda_\Delta \psi V^{\frac{1-\sigma}{2}}(0) = 2\theta(1-\psi)$, and $T^* = T_1^*$ for the condition $e\lambda_\Delta \psi V^{\frac{1-\sigma}{2}}(0) < 2\theta(1-\psi)$, respectively. Where λ_Δ is the biggest eigenvalue of Δ. $T_1^*$is smaller solution of the equation as follows:

$$V^{\frac{1-\sigma}{2}}(0) \exp\left\{ \lambda_\Delta \psi \left(\frac{1-\sigma}{2} \right) t \right\} = 2\theta(1-\psi)\left(\frac{1-\sigma}{2} \right) t,$$

where $\Delta = 2LI_{nN} + 2\sum_{k=1}^{m} c_k A_{(k)} \otimes \Gamma_k$.

4. Numerical simulation

To verify the validity of the proposed theorem in the section III, we give the numerical example.

Example 1 In this example, we choose 10 cities as the nodes of the complex transportation networks. Air transportation, railway transportation and highway transportation are three modes of transportation in complex transportation network. According to the level of the city and the busy degree of a certain vehicle, the weight is divided into three orders of magnitude. We abstract it as the complex transportation networks with 3-weighted and 2 coupling delays, which can be described as follows:

$$\begin{aligned} \dot{x}_i(t) \;=\; & f(x_i(t),t) + \sum_{r_0=1}^{3} c_{r_0} \sum_{j=1}^{3} a_{(r_0)ij} \Gamma_0 x_j(t) \\ & + \sum_{r_1=1}^{2} c_{r_1} \sum_{j=1}^{3} a_{(r_1)ij} \Gamma_1 x_j(t-\tau_1) \\ & + \sum_{r_2=1}^{2} c_{r_2} \sum_{j=1}^{3} a_{(r_2)ij} \Gamma_2 x_j(t-\tau_2) + u_i(t), \\ & (i = 1,2,\ldots,10), \end{aligned} \tag{33}$$

where $\Gamma_0 = \Gamma_1 = \Gamma_2 = \mathrm{diag}\{1,1,1\}$, $\tau_1 = 0.5$, $\tau_2 = 1.c_{1_0} = c_{1_1} = 0.5$, $c_{2_0} = c_{2_1} = 0.2$, $c_{3_0} = c_{1_2} = c_{2_2} = 0.3$,

$$A_{1_0} = A_{2_1} = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & -5 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & -7 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & -3 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -4 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & -5 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & -8 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -2 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & -3 \end{pmatrix},$$

$$A_{2_0} = A_{1_1} = A_{2_2} = \begin{pmatrix} -4 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & -9 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & -5 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -3 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & -6 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & -3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & -4 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & -4 \end{pmatrix},$$

$$A_{3_0} = A_{1_2} = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & -4 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & -7 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & -3 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & -9 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & -5 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & -6 \end{pmatrix}.$$

$A_{r_l} = (a_{(r_l)ij}) \in R^{10\times 10}$ represents the coupling matrices with zero-sum rows when the coupling strength is c_{r_l}.

Chen system as the single node of dynamical equation $f(\cdot)$ is described as follows:

$$\begin{cases} \dot{x}_{i1} = a(x_{i2} - x_{i1}), \\ \dot{x}_{i2} = bx_{i1} - x_{i1}x_{i3} - cx_{i2}, \\ \dot{x}_{i3} = x_{i1}x_{i2} - dx_{i3}. \end{cases} \tag{34}$$

When the parameters $a = 35$, $b = -7$, $c = 28d = -3$, the Chen system has a chaotic attractor. Similar to Ref. [25], there exists $L = 5.5619$ that makes Assumption 1 hold.

In this simulation, the initial values of states $x_i(0)(i = 1,2,\ldots,5)$ are given at random.$s(0) = (1,1,1)^{\mathrm{T}}$, $k_i(0) = 1$ and $\theta_i = 5$ $(i = 1,2,\ldots,10)$, $\sigma = 1/4$, $\theta = 6$. The aperiodically intermittent control period is randomly assigned to the interval.[1,2] The control time is randomly assigned to [0.5, 1.5]. Then $\Psi = 1/4$.

It can be obtained by verification that all the conditions of Theorem 1 are satisfied. Applying Theorem 1, the multi-weighted complex transportation networks with multiple delays (33) can be synchronized by applying the following aperiodically intermittent adaptive finite-time controllers (9) and the update laws (10) with a finite time $T^* = 2.88$.

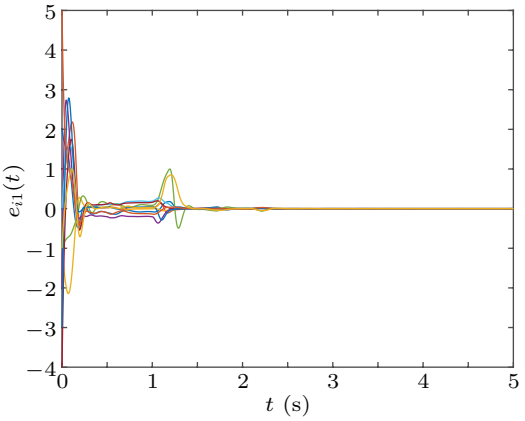


Fig. 1. Synchronization errors $e_{i1}(t)$ of network (33) under the aperiodically intermittent adaptive finite-time controllers (9)–(10).

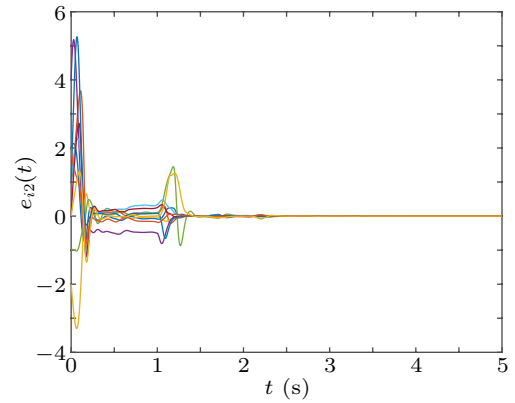


Fig. 2. Synchronization errors $e_{i2}(t)$ of network (33) under the aperiodically intermittent adaptive finite-time controllers (9)–(10).

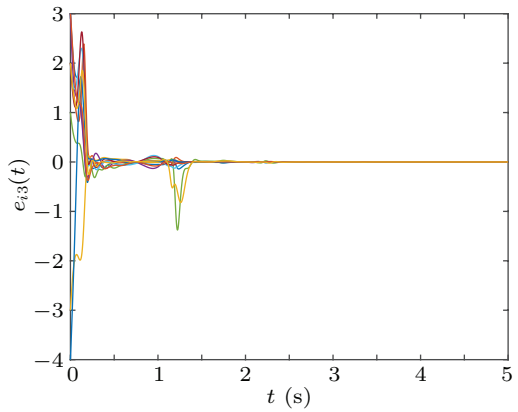


Fig. 3. Synchronization errors $e_{i3}(t)$ of network (33) under the aperiodically intermittent adaptive finite-time controllers (9)–(10).

Figures 1–3 show the synchronization errors of $e_{i1}(t)$, $e_{i2}(t)$, $e_{i3}(t)$ with aperiodically intermittent adaptive finite-time controllers (9)–(10). It is obvious that the synchronous errors of network (33) with adaptive feedback controller (9) under the updating laws (10) converge to zero after a short period of time. Because this paper uses intermittent control, the

error system is not immediately asymptotically stable. During the period of time that is not controlled, the system response curve may oscillate slightly and gradually stabilize, as shown in Figs. 1–3, from 1 second to 1.5 seconds, the error increases and then converges to zero. When the air transportation, railway transportation and road transportation reach the synchronization after a period of time, it can reduce the time of goods staying in the logistics center, reduce the empty rate of transport vehicles, and shorten the average inventory time.

Example 2 In the actual complex dynamic network system, the coupling strength and structure are often disturbed and change with time. However, the aperiodically intermittent adaptive finite-time controller designed by us still has strong robustness when the coupling strength or structure changes. For validating the result, we introduce the quality of the synchronization process $Q(t)=\sqrt{\frac{1}{n}\sum_{i=1}^{n}\|x_i(t)-s(t)\|^2}$. We still take Eq. (33) as an example. We assume that coupling matrix A_{1_0} is included mismatched disturbance. Let

$$A_{1_0}=\begin{pmatrix}
-2 & 1 & 0.1\sin t & 0 & 0 & 0.2\arctan t & 0 & 0 & 1 & 0\\
1 & -5 & 1 & -0.2\arctan t & 0 & 1 & 0 & 1+0.3\cos^2 t & 0 & 1\\
1 & 1 & -7 & 0 & 1 & 1 & 1 & 1 & 1 & 0\\
-0.1\operatorname{arccot} t & 1 & 0 & -3 & 0 & 0 & 1 & 0 & 0 & 1\\
0 & 0 & 1 & 1 & -4 & 0.1\sin^2 t & 0 & 1 & 1 & 0\\
1 & 1 & 0 & 0 & 1 & -5 & 1 & 0 & 0 & 1\\
0 & 0 & 1 & 0.1\sin t & 0 & 0 & -1 & 0 & 0 & 0\\
0 & 1 & 1 & 1 & 1 & 1 & 1 & -8 & 1 & 1\\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -2+0.2\cos t & 1\\
1 & 0 & 0 & 1 & 0.2\operatorname{arccot} t & 0 & 0 & 1 & 0 & -3
\end{pmatrix},$$

All other parameters are the same as Eq. (33). We plot the synchronization quality $Q(t)$ in the Fig. 4.

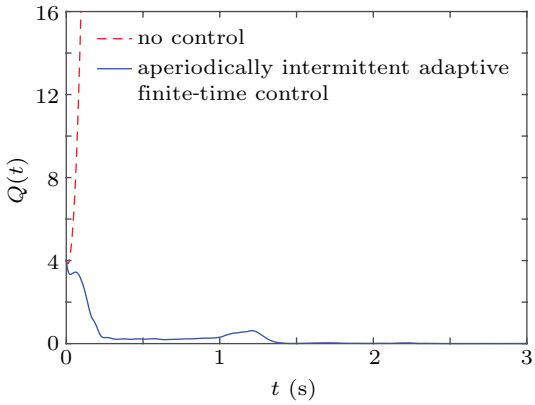


Fig. 4. The qualities $Q(t)$ with the disturbance signal under the aperiodically intermittent adaptive finite-time controllers.

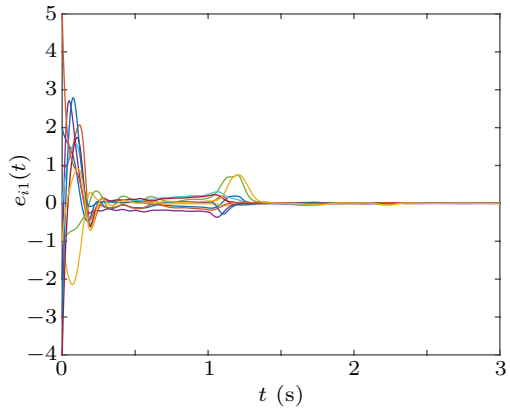


Fig. 5. Synchronization errors $e_{i1}(t)$ of network (33) with the disturbance signal under the aperiodically intermittent adaptive finite-time controllers (9)–(10).

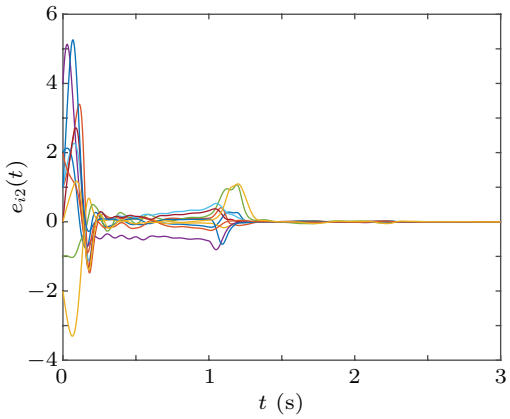


Fig. 6. Synchronization errors $e_{i2}(t)$ of network (33) with the disturbance signal under the aperiodically intermittent adaptive finite-time controllers (9)–(10).

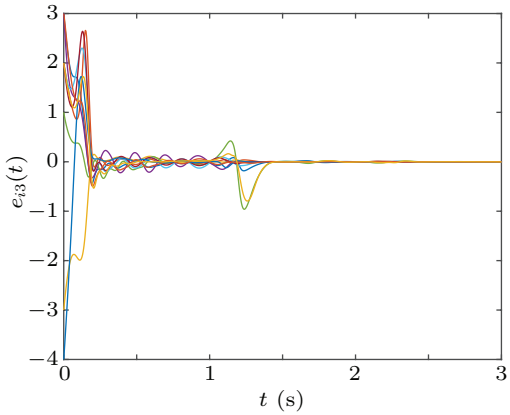


Fig. 7. Synchronization errors $e_{i3}(t)$ of network (33) with the disturbance signal under the aperiodically intermittent adaptive finite-time controllers (9)–(10).

Obviously, the synchronization quality $Q(t)$ can not achieve synchronization without controller. The state of the

error system diverges rapidly without controller, while the aperiodically intermittent adaptive finite-time controllers (9)–(10) can still guarantee the synchronization of the controlled network (33). It is shown that the aperiodically intermittent adaptive finite-time controllers have strong robustness against uncertainties.

5. Conclusion

In this paper, we have researched the adaptive finite-time synchronization of the multi-weighted complex transportation networks with multiple delays via aperiodically intermittent strategy based on the CPS. Using the technique of adaptive control, finite-time control, aperiodically intermittent control and the theory of Lyapunov stability, we design the adaptive aperiodically intermittent finite-time synchronization controllers. The controllers we designed are very useful for understanding the finite-time synchronization in the multi-weighted complex transportation networks with multiple delays. When the coupling strength or structure changes, the aperiodically intermittent adaptive finite-time controller designed still has strong robustness. Moreover, there is no requirement for the outer coupling configuration matrices to be irreducible or symmetric. Finally, the effectiveness of the adaptive aperiodically intermittent finite-time synchronization controllers is verified by numerical example. It shows that the controllers designed in this paper have correctness and effectiveness.

References

[1] Lee E A 2008 *IEEE Symposium on Object Oriented Real-time Distributed Computing*, May 5–7, 2008, Orlando, FL, USA, p. 363
[2] Stankovic J A, Lee I, Mok A and Rajkumar R 2005 *Computer* **38** 23
[3] Chen Z, Wu J, Xia Y and Zhang X 2018 *IEEE Trans. Circuits Syst. II: Exp. Briefs* **65** 115
[4] Mo H, Wagle N S and Zuba M 2009 *Computer* **42** 88
[5] Zhao W Y, Ma Y C, Chen A H, Fu L and Zhang Y T 2019 *Appl. Math. Comput.* **349** 81
[6] Tan F, Wu J J, Xia Y X and Tse C K 2014 *Phys. Rev. E* **89** 062813
[7] Albert R and Barabási A L 2002 *Rev. Mod. Phys.* **74** 47
[8] Newman M E J 2003 *SIAM Rev.* **45** 167
[9] Ma Y C, Jia X R and Liu D Y 2018 *Appl. Math. Model* **53** 49
[10] Lü L, Li C R, Liu S, Wang Z Y, Tian J and Gu J J 2015 *Nonlinear Dyn.* **81** 801
[11] Lü L, Xu Y Q, Chen L S and Li C R 2019 *Physica A* **521** 121
[12] Lü L, Li C R, Li G, Bai S Y, Gao Y, Yan Z and Rong T T 2018 *Physica A* **503** 355
[13] Deng X and Yang Y 2013 *IEEE Trans. Emerg. Top Com.* **1** 98
[14] Chen G R, Wang X F and Li X 2015 *Introduction to Complex Networks-Models, Structures and Dynamics*, 2nd edn. (Beijing: Higher Education Press) p. 289
[15] Wang X F, Li X and Chen G R 2006 *Complex network theory and its application* (Beijing: Tsinghua university press) p. 194
[16] Erdös P and Rényi A 1960 *Publicationes of the Mathematical Institute of the Hungarian Academy of Sciences* **5** 17
[17] Strogatz S H 2001 *Nature* **410** 268
[18] Wang J Y, Xu C, Feng J W, Chen Z Q, Wang X F and Zhao Y 2016 *IEEE Trans. Circuits Syst. I-Regul. Pap* **62** 2544
[19] Arenas A, Díaz-Guilera A, Kurths J, Moreno Y and Zhou C 2008 *Phys. Rep.* **469** 93
[20] Wang D, Che W W, Yu H and Li J Y 2018 *Int. J. Control Autom. Syst.* **16** 782
[21] Zhang W L, Li C D, He X and Li H F 2018 *Mod. Phys. Lett. B* **32** 1850002
[22] Li N, Sun H Y, Jing X and Zhang Q L 2013 *IET Control Theory Appl.* **7** 1725
[23] Zhang Q J, Lu J A, Lv J H and Tse C K 2008 *IEEE Trans. Circuits Syst. II: Exp. Briefs* **55** 183
[24] Feng J W, Sun S H, Xu C, Zhao Y and Wang J Y 2012 *Nonlinear Dyn* **67** 1623
[25] Yu W W, Chen G R and Lü J H 2009 *Automatica* **45** 439
[26] Li N, Sun H, Li Z T and Zhang Q L 2018 *IEEE Access* **6** 4742
[27] Liu M, Jiang H J and Hu C 2018 *Neurocomputing* **310** 1
[28] Zheng M W, Li L X, Peng H P, Xiao J H, Yang Y X, Zhao H and Ren J F 2016 *Eur. Phys. J. B* **89** 43
[29] Liu Y F, Ma Y C and Wang Y N 2018 *Appl. Math. Comput.* **320** 341
[30] Liu Y F, Ma Y C and Wang Y N 2018 *Int. J Robust and Nonlin.* **28** 381
[31] Chen C, Ding Z X, Li S and Wang L H 2020 *Chin. Phys. B* **29** 040202
[32] Wang W, Zeng H B and Teo K L 2017 *Chin. Phys. B* **26** 110503
[33] Wen G H, Yu W W, Yu X H and Lv J H 2017 *J. Syst. Sci. Complex* **30** 46
[34] Gao Y, Li L X, Peng H P, Yang Y X and Zhang X H 2008 *Acta Phys. Sin.* **57** 2801 (in Chinese)
[35] Sun H Y, Zhang Q L, Li N and Niu H 2010 *Journal of Systems Engineering* **25** 798 (in Chinese)
[36] Wang J L, Wei P C, Wu H N, Huang T and Xu M 2019 *IEEE Trans Syst Man Cybern* **49** 1357
[37] Sun H Y, Li N, Chen Z T and Jia X 2019 *Proceedings of the* 38*th Chinese Control Conference* July 27–30, 2019, Guangzhou, China, p. 5411
[38] Gong D W, Zhang H G, Huang B N and Ren Z Y 2013 *Neural Comput & Applic* **22** 151
[39] Zhu Q X, Cao J D and Rakkiyappan R 2015 *Nonlinear Dyn.* **79** 1085
[40] Cohen M A and Grossberg S 1983 *IEEE Trans Syst Man Cybern* **13** 815
[41] Abdurahman A, Jiang H J and Teng Z D 2015 *Neural Netw.* **69** 20