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Time-varying coupling-induced logical stochastic resonance in a periodically driven coupled bistable system*

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Coupling-induced logical stochastic resonance (LSR) can be observed in a noise-driven coupled bistable system where the behaviors of system can be interpreted consistently as a specific logic gate in an appropriate noise level. Here constant coupling is extended to time-varying coupling, and then we investigate the effect of time-varying coupling on LSR in a periodically driven coupled bistable system. When coupling intensity oscillates periodically with the same frequency with periodic force or relatively high frequency, the system successfully yields the desired logic output. When coupling intensity oscillates irregularly with phase disturbance, large phase disturbance reduces the area of optimal parameter region of coupling intensity and response speed of logic devices. Although the system behaves as a desired logic gate when the frequency of time-periodic coupling intensity is precisely equal to that of periodic force, the desired logic gate is not robust against tiny frequency difference and phase disturbance. Therefore, periodic coupling intensity with high frequency ratio is an optimal option to obtain a reliable and robust logic operation.

Keywords: logical stochastic resonance, coupled bistable system, time-varying coupling, Wiener process

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1. Introduction

It is increasingly difficult to suppress noise as logic devices continue to shrink in size. The presence of the noise has increased the instability of logic devices and seriously limit the computing speed of computation devices based logic gates.[1] More recently, more and more efforts have focused on obtaining reliable logic operations. Inspired by stochastic resonance (SR),[2–7] Murali *et al*. firstly put forward logical stochastic resonance (LSR), which characterizes that the reliability of logic gates can be realized in a proper window of noise intensity.[8] LSR has been successfully realized in electronic circuits,[9] optical systems,[10] synthetic genetic networks,[11–13] *etc*. As a potential mechanism for designing new-style logic gates, which can robustly work in noisy environment, LSR attracts more attention.

Besides the various types of noises,[14,15] other non-stochastic factors can also induce LSR-like phenomenon.[16–21] For example, Gupta and coworkers demonstrated that a bistable system subjected only to completely regular periodic forcing, despite having no stochastic influence, can also produce LSR-like phenomenon, and thus they termed this phenomenon noise-free LSR.[22] Furthermore, the constructive interplay of noise and periodic force can let systems adapt to varying noise level, and thus let systems can robustly operate in high noise conditions. This phenomenon is known as enhanced LSR.[23] In the real computational devices, such as integrated circuits, many logic gates are packed with wires into the surface of semi-conductor material to execute complex computation. Thus, coupling between different logic gates is a significant factor in designing computational devices.[24] Aravind *et al*. went beyond a single bistable system, and realized LSR in a coupled system. They termed this phenomenon coupling induced LSR. More specifically, when noise and coupling intensity are in optimal windows, the output of the coupled system can be mapped consistently to specific logic gate operations.[25]

However, the coupling intensity between two subsystems is assumed to be constant in the aforementioned study. Further, it has been demonstrated recently that time-periodic coupling intensity has significant influence on the dynamics of nonlinear systems.[26,27] Therefore, constant coupling is extended to time-varying coupling, and then investigate the effect of time-varying coupling on the reliability and response speed of logic operations in a periodically driven coupled bistable system via two measurement indexes: success probability and mean switching time.

2. Model and scheme

Consider a periodically driven coupled bistable system which consists of two bistable sub-systems coupled linearly and bi-directionally:[25]

$$\dot{x} = 4\left(x - 5x^3\right) + b + \kappa\left(y - x\right) + A\cos(\omega t) + I_1(t), \quad (1)$$

$$\dot{y} = 4\left(y - 5y^3\right) + b + \kappa\left(x - y\right) + A\cos\left(\omega t\right) + I_2\left(t\right). \quad (2)$$

Here the bias parameter b controls the symmetry of double potential wells (Fig. 1), and it takes value -0.28 for AND logic

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gate, and 0.28 for OR logic gate, respectively.[25] $A\cos(\omega t)$ denotes periodic force. Without loss of generality, the amplitude A and angular frequency ω are 0.5 and $10^{-0.5}$ respectively, so that in the case of weak coupling intensity, reliable logic output cannot be obtained under periodic force driving.

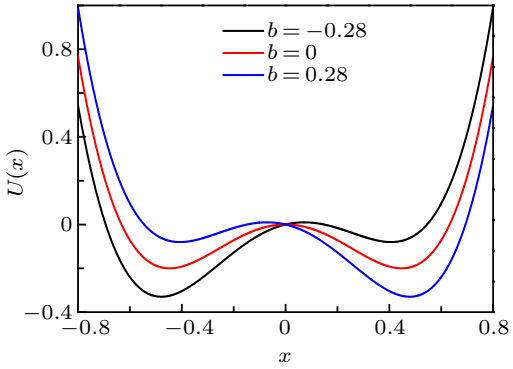


Fig. 1. Potential $U(x) = 5x^4 - 2x^2 + bx$, which is obtained by integrating $F(x) = 4\left(x - 5x^3\right) + b$.

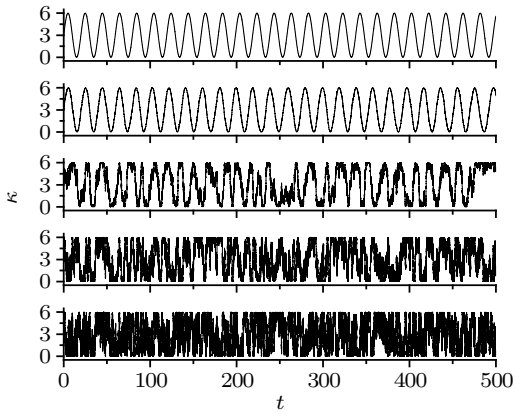


Fig. 2. The sampled time series of coupling intensity $\kappa = c[1+\sin(N\omega t + \sigma W(t))]W(t)$ is a standard Wiener process. Parameter σ stands for the intensity of the unit Wiener process $W(t)$, which governs the degree of deviation of coupling intensity oscillation from the perfect periodicity. From top to bottom, $\sigma = 0, 0.05, 0.5, 1, 1.5$. Larger σ means that coupling intensity oscillates more stochastically. $N = 1$, $c = 3$, and $\omega = 10^{-0.5}$.

Parameter κ indicates the coupling intensity between two subsystems. Coupling intensity is assumed to be time-varying, which is given by the following formula:[26] $\kappa = c[1+\sin(N\omega t + \sigma W(t)]$. Here c denotes the amplitude of coupling intensity. N indicates frequency ratio of coupling intensity to periodic force, which is not an integer and takes continuously value in an appropriate range. $W(t)$ is a standard Wiener process, while σ stands for the intensity of the unit Wiener process $W(t)$, which governs the degree of deviation of coupling intensity oscillation from the perfect periodicity (Fig. 2). Note that coupling intensity is time-periodic if σ=0. As previously mentioned,[28,29] the increment of standard Wiener process $W(t)$ is obtained according to the following formula: $W(t) = W(t-\Delta t) + \sqrt{-2\Delta t \ln(\chi_1)}\cos(2\pi\chi_2)$. Here χ_1 and χ_2 are two independent random numbers uniformly distributed between 0 and 1. Euler's algorithm with a fixed time step of $\Delta t = 5\times10^{-3}$ time unites is used to integrate the dynamic equations (1) and (2).

The $I_1(t)$ and $I_2(t)$ are two independent randomly switched 2-level square wave input signals (From top to bottom, panels 1 and 2 in Fig. 3). Without loss of generality, I_1 and I_2 take value 0.8 when logic input is 1, and -0.8 when logic input is 0. Logic output of system depends on the state of system. Logic output is 1 if x or y is greater than 0 (system lies in the positive well), or else logic output is 0 (system lies in the negative well) (Fig. 1). Therefore, logic gate can be defined according to logical input–output association (Table 1). In order to measure the reliability of a specific logic operation, a success probability P of getting the desired logic output is defined as the ratio of the number of successful runs to the total number of runs.[21] A successful run means that for all possible logic input sets (Table 1), each output of system matches the desired logic operation at each time step, allowing to exclude small transience. The small transience is defined as the 10% of each time interval.[14,30] The total number of runs R is 100 in our simulations. Note that increasing total number of runs does not lead to obvious differences in the results. In addition, mean switching time is obtained by averaging at least 500 switching time which the system takes averagely to switch from high to low states or *vice versa* after the switching of input signal. Note that increasing total number of switching time does not lead to obvious differences in the results. Mean switching time is used here to characterize the response speed of logic devices. In order to evaluate the degree of synchronization between two subsystems, a synchronization factor δ is defined as follows:

$$\delta = \frac{1}{R}\sum_{i=1}^{R}\delta_i, \quad \delta_i = \frac{1}{M}\sum_{j=1}^{M}\left|x_j(t) - y_j(t)\right|.$$

Here M is set to 4×10^6. Clearly, less δ indicates better synchronization.

Table 1. Truth table for logic AND & OR gates.

Values of I_1 and I_2	Input set	AND gate	OR gate
(−0.8, −0.8)	(0, 0)	0	0
(−0.8, 0.8)/(0.8, −0.8)	(0, 1)/(1, 0)	0	1
(0.8, 0.8)	(1, 1)	1	1

3. Results

When the frequency of coupling intensity is less than that of periodic force, such as $\log_{10}(N) = -1$ (from top to bottom, panels 3 and 4 in Fig. 3), the state (x or y) cannot be consistently mapped to AND or OR logic output according to logical input–output association (Table 1). Thus, the system cannot work robustly as a given logic AND or OR gate. However, when coupling intensity oscillates regularly with relatively high frequency than that of periodic force, such as $\log_{10}(N) = 1$ (from top to bottom, panels 5 and 6 in Fig. 3), the state (x or y) can be consistently mapped to AND or OR

logic output according to logical input–output association (Table 1). Consequently, the system can robustly operate as a specific logic gate. Furthermore, it is easy to switch from one logic gate to another by changing the bias b ($b = -0.28$ for logic AND gate, $b = 0.28$ for logic OR gate), so the logic gate is flexible (Fig. 3). It is noteworthy that for the AND and OR logic operations, the qualitative results are very similar, due to synchronization between two subsystems, so only x is used to obtain a logic output in the following sections.

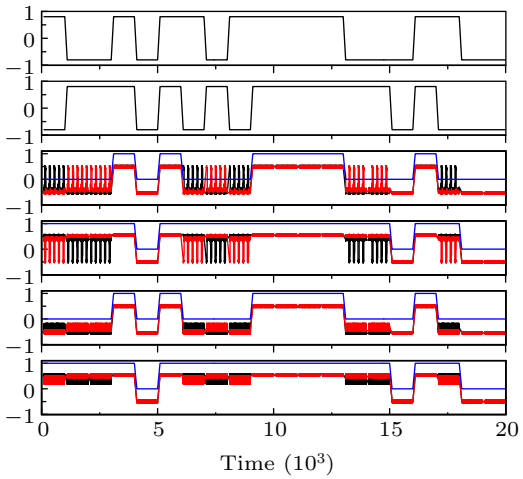


Fig. 3. The sampled time series of input signals I_1 and I_2, x (black), y (red), and logic output (blue) of logic AND or OR gate. From top to bottom, panels 1 and 2 indicate the randomly switched streams of input signals I_1 and I_2, panels 3 and 4 cannot yield consistent AND & OR logic outputs ($\log_{10}(N) = -1$), while panels 5 and 6 can yield consistent AND & OR logic outputs ($\log_{10}(N) = 1$). $c = 3$, and $\sigma = 0$.

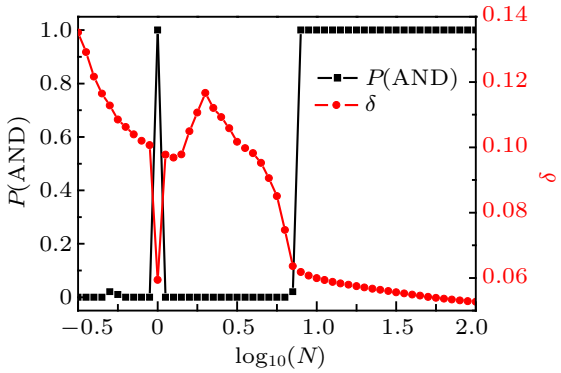


Fig. 4. The dependences of success probability P(AND) and synchronization factor δ on increasing $\log_{10}(N)$ for logic AND gate. $c = 3$, and $\sigma = 0$.

A quantitative measure of success probability P, which depicts how reliably this system performs as a logic device, can be used to further confirm the above observed results. $P = 1$ means that a reliable logic output is always available for each random stream of input signal ($I(t) = I_1(t) + I_2(t)$). As shown in Fig. 4, it is very obvious as the frequency of coupling intensity increases, one can observe a very sharp transition from $P \approx 0$ to $P = 1$ in the curve of P *versus* $\log_{10}(N)$ (Fig. 4), which indicates the occurrence of time-varying coupling-induced LSR. Interestingly, when the frequency of periodic coupling intensity is precisely equal to that of periodic force (*i.e.* $\log_{10}(N) = 0$), one can always obtain reliable logic operation ($P = 1$), but it is not robust and tiny frequency difference between coupling and periodic force will seriously destroy the reliability of logic operation (Fig. 4). Consequently, coupling intensity with high frequency ratio is an optimal option to obtain a reliable and robust logic operation. In addition, a synchronization factor δ is introduced to describe the degree of synchronization between two subsystems. It is quite clear that worse synchronization denoted by larger δ can account for why reliable logic operation cannot be obtained when $\log_{10}(N) < 0.9$ (Fig. 4).

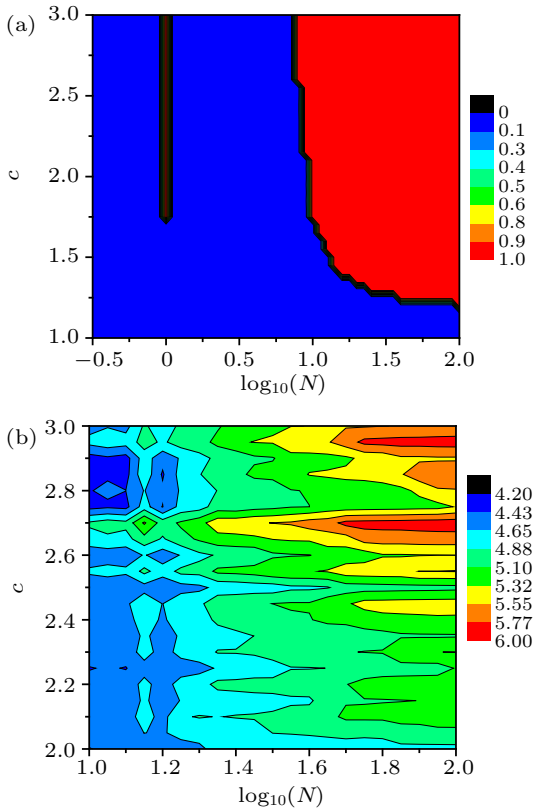


Fig. 5. Contour plots of success probability P(AND) (a) and mean switching time (b) in the c–$\log_{10}(N)$ plane. The red shaded region in (a) indicates where the success probability P(AND) takes maximum value of 1. Therefore, it shows the occurrence of time-periodic coupling-induced LSR phenomenon. $c = 3$, and $\sigma = 0$.

Furthermore, contour plot of success probability P (Fig. 5(a)) is used to illustrate the global dependences of success probability P on both amplitude $c$and the ratio of frequency $\log_{10}(N)$. As indicated by red shaded area where the success probability P takes the maximum value of 1 (Fig. 5(a)), time-varying coupling induced LSR can be achieved in a relatively wide range of the frequency ratio. In addition, mean switching time is used to characterize the response speed of the system to the change of input signal. In the premise of obtaining reliable logic outputs, the effect of varying frequency ratio on mean switching time is investigated by virtue of contour plot (Fig. 5(b)). For a given amplitude c or frequency ratio $\log_{10}(N)$ of coupling intensity, smaller frequency ratio $\log_{10}(N)$ or smaller amplitude c tends to reduce

mean switching time (Fig. 5(b)), and thus results in faster response of logic devices.

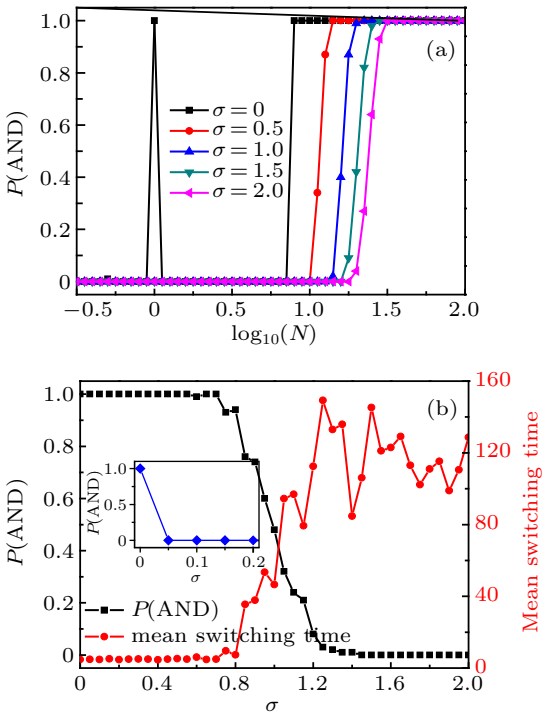


Fig. 6. The dependence of success probability P(AND) on increasing $\log_{10}(N)$ for various different σ (panel (a)). The dependences of success probability P(AND) and mean switching time on increasing σ for $\log_{10}(N) = 1.25$ (panel (b)). The insert in Fig. 6(b) indicates the dependences of success probability P(AND) on increasing σ for $\log_{10}(N) = 0$. Here c takes value 3.

In the examples above, coupling intensity oscillates periodically with time. In the following section, the effect of phase disturbance in the coupling intensity on the reliability and response speed of logic devices is investigated. Phase disturbance in coupling intensity is modeled by Wiener process $W(t)$ with the intensity σ. Larger σ means that more stochastically coupling intensity alters with time. For a given amplitude $c = 3$, the dependence of success probability P(AND) on increasing $\log_{10}(N)$ for different σ clearly show that larger σ makes the optimal window (where $P = 1$) of frequency ratio $\log_{10}(N)$ become narrower (Fig. 6(a)). In order to illustrate this conclusion more clearly, for a given amplitude $c = 3$ and a specific frequency ratio $\log_{10}(N) = 1.25$, the curves of success probability P(AND) and mean switching time *versus* increasing σ are drawn (Fig. 6(b)). It is evident that success probability P(AND) decreases rapidly if increasing σ is over a certain threshold, such as 0.9 (Fig. 6(b)). Note that when the frequency of coupling intensity is equal precisely to the frequency of periodic force, tiny phase disturbance may drastically destroy the reliability of logic operations as shown in the insert (Fig. 6(b)), thus logic gate is not robust against phase disturbance. In order to show the effect of phase disturbance degree σ from a global perspective, the boundary lines between reliable region (*i.e.* region above the curve) (where $P = 1$) and unreliable region (where $P < 1$) are plotted in the c–$\log_{10}(N)$ plane for different σ (Fig. 7). Clearly, with the increase of σ, the area of reliable region decreases drastically.

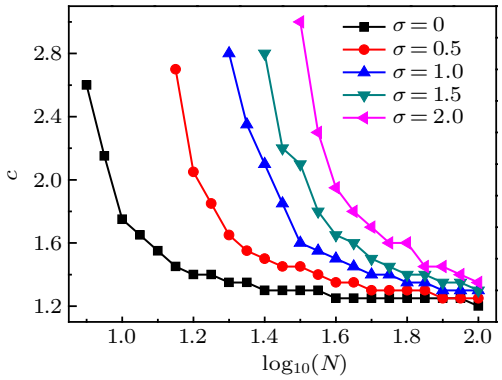


Fig. 7. Effect of phase disturbance degree σ on the area of reliable region. The region above the curve indicates reliable region (where $P = 1$). Obviously, with the increase of σ, the area of reliable region decreases drastically.

4. Conclusions

In summary, the effect of time-varying coupling intensity on reliable logic operation has been explicitly shown based on a coupled bistable system. It is possible to obtain a reliable logic operation in a periodically driven coupled bistable system with time-varying coupling intensity. In this study, two cases are considered: coupling intensity changes periodically or aperiodically with time. In the case of time-periodic coupling intensity, if coupling intensity oscillates periodically with relatively high frequency or the same frequency with periodic force, the system successfully yields the desired logical output. In the premise of obtaining reliable logic outputs, coupling intensity with smaller frequency ratio or smaller amplitude c tends to reduce mean switching time, and thus results in faster response of logic devices. In the case of time-varying coupling intensity with phase disturbance, large phase disturbance reduces the area of reliable parameter region of coupling intensity and response speed of logic devices. Although the system behaves as a desired logic gate when the frequency of time-periodic coupling intensity is precisely equal to that of periodic force, the desired logic gate is not robust against tiny frequency difference between periodic force and coupling intensity, and slight phase disturbance in coupling intensity. As a consequence, periodic coupling intensity with high frequency ratio is an optimal option to obtain a reliable and robust logic operation in a periodically driven coupled bistable system in the presence of time-varying coupling intensity.

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