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Speeding up generation of photon Fock state in a superconducting circuit via counterdiabatic driving*

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Optimal creation of photon Fock states is of importance for quantum information processing and state engineering. Here an efficient strategy is presented for speeding up generation of photon Fock state in a superconducting circuit via counterdiabatic driving. A transmon qubit is dispersively coupled to a quantized electrical field. We address a Λ-configuration interaction between the composite system and classical drivings. Based on two Gaussian-shaped drivings, a single-photon Fock state can be generated adiabatically. Instead of adding an auxiliary counterdiabatic driving, our concern is to modify these two Rabi drivings in the framework of shortcut to adiabaticity. Thus an accelerated operation with high efficiency can be realized in a much shorter time. Compared with the adiabatic counterpart, the shortcut-based operation is significantly insusceptible to decoherence effects. The scheme could offer a promising way to deterministically prepare photon Fock states with superconducting quantum circuits.

Keywords: photon Fock state, superconducting circuit, counterdiabatic driving

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1. Introduction

Analogous to conventional cavity quantum electrodynamics (QED),[1,2] a one-dimensional transmission line resonator (TLR) generates a quantized electromagnetic field, which can be strongly coupled to artificial atoms of superconducting quantum circuits.[3–9] The on-chip system consisting of atom–light interaction constitutes a device of circuit QED in the microwave range.[10,11] By taking advantage of circuit QED, the explorations of quantum information science and fundamental laws have attracted increasing attention in recent years.[12–18] In the context of quantum information processing, generating microwave photon Fock states in circuit QED is an interesting research subject.[19–25] Experimentally, by the method of stimulated Raman adiabatic passage (STIRAP), the generation of microwave photon Fock states has been demonstrated using the setup of superconducting circuit QED.[26] However, the adiabatic operations within relatively long evolution times were easily affected by decoherence sources.

To perform fast and robust quantum operations, a set of technique named as shortcuts to adiabaticity (STA)[27,28] has been exploited to coherently control over superconducting quantum circuits both theoretically and experimentally.[29–38] The method of STA mainly consists of invariant-based inverse engineering,[39] counterdiabatic driving,[40] and fast-forward approach.[41] The central idea of STA-based operation is that a quantum system can be rapidly evolved from a given initial state to a desired final state in an adiabatic-like process by engineering Hamiltonian. Without the restriction of adiabatic theorem, the same goal as the adiabatic case can be realized but in a shorter time. Quantum operation of interest can be completed in a fast way, which significantly mitigates the decoherence effects of noisy environment.[42] Meanwhile, within a given coherence time, the number of high-fidelity quantum operations could be increased considerably.

Very recently, a theoretical scheme has been proposed for the generation of microwave photon Fock states in a circuit QED by the method of invariant-based STA,[43] in which Rabi couplings were specially designed via polynomial ansatz for implementing the coherent process. Differently, although the pulse profile of counterdiabatic driving is generally continuous and smoothed, the counterdiabatic driving behaving as an auxiliary term needs to be added to a reference Hamiltonian.[44,45] In this case, the control variable of auxiliary driving could enhance the complexity of experimental manipulation. Thereby, the generation of photon states based on the technique of STA with feasible and effective Rabi drivings is highly preferable.

In this paper, we develop a theoretical scheme for speeding up adiabatic generation of photon Fock state via counterdiabatic driving. A transmon qubit is dispersively coupled to a cavity field of TLR. By applying classical drivings to the composite system, we deal with a Λ-configuration interaction. Based on two Gaussian-shaped Rabi drivings, a single-photon Fock state can be generated adiabatically by the method of STIRAP. To speed up the adiabatic operation, we adopt the

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technique of STA to obtain two modified Rabi pulses, making the process accelerated remarkably. In contrast to the adiabatic counterpart, the shortcut-based operation is significantly insensitive to the decoherence effects. Thus, the protocol could offer a potential approach to optimally create photon Fock states with superconducting quantum circuits.

This paper is organized as follows. We present a transmon qubit coupled to a TLR in Section 2. Section 3 briefly deals with the generation of single-photon Fock state by the method of STIRAP. Section 4 focuses on an accelerated operation by modifying the Rabi drivings in a shortcut manner. The decoherence effects are numerically analyzed in Section 5. Discussion and a conclusion are finally drawn in Section 6.

2. A composite system of a transmon qubit coupled to a TLR

As schematically depicted in Fig. 1(a), a Cooper-pair box (CPB) in the transmon regime is coupled to a one-dimensional TLR (of length L_r along the horizontal direction) via a capacitance C_r, in which the resonator induces a quantized electrical field with a single-mode frequency ω_r. Through two symmetric Josephson junctions (with same coupling energies E_J and junction capacitances C_J), the CPB including n excess Cooper-pairs is linked to another part of superconductor, forming a loop of superconducting quantum interference device. A static flux Φ_e penetrating the loop can adjust the effective Josephson coupling $\bar{E}_J$. A dc gate voltage V_g exerts on the box via a capacitance C_g. Apart from the static biases Φ_e and V_g, an ac microwave driving $\tilde{V}_s$ is applied to the CPB by means of a capacitance C_s. Besides, a microwave field (of frequency ω_p) is coupled to the resonator through an input capacitance C_i.[46] These two drivings are intended to cause the needed state transitions, as mentioned later.

With no radiations from the quantized field and classical driving, the Hamiltonian of the static CPB system, consisting of the charging energy and Josephson coupling, is described by

$$H_q = 4E_c(n-n_g)^2 - \bar{E}_J\cos\theta. \tag{1}$$

Here $E_c = e^2/2C_\Sigma$ stands for the scale of charging energy, with C_Σ $(= 2C_J + C_g + C_s + C_r)$ being the total capacitance of the CPB, and the offset gate charge induced by V_g is denoted as $n_g = C_gV_g/2e$. The effective Josephson coupling gives $\bar{E}_J = 2E_J\cos(\pi\Phi_e/\Phi_0)$, in which $\Phi_0 = h/2e$ is the flux quantum. The variable θ acts as the average phase difference of the two junctions, which is canonically conjugate to n. For the transmon-regime CPB with a typical ratio of $\bar{E}_J/E_c \sim 50$,[47] the eigenstates and eigenlevels of H_q can be obtained. The first two eigenstates $|g\rangle$ and $|e\rangle$ are chosen to constitute our qubit states having a transition frequency ω_q. Their eigenlevels E_g and E_e are well insensitive to the fluctuation of gate charge n_g, making the qubit states robust against charge noise.

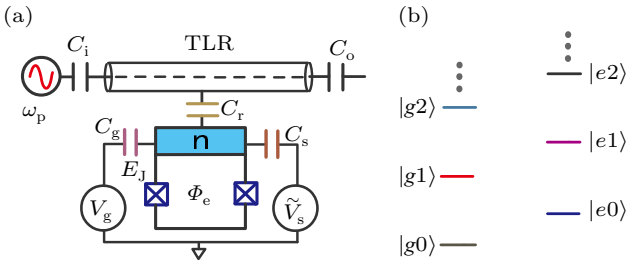


Fig. 1. (a) Schematic setup of a transmon-regime CPB coupled to a quantized electrical field of TLR and a classical driving $\tilde{V}_s$. (b) The energy level diagram of the composite system regarding the transmon qubit dispersively coupled to the cavity field.

The qubit is electrically coupled to the cavity mode of TLR, the coupling strength of which is λ, much smaller than the detuning $\delta = \omega_r - \omega_q$. In the dispersive regime, the qubit–cavity system can be characterized by the Jaynes–Cummings Hamiltonian[26]

$$H_{q-r} = \frac{1}{2}\hbar\omega_q\sigma_z + \hbar(\omega_r + \chi\sigma_z)a^\dagger a, \tag{2}$$

where σ_z $(= |e\rangle\langle e| - |g\rangle\langle g|)$ indicates the Pauli operator acting on states $|g\rangle$ and $|e\rangle$, $\chi = -\lambda^2/\delta$ is the frequency shift owing to the dispersive cavity–qubit coupling, $a^\dagger$ (a) denotes the creation (annihilation) operator regarding cavity photon. As schematically given in Fig. 1(b), the energy level structure of the composite system has two ladders, in which the left (right) one is associated with $|g\rangle$ $(|e\rangle)$. And the rungs of each ladder have a different photon number n_r of cavity field (with $n_r = 0, 1, 2, \ldots$).

3. Single-photon Fock state generation via STIRAP

From the level states shown in Fig. 1(b), we select three product states $|e0\rangle$, $|e1\rangle$, and $|g1\rangle$ to form a three-state system. A driving $\tilde{V}_s = V_s\cos(\omega_s t)$ with microwave frequency ω_s is applied to the CPB, giving rise to the coupling between $|g\rangle$ and $|e\rangle$ of interest. The interaction Hamiltonian between the qubit and microwave field $\tilde{V}_s$ is expressed as[43]

$$H_{q-s} = \hbar\Omega_s\cos(\omega_s t)(|g\rangle\langle e| + |e\rangle\langle g|), \tag{3}$$

in which $\Omega_s = -8E_cO_{ge}n_s$ denotes the Rabi coupling strength, with O_{ge} being the overlap between $|g\rangle$ and $|e\rangle$, and $n_s = C_sV_s/2e$ is the V_s-induced gate charge number. Another microwave driving with frequency ω_p exerts on the cavity field of TLR via input capacitance C_i. The Hamiltonian describing the interaction between the classical driving and cavity mode reads[10]

$$H_{p-r} = \hbar\Omega_p(a\,e^{i\omega_p t} + a^\dagger e^{-i\omega_p t}), \tag{4}$$

where Ω_p acts as the coupling rate that is dependent on the microwave strength. As a result, the desired transition between photon Fock states $|0\rangle$ and $|1\rangle$ can be effectively triggered by

virtue of $H_{\rm p-r}$. Note that the Rabi couplings $\Omega_{\rm s}$ and $\Omega_{\rm p}$ are adjustable in our scheme.

At two-photon resonance, $\omega_{\rm q}+2\chi-\omega_{\rm s}=\omega_{\rm r}+\chi-\omega_{\rm p}=\Delta$, a Λ-configuration interaction within the subspace $\{|e0\rangle,|e1\rangle,|g1\rangle\}$ can be obtained using two coherent drivings, as demonstrated in Fig. 2(a). The Rabi frequencies with respect to the transitions $|e0\rangle\leftrightarrow|e1\rangle$ and $|g1\rangle\leftrightarrow|e1\rangle$ are given by $\Omega_{\rm p}$ and $\Omega_{\rm s}$, respectively. By taking a rotating reference, an interaction Hamiltonian under the rotating wave approximation (RWA) is expressed as

$$H_1=\frac{\hbar}{2}\begin{pmatrix}0&\Omega_{\rm p}&0\\ \Omega_{\rm p}&2\Delta&\Omega_{\rm s}\\ 0&\Omega_{\rm s}&0\end{pmatrix}. \tag{5}$$

As mentioned in Ref. [26], population transfer from an initial state $|e0\rangle$ to a prescribed state $|g1\rangle$ can be realized by the technique of STIRAP, leading to a single-photon excitation of $|0\rangle\rightarrow|1\rangle$. To obtain this evolution trajectory, the profiles of Rabi drivings are designed to be Gaussian functions of time $t\in[0,t_{\rm f}]$

$$\Omega_{\rm p}(t) = \Omega_0\,{\rm e}^{-(t-t_{\rm f}/2-\tau)^2/\sigma^2}, \tag{6}$$
$$\Omega_{\rm s}(t) = \Omega_0\,{\rm e}^{-(t-t_{\rm f}/2+\tau)^2/\sigma^2}, \tag{7}$$

where $t_{\rm f}$ is the final time, Ω_0 indicates the amplitude peak of Rabi drivings, τ and σ represents the pulse-related parameters. As demonstrated in Fig. 2(b), these two drivings are applied to the composite system in a STIRAP-like sequence.

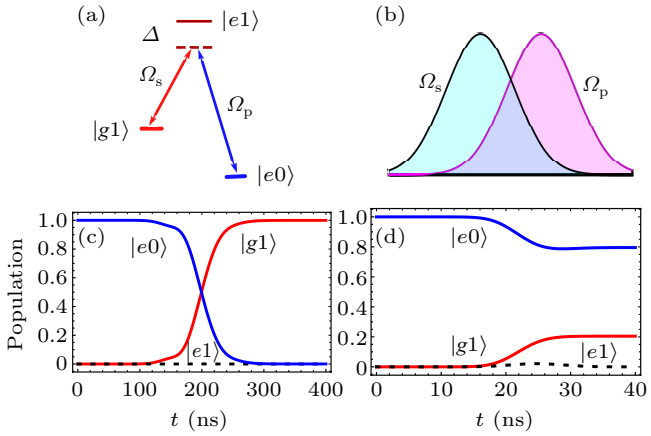


Fig. 2. (a) With a common detuning Δ, two microwave drivings induce the state transitions between $|e1\rangle$ and $|g1\rangle$ ($|e0\rangle$) with Rabi rates $\Omega_{\rm s}$ ($\Omega_{\rm p}$), respectively. (b) Two Gaussian-profile Rabi drivings $\Omega_{\rm s}$ and $\Omega_{\rm p}$ are applied to the three-state system in a STIRAP-pulse order. (c) In the adiabatic case, population transfer from $|e0\rangle$ to $|g1\rangle$ can be realized in a time $t=400$ ns. (d) An incomplete transfer from $|e0\rangle$ to $|g1\rangle$ induced by two non-adiabatic drivings $\Omega_{\rm s}(t)$ and $\Omega_{\rm p}(t)$ with $t\in[0,40$ ns$]$.

Within the subspace $\{|e0\rangle,|e1\rangle,|g1\rangle\}$, the time evolution of an arbitrary state $|\psi\rangle$ is governed by the Schrödinger equation $\mathrm{i}\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi\rangle=H_1|\psi\rangle$. Under the adiabatic condition $\int_0^{t_{\rm f}}\Omega_{\rm p,s}(t)\,\mathrm{d}t\gg1$, state transfer from $|e0\rangle$ to $|g1\rangle$ can be well performed in a duration time $t_{\rm f}=400$ ns, see Fig. 2(c), in which the detuning takes $\Delta/2\pi=0.65$ GHz. And the pulse parameters are chosen as $\Omega_0/2\pi=0.21$ GHz, $\tau=t_{\rm f}/8$, and $\sigma=t_{\rm f}/5$. For the adiabatic operation, the evolution time is relatively long, which easily makes the quantum operation susceptible to noise effects. In the context of quantum information processing, significantly shortening the operation time is an attractive approach to mitigate decoherence effects. Based on two Gaussian-type pulses $\Omega_{\rm p}(t)$ and $\Omega_{\rm s}(t)$ but within a shorter duration time $t\in[0,t_{\rm f}]$ ($t_{\rm f}=40$ ns, $\tau=t_{\rm f}/8$ ns, and $\sigma=t_{\rm f}/5$), a complete transfer from initial state $|e0\rangle$ to $|g1\rangle$ cannot be achieved effectively, as shown in Fig. 2(d). Numerically, it is found that the probabilities occupied by $|e0\rangle$ and $|g1\rangle$ (at the final time $t_{\rm f}=40$ ns) are 79.54% and 20.46%, respectively, meaning that directly shortening the pulse time does not induce our desired state inversion. This is due to the considerable non-adiabatic coupling between the eigenstates of H_1 in the present condition. Thereby, how to realize a full population transfer from $|e0\rangle$ to $|g1\rangle$ within the shorter time is the following concern.

4. Accelerated generation of single-photon Fock state via STA

In what follows, we speed up the adiabatic operation for rapidly creating photon Fock state by the technique of STA with effective counterdiabatic driving. Generally, an auxiliary driving needs to be applied to the three-state system for causing the target transition from $|e0\rangle$ to $|g1\rangle$. In this case, system evolution driven by the total Hamiltonian can exactly follow an eigenstate of H_1, owing to the effective elimination of non-adiabatic transition between eigenstates. Despite the permission by level transition rule, the counterdiabatic driving will unavoidably increase the number of control variables, enhancing the possible complexity of experimental manipulation. Therefore, as an interesting issue for coherent control in experiment, the STA-based operation with a minimized number of drivings is our focus.

In the large-detuning regime $\Delta\gg\Omega_{\rm p,s}$, the occupied probability of intermediate state $|e1\rangle$ can be safely neglected during the transfer process $|e0\rangle\rightarrow|g1\rangle$. As a result, the three-state system $\{|e0\rangle,|e1\rangle,|g1\rangle\}$ can be reduced to a two-state system $\{|e0\rangle,|g1\rangle\}$ after adiabatically removing $|e1\rangle$.[48] The two-state system is described by a reference Hamiltonian

$$H_{\rm ts}=\frac{\hbar}{2}\begin{pmatrix}-\Delta_{\rm ts}&\Omega_{\rm ts}\\ \Omega_{\rm ts}&\Delta_{\rm ts}\end{pmatrix}, \tag{8}$$

where $\Delta_{\rm ts}$ and $\Omega_{\rm ts}$ are related to Δ and $\Omega_{\rm p,s}$ by

$$\Delta_{\rm ts}=\frac{\Omega_{\rm p}^2-\Omega_{\rm s}^2}{4\Delta},\quad \Omega_{\rm ts}=-\frac{\Omega_{\rm p}\Omega_{\rm s}}{2\Delta}. \tag{9}$$

To the reference Hamiltonian $H_{\rm ts}$, a counterdiabatic driv-

ing $H_{\rm cd}$ is given by[44,49]

$$H_{\rm cd}=\frac{\hbar}{2}\begin{pmatrix}0 & -{\rm i}\Omega_{\rm cd}\\ {\rm i}\Omega_{\rm cd} & 0\end{pmatrix}, \tag{10}$$

with $\Omega_{\rm cd}=(\Omega_{\rm ts}\dot{\Delta}_{\rm ts}-\dot{\Omega}_{\rm ts}\Delta_{\rm ts})/(\Delta_{\rm ts}^2+\Omega_{\rm ts}^2)$. The total Hamiltonian, consisting of $H_{\rm ts}$ and $H_{\rm cd}$, can thus be expressed as

$$H=\frac{\hbar}{2}\begin{pmatrix}-\Delta_{\rm ts} & \sqrt{\Omega_{\rm ts}^2+\Omega_{\rm cd}^2}\,{\rm e}^{-{\rm i}\phi}\\ \sqrt{\Omega_{\rm ts}^2+\Omega_{\rm cd}^2}\,{\rm e}^{{\rm i}\phi} & \Delta_{\rm ts}\end{pmatrix}, \tag{11}$$

in which the phase angle satisfies $\phi=\arctan(\Omega_{\rm cd}/\Omega_{\rm ts})$. After making a unitary transformation on H, we have an effective Hamiltonian

$$\begin{aligned}H_{\rm eff}&=U^{\dagger}HU-{\rm i}\hbar U^{\dagger}\dot{U}\\&=\frac{\hbar}{2}\begin{pmatrix}-\Delta_{\rm eff} & \Omega_{\rm eff}\\ \Omega_{\rm eff} & \Delta_{\rm eff}\end{pmatrix},\end{aligned} \tag{12}$$

where the unitary operator is

$$U=\begin{pmatrix}{\rm e}^{-{\rm i}\phi/2} & 0\\ 0 & {\rm e}^{{\rm i}\phi/2}\end{pmatrix}, \tag{13}$$

and the relevant variables are $\Delta_{\rm eff}=\Delta_{\rm ts}+\dot{\phi}$ and $\Omega_{\rm eff}=\sqrt{\Omega_{\rm ts}^2+\Omega_{\rm cd}^2}$. Obviously, the Hamiltonian $H_{\rm eff}$ has a form similar to that of $H_{\rm ts}$ in Eq. (8). By an analogy of $H_{\rm eff}$ with $H_{\rm ts}$, we obtain the following relations:

$$\Delta_{\rm eff}=\frac{\tilde{\Omega}_{\rm p}^2-\tilde{\Omega}_{\rm s}^2}{4\tilde{\Delta}},\quad \Omega_{\rm eff}=-\frac{\tilde{\Omega}_{\rm p}\tilde{\Omega}_{\rm s}}{2\tilde{\Delta}}, \tag{14}$$

where $\tilde{\Omega}_{\rm p}$ and $\tilde{\Omega}_{\rm s}$ are the modified pump and Stokes drivings that contain the contribution of $\Omega_{\rm cd}$. And $\tilde{\Delta}$ acts as a frequency, which is assumed to be Δ in our scenario.[49] $\tilde{\Omega}_{\rm p}$ and $\tilde{\Omega}_{\rm s}$ can be inversely obtained as follows:

$$\tilde{\Omega}_{\rm p}=\sqrt{2\tilde{\Delta}[\sqrt{\Delta_{\rm eff}^2+\Omega_{\rm eff}^2}+\Delta_{\rm eff}]}, \tag{15}$$

$$\tilde{\Omega}_{\rm s}=\sqrt{2\tilde{\Delta}[\sqrt{\Delta_{\rm eff}^2+\Omega_{\rm eff}^2}-\Delta_{\rm eff}]}. \tag{16}$$

After going back into the subspace $\{|e0\rangle,|e1\rangle,|g1\rangle\}$, we focus on the interaction of the three-state system with two modified Rabi drivings $\tilde{\Omega}_{\rm p}$ and $\tilde{\Omega}_{\rm s}$, as shown in Fig. 3(a). Thus the interaction Hamiltonian is given by

$$H_2=\frac{\hbar}{2}\begin{pmatrix}0 & \tilde{\Omega}_{\rm p} & 0\\ \tilde{\Omega}_{\rm p} & 2\Delta & \tilde{\Omega}_{\rm s}\\ 0 & \tilde{\Omega}_{\rm s} & 0\end{pmatrix}. \tag{17}$$

As mentioned before, the large detuning Δ guarantees a negligible population of $|e1\rangle$. Compared with the drivings indicated in Fig. 2(a), $\tilde{\Omega}_{\rm p}$ and $\tilde{\Omega}_{\rm s}$, integrating the contribution of counterdiabatic driving, are capable of inducing an accelerated population transfer from $|e0\rangle$ to $|g1\rangle$ in a shortcut manner. Without the restriction of adiabatic theorem, the operation can be fully completed within a much shorter time and remains high robustness against the imperfection of control variables. In contrast to two Gaussian-type drivings $\Omega_{\rm p}$ and $\Omega_{\rm s}$ that cause an adiabatic evolution demonstrated in Fig. 2(d), we give the modified drivings $\tilde{\Omega}_{\rm p}$ and $\tilde{\Omega}_{\rm s}$ as functions of time $t\in[0,40\ {\rm ns}]$, see Fig. 3(b). The amplitude peaks of $\tilde{\Omega}_{\rm p,s}/2\pi$ are about 0.25 GHz, which are accessible in superconducting artificial atoms.[43] Meanwhile, the smooth profiles of $\tilde{\Omega}_{\rm p,s}$ are beneficial to experimental realization. Note that in the current condition of pulse calibration technology, the modified drivings $\tilde{\Omega}_{\rm s}$ ($\tilde{\Omega}_{\rm p}$) having a non-zero value at the initial (final) time $t=0$ ($t_{\rm f}=40$ ns) could be accessible.[32,50]

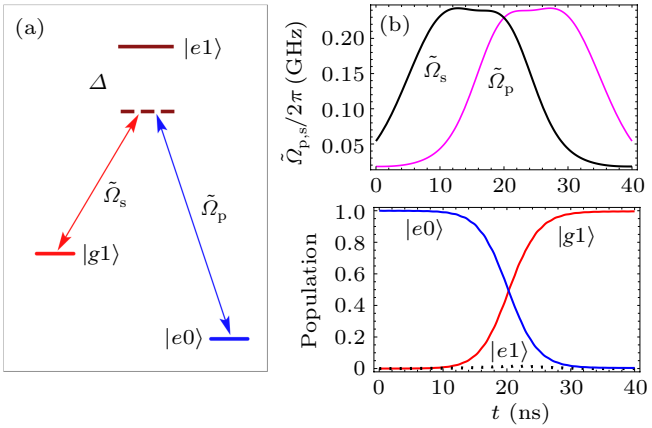


Fig. 3. (a) According to the STA for eliminating non-adiabatic transition, two modified Rabi couplings $\tilde{\Omega}_{\rm s}$ and $\tilde{\Omega}_{\rm p}$ are utilized to drive the three-state system $\{|e0\rangle,|e1\rangle,|g1\rangle\}$. $\tilde{\Omega}_{\rm s}$ and $\tilde{\Omega}_{\rm p}$ as functions of time $t\in[0,40\ {\rm ns}]$ in (b), and the resulting fast population transfer from $|e0\rangle$ to $|g1\rangle$ in (c).

Driven by H_2 with the modified drivings $\tilde{\Omega}_{\rm p,s}$, a rapid state transfer from $|e0\rangle$ to $|g1\rangle$ can be implemented in a time as short as $t=40$ ns, leading to the creation of single-photon Fock state. Numerically, it is found that the occupied probability of $|g1\rangle$ at the final time $t_{\rm f}=40$ ns reaches up to 99.58%. Note that the probability occupied by $|e1\rangle$ is not more than 1.9% during the whole process, and the probability gets vanishingly small when the transfer is finished, as plotted in Fig. 3(c). These results also indicate that the adiabatical elimination of the intermediate state $|e1\rangle$ is valid enough in the present subspace.

5. Decoherence mitigation and negligible leakage errors

Practically, the qubit and cavity field are inevitably coupled to noisy environment, resulting in decoherence effects on quantum operation under consideration. Different from the adiabatic case, the creation of photon state in a shortcut way needs a shorter time, which contributes to reduce decoherence effects and then enhance fidelity. For the considered operations, we next analyze the effects of qubit decoherence and cavity decay. The reduced density matrix ρ (regarding $|e0\rangle$, $|e1\rangle$, and $|g1\rangle$) can be obtained by tracing over the environmental degrees of freedom. In the Born–Markov approxima-

tion, the time evolution of ρ is described by the Lindblad master equation[43]

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\mathrm{i}\,[H_j,\rho] + \kappa D[a]\rho + \gamma D[\sigma_-]\rho + \frac{\gamma_\phi}{2} D[\sigma_z]\rho, \quad (18)$$

where the first term of right side denotes the decoherence-free system evolution subject to H_j (with $j = 1$ and 2), and the last three terms represent the cavity decay and qubit relaxation (dephasing) effects, respectively. Here κ is the decay rate of the cavity photon, γ (γ_ϕ) stands for the relaxation (dephasing) rate between states $|g\rangle$ and $|e\rangle$. The Lindblad superoperators take $D[L]\rho = (2L\rho L^\dagger - L^\dagger L\rho - \rho L^\dagger L)/2$, with $L = a$, σ_-, and σ_z. The lowering operator is given by $\sigma_- = |g\rangle\langle e|$, and $\sigma_z (= |e\rangle\langle e| - |g\rangle\langle g|)$ is defined as the Pauli operator.

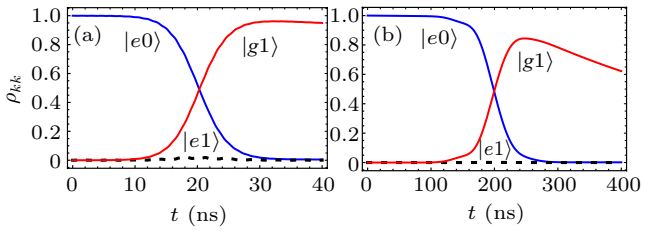


Fig. 4. In the presence of decoherence effects, the density matrix elements ρ_{kk} versus time for the cases of the shortcut-based operation (a) and the STIRAP-like process (b).

By numerically solving Eq. (18) with the accessible decoherence rates $\kappa/2\pi = 0.37$ MHz,[51] $\gamma/2\pi = 6.6\times10^{-3}$ MHz, and $\gamma_\phi/2\pi = 7.2\times10^{-3}$ MHz,[26] we first address a time-dependent ρ in the case of shortcut-based evolution with an initial state $|e0\rangle$. The matrix elements ρ_{kk} (with $k = e0$, $e1$, and $g1$), which represent the occupation probabilities of composite states $|e0\rangle$, $|e1\rangle$, and $|g1\rangle$, are illustrated in Fig. 4(a). Clearly, in spite of decoherence effects, the occupied probability of target state $|g1\rangle$ (at the final time $t_\mathrm{f} = 40$ ns) could approach $\rho_{g1g1} \simeq 0.949$. As an adiabatic counterpart, we comparatively consider the decoherence effects on the STIRAP-based state transfer, as indicated in Fig. 4(b). Due to a much longer duration time, the adiabatic operation is greatly sensitive to the decoherence effects. The occupation probability of $|g1\rangle$ remarkably drops down to $\rho_{g1g1} \simeq 0.622$ at the final time $t_\mathrm{f} = 400$ ns. As a consequence, benefited from the shortened time of evolution subject to the counterdiabatic driving, the decoherence effects can be reduced significantly. In this way, high-fidelity generation of photon states could be effectively explored with the superconducting circuit QED.

In addition, leakage transitions caused by the modified Rabi drivings $\tilde{\Omega}_{\mathrm{s,p}}$ can be safely neglected in the regime of large detuning. First, the $\tilde{\Omega}_\mathrm{s}$-induced leakage beyond the considered subspace $\{|g\rangle, |e\rangle\}$ is vanishingly small. Assume that the characteristic parameters of the transmon-type CPB satisfy $\bar{E}_\mathrm{J}/E_\mathrm{c} = 50$, with $E_\mathrm{c}/\hbar = 2\pi\times321$ MHz.[43] We address a dominant leakage from $|e\rangle$ to $|f\rangle$, where $|f\rangle$ is the second excited state of the transmon system. Here the microwave driving (with a frequency ω_s) is largely detuned to transition frequencies ω_{fe} and ω_{eg}. In spite of the system levels with weak anharmonicity, the driving scarcely induces the leakage transition between $|e\rangle$ and $|f\rangle$, as mentioned in Ref. [52]. At the same time, $\tilde{\Omega}_\mathrm{p}$ (with an amplitude peak $\tilde{\Omega}_\mathrm{p}/2\pi = 0.25$ GHz) causes a negligible leakage outside the subspace $\{|e0\rangle, |e1\rangle, |g1\rangle\}$. For a potential leakage of $|e1\rangle \leftrightarrow |e2\rangle$ triggered by $\tilde{\Omega}_\mathrm{p}$, the probability occupied by $|e2\rangle$ during the desired transfer process from $|e0\rangle$ to $|g1\rangle$ is not higher than 0.5%. Thereby, benefited from the prohibition of large detuning Δ, the leakage transition $|e1\rangle \leftrightarrow |e2\rangle$ can be omitted as well.

6. Discussion and conclusion

Our strategy could have the following advantages and features. (i) Different from the general case of adding an auxiliary counterdiabatic driving to a reference Hamiltonian,[44] our concentration is to modify the original drivings by taking the core idea of STA. Thereby, the number of drivings will not be increased, contributing to reduce the complexity of coherent control experimentally. (ii) Compared to the invariant-based pulses with specified time profiles,[43] the drivings adopted in the present protocol are derived from the modification of two commonly utilized Gaussian-type pulses, which are helpful to physical realization in the condition of current technology. (iii) Owing to the dispersive interaction of the qubit with cavity mode, the direct transition $|e0\rangle \leftrightarrow |g1\rangle$ caused by the cavity field will be suppressed greatly. In this case, we can address an effective Λ-configuration three-state system through two individual drivings $\tilde{\Omega}_\mathrm{p}$ and $\tilde{\Omega}_\mathrm{s}$. (iv) With the assistance of a necessary process of qubit initialization, as mentioned in Refs. [26,43], our scheme could also be generalized to sequently induce multi-photon Fock states, which are desirable to quantum state engineering. (v) Based on the method of STA with modified Rabi drivings, Ref. [49] considered state transfers within a single cold atom and an electron spin of nitrogen-vacancy center. Differently, our protocol focuses on coherent control over a composite system of a transmon qubit coupled to a cavity field, thus providing a potential possibility of investigating many-body dynamics in a shortcut way. (vi) With the conventional method of vacuum Rabi coupling, one can generate photon Fock state in a rapid manner. However, such operation is typically sensitive to timing error and variable fluctuations. Differently, subject to the designed coherent drivings, the shortcut-based operation circumvents these error effects while requiring evolution time much shorter than that of adiabatic case.

In conclusion, we develop a theoretical scheme for speeding up adiabatic generation of photon Fock state via effective

counterdiabatic driving. A transmon-regime CPB qubit is dispersively coupled to a cavity field of TLR. By applying two classical drivings, we address a Λ-configuration interaction in the composite system. First, based on two Gaussian-shaped Rabi drivings, a single-photon Fock state can be adiabatically generated by the method of STIRAP. In the framework of STA, we modify these two Rabi drivings, instead of directly adding an auxiliary counterdiabtic driving, to achieve a remarkably accelerated operation. Due to the shortened evolution time, the decoherence effects on the shortcut-based operation are significantly mitigated in comparison with the adiabatic counterpart. Compared with the conventional methods, the strategy could pave a promising avenue towards fast and robustly preparing photon Fock states with superconducting quantum circuits.

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