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Citation: Chin. Phys. B, 2021, 30 (**7**): 070303. DOI: 10.1088/1674-1056/abe1a9
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Steered coherence and entanglement in the Heisenberg *XX* chain under twisted boundary conditions*

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(Received 31 October 2020; revised manuscript received 8 January 2021; accepted manuscript online 1 February 2021)

We study steered coherence (SC) and entanglement in a three-spin Heisenberg *XX* model under twisted boundary conditions and show that their strengths can be significantly enhanced by tuning the twist angle. The optimal twist angle $\theta_{\rm opt}$ for achieving the maximum l_1 norm of SC is π in the region of weak field B and decreases gradually from π to 0 when B increases after a critical value, while for the relative entropy of SC, $\theta_{\rm opt}$ equals π in the weak field region and 0 otherwise. The entanglement and the critical temperature above which the entanglement vanishes can also be significantly enhanced by tuning the twist angle from 0 to π.

Keywords: quantum coherence, entanglement, twisted boundary conditions

PACS: 03.65.Yz, 64.70.Tg, 75.10.Pq **DOI:** 10.1088/1674-1056/abe1a9

1. Introduction

Quantum spin systems provide the natural playground for studying nontrivial physical phenomena in many-body physics. Traditionally, these phenomena were studied by calculating various thermal quantities such as the entropy, magnetization, magnetic susceptibility, and specific heat.[1] With the development of quantum information science, a variety of quantum correlations have also been proven to be efficient tools to study quantum characters of a system in a physically meaningful and mathematically rigorous fashion. Among them, entanglement which was first noted in 1935[2] and experienced rapid development in the past few decades, has been extensively investigated.[3] Some more general quantum correlations, such as quantum discord[4] and measurement-induced nonlocality,[5–7] also provide intriguing perspectives for understanding quantumness of a many-body system.[8]

While quantum correlations describe quantum characters of the bipartite or multipartite states, there is a more fundamental notion called quantum coherence. It stems from the superposition principle of the basis states and characterizes quantumness of a single-partite system. Although it has been perceived earlier than those of different quantum correlations, a rigorous framework for quantifying coherence was formulated only in 2014.[9] It is the resource theory of coherence, within which a variety of coherence measures were proposed in the past few years.[9–17] Inspired by these coherence measures, researches focused on the evolution[18–25] and freezing[26–29] of coherence, as well as its fundamental properties,[30–33] operational interpretation,[34–39] and role in quantum information tasks,[40–45] have been carried out. Moreover, quantum coherence in various spin-chain models was analyzed in Refs. [46–51] using these measures.

When considering the multipartite systems, coherence is also helpful in interpreting quantum correlations such as entanglement[10,16,52] and quantum discord.[53–57] In particular, an operational connection between steered coherence (SC) and nonlocal quantum resources (e.g., entanglement[58,59] and Bell nonlocality[60]) has been revealed. When the spin-chain systems are considered, the control of SC has been studied in Refs. [61–64], and the role of SC in detecting quantum phase transitions can be found in Ref. [65]. Different from the previous works, in this work we investigate the Heisenberg *XX* model with a twisted boundary. We will show that both the SC and entanglement for thermal equilibrium states of this model can be significantly enhanced by tuning the twist angle θ to an optimal value.

This paper is organized as follows. In Section 2 we recall how to measure the SC and entanglement. In Section 3 we introduce the physical model. In Section 4, the SC and entanglement for thermal states of the considered model with varying twist angle are analyzed. Finally, Section 5 is devoted to a summary of the main finding of this paper.

2. Measures of the SC and entanglement

To start with, we recall in short how to measure the amount of SC in the bipartite states. As such a measure is based on the resource theory of coherence, we first review two extensively used measures of coherence.[9] The first one is the l_1 norm of coherence defined by $\|\rho-\delta\|_{l_1}$ minimized over all $\delta\in\mathcal{I}$, where ρ is the state of interest and $\mathcal{I}$ represents the set of incoherent states. In the reference basis $\{|i\rangle\}$, it can be

*Project supported by the National Natural Science Foundation of China (Grant No. 11675129).

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obtained as[9]

$$C_{l_1}(\rho)=\sum_{i\neq j}|\langle i|\rho|j\rangle|, \tag{1}$$

which is just the sum of the absolute values of the off-diagonal elements of ρ. The second one is the relative entropy of coherence defined by $S(\rho\|\delta)$ minimized over all $\delta\in\mathcal{I}$, and can be obtained analytically as[9]

$$C_{\rm re}(\rho)=S(\rho_{\rm diag})-S(\rho), \tag{2}$$

where $\rho_{\rm diag}$ represents the density operator obtained by deleting all the off-diagonal elements of ρ, and $S(\rho)=-{\rm tr}(\rho\log_2\rho)$ is the von Neumann entropy of ρ.

Based on these coherence measures, one can introduce the SC for a two-qubit state ρ_{AB}. It was defined by first performing the local measurements $\Pi_i^{\pm}=(I\pm\sigma_i)/2$ ($\sigma_{1,2,3}$ are the standard Pauli operators) on qubit A and then calculate the average coherence of the collapsed states for qubit B. Depending on the framework adopted, the SC can be defined in two different manners. The first one is given by[58]

$$C_\alpha^{\rm sc}(\rho_{AB})=\frac{1}{2}\sum_{\substack{i,j,a\\ i\neq j}}p_{a|\sigma_i}C_\alpha^{\sigma_j}(\rho_{B|\sigma_i^a}), \tag{3}$$

where $p_{a|\sigma_i}$ represents the probability of the measurement outcome a, $\rho_{B|\sigma_i^a}$ is the corresponding collapsed state on qubit B, and $\{\sigma_{1,2,3}\}$ constitute a set of mutually unbiased observables in the two-dimensional Hilbert space.[66,67] Moreover, $C_\alpha^{\sigma_j}(\cdots)$ represents the coherence with respect to the eigenbasis of σ_j, and $\alpha\in\{l_1,re\}$ was introduced for distinguishing the SC related to the two coherence measures.

The SC given in Eq. (3) corresponds to B's coherence averaged over $C_\alpha^{\sigma_j}(\rho_{B|\sigma_i^a})$ and $C_\alpha^{\sigma_k}(\rho_{B|\sigma_i^a})$ $(j,k\neq i)$ after each local measurements $\Pi_i^{\pm}$ with equal probability. Apart from this, it can also be formulated by first averaging B's coherence over $C_\alpha^{\sigma_{\nu_i}}(\rho_{B|\sigma_i^a})$ after each measurement $\Pi_i^{\pm}$ with a one-to-one correspondence between i and ν_i, and then maximizing it over the six possible permutations of $\{\nu_1,\nu_2,\nu_3\}$. For this case, the SC can be written as[59]

$$\tilde{C}_\alpha^{\rm sc}(\rho_{AB})=\max_{\{\nu_i\}}\sum_{i,a}p_{a|\sigma_i}C_\alpha^{\sigma_{\nu_i}}(\rho_{B|\sigma_i^a}). \tag{4}$$

As it has been proven that $\tilde{C}_\alpha^{\rm sc}(\rho_{AB})\geqslant C_\alpha^{\rm sc}(\rho_{AB})$,[59] we consider in the following only the SC given in Eq. (4). Moreover, as the SC is intimately related to entanglement,[58,59] we will further compare behaviors of the SC with pairwise entanglement in ρ_{AB}. To this end, we consider the entanglement of formation defined as[68,69]

$$E_{\rm f}(\rho_{AB})=H_2\left(\frac{1+\sqrt{1-C^2(\rho_{AB})}}{2}\right), \tag{5}$$

where $H_2(\cdots)$ denotes the binary Shannon entropy function, and $C(\rho_{AB})$ is the concurrence given by[69]

$$C(\rho_{AB})=\max\{0,\lambda_1-\lambda_2-\lambda_3-\lambda_4\}, \tag{6}$$

with λ_i's being the square roots of the eigenvalues of the product matrix $R=\rho_{AB}(\sigma_2\otimes\sigma_2)\rho_{AB}^*(\sigma_2\otimes\sigma_2)$ arranged in non-increasing order.

3. The Heisenberg *XX* model

We consider the Heisenberg *XX* model with twisted boundary conditions. The corresponding Hamiltonian is given by

$$\hat{H}=\sum_{i=1}^{3}\left[J\left(s_i^xs_{i+1}^x+s_i^ys_{i+1}^y\right)+Bs_i^z\right], \tag{7}$$

where $s_i^{x,y,z}$ denote the spin-1/2 operators at site i ($s_i^x=\sigma_1/2$, $s_i^y=\sigma_2/2$, $s_i^z=\sigma_3/2$), J is the coupling constant of the neighboring spins, and B is the transverse magnetic field applied to the system. In this paper, we consider the twisted boundary conditions, which corresponds to[70,71]

$$s_4^x=\cos\theta s_1^x-\sin\theta s_1^y,\quad s_4^y=\sin\theta s_1^x+\cos\theta s_1^y, \tag{8}$$

that is, the third spin interacts with the first spin after rotating an angle θ along the z-axis clockwise. Clearly, Eq. (8) reduces to the usual periodic boundary conditions when $\theta=0$ and to the anti-periodic boundary conditions when $\theta=\pi$.

For this model, the thermal equilibrium state at temperature T can be described by the density operator $\rho(T)=Z^{-1}\,{\rm e}^{-\hat{H}/k_{\rm B}T}$, with $Z={\rm tr}({\rm e}^{-\hat{H}/k_{\rm B}T})$ being the partition function and $k_{\rm B}$ is Boltzman's constant. By virtue of the eigenvalues $\{\varepsilon_i\}$ and the eigenstates $\{|\psi_i\rangle\}$ of $\hat{H}$, $\rho(T)$ can be written explicitly as

$$\rho(T)=\frac{1}{Z}\sum_{i=1}^{8}{\rm e}^{-\varepsilon_i/k_{\rm B}T}|\psi_i\rangle\langle\psi_i|, \tag{9}$$

where the partition function $Z=\sum_i{\rm e}^{-\varepsilon_i/k_{\rm B}T}$.

Because the form of $\hat{H}$ is complicated, it is difficult to derive $\{\varepsilon_i\}$ and $\{|\psi_i\rangle\}$ analytically, hence in the following we solve them numerically. As we consider in this paper the SC and the pairwise entanglement, we further derive the reduced density operator $\rho_{ij}(T)={\rm tr}_k\rho(T)$, which is in the following form:

$$\rho_{ij}(T)=\begin{pmatrix}a&0&0&0\\0&b&z_{ij}&0\\0&z_{ij}^*&b&0\\0&0&0&c\end{pmatrix}, \tag{10}$$

where the matrix elements a, b, and c are the same for different ρ_{ij}, while z_{ij} are different for different ρ_{ij}, but their absolute values are also the same. All these elements can be determined

numerically, then $\tilde{C}_{\alpha}^{\rm sc}(\rho_{ij})$ and $E_{\rm f}(\rho_{ij})$ can be obtained using the formulae in Section 2. In particular, the concurrence can be obtained as

$$C(\rho_{ij}) = 2\max\left\{0, |z_{ij}| - \sqrt{ac}\right\}, \tag{11}$$

hence we always have $E_{\rm f}(\rho_{12}) = E_{\rm f}(\rho_{13})$.

In the following, we will assume $J = 1$ and $k_{\rm B} = 1$, i.e., all our subsequent discussions are in units of J and $k_{\rm B}$.

4. Improving SC and entanglement by tuning the twist angle

Based on the above preliminaries, we start to study how the twist angle θ and the transverse magnetic field B affect the SC and entanglement in the XX model. Because it can be directly obtained from Eq. (7) that both $\tilde{C}_{\alpha}^{\rm sc}(\rho_{ij})$ and $E_{\rm f}(\rho_{ij})$ are symmetric with respect to $B = 0$ and $\theta = \pi$, we present only those results for $B \geqslant 0$ and $\theta \in [0, \pi]$. In fact, entanglement in the cyclic XX[72] and XXX chains[73,74] have been studied. Moreover, by assuming the topological boundary conditions, entanglement and quantum discord in the Heisenberg XXZ chain[75] and nonlocal advantage of quantum coherence in the Heisenberg XXX chain[63] have also been studied recently.

4.1. The steered coherence

First, we consider the l_1 norm of SC for the thermal states at finite temperature. In Fig. 1, we display the three-dimensional plots of the dependence of $\tilde{C}_{l_1}^{\rm sc}(\rho_{12})$ and $\tilde{C}_{l_1}^{\rm sc}(\rho_{13})$ on B and θ/π with $T = 0.1$. One can observe that in the weak field region, $\tilde{C}_{l_1}^{\rm sc}(\rho_{12})$ increases monotonically with the increase of θ, while in the strong field region, both $\tilde{C}_{l_1}^{\rm sc}(\rho_{12})$ and $\tilde{C}_{l_1}^{\rm sc}(\rho_{13})$ decrease slightly with the increase of θ. In the intermediate region of B, they are not monotonic functions of θ [$\tilde{C}_{l_1}^{\rm sc}(\rho_{13})$ is not a monotonic function of θ even in the weak field region]. Moreover, the relation between $\tilde{C}_{l_1}^{\rm sc}(\rho_{12})$ and $\tilde{C}_{l_1}^{\rm sc}(\rho_{13})$ is parameter dependent. For $T = 0.1$, their strengths are approximately the same in the strong field region, while in the weak field region, the former is smaller than the latter for θ taking the intermediate value and larger than the latter otherwise.

Moreover, one can see from Fig. 1(a) that in the weak field region, the optimal twist angle at which $\tilde{C}_{l_1}^{\rm sc}(\rho_{12})$ takes its maximum is $\theta_{\rm opt} = \pi$, that is, the chain under anti-periodic boundary conditions is beneficial for creating SC. In the intermediate region of B, however, $\theta_{\rm opt}$ becomes smaller than π, and when the strength of B is strong enough, $\theta_{\rm opt}$ will decreases to 0. When considering $\tilde{C}_{l_1}^{\rm sc}(\rho_{13})$, as can be seen from Fig. 1(b), the optimal twist angle $\theta_{\rm opt}$ shows a similar dependence on B, and the only difference is that the critical magnetic field after which $\theta_{\rm opt}$ becomes smaller than π is slightly different.

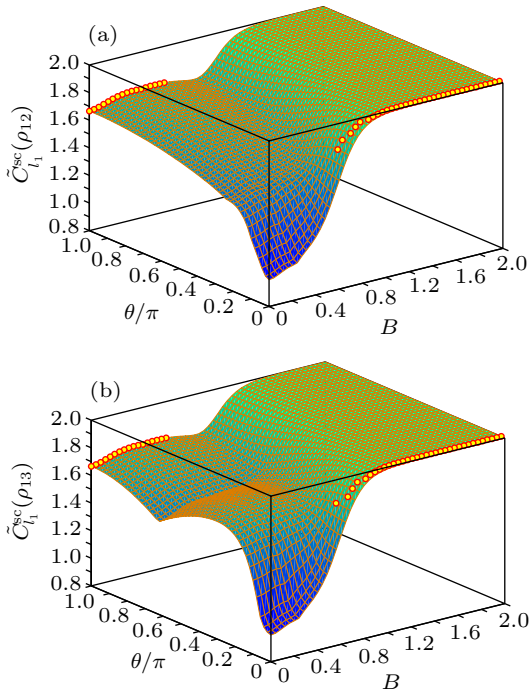


Fig. 1. $\tilde{C}_{l_1}^{\rm sc}(\rho_{12})$ and $\tilde{C}_{l_1}^{\rm sc}(\rho_{13})$ versus B and θ/π for the Heisenberg XX model with $T = 0.1$. The circles denote the critical points at which the corresponding SC takes its maximum.

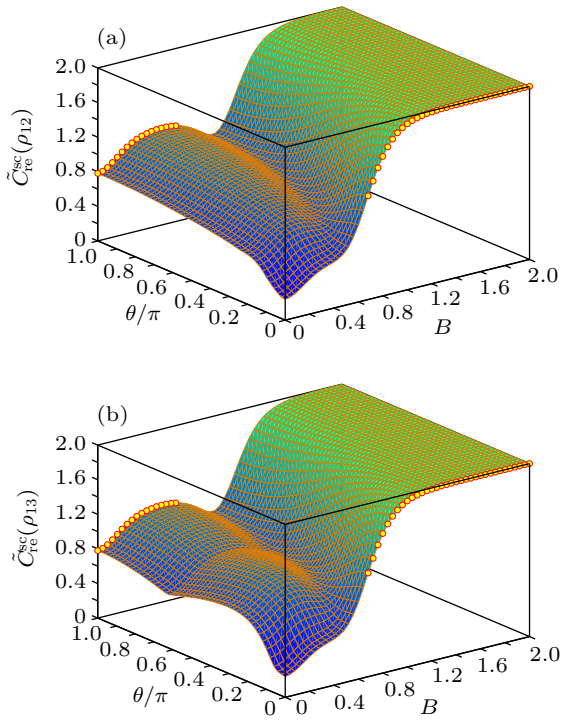


Fig. 2. $\tilde{C}_{\rm re}^{\rm sc}(\rho_{12})$ and $\tilde{C}_{\rm re}^{\rm sc}(\rho_{13})$ versus B and θ/π for the Heisenberg XX model with $T = 0.1$. The circles denote the critical points at which the corresponding SC takes its maximum.

Next, we consider the relative entropy of SC for the reduced thermal states at finite temperature $T = 0.1$. The corresponding three-dimensional plots are shown in Fig. 2, from which one can observe that they show similar behaviors to that of the l_1 norm of SC. That is, $\tilde{C}_{\rm re}^{\rm sc}(\rho_{12})$ [both $\tilde{C}_{\rm re}^{\rm sc}(\rho_{12})$ and $\tilde{C}_{\rm re}^{\rm sc}(\rho_{13})$] behaves as a monotonically increasing (decreasing) function of θ in the weak (strong) field region, and they do not

behave as monotonic functions of θ in the intermediate region of B. Moreover, the relation between $\tilde{C}^{\rm sc}_{\rm re}(\rho_{12})$ and $\tilde{C}^{\rm sc}_{\rm re}(\rho_{13})$ is also similar to that of the l_1 norm of SC. However, the optimal twist angle at which the relative entropy of SC takes its maximum is different from that of the l_1 norm of SC. From the circles shown in Fig. 2, one can see that $\theta_{\rm opt}=\pi$ when B is weaker than a critical value, and $\theta_{\rm opt}=0$ when B becomes stronger than this critical value.

From the above analysis, one can see that the SC can be noticeably enhanced by tuning the twist angle θ of the boundary spins. Physically, such an enhancement is due to the change of the energy level structure of the system as twisting the boundary spins breaks the translational invariance of $\hat{H}$, and the form of $\{\varepsilon_i\}$ and $\{|\psi_i\rangle\}$ will be changed. Such a change will affect the SC of each $|\psi_i\rangle$ and their probabilities in $\rho(T)$, thus results in the improvement of the SC of $\rho(T)$ for specific twist angle. For $\theta=0$ and π, one can derive analytically $\{\varepsilon_i\}$ and $\{|\psi_i\rangle\}$ of $\hat{H}$ and show directly that the SC of $\rho(T)$ with $\theta=\pi$ is larger than that with $\theta=0$ in the weak field region, whereas the opposite case occurs in the strong field region.

In Fig. 3, we show the temperature dependence of the critical magnetic field $B_{c,\alpha}(\rho_{ij})$ below which the optimal twist angle $\theta_{\rm opt}=\pi$. When considering the l_1 norm of SC, $B_{c,l_1}(\rho_{ij})$'s for different spin pairs are different. They both increase rapidly with an increase in the temperature T, and when $T\gtrsim 0.782$, their values become very large. For the relative entropy of SC, however, $B_{c,re}(\rho_{ij})$ is independent of the different spin pairs, and it increases smoothly with the increase of T. In particular, in the high temperature region, $B_{c,re}(\rho_{ij})$ increases nearly linearly with the increase of T (e.g., by using the numerical data obtained in the region of $T\in[0.8,2.0]$, their relation can be fitted approximately as $B_{c,re}(\rho_{ij})\approx 1.501T+0.1822$). The physical origin of the difference between B_{c,l_1} and $B_{c,re}$ is the difference between the l_1 norm of coherence and the relative entropy of coherence which are at the root of the SC given in Eq. (4), as $C_{l_1}(\rho)$ contains only the magnitude information of the off-diagonal elements of a density operator ρ, while $C_{\rm re}(\rho)$ contains both the magnitude and phase information of all the elements of ρ, see Eqs. (1) and (2).

Finally, we give a short analysis on how the transverse magnetic field B affects the SC. In the low temperature region, one can see from Figs. 1 and 2 that both the measures of SC can be noticeably enhanced when the magnetic field is strong enough, although they may experience slight decrease in the intermediate region of B. In the high temperature region (we do not list the plots here for conciseness of this paper), the SC increases gradually with the increasing B, that is, the presence of the transverse magnetic field is beneficial for enhancing the SC between any two spins.

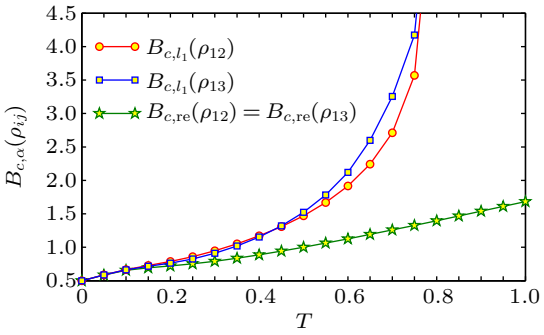


Fig. 3. The critical $B_{c,\alpha}(\rho_{ij})$ below which $\theta_{\rm opt}=\pi$ versus T for the Heisenberg XX model. When $T\gtrsim 0.782$, both $B_{c,l_1}(\rho_{12})$ and $B_{c,l_1}(\rho_{13})$ will be very large and the figure is cut to better visual for the temperature dependence of $B_{c,\rm re}(\rho_{12})$.

4.2. The pairwise entanglement

In this subsection, we investigate how the pairwise entanglement is affected by tuning the twist angle of the Heisenberg XX chain. As pointed out in Section 3, we always have $E_{\rm f}(\rho_{12})=E_{\rm f}(\rho_{13})$, so Fig. 4 only shows the dependence of $E_{\rm f}(\rho_{12})$ on B and θ/π, from which one can observe that it exhibits different behaviors from those of the SC. Firstly, $E_{\rm f}(\rho_{12})$ always increases monotonically with the increase of θ and takes its maximum at $\theta_{\rm opt}=\pi$. This indicates that the Heisenberg XX model with anti-periodic boundary conditions is beneficial for creating entanglement. Secondly, one can see from Fig. 4 that for fixed twist angle θ, $E_{\rm f}(\rho_{12})$ first increases with the increase of B and reaches its maximum at a critical magnetic field $B_{c,E}$, after which it turns out to be decreased gradually. A further numerical calculation shows that when θ increases from 0 to π with fixed $T=0.1$, $B_{c,E}$ first decreases from the value of about 0.398 to 0.379 and then increases monotonically to 0.622.

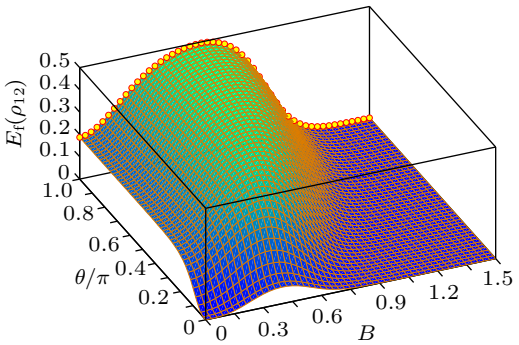


Fig. 4. $E_{\rm f}(\rho_{12})$ versus B and θ/π for the Heisenberg XX model with $T=0.1$. The circles denote critical points at which $E_{\rm f}(\rho_{12})$ takes its maximum.

By increasing the temperature T, the pairwise entanglement may undergo sudden death in the whole parameter region of (B,θ). In Fig. 5, we show the dependence of the critical temperature $T_{\rm c}$ above which the pairwise entanglement disappears on θ/π in the weak field region (the entanglement is infinitesimal in the strong field region even when the temperature T is very low, therefore it is meaningless to consider it).

From this figure one can see that T_c increases with an increase in θ. This implies that the temperature region in which the entanglement exists can also be enhanced by increasing the twist angle, and the anti-periodic boundary conditions (i.e., $\theta = \pi$) is the most suitable boundary conditions both for creating pairwise entanglement and for extending temperature region of non-vanishing entanglement.

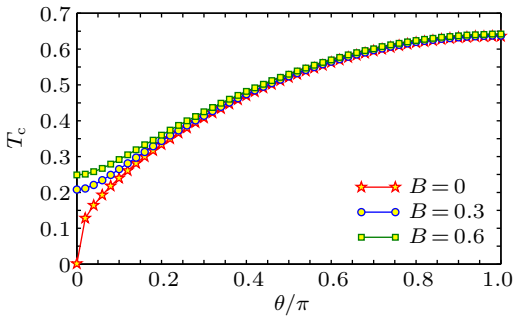


Fig. 5. The critical temperature T_c above which $E_f(\rho_{12}) = 0$ versus θ/π for the Heisenberg XX model with different B.

5. Summary

To summarize, we have examined SC and entanglement for thermal states of a three-spin XX model under twisted boundary conditions. Different from the previous works which focused on the periodic boundary conditions, our results show that both the SC and the entanglement can be noticeably enhanced by tuning the twist angle θ. For the l_1 norm of SC, the optimal twist angle θ_{opt} equals π in the weak field region and decreases from π to 0 with an increase in the strength of the magnetic field, while for the relative entropy of SC, $\theta_{opt} = \pi$ in the weak field region and $\theta_{opt} = 0$ otherwise. As for the entanglement of any two spins, it increases monotonically when the twist angle increases from 0 to π and therefore we always have $\theta_{opt} = \pi$ for this case. Moreover, the critical temperature T_c above which the entanglement vanishes can also be noticeably enhanced by increasing the twist angle. These show evidently that the twist angle can serve as an efficient parameter for tuning the SC and entanglement in the spin-chain systems. As the SC characterizes the ability of one party to control the coherence of another party by local operations and classical communication, the present results may be useful in coherence-based quantum information processing tasks such as the remote preparation of coherent states.

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