

Phase- and spin-dependent manipulation of leakage of Majorana mode into double quantum dot

Fu-Bin Yang(羊富彬), Gan Ren(任淦), and Lin-Guo Xie(谢林果)

Citation: Chin. Phys. B, 2021, 30 (**7**): 078505. DOI: 10.1088/1674-1056/abe1a8
Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Spin-filter effect and spin-polarized optoelectronic properties in annulene-based molecular spintronic devices

Zhiyuan Ma(马志远), Ying Li(李莹), Xian-Jiang Song(宋贤江), Zhi Yang(杨致), Li-Chun Xu(徐利春), Ruiping Liu(刘瑞萍), Xuguang Liu(刘旭光), Dianyin Hu(胡殿印)

Chin. Phys. B, 2017, 26 (**6**): 067201. DOI: 10.1088/1674-1056/26/6/067201

Detection of Majorana fermions in an Aharonov-Bohm interferometer

Shang En-Ming, Pan Yi-Ming, Shao Lu-Bing, Wang Bai-Gen

Chin. Phys. B, 2014, 23 (**5**): 057201. DOI: 10.1088/1674-1056/23/5/057201

Oxygen vacancy formation and migration in Sr- and Mg-doped $LaGaO_3$: a density functional theory study

Zhang Jie,Liang Er-Jun,Sun Qiang,Jia Yu

Chin. Phys. B, 2012, 21 (**4**): 047201. DOI: 10.1088/1674-1056/21/4/047201

Phase- and spin-dependent manipulation of leakage of Majorana mode into double quantum dot*

Fu-Bin Yang(羊富彬)†, Gan Ren(任淦), and Lin-Guo Xie(谢林果)

Department of Physics and Key Laboratory of Photonic and Optical Detection in Civil Aviation, Civil Aviation Flight University of China, Guanghan 618307, *China*

(Received 31 October 2020; revised manuscript received 17 January 2021; accepted manuscript online 1 February 2021)

We present a phase- and spin-dependent manipulation of leakage of a Majorana mode into a double quantum dot. We study the density of states (DOS) to show the effect of phase change factor on the Majorana leakage into (out) of a double quantum dot. The DOS is derived from the Green's function of the quantum dot by the equation of motion method, and exhibits a formant structure when $\phi = 0, 2\pi$ and a resonance shape when $\phi = 0.5\pi$ and 1.5π. Also, it changes more strongly under the spin-polarized coefficient than the non-polarized lead. Such a theoretical model can be modified to explore the spin-dependent effect in the hybrid Majorana quantum dot system.

Keywords: spin-dependent coupling, Majorana bound states, double quantum dots

PACS: 85.75.Mm, 91.62.Bf, 72.10.Fk **DOI:** 10.1088/1674-1056/abe1a8

1. Introduction

Majorana bound states[1–5] are at the forefront of condensed matter physics in the topological quantum computation. In fact, Majorana bound state can be realized by detecting the hybrid Majorana quantum dot (QD) system,[6,7] which is used as a spectrometer of Majorana by performing the bias-dependent spectroscopy.[8] Specifically, Majorana bound states can be realized at the ends of a one-dimensional semiconducting wire with spin–orbit coupling.[9] The recent focus on the Majorana bound states is the perspective of interferometer characteristics in Majorana quasiparticles.[10–15] An Aharonov–Bohm interferometer has been studied through controlling the phase factor to achieve the perspective of single source excitation.[15,16] The interference effects were studied[17,18] in the hybrid Majorana QD system through Green's function method. These results undoubtedly give a method to elaborate experimental realizations in the hybrid Majorana QD system. Also, recent literature gives the spin-dependent effect of such an interferometer to verify the correlation between the Kondo–Coulomb repulsion and Majorana bound states.[19–21] The QD-coupled Majorana wire was used to detect the Majorana bound states under the spin-polarized condition.[22–26] They explored the spin properties inside the Majorana by constructing the coupling between the electrode and the Majorana QD system.[27–29]

Studying the spin-dependent properties plays an important role in the hybrid Majorana QD system. However, the research is still insufficient in explaining the spin properties of such hybrid Majorana QD system. The Majorana zero mode can be detected in recent setup through the QD density of states (DOS). Here, we show that the multi-QD system can also be used to manipulate the Majorana zero mode.

2. Model and theory

As shown in Fig. 1, we consider an interferometer which consists of double quantum dots coupled to a Majorana topological superconducting wire and a metallic (or a ferromagnetic) lead. When the tunnel couplings between the Majorana bound states and the QDs are fixed to a specific phase, the DOS becomes unchangeable irrespective to the coupling between the Majorana and the double quantum dots. The detection of Majorana mode in Majorana wire without electrode coupling has been discussed.[9,15] In the ring structure, it allows us to analytically explore the effect of phase change factor on the Majorana leakage into (out) of the double quantum dots. Also, we incorporate the situation of spin dependence into the lead, which is sensitive to the unique interaction of the double quantum dots. We consider the noninteracting regimes of the double quantum dots. We provide an exact retarded Green's function to calculate the DOS of the two dots by using the equation of motion (EOM) method.[30,31] While the noninteracting quantum dot is enough for obtaining the exact expression for the Green's function, it can also show the information about the Majorana signature co-existing together with some related phenomena.[32,33] Hence, the Hamiltonian can be expressed as

$$H = H_{\rm QD} + H_{\rm M} + H_{\rm DM}, \tag{1}$$

where

$$H_{\rm QD} = \sum_{n=1,2} \varepsilon_{n\sigma} d^{\dagger}_{n\sigma} d_{n\sigma} + \sum_{k,\sigma} (V_{n\sigma} d^{\dagger}_{n\sigma} c_{k\sigma} + V^{*}_{n\sigma} c^{\dagger}_{k\sigma} d_{n\sigma})$$

*Project supported by the Science Foundation of Civil Aviation Flight University of China (Grant No. JG2019-19).

†Corresponding author. E-mail: fbinyang@qq.com

© 2021 Chinese Physical Society and IOP Publishing Ltd

$$+\sum_{k,\sigma}\varepsilon_{k\sigma}c^{\dagger}_{k\sigma}c_{k\sigma} \tag{2}$$

is the QD's Hamiltonian which connects with the lead. Here, $\varepsilon_{n\sigma}$ is the energy level of dot, $d^{\dagger}_{n\sigma}$ ($d_{n\sigma}$) denotes creating (annihilating) a particle in the dot, $c^{\dagger}_{k\sigma}$ is the creation operator of a particle with momentum $\boldsymbol{k}$, and spin σ in the lead with the band energy $\varepsilon_{k\sigma}$, $V_{n\sigma}$ is described by $V_{1\sigma}=V_1\,\mathrm{e}^{-\mathrm{i}\phi/4}$, and $V_{2\sigma}=V_2\,\mathrm{e}^{\mathrm{i}\phi/4}$ to represent the coupling effect depending on the broadening parameter, which can be finally given by an effective coupling in the form of

$$\Gamma_{n\sigma}=\pi\sum_{k\sigma}|V_{n\sigma}|^2\,\delta(\varepsilon-\varepsilon_{k\sigma})$$

in the wide band limit $\phi=\Phi/\Phi_0$, $\Phi_0=h/2e$ is the magnetic flux phase factor threading through the ring of the system.

$$H_{\mathrm{M}}=\mathrm{i}\varepsilon_{\mathrm{M}}\gamma_1\gamma_2 \tag{3}$$

represents the Majorana Hamiltonian with $\gamma_{1(2)}$, the corresponding Majorana fermion operators which fulfill $\{\gamma_i,\gamma_j\}=\delta_{ij}$ at the two ends of the Majorana wire, ε_{M} is the overlap amplitude depending on the wire parameters: $\varepsilon_{\mathrm{M}}\propto\exp[-L/l]$ with L being the length of the wire

$$H_{\mathrm{DM}}=\sum_{n=1,2}(\lambda_n d_{n\sigma}-\lambda_n^{*}d^{\dagger}_{n\sigma})\gamma_1 \tag{4}$$

is the coupling Hamiltonian between the double QDs and the Majorana wire $\lambda_1=|\lambda_1|\,\mathrm{e}^{\mathrm{i}\phi/4}$, $\lambda_2=|\lambda_2|\,\mathrm{e}^{-\mathrm{i}\phi/4}$ denote the respective phase-dependent coupling strength around them. The phase selection of $V_{1\sigma(2\sigma)}$ and $\lambda_{1(2)}$ depends on what case they are in: the unitary case or antiunitary case. For example, we consider the antiunitary case and choose the phase parameters as before. We write γ_1 and γ_2 as a superposition of the creation f^{+}_{M} operator and annihilation f_{M} operator by defining $\gamma_1=(f_{\mathrm{M}}+f^{\dagger}_{\mathrm{M}})/\sqrt{2}$ and $\gamma_2=-\mathrm{i}(f_{\mathrm{M}}-f^{\dagger}_{\mathrm{M}})/\sqrt{2}$, where f_{M} and $f^{\dagger}_{\mathrm{M}}$ are the regular fermion operators satisfying $\left\{f^{\dagger}_{\mathrm{M}},f_{\mathrm{M}}\right\}=1$, which transforms the Majorana Hamiltonian into[34,35]

$$H_{\mathrm{MB}}=\varepsilon_{\mathrm{M}}\left(f^{\dagger}_{\mathrm{M}}f_{\mathrm{M}}-\frac{1}{2}\right). \tag{5}$$

In this representation, the effective coupling between the Majorana wire and the quantum dots in Eq. (4) becomes

$$H_{\mathrm{DM}}=\sum_{n}\frac{1}{\sqrt{2}}(\lambda_n d_{n\sigma}-\lambda_n^{*}d^{\dagger}_{n\sigma})(f_{\mathrm{M}}+f^{\dagger}_{\mathrm{M}}). \tag{6}$$

We describe the DOS in order to describe the phase-dependent characteristics of the system. We focus on these quantities to explore the coexistence and interplay of phase and spin couplings of the system. We use the EOM procedure to obtain the full retarded Green's function, which can be written as the following form:

$$[G^{\mathrm{r}}_{\sigma}]^{-1}=\begin{pmatrix}
\varepsilon-\varepsilon_{1\sigma}+\mathrm{i}\Gamma_{1\sigma} & 0 & \mathrm{i}\sqrt{\Gamma_{1\sigma}\Gamma_{2\sigma}}\exp[-\mathrm{i}\phi/2] & 0 & \lambda_1^{*}/\sqrt{2} & \lambda_1^{*}/\sqrt{2}\\
0 & \varepsilon+\varepsilon_{1\sigma}+\mathrm{i}\Gamma_{1\sigma} & 0 & \mathrm{i}\sqrt{\Gamma_{1\sigma}\Gamma_{2\sigma}}\exp[\mathrm{i}\phi/2] & -\lambda_1/\sqrt{2} & -\lambda_1/\sqrt{2}\\
\mathrm{i}\sqrt{\Gamma_{1\sigma}\Gamma_{2\sigma}}\exp[\mathrm{i}\phi/2] & 0 & \varepsilon-\varepsilon_{2\sigma}+\mathrm{i}\Gamma_{2\sigma} & 0 & \lambda_2^{*}/\sqrt{2} & \lambda_2^{*}/\sqrt{2}\\
0 & \mathrm{i}\sqrt{\Gamma_{1\sigma}\Gamma_{2\sigma}}\exp[-\mathrm{i}\phi/2] & 0 & \varepsilon+\varepsilon_{2\sigma}+\mathrm{i}\Gamma_{2\sigma} & -\lambda_2/\sqrt{2} & -\lambda_2/\sqrt{2}\\
\lambda_1/\sqrt{2} & -\lambda_1^{*}/\sqrt{2} & \lambda_2/\sqrt{2} & -\lambda_2^{*}/\sqrt{2} & \varepsilon-\varepsilon_{\mathrm{M}} & 0\\
\lambda_1/\sqrt{2} & -\lambda_1^{*}/\sqrt{2} & \lambda_2/\sqrt{2} & -\lambda_2^{*}/\sqrt{2} & 0 & \varepsilon+\varepsilon_{\mathrm{M}}
\end{pmatrix}. \tag{7}$$

The density of states can be written as $\rho_{n\sigma}=-(1/\pi)\,\mathrm{Im}\,G^{\mathrm{r}}_{n\sigma}$ where $G^{\mathrm{r}}_{n\sigma}$ is the matrix element of the full retarded Green's function G^{r}_{σ}.

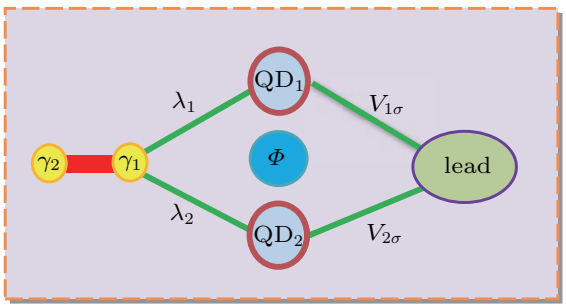


Fig. 1. Schematic diagram of Majorana-double quantum dots system in which single Majorana mode γ_1 is located at the edge of a one-dimensional Majorana topological superconducting wire which is coupled to double quantum dots attached to a single metallic lead, $\lambda_{1(2)}$ is the effective coupling strength between Majorana bound states and the quantum dot$_{1(2)}$, $V_{1\sigma(2\sigma)}$ represents the coupling strength between quantum dot$_{1(2)}$ with metallic lead, and Φ is the magnetic flux threading through the ring of the system.

3. Results and discussion

In this section, we give the numerical results under different coupling effects on the phase. We first study the properties of DOS under different values of phase ϕ in Fig. 2. We choose the same coupling strength as $\Gamma_{1\sigma}=\Gamma_{2\sigma}=\Gamma$ and $|\lambda_1|=|\lambda_2|=\lambda$ for the same DOS of the two QDs. The DOS exhibits a relatively flat region at $\varepsilon=0$ and a resonance around $\varepsilon=\pm1.0$ in Fig. 2(a). The bimodal structure gradually gathers towards $\varepsilon=0$ and a more pronounced resonance peak structure appears in Fig. 2(b). The bimodal structure disappears instead of a single resonance peak structure in Fig. 2(c). The characteristics of phase dependence can be explained by the diagonalization of the central Majorana QD Hamiltonian. The resonance of DOS is found to be related to the matrix of the retarded Green's function. Namely, the position of resonance is determined by the zero point of the

Green's function $G^{\mathrm{r}}_{n\sigma}$, which has the real form as $\mathrm{Re}[G^{\mathrm{r}}_{n\sigma}] = \varepsilon^2([\varepsilon^2 - 2(\Gamma^2+\lambda^2)]^2 - \sqrt{4(\Gamma^2+\lambda^2)^2 + 8\Gamma^2\lambda^2(1-\cos\phi)})$ and the imaginary part as $\mathrm{Im}[G^{\mathrm{r}}_{n\sigma}] = 4\Gamma\varepsilon^3[(\varepsilon^2 - 3\lambda^2) + \lambda^2\cos\phi]$. The change of phase leads to the DOS symmetry by taking a specific value of Γ and λ. For example, the shape of the DOS repeats when $\cos\phi = 1,\ 0,\ -1$. We write the analytical form for the zero energy dispersion as $\varepsilon^2 = 2(\Gamma^2+\lambda^2)]^2 \pm \sqrt{4(\Gamma^2+\lambda^2)^2 + 8\Gamma^2\lambda^2(1-\cos\phi)}$, which determines the peak resonance curve located at $\varepsilon^2 = \pm 2\sqrt{(\Gamma^2+\lambda^2)}] = \pm 1$.[18] This formation and variation of the DOS probability results from the interference process between the two quantum dots and the lead. The energy oscillation shows 2π periodicity.[15] Therefore, it opens up possibilities for manipulating the Majorana bound states in a nontrivial way by the varying phase.

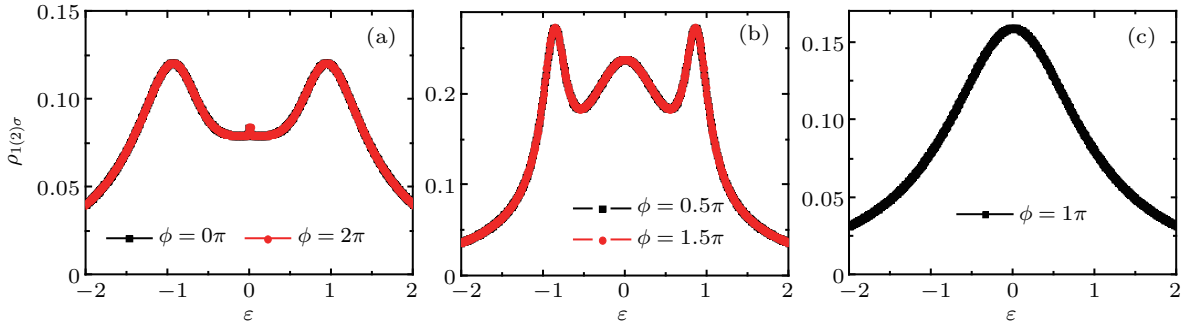


Fig. 2. Densities of states under different values of phase ϕ threading through the ring of the system. The other parameters are chosen as follows: $\varepsilon_{\mathrm{M}} = 0$, $\varepsilon_{1\sigma} = \varepsilon_{2\sigma} = 0$, $\Gamma_{1\sigma} = \Gamma_{2\sigma} = 0.5\Gamma_0$, $\lambda_1 = \lambda_2 = 0.5\Gamma_0$, where Γ_0 is chosen as the unit of the energy.

Next, we study the specific DOSs (*e.g.* $\rho(0.05)$) *versus* variable ϕ under different coupling strengths in Fig. 3. Figure 3(a) shows the case of changing dot–lead coupling (Γ) and constant Majorana QD coupling (λ). Figure 3(b) displays the case of changing λ and constant Γ. When Γ is small, the DOS shows a resonance around $\phi = \pi$, therefore we can ignore the effect of the electrode on the quantum dots. However, it changes to an anti-resonance case and two symmetrical peaks form when Γ increases from $\Gamma = 0.1$ to $\Gamma = 0.5$. The DOS (Fig. 3(b)) shows a minor change when λ is varied. In order to further verify this different coupling effect, we give the DOSs under different values of λ_2 in Figs. 3(c) and 3(d). When λ_2 is small, the DOSs for QD_1 and QD_2 are both exhibiting anti-resonant structures at $\phi = 0$. The periodic characteristics are consistent with that presented in Fig. 2. If we consider the case of uniform coupling changing of the system, it can be described by the same rule as that described in Fig. 2. We can then determine where the resonance will appear by solving the retarded Green's function denominator from the change of λ and Γ. When considering the unequal couplings λ and Γ, it will be reflected in different DOSs of the two dots. The $\rho_{1\sigma}$ changes from the anti-resonance structure to the resonance structure at $\phi = 0$, while $\rho_{2\sigma}$ decreases with λ_2 increasing. The reason is related to the coupling between the Majorana QDs by considering the self-adjoint characteristics of Majorana bound states. The self-adjoint behavior of Majorana fermions results in the characteristic of DOS, which is a remarkable signature of the presence of the Majorana zero mode leaking into the lead and the QD. We consider QD_1 to have the same coupling constant, but QD_2 to possess a different coupling coefficient. λ_2 increases the antiresonance point in the DOS of QD_1, but it reduces the value in the DOS of QD_2. The interplay is manifested in the QD DOS, which will destroy the symmetry property. When λ_2 is increased, the influence of its interaction on QD_2 is significantly greater than that on QD_2: λ_2 will cause its resonance energy to increase due to the relation $\varepsilon^2 = \pm 2\sqrt{(\Gamma^2+\lambda^2)}$. The DOS resonance energy shifts toward higher energy level and it will split as the energy level increases. Therefore, we see the result of the transition from anti-resonance to resonance in Fig. 3(d). For the detection of the Majorana zero mode of the system, we readily know that in the presence of Majorana bound states, the change of DOS is helpful for the relevant experiment. The main cause should be attributed to the weakened transport in the presence of Majorana QD coupling. The increase of λ enhances (suppresses) the amplitude of $\rho_{1\sigma}$ ($\rho_{2\sigma}$). The DOS is a probe of the presence of the Majorana bound states.

It should be noted that that in addition to the values of Γ and ϕ, the formant of DOS is also related to the increase of the coupling constant. This coupling effect leads to an asymmetric structure of the energy levels. From the perspective of the real and imaginary parts of the specific Green's function of the two QDs, we will find the influence of such an asymmetric structure on Majorana bound state if $\lambda_1 \neq \lambda_2$. For comparisons, we present the DOS under different values of the coupling strength λ_2 in Fig. 4. The DOS shows different behaviors, based on which one can see whether there are Majorana bound states in this system. A resonance structure still exists around $\varepsilon = 0$ with the value of λ_2 changing as shown in Fig. 4(a). But λ_2 induces the Majorana modes to change $\rho_{2\sigma}$ around $\varepsilon = 0$ and shifts toward the large energy and it finally shows a transition from antiresonance to resonance as indicated in Fig. 4(b). The presence of the Majorana modes

efficiently dissolves the antiresonance point in the DOS while inducing a resonance to appear at the same energy position. Therefore, we can conclude that such a relationship is sufficient to show the importance of the coupling relationship between Majorana and QDs. We believe that this work can be helpful in the relevant experiments.

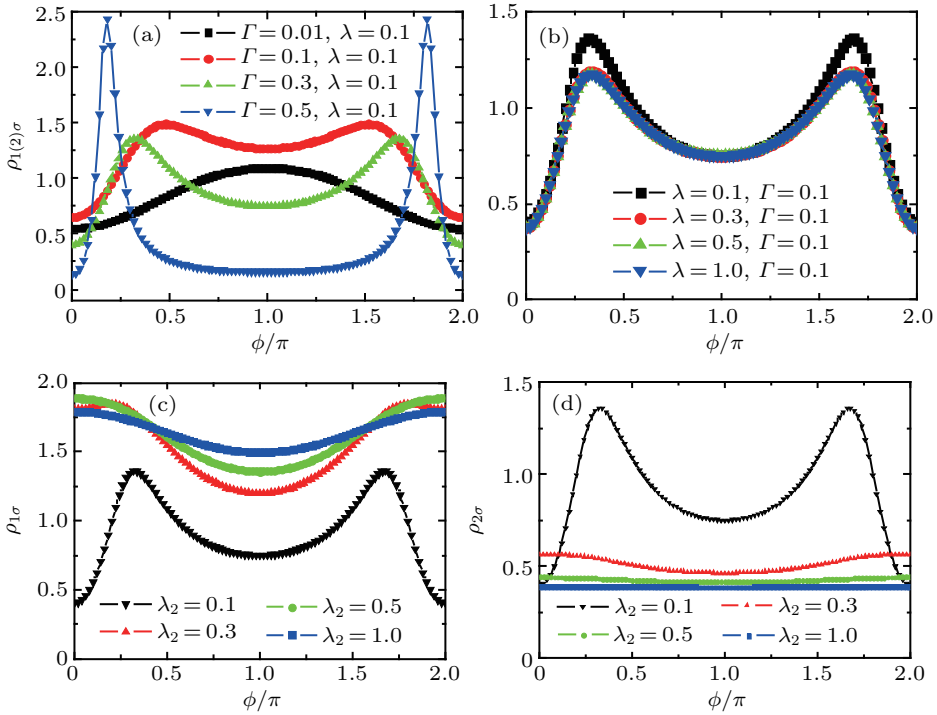


Fig. 3. Specific DOSs varying with ϕ for the cases of [(a), (b)] the same coupling strength: $\Gamma_{1\sigma}=\Gamma_{2\sigma}=\Gamma$, $\lambda_1=\lambda_2=\lambda$; [(c), (d)] different values of coupling λ_2. The other parameters are $\varepsilon_M=0.0$, $\varepsilon_{1\sigma}=\varepsilon_{2\sigma}=0$, $\lambda_1=0.1$, Γ_0, $\Gamma_{1\sigma}=\Gamma_{2\sigma}=0.1\Gamma_0$.

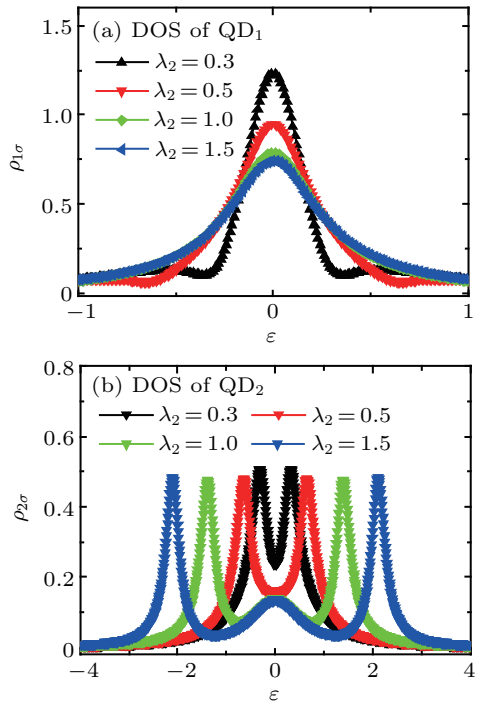


Fig. 4. DOSs under different coupling strengths, with parameters being $\Gamma_1=\Gamma_2=0.5$, $\lambda_1=0.5$, and $\phi=0$.

Finally, we consider the spin-dependent lead by introducing a definition of spin polarization strength p of the lead–dot coupling ($p=(\Gamma^{-}_{n\sigma}\Gamma_{n\bar{\sigma}})/(\Gamma^{+}_{n\sigma}\Gamma_{n\bar{\sigma}})$). We then define the parallel case in Figs. 5(a) and 5(b). Namely, $\Gamma_{1\uparrow\downarrow}=\Gamma_{2\uparrow\downarrow}=1\pm p$ is the spin-up and spin-down coefficients of the lead. Each $\rho_{1(2)\uparrow}$ at $\varepsilon=0$ decreases to a minimum value and the resonance is weakened as the value of p increases (Fig. 5(a)). However, each increases with the value of p increasing (Fig. 5(b)). The DOS at $\varepsilon=0$ shows anti-resonance (resonance) structure at small (large) p. The anti-resonance varies with resonance transformation around $\varepsilon=0$ when p increases from $p=0.6$ to $p=0.9$. The value for the spin-down DOS is larger than that for the spin-up DOS as indicated in Figs. 5(a) and 5(b). Namely, the spin-down mainly contributes to the total DOS in the parallel configuration. Unlike the coupling between electrodes and quantum dots, the spin-down is the main spin coupling between Majorana bound states and quantum dots. The Majorana mode is assumed to be provided by a fully polarized spin-down.[24] When we consider the spin-dependent coupling, the quantum dot will be changed by the spin coupling in the electrode. When p increases, the corresponding spin-down coupling is strengthened. The spin-down characteristics are significantly affected, which is beneficial to the system to measure the spin properties of the Majorana hybridized quantum dot system.

We further explore the DOS in the antiparallel couple case ($\Gamma_{1\uparrow\downarrow}=1\pm p, \Gamma_{2\uparrow\downarrow}=1\mp p$), and the results are shown in Figs. 5(c) and 5(d), respectively. Unlike the parallel cou-

ple case, ρ_1 increases with p increasing, the value of anti-resonance decreases around $\varepsilon = 0$, and the peak structure at other energy levels increases. The $\Gamma_{1\uparrow}$ increases with p increasing, while $\Gamma_{2\uparrow}$ decreases as p increases. Meanwhile, ρ_2 exhibits an inversion when p reaches a large value due to the effects of down-spin. The original resonance transforms into an anti-resonance structure, which can be used to explain the origin of the major spin-down properties.

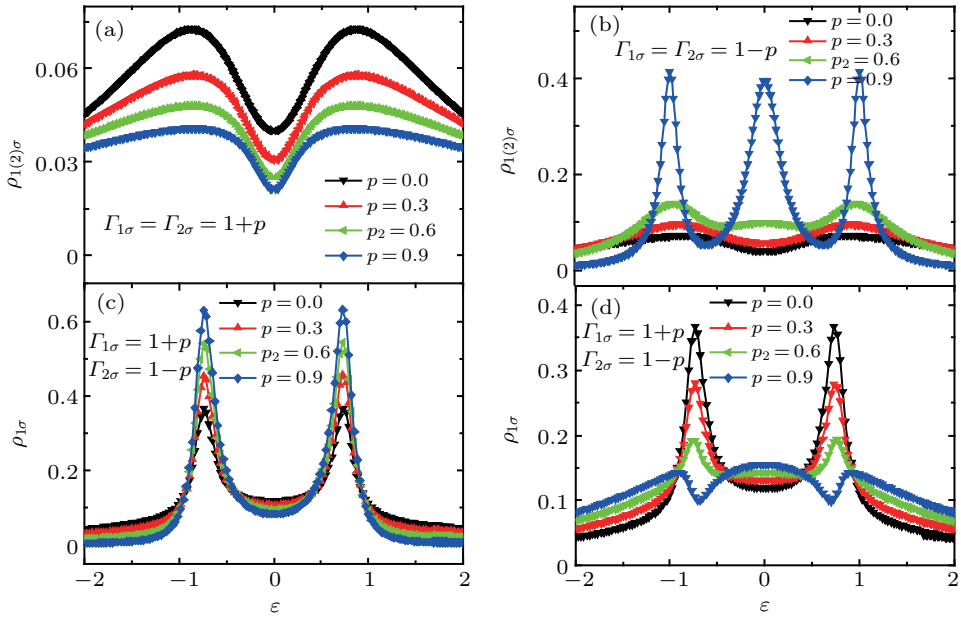


Fig. 5. DOSs with different values of spin polarization strength p, showing [(a), (b)] parallel lead alignment, and [(c), (d)] anti-parallel coupling state between lead and two QDs, with other parameters being $\varepsilon_{\rm M} = 0$, $\varepsilon_{1\sigma} = \varepsilon_{2\sigma} = 0$, $\lambda_1 = \lambda_2 = 0.5\Gamma_0$, and $\phi = 0$.

4. Conclusions

In summary, we have studied the physical phase-dependent properties of the double quantum dots coupled to a Majorana nanowire in a controlled parameter regime. The DOSs of the two quantum dots exhibit different resonance and anti-resonance behaviors driven by different phases and different coupling competitions. The DOS exhibits a formant when $\phi = 2\pi$. But the resonance structure shields and the bimodal structure gradually gathers when $\phi = 0.5\pi$ and 1.5π. We also studied the DOSs of electrons under different spin polarization intensities. A characteristic of the spin-down DOS panel changes more strongly than that of the spin-up DOS. We believe that the model discussed presents a good way to effectively verify the spin-dependent mode in the hybrid Majorana QD device.

References

[1] Wang J and Zhang S C 2017 *Nat. Mater.* **16** 1062
[2] Mourik V, Zuo K, Frolov S M, Plissard S R, Bakkers E P A M and Kouwenhoven L P 2012 *Science* **336** 1003
[3] Ruiz-Tijerina D A, Vernek E, Dias da Silva L G G V and Egues J C 2015 *Phys. Rev. B* **91** 115435
[4] Nayak C, Simon S H, Stern A, Freedman M and Das Sarma S 2008 *Rev. Mod. Phys.* **80** 1083
[5] Alicea J 2012 *Rep. Prog. Phys.* **75** 076501
[6] Deng M T, Vaitiekėnas S, Hansen E B, Danon J, Leijnse M, Flensberg K, Nygård J, Krogstrup P and Marcus C M 2016 *Science* **354** 1557
[7] Jiang Z T and Zhong C C 2016 *Chin. Phys. B* **25** 067302
[8] Ptok A, Kobiałka A and Domański T 2017 *Phys. Rev. B* **96** 195430
[9] Fu L and Kane C L 2009 *Phys. Rev. B* **79** 161408
[10] Cifuentes J D and Dias da Silva L G G V 2019 *Phys. Rev. B* **100** 085429
[11] Li Ch An, Li J and Shen Sh Q 2019 *Phys. Rev. B* **99** 100504
[12] Li Y, Li S X, Lidag H O, Deng G W, Cao G, Xiao M and Guo G P 2018 *Chin. Phys. B* **27** 76105
[13] Røising H S and Simon S H 2018 *Phys. Rev. B* **97** 115424
[14] Ricco L S, Marques Y, Dessotti F A, de Souza M and Seridonio A C 2016 *Physica E* **78** 25
[15] Ueda A and Yokoyama T 2014 *Phys. Rev. B* **90** 081405
[16] Ramos-Andrade J P, Orellana P A and Ulloa S E 2018 *J. Phys.: Condens. Matter* **30** 045301
[17] Barański J, Kobiałka A and Domański T 2017 *J. Phys.: Condens. Matter* **29** 075603
[18] Zeng Q B, Chen Sh and Lü R 2016 *Phys. Lett. A* **380** 951
[19] Silva J F, Dias da Silva L G G V and Vernek E 2020 *Phys. Rev. B* **101** 075428
[20] Ricco L S, de Souza M, Figueira M S, Shelykh I A and Seridonio A C 2019 *Phys. Rev. B* **99** 155159
[21] Wu S Q, He Z, Yan C H, *et al.* 2006 *Acta Phys. Sin.* **55** 1413 (in Chinese)
[22] Yang F B 2019 *Physica E* **109** 164
[23] Liu J, Li K M, Chi F, *et al.* 2020 *Chin. Phys. B* **29** 077302
[24] Ramos-Andrade J P, Orellana P A and Vernek E 2020 *Phys. Rev. B* **101** 115403
[25] Yang F B 2019 *Commun. Theor. Phys.* **71** 1024
[26] Yang F B 2020 *Phys. Lett. A* **384** 126424
[27] Schuray A, Weithofer L and Recher P 2017 *Phys. Rev. B* **96** 085417
[28] Gorski G, Baranski J, Weymann I and Domanski T 2018 *Sci. Rep.* **8** 15717
[29] Cheng M, Becker M, Bauer B and Lutchyn R M 2014 *Phys. Rev. X* **4** 031051
[30] Yang F, Cheng Y, Liu F and Chen X 2013 *Appl. Phys. Lett.* **102** 011911
[31] Yang F, Cheng Y, Liu F and Chen X 2013 *Appl. Phys. Lett.* **103** 033513
[32] Goldhaber-Gordon D, Shtrikman H, Mahalu D, AbuschMagder D, Meirav U and Kastner M A 1998 *Nature* **391** 156
[33] Eriksson E, Zazunov A, Sodano P and Egger R 2015 *Phys. Rev. B* **91** 064501
[34] Gorski G, Baranski J, Weymann I and Domanski T 2018 *Sci. Rep.* **8** 15717
[35] Alicea J 2012 *Rep. Prog. Phys.* **75** 076501