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Signal-recycled weak measurement for ultrasensitive velocity estimation*

Sen-Zhi Fang(方森智), Yang Dai(戴阳), Qian-Wen Jiang(姜倩文),
Hua-Tang Tan(谭华堂), Gao-Xiang Li(李高翔), and Qing-Lin Wu(吴青林)†

Department of Physics, Central China Normal University, Wuhan 430079, *China*

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Weak value amplification has shown its superiority in measurement of small physical effects. Here we introduce a signal-recycled weak-value-based velocity measurement strategy to decrease the attenuation of detected photons during the post-selection. Like the power-recycled scheme, we can improve the number of detected photons and signal-to-noise ratio of velocity by forming a cavity. However, optimal improvements of number of detected photons and signal-to-noise ratio cannot be obtained simultaneously in our signal-recycled scheme owing to the walk-off effect. Furthermore, we find that the reflected light is relatively strong compared with the power-recycled scheme, which may increase the collection-detection efficiency in prospective relevant experiment.

Keywords: quantum optics, weak value, recycling

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1. Introduction

Weak value amplification, which was originally introduced by Aharonov, Albert, and Vaidman in 1988,[1] has shown its outstanding performance in precise measurements. By preparing appropriate pre- and post-selected states of a system, one can obtain a significantly amplifying signal in meter distribution. It has been successfully used to achieve many sensitive physical effects, such as beam deflections,[2,3] phase shift,[4–7] spin Hall effect of light,[8] ultralow velocity,[10,11] angular rotations,[12–14] and temperature measurements.[15–17] In this article, we focus on the weak value amplification setup used in Ref. [10], which couples the velocity of a piezo-driven mirror to the "which-path" modes of a Michelson interferometer. The meter corresponds to the arrival time of output light from the dark port of the interferometer.

The common disadvantage of standard weak-value-based metrology is that only a small part of information of the incident beam is used, while most information is lost during the post-selection process. Therefore, this technique is also thought as a method that concentrates finite Fisher information of measured parameters on a fraction of events.[18–20] There is plenty of doubt whether the weak value amplification can increase the ultimate limit of measurement precision.[18,21,22] With regard to probe loss by post selection, researchers have put forward the power recycling technique, which is initially applied in the detection of gravitational waves.[23,24] The aim is to reuse the discarded events by placing a partially transmitting mirror at the bright port of interferometer. This has also been demonstrated and can be used to effectively improve the ultimate signal-to-noise ratio of beam deflections in both theory and experiment.[25–27] Here we propose a signal recycling scheme, which places a partially transmitting mirror at the dark port of an interferometer, to overcome the shortcoming of post-selection.

2. Interferometric weak value measurement

We have already known from Ref. [10] (see Fig. 1, in the absence of partially transmitting mirror $M_{\rm p}$) that light entering into the Michelson interferometer is a non-Fourier limited Gaussian pulse,

$$I_0(t) = \left(\frac{N^2}{2\pi\tau^2}\right)^{1/2} \exp\left(-\frac{t^2}{2\tau^2}\right), \tag{1}$$

and the outgoing light from interferometer is given by

$$I_{\rm out}(t) \approx \left(\frac{N^2}{2\pi\tau^2}\right)^{1/2} \sin^2\phi \exp\left[-\frac{1}{2\tau^2}\left(t - \frac{2kv\tau^2}{\phi}\right)^2\right], \tag{2}$$

where we have assumed $kv\tau \ll \phi \ll 1$, N is the normalized average photon numbers per unit time, τ is the modulated pulse width, v is the velocity of moving mirror, ϕ is the near destructive interferometric angle, and $k = 2\pi/\lambda$ is the wave number of laser.

We can know from Eq. (2) compared with Eq. (1) that the output light has a time shift $\delta t = 2kv\tau^2/\phi$ and is attenuated by a factor $\sin^2\phi$, which is also the possibility for a single photon detected by the detector passing through the interferometer. In this weak-value velocity measurement scheme, we convert the measurement of Doppler spectral shift induced by

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†Corresponding author. E-mail: qlwu@mail.ccnu.edu.cn

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the moving mirror to the time shift in time domain. Thus, the information about velocity of mirror is determined by the time shift $v = \delta t\phi/2k\tau^2$. Based on the quantum Cramer–Rao theory,[28–30] the sensitivity in the determination of δt is bounded by $\Delta(\delta t) \geq \tau/\phi\sqrt{N}$. Thus the ultimate signal-to-noise ratio (SNR) of velocity is given by[10]

$$R = \frac{v}{\Delta v} = \frac{\delta t}{\Delta(\delta t)} = \frac{\delta t}{\tau}\phi\sqrt{N} = \frac{\delta t}{\tau}\sqrt{N_{\rm det}}, \tag{3}$$

where $N_{\rm det} = N\phi^2$ is the number of detected photons.

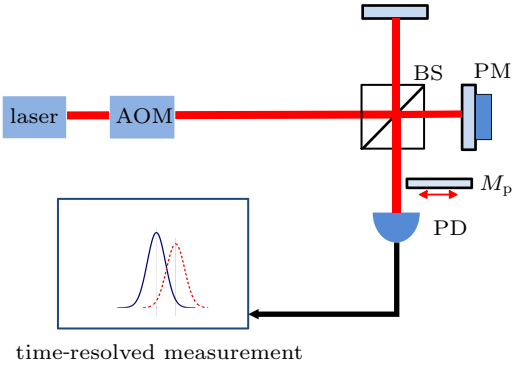


Fig. 1. Schematic of weak-value velocity measurement with signal recycling technique. The beam is modulated as a Gaussian shaped pulse by an AOM (acoustic optical modulator) and then enters into the Michelson interferometer, which is turned slightly off destructive interference by a small angle 2ϕ. The piezo-drven mirror (PM) is moving with a low velocity v, which is hopeful for precise measurement. The majority of light would be reflected back and a small amount is collected by thephoto detector (PD), and the arrival time of light is measured with a time-resolved measurement module. We note the orientations of reflected and detected light as bright and dark port of the interferometer, respectively. In the presence of the partially transmitting mirror $M_{\rm p}$, the light at the dark port would be reflected and cycled.

3. Signal recycling

In our signal recycling scheme, we form a resonant cavity by introducing a partially transmitting mirror. Unlike the power recycling weak value scheme[25–27,31] in which the partially transmitting mirror is at the bright port, here we place the mirror at the dark port of the Michelson interferometer, i.e., in front of the detector. We will show that this cavity in our signal recycling scheme can also improve the number of detected photons and the corresponding SNR like the power recycling scheme.

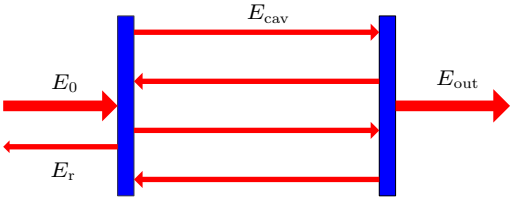


Fig. 2. Schematic of a resonant optical cavity formed by two partially transmitting mirrors. Under the condition of the resonant cavity, the reflected amplitude $E_{\rm r}$ is exactly zero and the amplitude leaving the cavity through the second mirror $E_{\rm out}$ becomes equal to the input amplitude E_0.

Now we construct a resonant cavity to explain why the intensity of light can be enhanced. Imagine an initial light with amplitude E_0 incidences into a cavity consisting of two partially transmitting mirrors with same reflection r and transmission $p = \sqrt{1-r^2}$, as shown in Fig. 2. Here we assume that each beam of light has a additional phase θ after each cycle. Thus the amplitude of light in the cavity is the sum of each beam of light going through multiple cycles,

$$\begin{aligned} E_{\rm cav} &= E_0 p[1 + rr\mathrm{e}^{\mathrm{i}\theta} + (rr\mathrm{e}^{\mathrm{i}\theta})^2 + \cdots] \\ &= \frac{p}{1 - r^2\mathrm{e}^{\mathrm{i}\theta}}E_0, \end{aligned} \tag{4}$$

and the amplitude of light reflected back the laser is the sum of light reflected by the first mirror and reflections inside the cavity

$$E_{\rm r} = \left[-r + \frac{p^2 r\mathrm{e}^{\mathrm{i}\theta}}{1 - r^2\mathrm{e}^{\mathrm{i}\theta}}\right]E_0. \tag{5}$$

We note that the amplitude of outgoing light from the cavity is

$$E_{\rm out} = pE_{\rm cav} = \frac{p^2}{1 - r^2\mathrm{e}^{\mathrm{i}\theta}}E_0. \tag{6}$$

Under the condition of resonant cavity, the phase addition every cycle is $\theta = 2\pi n$, where n is an integer. Thus we find the amplitude of reflected light is zero and the outgoing light is equal to the initial incident light. The light in the cavity has a gain

$$G = \frac{|E_{\rm cav}|^2}{|E_0|^2} = \frac{p^2}{(1-r^2)^2} = \frac{1}{p^2}. \tag{7}$$

These general results, as we will show soon, can be applied to our weak value amplification cavity formed with one partially transmitting mirror and the Michelson interferometer. In this weak-value velocity measurement scheme, the light is the meter which can be expressed as $|\varphi_0\rangle = \sqrt{I_0(t)}|\alpha\rangle = (N^2/2\pi\tau^2)^{1/4}\exp(-t^2/4\tau^2)|\alpha\rangle$, where $|\alpha\rangle$ is the coherent light state representation, and the system is the which path of light in this experimental schematic. We couple the system with the meter via the piezo-driven mirror. Here we introduce a unitary operator to describe this effect, $\widehat{U}_{\rm PM} = \exp(\mathrm{i}W_d t\widehat{A}/2)$, where $W_d = 2kv$ is the Doppler spectral shift induced by the piezo-driven mirror, $\widehat{A} = |M\rangle\langle M| - |P\rangle\langle P|$ is defined with the path states $|M\rangle$ and $|P\rangle$ corresponding to the which arm (mirror or piezo-driven mirror). The path states at the bright and dark port of interferometer are equal superposition of the path states $|M\rangle$ and $|P\rangle$ and can be written as $|\psi_{\rm B}\rangle = (\mathrm{i}|M\rangle + |P\rangle)/\sqrt{2}$ and $|\psi_{\rm D}\rangle = (|M\rangle + \mathrm{i}|P\rangle)/\sqrt{2}$. Thus the pre- and post selected states are respectively $|\psi_i\rangle = \widehat{U}_\phi|\psi_{\rm B}\rangle = (\mathrm{i}\exp(\mathrm{i}\phi)|M\rangle + \exp(-\mathrm{i}\phi)|P\rangle)/\sqrt{2}$ and $|\psi_f\rangle = |\psi_{\rm D}\rangle = (|M\rangle + \mathrm{i}|P\rangle)/\sqrt{2}$, where $\widehat{U}_\phi = \exp(\mathrm{i}\phi\widehat{A})$ is a unitary operator to express the near destructive interference. Pre- and post-selected by the dark and bright port, the meter lights exiting to the detector and reflected back towards the laser are respectively

$|\varphi_{\rm D}\rangle = \langle\psi_{\rm D}|\widehat{U}_{\rm PM}|\psi_i\rangle|\varphi_0\rangle$ and $|\varphi_{\rm B}\rangle = \langle\psi_{\rm B}|\widehat{U}_{\rm PM}|\psi_i\rangle|\varphi_0\rangle$, which produce two measurement operators

$$\widehat{M}_{\rm D} = \langle\psi_{\rm D}|\widehat{U}_{\rm PM}|\psi_i\rangle = -\sin(\phi + kvt), \tag{8}$$

and $\widehat{M}_{\rm B} = \langle\psi_{\rm B}|\widehat{U}_{\rm PM}|\psi_i\rangle = \cos(\phi + kvt)$. We find that the intensity of outgoing light in Eq. (2) is exactly $I_{\rm out} = |\langle\alpha|\varphi_{\rm D}\rangle|^2$. The probability of post-selection is $P = |\langle\psi_f|\psi_i\rangle|^2 = \sin^2\phi \approx \phi^2$, which is the attenuation factor in the above expression of detected photons, and the so-called weak value is $\widehat{A}_w = \langle\psi_f|\widehat{A}|\psi_i\rangle/\langle\psi_f|\psi_i\rangle = -{\rm i}\cot\phi \approx -{\rm i}/\phi$, which is related to the time shift.

When we insert the partially transmitting mirror $M_{\rm p}$ whose reflection is r and transmission is $p = \sqrt{1-r^2}$, the outgoing light exiting to detector will be reflected back and reenter into the interferometer. Thus this mirror and interferometer form a cavity in which the light will be cycled. After reentering to the interferometer, the reflected light must also experience the effect of near destructive interference $\widehat{U}_\phi$ and weak interaction $\widehat{U}_{\rm PM}$, and then comes out at the dark and bright port again, which can be expressed as $|\varphi_{\rm D1}\rangle = \langle\psi_{\rm D}|\widehat{U}_{\rm PM}\widehat{U}_\phi|\psi_{\rm D}\rangle r|\varphi_{\rm D}\rangle$ and $|\varphi_{\rm B1}\rangle = \langle\psi_{\rm B}|\widehat{U}_{\rm PM}\widehat{U}_\phi|\psi_{\rm D}\rangle r|\varphi_{\rm D}\rangle$, which produces another set of measurement operators

$$\widehat{N}_{\rm D} = \langle\psi_{\rm D}|\widehat{U}_{\rm PM}\widehat{U}_\phi|\psi_{\rm D}\rangle = \cos(\phi + kvt), \tag{9}$$

and $\widehat{N}_{\rm B} = \langle\psi_{\rm B}|\widehat{U}_{\rm PM}\widehat{U}_\phi|\psi_{\rm D}\rangle = \sin(\phi + kvt)$. To take into account small but constant losses of imperfect experimental apparatus, we introduce a non-unitary operator $\widehat{L} = \sqrt{1-\gamma}\cdot\widehat{1}$, where γ is the probability of power loss per traversal. The steady state amplitude of light in cavity is the sum of amplitudes from all longitudinal numbers

$$\begin{aligned}\langle\alpha|\varphi_c\rangle &= \sum_{n=0}^{\infty}\langle\alpha|(r\widehat{L})^n\widehat{N}_{\rm D0}\widehat{N}_{\rm D1}\cdots\widehat{N}_{{\rm D}n}\widehat{M}_{\rm D}\widehat{L}|\varphi_n\rangle \\ &= \sqrt{1-\gamma}\sum_{n=0}^{\infty}(r\sqrt{1-\gamma})^n\widehat{N}_{\rm D0}\widehat{N}_{\rm D1}\cdots\widehat{N}_{{\rm D}n}\widehat{M}_{\rm D} \\ &\quad\times\left(\frac{N^2}{2\pi\tau^2}\right)^{1/4}\exp\left[-\frac{1}{4\tau^2}(t+nt_{\rm cav})^2\right],\end{aligned} \tag{10}$$

where $|\varphi_n\rangle = (N^2/2\pi\tau^2)^{1/4}\exp[-(t+nt_{\rm cav})^2/4\tau^2]|\alpha\rangle$, $t_{\rm cav} = 2l_{\rm cav}/c$ is the one transition time, and $l_{\rm cav}$ is the length of cavity; $\widehat{N}_{\rm D0} = \widehat{1}$ and $\widehat{N}_{{\rm D}n} = \cos[\phi + kv(t - nt_{\rm cav})]$ $(n = 1,2,\ldots)$, corresponding measurement operators when the light is reflected by the partially transmitting mirror and reentered into the interferometer $nt_{\rm cav}$ ago. In fact, we do not need to calculate all items in Eq. (10) because every reflected light has some attenuation. The sum of top $n_{\max}$ items is nearly equal to all of them if parameter $(r\sqrt{1-\gamma})^{n_{\max}}$ is far small. On the other hand, the coherent length of light should be longer than the maximum optical path difference, thus we have relation $c\tau \gg l_\lambda > 2n_{\max}l_{\rm cav}$, which can result in the relations: $nkvt_{\rm cav} = kv\tau\frac{2nl_{\rm cav}}{c\tau} \ll kv\tau \ll \phi$ and $\exp(-\frac{n^2l_{\rm cav}^2}{c^2\tau^2}) \approx 1$. Therefore, Eq. (10) is

$$\begin{aligned}\langle\alpha|\varphi_c\rangle &\approx -\sqrt{1-\gamma}\sin(\phi+kvt)\sum_{n=0}^{n_{\max}}[r\sqrt{1-\gamma}\cos(\phi+kvt)]^n\left(\frac{N^2}{2\pi\tau^2}\right)^{1/4}\exp\left(-\frac{t^2}{4\tau^2}\right)\exp\left(-\frac{nl_{\rm cav}t}{c\tau^2}\right)\exp\left(-\frac{n^2l_{\rm cav}^2}{c^2\tau^2}\right) \\ &\approx -\frac{\sqrt{1-\gamma}\sin(\phi+kvt)\left(\frac{N^2}{2\pi\tau^2}\right)^{1/4}\exp\left(-\frac{t^2}{4\tau^2}\right)}{1-r\sqrt{1-\gamma}\cos(\phi+kvt)\exp\left(-\frac{l_{\rm cav}t}{c\tau^2}\right)} \approx -\frac{\left(\frac{N^2}{2\pi\tau^2}\right)^{1/4}\sqrt{1-\gamma}\sin\phi\left[1+\frac{kvt}{\phi_1}+O\left(\frac{kvt}{\phi_1}\right)\right]\exp\left(-\frac{t^2}{4\tau^2}\right)}{1-r\sqrt{1-\gamma}\cos\phi},\end{aligned} \tag{11}$$

where we have made a series expansion of

$$\frac{\sin(\phi+kvt)}{1-r\sqrt{1-\gamma}\cos(\phi+kvt)\exp\left(-\frac{l_{\rm cav}t}{c\tau^2}\right)}$$

and abandoned higher orders under the condition $kv\tau \ll \phi \ll 1$ and $c\tau \gg l_\lambda > 2n_{\max}l_{\rm cav}$. The parameter ϕ_1 is

$$\begin{aligned}\frac{1}{\phi_1} &= \frac{\cos\phi - r\sqrt{1-\gamma}}{\sin\phi(1-r\sqrt{1-\gamma}\cos\phi)} \\ &\quad - \frac{l_{\rm cav}r\sqrt{1-\gamma}\cos\phi}{kvc\tau^2(1-r\sqrt{1-\gamma}\cos\phi)}.\end{aligned} \tag{12}$$

Thus the steady state amplitude of detected light is

$$\begin{aligned}\langle\alpha|\varphi_d\rangle &= p\langle\alpha|\varphi_c\rangle \approx -\left(\frac{N^2}{2\pi\tau^2}\right)^{1/4}M\sqrt{1-\gamma}\sin\phi \\ &\quad\times\exp\left[-\frac{1}{4\tau^2}(t-\delta t_2-\delta t_3)^2\right],\end{aligned} \tag{13}$$

where

$$M = \frac{\sqrt{1-r^2}}{1-r\sqrt{1-\gamma}\cos\phi}, \tag{14}$$

$$\delta t_2 = \frac{\cos\phi - r\sqrt{1-\gamma}}{1-r\sqrt{1-\gamma}\cos\phi}\cdot\frac{2kv\tau^2}{\phi}, \tag{15}$$

$$\delta t_3 = -\frac{2l_{\rm cav}r\sqrt{1-\gamma}\cos\phi}{c(1-r\sqrt{1-\gamma}\cos\phi)}. \tag{16}$$

We are interested in the time shift δt_2 where the velocity is included, and δt_3 is a constant time delay which does not influence our measurement. Thus the intensity of detected light is

$$I_d = |\langle\alpha|\varphi_d\rangle|^2 = \left(\frac{N^2}{2\pi\tau^2}\right)^{1/2}M^2(1-\gamma)\sin^2\phi$$

$$\times \exp\left[-\frac{1}{2\tau^2}(t-\delta t_2-\delta t_3)^2\right]. \tag{17}$$

4. Comparison with standard weak measurement

We now compare these results with standard weak measurement. From Eq. (17) we can easily obtain the number of detected photons and the signal-to-noise ratio of velocity in this signal-recycled scheme

$$N_{\rm s}=\frac{1-r^2}{(1-r\sqrt{1-\gamma}\cos\phi)^2}(1-\gamma)N\phi^2, \tag{18}$$

$$R_{\rm s}=\frac{(\cos\phi-r\sqrt{1-\gamma})\sqrt{1-r^2}}{(1-r\sqrt{1-\gamma}\cos\phi)^2}\sqrt{1-\gamma}\frac{\delta t}{\tau}\phi\sqrt{N}. \tag{19}$$

If the loss of system is also considered in the standard scheme, we can have the factors about improvements of detected photons and SNR as follows:

$$G=M^2=\frac{1-r^2}{(1-r\sqrt{1-\gamma}\cos\phi)^2}, \tag{20}$$

$$Q=\frac{(\cos\phi-r\sqrt{1-\gamma})\sqrt{1-r^2}}{(1-r\sqrt{1-\gamma}\cos\phi)^2}. \tag{21}$$

In Figs. 3(a)–3(c), we plot parameters G and Q against post-selection related factor ϕ, loss of system (γ) and reflectivity of partially transmitting mirror (r) under some conditions, which demonstrate that the cavity can increase the number of detected photons and SNR. From Fig. 3(c), the parameter G reaches the maximum when $r=r_{\rm G}=\sqrt{1-\gamma}\cos\phi=0.8766$ and Q has the maximum when

$$r=r_{\rm Q}=\frac{\cos\phi(2-\gamma)-\sqrt{\cos^2\phi[1+(1-\gamma)^2]+(8\cos^4\phi-18\cos^2\phi+8)(1-\gamma)}}{2\sqrt{1-\gamma}(2-\cos^2\phi)}=0.8304,$$

which indicate that we cannot simultaneously reach the optimal improvements of detected photons and information about the velocity. Equation (21) can be written as

$$Q=\frac{\cos\phi-r\sqrt{1-\gamma}}{1-r\sqrt{1-\gamma}\cos\phi}\sqrt{G}.$$

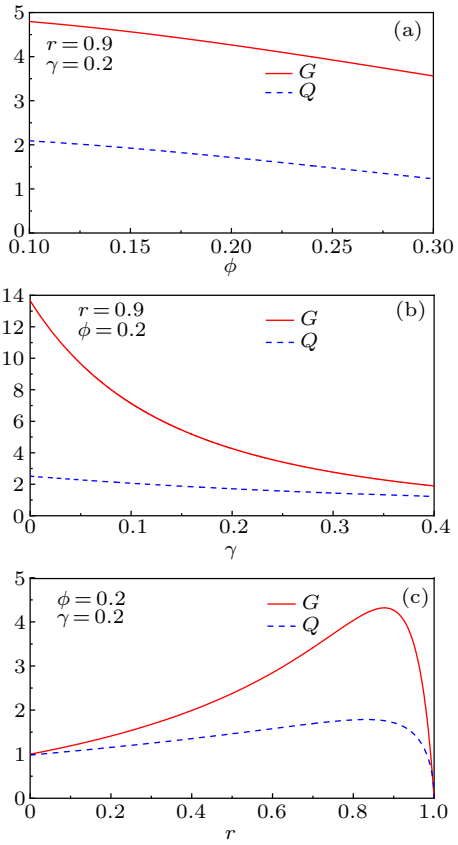


Fig. 3. The parameters G (red, solid) and Q (blue, dashed) versus ϕ, γ and r: (a) ϕ is between 0.1 and 0.3, and $r=0.9$, $\gamma=0.2$; (b) γ is between 0 and 0.4, and $r=0.9$, $\phi=0.2$; (c) r is between 0 and 1, and $\phi=0.2$, $\gamma=0.2$.

The coefficient

$$\frac{\cos\phi-r\sqrt{1-\gamma}}{1-r\sqrt{1-\gamma}\cos\phi}=\cos\phi\frac{1-r\sqrt{1-\gamma}/\cos\phi}{1-r\sqrt{1-\gamma}\cos\phi}=\frac{\delta t_2}{\delta t}<1,$$

which means that the time shift δt_2 in the signal-recycled scheme is smaller than that in the standard weak value measurement. This is owing to the walk-off effect because the time shift of a small fraction of light exiting to the dark port is forward or backward, which causes a complementary shift in the opposing direction for the light exiting to the bright port. We now simply explain this effect.

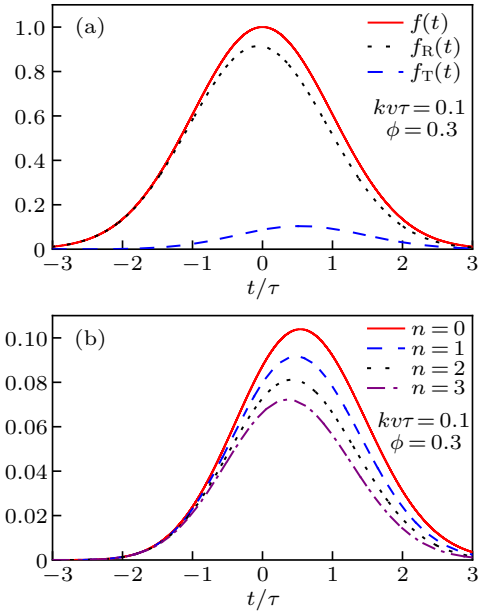


Fig. 4. Walk-off effect in the signal-recycled scheme: (a) $f(t)$ (red, solid), $f_{\rm T}(t)$ (blue, dashed) and $f_{\rm R}(t)$ (black, dotted); (b) $n=0$ (red, solid), $n=1$ (blue, dashed), $n=2$ (black, dotted), $n=3$ (purple, dash-dotted).

Consider that the initial input light is $f(t)=\exp(-\frac{t^2}{2\tau^2})$, and the lights exiting to the dark and bright port are, respectively, $f_{\rm T}(t)=\widehat{M}_{\rm D}\widehat{M}_{\rm D}^{*}f(t)=\sin^2(\phi+kvt)\exp(-\frac{t^2}{2\tau^2})$ and $f_{\rm R}(t)=\widehat{M}_{\rm B}\widehat{M}_{\rm B}^{*}f(t)=\sin^2(\phi+kvt)\exp(-\frac{t^2}{2\tau^2})$. We plot $f(t)$, $f_{\rm T}(t)$ and $f_{\rm R}(t)$ in Fig. 4(a) for $kv\tau=0.1$ and $\phi=0.3$. We can see that the transmission light $f_{\rm T}(t)$ has a time shift forward and the reflected light $f_{\rm R}(t)$ has a time shift backward. On the other hand, for the transmission light that is reflected by the partially transmitting mirror and reenter into the interferometer, the port exiting to the detector becomes the bright port and the port exiting to the laser is the dark port. We note that the light is $f_{\rm Tn}(t)=|\widehat{N}_{\rm D}\widehat{N}_{\rm D}^{*}|^n f_{\rm T}(t)=\cos^{2n}(\phi+kvt)\sin^2(\phi+kvt)\exp(-\frac{t^2}{2\tau^2})$, which has been reflected by the interferometer for n times. We plot $f_{\rm Tn}(t)$ in Fig. 4(b) for $n=0,1,2,3$. The larger the n is, the smaller the time shift is. The time shift δt_2 is the weighted average of the top $n_{\max}$ items in Eq. (10).

In the deviation of Eq. (11), we has abandoned high orders except for the top $n_{\max}$ items. Here we have used the conditions $c\tau\gg l_\lambda>2n_{\max}l_{\rm cav}$ and $(r\sqrt{1-\gamma})^n_{\max}\ll 1$. When $r\sqrt{1-\gamma}=0.8$ and $n_{\max}=20$, thus $(r\sqrt{1-\gamma})^{20}\approx 0.01$. If the length of cavity is $l_{\rm cav}=1m$, the coherent length of light should be larger than 40 m and $c\tau$ should be greater. These are easily realized under the current technical conditions.

5. Comparison with power recycling weak measurement

We now compare the signal recycling scheme with the power recycling scheme.[31] Since the detected light in the signal recycling scheme is the same as that in the power recycling scheme, we now focus the light reflected back to the laser and in the cavity. It is easy to calculate the reflected light in the signal recycling scheme as follows:

$$\begin{aligned}\langle\alpha|\varphi_{\rm r}\rangle_{\rm s} &= \langle\alpha|\widehat{L}\widehat{M}_{\rm B}|\varphi_0\rangle+\langle\alpha|r\widehat{L}\widehat{N}_{\rm B}|\varphi_c\rangle\\ &\approx\Bigg[\sqrt{1-\gamma}\cos(\phi+kvt)\\ &\quad-\frac{(1-\gamma)r\sin^2(\phi+kvt)}{1-r\sqrt{1-\gamma}\cos(\phi+kvt)\exp(-\frac{l_{\rm cav}t}{c\tau^2})}\Bigg]\\ &\quad\times\left(\frac{N^2}{2\pi\tau^2}\right)^{1/4}\exp\left(-\frac{t^2}{4\tau^2}\right)\\ &\approx\Bigg[\sqrt{1-\gamma}(\cos\phi-kvt\sin\phi)\\ &\quad-\frac{r(1-\gamma)\sin^2\phi}{1-r\sqrt{1-\gamma}\cos\phi}\left(1+\frac{kvt}{\phi_1}\right)\left(1+\frac{kvt\cos\phi}{\sin\phi}\right)\Bigg]\\ &\quad\times\left(\frac{N^2}{2\pi\tau^2}\right)^{1/4}\exp\left(-\frac{t^2}{4\tau^2}\right),\end{aligned}\tag{22}$$

and the reflected light in the power recycling scheme is

$$\begin{aligned}\langle\alpha|\varphi_{\rm r}\rangle_{\rm p}&\approx\left[r-\frac{(1-r^2)\sqrt{1-\gamma}\cos(\phi+kvt)}{1-r\sqrt{1-\gamma}\cos(\phi+kvt)\exp\left(-\frac{l_{\rm cav}t}{c\tau^2}\right)}\right]\cdot\left(\frac{N^2}{2\pi\tau^2}\right)^{1/4}\exp\left(-\frac{t^2}{4\tau^2}\right)\\ &\approx\left\{r-\frac{(1-r^2)\sqrt{1-\gamma}\cos\phi}{1-r\sqrt{1-\gamma}\cos\phi}\left[1+\frac{-\tan\phi+r\sqrt{1-\gamma}\sin\phi+r\sqrt{1-\gamma}\left(\sin\phi+\frac{\cos\phi l_{\rm cav}}{kvc\tau^2}\right)}{1-r\sqrt{1-\gamma}\cos\phi}kvt\right]\right\}\left(\frac{N^2}{2\pi\tau^2}\right)^{\frac{1}{4}}\exp\left(-\frac{t^2}{4\tau^2}\right).\end{aligned}\tag{23}$$

Under the condition $kv\tau\ll\phi\ll 1$, for example, in Ref. [10], $kv\tau\approx 4\times10^{-4}\ll\phi=0.31$ when $v=60$ pm/s and $\tau=833$ ms, the first order term (containing kvt) is far smaller than the constant term in Eqs. (22) and (23). Thus the amplitudes of the reflected light except for the same part $(\frac{N^2}{2\pi\tau^2})^{1/4}\exp(-\frac{t^2}{4\tau^2})$ are, respectively,

$$Y_{\rm rs}=\left|\sqrt{1-\gamma}\cos\phi-\frac{r(1-\gamma)\sin^2\phi}{1-r\sqrt{1-\gamma}\cos\phi}\right|,\tag{24}$$

$$Y_{\rm rp}=\left|r-\frac{(1-r^2)\sqrt{1-\gamma}\cos\phi}{1-r\sqrt{1-\gamma}\cos\phi}\right|,\tag{25}$$

in the signal recycling and power recycling schemes. Similarly, the amplitudes of light in the cavity are, respectively,

$$Y_{\rm cs}=\left|\frac{\sqrt{1-\gamma}\sin\phi}{1-r\sqrt{1-\gamma}\cos\phi}\right|,\tag{26}$$

$$Y_{\rm cp}=\left|\frac{\sqrt{1-r^2}}{1-r\sqrt{1-\gamma}\cos\phi}\right|,\tag{27}$$

in the signal recycling and power recycling schemes.

When the reflectivity of the partially transmitting mirror is $r=r_{\rm G}$, i.e., the intensity of detected light reaches the maximum, we can obtain $Y_{\rm rp}=0$ and $Y_{\rm rs}>0$. We plot $Y_{\rm rs}$, $Y_{\rm cs}$, $Y_{\rm cp}$ in Fig. 5 for $r=r_{\rm G}$ under some conditions. When the reflectivity of the partially transmitting mirror is $r=r_{\rm Q}$, i.e., the signal-to-noise ratio of detected light is maximal, we show $Y_{\rm rs}$, $Y_{\rm rp}$, $Y_{\rm cs}$, $Y_{\rm cp}$ in Fig. 6 under some conditions. It is easy to see that the intensity of the reflected light is stronger and the light in the cavity is weaker in the signal recycling scheme than those in the power recycling scheme.

Now we consider the reflected light as the cavity locking signal like Ref. [32], where the Pound–Drever–Hall technique plays a critical role.[33–35] Thus we need not place a beam splitter at the detecting port like Ref. [27] and the collection-detection efficiency can be improved. Equations (24)–(27) are an approximation for a Gaussian pulse but exactly accurate for

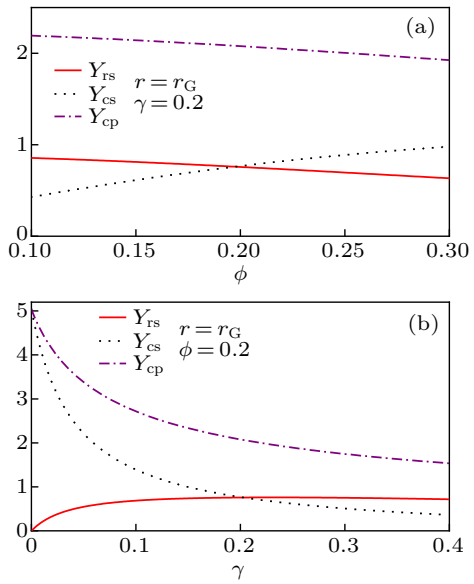


Fig. 5. Reflectivity of the partially transmitting mirror, $r = r_G = \sqrt{1-\gamma}\cos\phi$: (a) Y_{rs} (red solid), Y_{cs} (black dotted), Y_{cp} (purple dash-dotted) versus ϕ for $\gamma = 0.2$; (b) Y_{rs} (red solid), Y_{cs} (black dotted), Y_{cp} (purple dash-dotted) versus γ for $\phi = 0.2$.

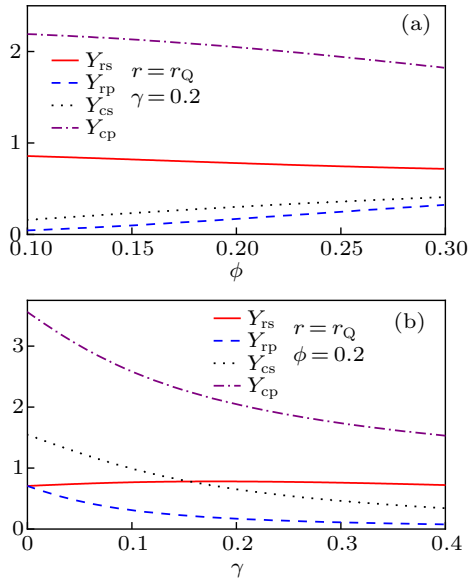


Fig. 6. Reflectivity of the partially transmitting mirror, $r = r_Q = \dfrac{(2-\gamma)\cos\phi - \sqrt{\cos^2\phi[1+(1-\gamma)^2] + (8\cos^4\phi - 18\cos^2\phi + 8)(1-\gamma)}}{2\sqrt{1-\gamma}(2-\cos^2\phi)}$: (a) Y_{rs} (red solid), Y_{rp} (blue dashed), Y_{cs} (black dotted), Y_{cp} (purple dash-dotted) versus ϕ for $\gamma = 0.2$; (b) Y_{rs} (red solid), Y_{rp} (blue dashed), Y_{cs} (black dotted), Y_{cp} (purple dash-dotted) versus γ for $\phi = 0.2$.

a stable wave. Here we directly use the Pound–Drever–Hall analysis in the supplemental material of Ref. [27]. If a reference beam with a phase modulation by the angular frequency of Ω is used to control the cavity length, then the reflected light is

$$\begin{aligned} E_r \approx\ & E_0\{[a - g(\omega_0)]J_0(\alpha)\,e^{i\omega_0 t} \\ & + [a - g(\omega_0+\Omega)]J_1(\alpha)\,e^{i(\omega_0+\Omega)t} \\ & - [a - g(\omega_0-\Omega)]J_1(\alpha)\,e^{i(\omega_0-\Omega)t}\}, \end{aligned} \tag{28}$$

where a is a constant, E_0 is the amplitude of laser, ω_0 is the angular frequency of the laser, α is the modulation depth, and

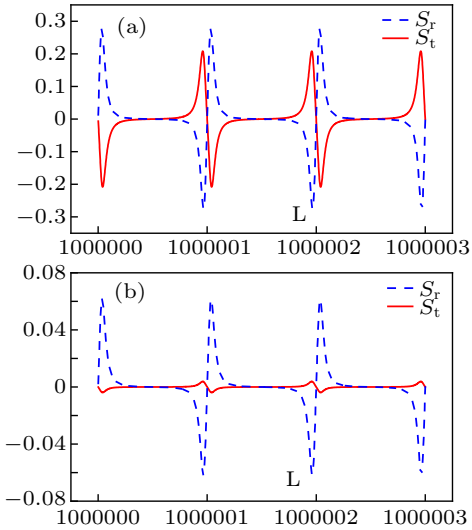


Fig. 7. The error signal S_r (blue dashed) and S_t (red solid), in units of wavelength λ in the horizontal axis: (a) the power recycling scheme, (b) the signal recycling scheme.

$g(\omega)$ is denoted as

$$g(\omega) = \frac{c}{1 - b\exp(i\omega L/c)}, \tag{29}$$

where $L = 2l_{cav}$ is the optical length for one traversal in the cavity. We can see $a = r$, $b = r\sqrt{1-\gamma}\cos\phi$, $c = (1-r^2)\sqrt{1-\gamma}\cos\phi$ in the power recycling scheme and $a = \sqrt{1-\gamma}\cos\phi$, $b = r\sqrt{1-\gamma}\cos\phi$, $c = r\sqrt{1-\gamma}\sin^2\phi$ in the signal recycling scheme. The power of reflected light can be written as

$$\begin{aligned} P_r \approx\ & P_c|a - g(\omega_0)|^2 + P_s|a - g(\omega_0+\Omega)|^2 \\ & + P_s|a - g(\omega_0-\Omega)|^2 + 2\sqrt{P_c P_s}\{\mathrm{Re}[(a - g(\omega_0)) \\ & \times (a - g(\omega_0+\Omega))^* - (a - g(\omega_0))^*(a - g(\omega_0-\Omega))]\cos(\Omega t) \\ & + \mathrm{Im}[(a - g(\omega_0))(a - g(\omega_0+\Omega))^* - (a - g(\omega_0))^* \\ & \times (a - g(\omega_0-\Omega))]\sin\Omega t\} + (2\Omega \text{ terms}), \end{aligned} \tag{30}$$

where $P_c = J_0(\alpha)^2|E_0|^2$ and $P_s = J_1(\alpha)^2|E_0|^2$. We note the error signal from the reflected light

$$\begin{aligned} S \propto S_r =\ & \mathrm{Re}\{[a - g(\omega_0)][a - g(\omega_0+\Omega)]^* \\ & - [a - g(\omega_0)]^*[a - g(\omega_0-\Omega)]\}, \end{aligned} \tag{31}$$

while the error signal from the detected light is

$$S \propto S_t = \mathrm{Re}[g(\omega_0)g^*(\omega_0+\Omega)] - g^*(\omega_0)g(\omega_0-\Omega)], \tag{32}$$

where we have not considered the constant coefficient for simplicity. In Fig. 7 we show S_r and S_t versus L for $L \in [1000000\lambda, 1000003\lambda]$, $\phi = 0.2$, $\gamma = 0.2$, $r = \sqrt{1-\gamma}\cos\phi$, $\Omega = 2\pi \times 12$ MHz, and $\lambda = 780$ nm. We can see that the reflected light can also be used to control the length of cavity. However, the reflected light is zero when the reflectivity of the partially transmitting mirror is $r = \sqrt{1-\gamma}\cos\phi$ in the power recycling scheme. The sum of first three terms,

$P_c|a-g(\omega_0)|^2+P_s|a-g(\omega_0+\Omega)|^2+P_s|a-g(\omega_0-\Omega)|^2=0$, in Eq. (30) for $L=m\lambda$ where m is an integer and the detector can not sense the negative signals. Therefore, the reflected light cannot be used as the feedback signals in this situation while there is not this condition in the signal recycling scheme unless the loss $\gamma=0$, which is impossible in real experiment.

6. Conclusions

We have proposed a signal-recycled technique to increase the intensity of the outgoing light of the standard-weak-value-based velocity measurement system. Like the power-recycled scheme, the cavity formed by the Michelson interferometer and the partially transmitting mirror can increase the SNR of velocity estimation compared to the standard weak measurement. However, optimal improvements of detected photons and SNR cannot be obtained simultaneously owing to the walk effect. Moreover, we find that the reflected light is stronger and the light in cavity is weaker compared with those in the power recycling scheme, which may increase the collection-detection efficiency in prospective relevant experiment.

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