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# Fast generation of W state via superadiabatic-based shortcut in circuit quantum electrodynamics*

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We propose a scheme to fast prepare the three-qubit W state via superadiabatic-based shortcuts in a circuit quantum electrodynamics (circuit QED) system. We derive the effective Hamiltonian to suppress the unwanted transitions between different eigenstates by counterdiabatic driving, and obtain the W state with high-fidelity based on the superadiabatic passage. The numerical simulation results demonstrate that the proposed scheme can accelerate the evolution, and is more efficient than that with the adiabatic passage. In addition, the proposed scheme is robust to the decoherence caused by the resonator decay and qubit relaxation, and does not need additional parameters, which could be feasible in experiment.

**Keywords:** W state, superadiabatic-based shortcut, circuit quantum electrodynamics

**PACS:** 03.67.Lx, 03.67.Bg, 42.50.Dv **DOI:** 10.1088/1674-1056/abd75b

## 1. Introduction

During the development of quantum communication and quantum computation, quantum entanglement plays an increasingly important role in quantum information processing. Correspondingly, the generation of entanglement states has also been widely investigated in literature both theoretically[1–16] and experimentally.[17–19] For multi-particle entanglement, there are two major types of entangled states, the W states and the Greenberger–Horne–Zeilinger (GHZ) states. They can not be converted to each other by local operations and classical communications. The W states of three or multi-particle are more stable when one of the particles is lost. Additionally, the W states are robust to the global dephasing and bit flip noises. Therefore, the W states have been widely exploited in quantum key distribution (QKD), quantum teleportation, quantum conversation, etc.[20–27]

Till now, the preparation of W states was demonstrated in different physical systems, such as, the ion trap system,[28] the cavity quantum electrodynamics (cavity QED),[29,30] circuit quantum electrodynamics (circuit QED),[31,32] and many other physical systems.[33,34] In different physical systems, due to their different devices and protocols, the time taken to prepare W states is also very different. Among them, the time used to prepare W state in Rydberg atomic system is 0.6 μs.[33] Among the different physical systems, the circuit QED has become the promising candidate due to its long coherent time, easily being coupled with other physical system and scaled up to large scale. In the circuit QED, the widely used qubit is the superconducting transmon qubit.[35,36] Compared with the superconducting charge qubit, the transmon qubit is immune to charge noise, and has a longer coherent time.

Many evolution processes to prepare the W states followed the adiabatic passage,[37–39] so that they required a long enough time to meet the adiabatic condition, which severely limits the applications of the W states in quantum information processing. One possible way to overcome the above drawback is the shortcut to adiabatic passage.[40–45] Several schemes have been proposed for the generation of the W states via the shortcut adiabatic passage. For example, Kang *et al.*[46] proposed a scheme for the fast preparation of W state with the superconducting quantum interference devices by choosing a set of dressed states suitably, which could be used to accelerate the adiabatic passage with an auxiliary qubit. On the other hand, Yu *et al.*[47] prepared the W states of three superconducting qubits via Lewis–Riesenfeld (LR) invariants, which needed an additional parameter and made it difficult to realize in experiment.

In this paper, we propose a simple scheme to achieve the fast generation of three-qubit W state based on the circuit QED system via the superadiabatic-based shortcut. First, the effective Hamiltonian is obtained by unitary transformation and supposed to ignore the high frequency term. Then, two iterations are carried out to obtain the counterdiabatic driving for suppressing the unwanted transitions between the different eigenstates for acceleration. Finally, the W state is achieved through the superadiabatic passage with high-fidelity. In addition, the pulse form in the Hamiltonian is designed to be re-

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placed by the alternative superpositions of Gaussian pulses for the feasibility. And the performance of the proposed scheme is discussed by numerical simulations.

The paper is organized as follows. Section 2 presents the physical model based on the circuit QED system, and an effective Hamiltonian is deduced to realize the fast generation of the three-qubit high-fidelity W state. In Section 3, we describe the superadiabatic scheme and investigate the conditions for the fast generation of W state analytically. In Section 4, numerical simulations are performed with selectively controllable parameters which prove that the protocol is feasible. Finally, conclusions are given in Section 5.

## 2. Physical model and effective dynamics

We consider a superconducting architecture where three identical transmon qubits are fabricated in a transmission line resonator. Figure 1 shows the schematic diagram of the superconducting architecture, including the circuit QED and the energy-level of the transmon qubit. All the qubits are truncated to three-level, i.e., $|0\rangle_l$, $|e\rangle_l$, $|1\rangle_l$, here, $|e\rangle_l$ is an auxiliary state, and the subscript $l = 1,2,3$ label different transmon qubits. The information is encoded on the states $|0\rangle_l$ and $|1\rangle_l$. The transition between states $|0\rangle_l$ and $|e\rangle_l$ is resonantly coupled to the resonator with coupling strength $g_l$. And states $|1\rangle_l$ and $|e\rangle_l$ are resonantly driven by a classical pulse with time-dependent Rabi frequency $\Omega_l(t)$. Both the qubit–resonator and qubit–pulse are considered on one-photon resonance and the $|0\rangle_l \leftrightarrow |1\rangle_l$ transition is forbidden.

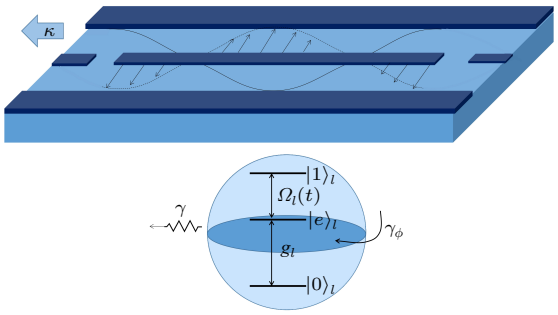



**Fig. 1.** The schematic diagram of the superconducting architecture, including the circuit QED and the energy-level of the transmon qubit. The $\kappa$ is the decay rete of the resonator. The $\gamma$ and $\gamma_\phi$ denote the spontaneous emission rate and the dephasing rate of the qubit, respectively.

The transmission line resonator possesses many modes, and each of them can be modelled as an LC resonator. Here, we just consider that one mode is closely related to the qubit–resonator interation, and the Hamiltonian of the resonator can be written as (here in after $\hbar = 1$)

$$H = \omega_{\rm r}\left(a^{\dagger}a + \frac{1}{2}\right), \tag{1}$$

where $\omega_{\rm r}$ is the frequency of the resonator, and $a$ ($a^{\dagger}$) is the associated annihilation (creation) operator for the resonator. Under the rotating-wave approximation, the effective Hamiltonian of the circuit QED system can be described as

$$H_{\rm total} = \sum_{l=1}^{3}\left[g_l a|e\rangle_l\langle 0| + \Omega_l(t)|1\rangle_l\langle e|\right] + {\rm h.c.} \tag{2}$$

The total excitation operator $N_{\rm e} = \sum_{l=1}^{3}(|e\rangle_l\langle e| + |1\rangle_l\langle 1|) + a^{\dagger}a$, and $N_{\rm e}$ commutes with $H_{\rm total}(t)$ which demonstrates that the excitation number is conserved during the whole dynamical evolution.

Assume that the system is initialized on the state $|0\rangle_c|1\rangle_1|0\rangle_2|0\rangle_3$, where the resonator is in the vacuum state $|0\rangle$, the qubit-1 is prepared in the state $|1\rangle$, and the qubits 2 and 3 are in the ground state $|0\rangle$. According to the effective Hamiltonian, the system evolves in a subspace spanned by the following seven bases:

$$\begin{aligned}
&|\phi_1\rangle = |0\rangle_c|1\rangle_1|0\rangle_2|0\rangle_3, \quad |\phi_2\rangle = |0\rangle_c|0\rangle_1|1\rangle_2|0\rangle_3,\\
&|\phi_3\rangle = |0\rangle_c|0\rangle_1|0\rangle_2|1\rangle_3, \quad |\phi_4\rangle = |1\rangle_c|0\rangle_1|0\rangle_2|0\rangle_3,\\
&|\phi_5\rangle = |0\rangle_c|e\rangle_1|0\rangle_2|0\rangle_3, \quad |\phi_6\rangle = |0\rangle_c|0\rangle_1|e\rangle_2|0\rangle_3,\\
&|\phi_7\rangle = |0\rangle_c|0\rangle_1|0\rangle_2|e\rangle_3.
\end{aligned} \tag{3}$$

In order to realize the fast generation of three-qubit W state under the superadiabatic evolution, the assumptions $g_1 = \sqrt{2}g_2 = \sqrt{2}g_3 = \sqrt{2}g$ ($g$ is real), and $\Omega_2(t) = \Omega_3(t)$ are given. Then the effective Hamiltonian in Eq. (2) can be rewritten as

$$\begin{aligned}
H_{\rm total} &= \sqrt{2}g|\phi_5\rangle\langle\phi_4| + g(|\phi_6\rangle + |\phi_7\rangle)\langle\phi_4| + \Omega_1(t)|\phi_1\rangle\langle\phi_5|\\
&\quad + \Omega_2(t)(|\phi_2\rangle\langle\phi_6| + |\phi_3\rangle\langle\phi_7|) + {\rm h.c.}\\
&= \sqrt{2}g|\phi_5\rangle\langle\phi_4| + \sqrt{2}g|\zeta\rangle\langle\phi_4| + \Omega_1(t)|\phi_1\rangle\langle\phi_5|\\
&\quad + \Omega_2(t)(|\Phi\rangle\langle\zeta| + |\Phi^{-}\rangle\langle\zeta^{-}|) + {\rm h.c.},
\end{aligned} \tag{4}$$

for which $|\zeta\rangle = \frac{1}{\sqrt{2}}(|\phi_6\rangle + |\phi_7\rangle)$, $|\zeta^{-}\rangle = \frac{1}{\sqrt{2}}(|\phi_6\rangle - |\phi_7\rangle)$, $|\Phi\rangle = \frac{1}{\sqrt{2}}(|\phi_2\rangle + |\phi_3\rangle)$, and $|\Phi^{-}\rangle = \frac{1}{\sqrt{2}}(|\phi_2\rangle - |\phi_3\rangle)$. They are the two set of orthogonal states constructed from Eq. (3). If the initial state is $|\phi_1\rangle$, it is obviously noted that the states $|\zeta^{-}\rangle$ and $|\Phi^{-}\rangle$ are not populated during the whole evolution.[48] Thus the Hamiltonian in Eq. (4) can be simplified as

$$\begin{aligned}
&H_{\rm total} = H_{\rm C} + H_{\rm L}(t),\\
&H_{\rm C} = \sqrt{2}g|\phi_5\rangle\langle\phi_4| + \sqrt{2}g|\zeta\rangle\langle\phi_4| + {\rm h.c.},\\
&H_{\rm L}(t) = \Omega_1(t)|\phi_1\rangle\langle\phi_5| + \Omega_2(t)|\Phi\rangle\langle\zeta| + {\rm h.c.}
\end{aligned} \tag{5}$$

And the eigenstates of Hamiltonian $H_{\rm C}$ are

$$\begin{aligned}
&|\psi_0\rangle = \frac{1}{\sqrt{2}}(|\zeta\rangle - |\phi_5\rangle), \ |\psi_1\rangle = \frac{1}{2}\left(\sqrt{2}|\phi_4\rangle + |\phi_5\rangle + |\zeta\rangle\right),\\
&|\psi_2\rangle = \frac{1}{2}\left(-\sqrt{2}|\phi_4\rangle + |\phi_5\rangle + |\zeta\rangle\right),
\end{aligned} \tag{6}$$

with the corresponding eigenvalues of $E_0 = 0$, $E_1 = 2g$, and $E_2 = -2g$.

Therefore, the Hamiltonian in Eq. (5) can be further rewritten in the eigenstates of $H_{\rm C}$ as

$$H_{\rm total} = H_{\rm C} + H_{\rm L}(t),$$

$$H_{\rm C} = 2g|\psi_1\rangle\langle\psi_1| - 2g|\psi_2\rangle\langle\psi_2| + \text{h.c.},$$
$$H_{\rm L}(t) = \Omega_1(t)|\phi_1\rangle\left[-\frac{1}{\sqrt{2}}\langle\psi_0| + \frac{1}{2}(\langle\psi_1| + \langle\psi_2|)\right] + \Omega_2(t)|\Phi\rangle\left[\frac{1}{\sqrt{2}}\langle\psi_0| + \frac{1}{2}(\langle\psi_1| + \langle\psi_2|)\right] + \text{h.c.} \quad (7)$$

After the unitary transformation $U_{\rm C} = \exp(-{\rm i}H_{\rm C}t)$ performed on the above Hamiltonian and neglecting the high-frequency oscillating terms under the condition $\Omega_1(t),\ \Omega_2(t) \ll g$, the effective Hamiltonian of the system becomes

$$\begin{aligned} H_{\rm eff}(t) &= -\frac{\Omega_1(t)}{\sqrt{2}}|\phi_1\rangle\langle\psi_0| + \frac{\Omega_2(t)}{\sqrt{2}}|\Phi\rangle\langle\psi_0| + \text{h.c.} \\ &= \Omega_{\rm p}(t)|\phi_1\rangle\langle\psi_0| + \Omega_{\rm s}(t)|\Phi\rangle\langle\psi_0| + \text{h.c.}, \end{aligned} \quad (8)$$

where $\Omega_{\rm p}(t) = -\Omega_1(t)/\sqrt{2}$ and $\Omega_{\rm s}(t) = \Omega_2(t)/\sqrt{2}$. Now, the Hamiltonian in Eq. (8) can be simply considered as a three-level quantum system that consists of two ground states $|\phi_1\rangle$ and $|\Phi\rangle$, and an excited state $|\psi_0\rangle$.

## 3. Generation of three-qubit W state via superadiabatic passgae

In this section, we discuss the fast generation of the three-qubit W state via the superadiabatic-based shortcut path.

By defining $\Omega(t) = \sqrt{\Omega_{\rm p}^2(t) + \Omega_{\rm s}^2(t)}$ and $\tan\theta_0(t) = \Omega_{\rm p}(t)/\Omega_{\rm s}(t)$, the instantaneous eigenstates of the effective Hamiltonian in Eq. (8) are

$$\begin{aligned} |\varphi_0(t)\rangle &= \cos\theta_0(t)|\phi_1\rangle - \sin\theta_0(t)|\Phi\rangle, \\ |\varphi_+(t)\rangle &= \frac{1}{\sqrt{2}}[\sin\theta_0(t)|\phi_1\rangle + |\psi_0\rangle + \cos\theta_0(t)|\Phi\rangle], \\ |\varphi_-(t)\rangle &= \frac{1}{\sqrt{2}}[\sin\theta_0(t)|\phi_1\rangle - |\psi_0\rangle + \cos\theta_0(t)|\Phi\rangle], \end{aligned} \quad (9)$$

with the corresponding eigenvalues being $\varepsilon_0 = 0$, $\varepsilon_+ = \Omega(t)$, and $\varepsilon_- = -\Omega(t)$, respectively. Then, the unitary transformation $U_0(t) = \sum_{k=0,\pm}|\varphi_k\rangle\langle\varphi_k(t)|$ is performed on the Hamiltonian in Eq. (8) under the adiabatic frame. After the transformation, the Hamiltonian becomes

$$\begin{aligned} H_1(t) &= U_0(t)H_{\rm eff}(t)U_0^\dagger(t) + {\rm i}\dot{U}_0(t)U_0^\dagger(t) \\ &= \Omega(t)(|\varphi_+\rangle\langle\varphi_+| - |\varphi_-\rangle\langle\varphi_-|) \\ &\quad + \frac{\dot{\theta}_0(t)}{\sqrt{2}}({\rm i}|\varphi_+\rangle\langle\varphi_0| + {\rm i}|\varphi_-\rangle\langle\varphi_0| + \text{h.c.}). \end{aligned} \quad (10)$$

It is indicated that the system evolves adiabatically following one of the states $|\varphi_{0,\pm}(t)\rangle$ on the condition of adiabatic approximation $|\dot{\theta}(t)| \ll \Omega(t)$.[45] Here, the last term in Eq. (10) represents the transitions between different eigenstates. Hence, with slow enough change of the parameters of the Hamiltonian, the evolution of the system would just follow one of the eigenstates but it needs a long time.[11] Otherwise, the transitions between different eigenstates would be induced with a little faster change of the system's Hamiltonian. However, in Refs. [49,50], a suitable counterdiabatic (CD) Hamiltonian was proposed to suppress the transitions between different eigenstates. Inspired by their works, the CD Hamiltonian in Eq. (10) should have the form of $-{\rm i}\dot{U}_0(t)U_0^\dagger(t)$ within the basis $\{|\phi_1\rangle, |\psi_0\rangle, |\Phi\rangle\}$. Hence, in the adiabatic frame, the CD Hamiltonian can be written as

$$\begin{aligned} H_{\rm CD}^1(t) &= -{\rm i}U_0^\dagger(t)\dot{U}_0(t) \\ &= {\rm i}\dot{\theta}_0(t)(|\phi_1\rangle\langle\Phi| - |\Phi\rangle\langle\phi_1|). \end{aligned} \quad (11)$$

Compared with the Hamiltonian in Eq. (8), this CD Hamiltonian needs the direct coupling between the states $|\phi_1\rangle$ and $|\Phi\rangle$, which is hard to realize in practice. In order to suppress the transitions between different eigenstates, another CD Hamiltonian for Eq. (10) should be achieved which is easy to implement. We adopt the method proposed in Ref. [51] to obtain the CD Hamiltonian by only modifying the shapes of pulses as the following.

Firstly, we get the instantaneous eigenstates of $H_1(t)$ with the corresponding eigenvalues $\varepsilon_0' = 0$, $\varepsilon_+' = \Omega'(t)$, and $\varepsilon_-' = -\Omega'(t)$ as

$$\begin{aligned} |\varphi_0'(t)\rangle &= \frac{1}{\sqrt{2}}\left[-{\rm i}\sin\theta_1(t)|\varphi_+\rangle + \sqrt{2}\cos\theta_1(t)|\varphi_0\rangle + {\rm i}\sin\theta_1(t)|\varphi_-\rangle\right], \\ |\varphi_+'(t)\rangle &= \frac{1}{2}{\rm i}[1+\cos\theta_1(t)]|\varphi_+\rangle + \frac{1}{\sqrt{2}}\sin\theta_1(t)|\varphi_0\rangle + \frac{1}{2}{\rm i}[1-\cos\theta_1(t)]|\varphi_-\rangle, \\ |\varphi_-'(t)\rangle &= \frac{1}{2}{\rm i}[1-\cos\theta_1(t)]|\varphi_+\rangle - \frac{1}{\sqrt{2}}\sin\theta_1(t)|\varphi_0\rangle + \frac{1}{2}{\rm i}[1+\cos\theta_1(t)]|\varphi_-\rangle, \end{aligned} \quad (12)$$

where $\Omega'(t) = \sqrt{\dot{\theta}_0(t)^2 + \Omega^2(t)}$ and $\tan\theta_1(t) = \dot{\theta}_0(t)/\Omega(t)$. Then we move the Hamiltonian $H_1(t)$ to the instantaneous eigenstates [shown in Eq. (12)] with respect to $U_1(t) = \sum_{k=0,\pm}|\varphi_k'\rangle\langle\varphi_k'(t)|$. After the transformation, the Hamiltonian becomes

$$\begin{aligned} H_2(t) &= U_1(t)H_1(t)U_1^\dagger(t) + {\rm i}\dot{U}_1(t)U_1^\dagger(t) \\ &= \Omega'(t)(|\varphi_+'\rangle\langle\varphi_+'| - |\varphi_-'\rangle\langle\varphi_-'|) \\ &\quad + \frac{\dot{\theta}_1(t)}{\sqrt{2}}({\rm i}|\varphi_+'\rangle\langle\varphi_0'| - {\rm i}|\varphi_-'\rangle\langle\varphi_0'| + \text{h.c.}). \end{aligned} \quad (13)$$

Like the CD Hamiltonian in Eq. (11) in the basis $\{|\phi_1\rangle, |\psi_0\rangle, |\Phi\rangle\}$, the new version CD Hamiltonian $-{\rm i}\dot{U}_1(t)U_1^\dagger(t)$ can be expanded as

$$\begin{aligned} H_{\rm CD}^2(t) &= {\rm i}U_0^\dagger(t)\dot{U}_1^\dagger(t)U_1(t)U_0(t) \\ &= \dot{\theta}_1(t)[-\cos\theta_0(t)|\phi_1\rangle\langle\psi_0| + \sin\theta_0(t)|\Phi\rangle\langle\psi_0|] + \text{h.c.} \end{aligned} \quad (14)$$

The new version $H_{\rm CD}^2(t)$ is feasible since it has the same form as the effective Hamiltonian $H_{\rm eff}(t)$. Meanwhile, the system evolves along the eigenstates in Eq. (12). Finally, the total

Hamiltonian of the system can be expressed as

$$\begin{aligned} H_{\rm mod}(t) &= H_{\rm eff}(t)+H_{\rm CD}^{2}(t) \\ &= \left[\Omega_{\rm p}(t)-\dot{\theta}_1(t)\cos\theta_0(t)\right]|\phi_1\rangle\langle\psi_0| \\ &\quad+\left[\Omega_{\rm s}(t)+\dot{\theta}_1(t)\sin\theta_0(t)\right]|\Phi\rangle\langle\psi_0|+{\rm h.c.} \\ &= \Omega_{\rm p}^{\rm A}(t)\,|\phi_1\rangle\langle\psi_0|+\Omega_{\rm s}^{\rm A}(t)|\Phi\rangle\langle\psi_0|+{\rm h.c.} \quad (15) \end{aligned}$$

for which $\Omega_{\rm p}^{\rm A}(t)=\Omega_{\rm p}(t)-\dot{\theta}_1(t)\cos\theta_0(t)$ and $\Omega_{\rm s}^{\rm A}(t)=\Omega_{\rm s}(t)+\dot{\theta}_1(t)\sin\theta_0(t)$. Without loss of generality, we assume that the time-dependent pulses $\Omega_{\rm p}^{\rm A}(t)$ and $\Omega_{\rm s}^{\rm A}(t)$ are Gaussian envelopes which are easy to fit through numerical calculation.

From the above discussion, the Hamiltonian $H_{\rm CD}^{2}(t)$ is achieved to suppress the unwanted transitions, and the parameters $\theta_0(t)$, $\theta_1(t)$ should follow the conditions $\theta_{0,1}(0)=\theta_1(t_{\rm f})=0$, $\theta_0(t_{\rm f})=\arctan(-\sqrt{2})$, where $t_{\rm f}$ is the final time of the dynamical evolution. The reasons are the following. If $\theta_1(0)=\theta_1(t_{\rm f})=0$, then $|\varphi_0'(0)\rangle=|\varphi_0'(t_{\rm f})\rangle=|\varphi_0\rangle$, the instantaneous eigenstates of $H_1(t)$ are coincided with those of $H_{\rm eff}(t)$. When the initial state is $|\phi_1\rangle$ and the target state is $|W_3\rangle$, the boundary conditions $\theta_0(0)=0$ and $\theta_0(t_{\rm f})=\arctan(-\sqrt{2})$ are obtained according to $|\varphi_0(t)\rangle$.

With the above conditions, the three-qubit W state $|W_3\rangle=\frac{1}{\sqrt{3}}(|1\rangle_1|0\rangle_2|0\rangle_3+|0\rangle_1|1\rangle_2|0\rangle_3+|0\rangle_1|0\rangle_2|1\rangle_3)$ could be obtained via the superadiabatic protocol.

## 4. The numerical simulation

In this section, we do some numerical simulations to verify the proposed scheme.

In order to satisfy the conditions $\theta_{0,1}(0)=\theta_1(t_{\rm f})=0$, $\theta_0(t_{\rm f})=\arctan(-\sqrt{2})$, the pulses $\Omega_{\rm p}^{\rm A}(t)$ and $\Omega_{\rm s}^{\rm A}(t)$ with Gaussian envelopes are chosen as[11]

$$\begin{aligned} \Omega_{\rm p}(t) &= \sqrt{2}\Omega_0\exp\left[-(t-\tau_{11})^2/T^2\right], \\ \Omega_{\rm s}(t) &= -\sqrt{2}\Omega_0\exp\left[-(t-\tau_{21})^2/T^2\right] \\ &\quad-\Omega_0\exp\left[-(t-\tau_{22})^2/T^2\right], \end{aligned} \quad (16)$$

where $\Omega_0$ is the pulse amplitude, $t_{\rm f}$ is the total evolution time, $\tau_{11}=0.5t_{\rm f}$, $\tau_{21}=0.05t_{\rm f}$ and $\tau_{22}=0.5t_{\rm f}$ are the time delay, and $T=0.2t_{\rm f}$ is the operation duration. Therefore, the $\theta_0(t)$ and $\theta_1(t)$ can be determined by engineering the shape of the driven pulses $\Omega_{\rm p}(t)$ and $\Omega_{\rm s}(t)$. In Fig. 2, we plot the time-dependent $\theta_0(t)$ and $\theta_1(t)$ versus different pulses $\Omega_{\rm p}(t)$ and $\Omega_{\rm s}(t)$. As shown in Fig. 2(a), one can easily find that the $\theta_0(t)$ is independent of the pulse amplitude $\Omega_0$ when the conditions $\theta_0(0)=0$, $\theta_0(t_{\rm f})=\arctan(-\sqrt{2})$ are satisfied all the time. However, for the time-dependent parameter $\theta_1(t)$, the limitations $\theta_1(0)=\theta_1(t_{\rm f})=0$ can only be satisfied on the condition that the value of $\Omega_0$ is large enough, as seen from Fig. 2(b). During the superadiabatic evolution process, $\Omega_k(t)\ll g$ $(k=1,2,3)$ should be ensured, which implies that the value of $\Omega_0$ can not be taken too large. Therefore, we adopt $\Omega_0=6.75t_{\rm f}^{-1}$ and $g=36t_{\rm f}^{-1}$ for the following discussions.

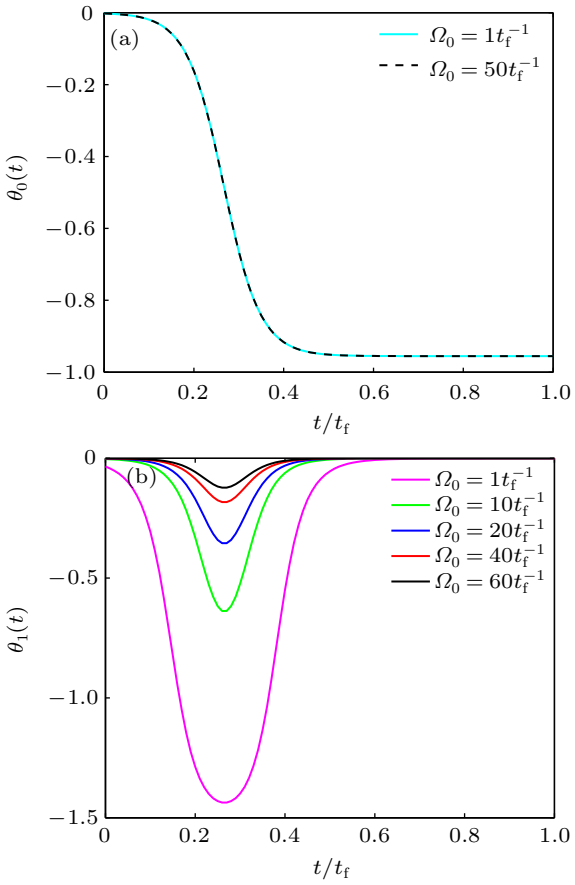



**Fig. 2.** (a) Time-dependent parameter $\theta_0(t)$ versus different pulse amplitude. (b) Time-dependent parameter $\theta_1(t)$ versus different values of pulse amplitude.

Since the analytic results of the pulses $\Omega_{\rm p}^{\rm A}(t)$ and $\Omega_{\rm s}^{\rm A}(t)$ are too complicated to realize in experiment, we replace them with the superpositions of Gaussian pulses by nonlinear least square fitting.[52] The form of the related Gaussian pulses can be written as

$$\begin{aligned} \Omega_{\rm p}^{0}(t) &= \sum_{i=1}^{4}\Omega_{1i}\exp\left[-(t-\tau_{1i})^2/T_{1i}^2\right], \\ \Omega_{\rm s}^{0}(t) &= \sum_{i=1}^{4}\Omega_{2i}\exp\left[-(t-\tau_{2i})^2/T_{2i}^2\right]. \end{aligned} \quad (17)$$

The related parameters are shown in the following:

$$\begin{aligned} &\Omega_{11}=4.807/t_{\rm f}, \quad \Omega_{12}=5.055/t_{\rm f}, \quad \Omega_{13}=12.75/t_{\rm f}, \\ &\Omega_{14}=-11.05/t_{\rm f}, \quad \tau_{11}=0.4385t_{\rm f}, \quad \tau_{12}=0.5319t_{\rm f}, \\ &\tau_{13}=0.227t_{\rm f}, \quad \tau_{14}=0.2829t_{\rm f}, \quad T_{11}=-0.2487t_{\rm f}, \\ &T_{12}=0.1699t_{\rm f}, \quad T_{13}=0.07043t_{\rm f}, \quad T_{14}=0.0818t_{\rm f}, \quad (18) \\ &\Omega_{21}=18.51/t_{\rm f}, \quad \Omega_{22}=-8.889/t_{\rm f}, \quad \Omega_{23}=-19.78/t_{\rm f}, \\ &\Omega_{24}=0.8233/t_{\rm f}, \quad \tau_{21}=0.2797t_{\rm f}, \quad \tau_{22}=0.3271t_{\rm f}, \\ &\tau_{23}=0.2521t_{\rm f}, \quad \tau_{24}=0.08117t_{\rm f}, \quad T_{21}=0.1655t_{\rm f}, \\ &T_{22}=0.06405t_{\rm f}, \quad T_{23}=0.309t_{\rm f}, \quad T_{24}=0.0581t_{\rm f}. \quad (19) \end{aligned}$$

To explore the difference between the alternative Gaussian pules and the original ones, we plot the superadiabatic pulses $\Omega^{\rm A}_{\rm p,s}(t)$ and their corresponding fitted pulses $\Omega^{0}_{\rm p,s}(t)$ in Fig. 3. The results show that the alternative superpositions of Gaussian pulses are almost the same as the original pulses $\Omega^{\rm A}_{\rm p,s}(t)$. The approximate coincidence of the two curves (as shown in Fig. 3) illustrates that the pulses $\Omega^{\rm A}_{\rm p,s}(t)$ can be replaced by the alternative Gaussian pulses.

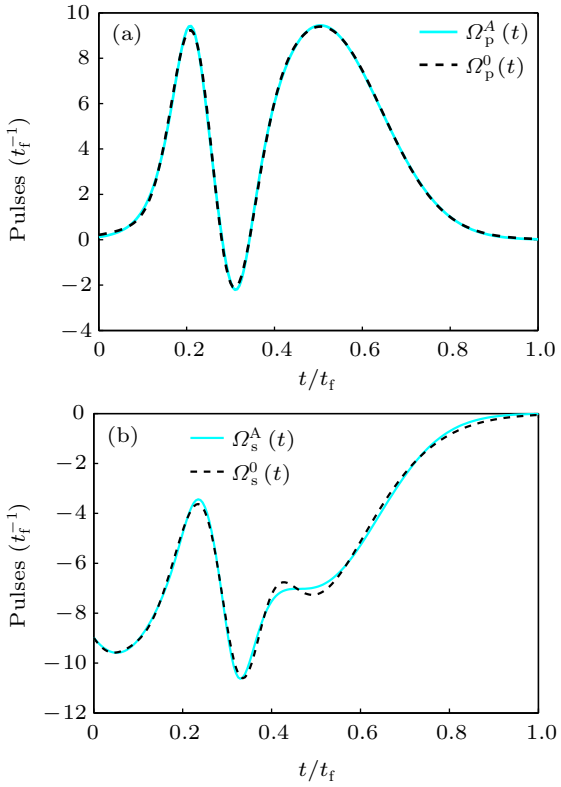



**Fig. 3.** (a) Time dependence of pulse $\Omega^{\rm A}_{\rm p}(t)$ and its fitted pulse $\Omega^{0}_{\rm p}(t)$ shown by blue and dotted black curves, respectively. (b) Time dependence of pulse $\Omega^{\rm A}_{\rm s}(t)$ and the corresponding fitted pulse $\Omega^{0}_{\rm s}(t)$ are indicated by blue and dotted black curves.

To further check the effectiveness of the two alternative pulses, we present the fidelity $F(t) = |\langle W|\Psi(t)\rangle|^2$ of the system state for the superadiabatic-based shortcut protocol with the pulses $\Omega^{0}_{\rm p,s}(t)$ in Fig. 4, where the $|W\rangle$ and $|\Psi(t)\rangle$ denote the ideal state and evolved state, respectively. During the dynamical evolution, one can find that the fidelity can almost approach one after $t/t_{\rm f}$ is greater than 0.4.

In Fig. 5, we plot the populations for all the states in Eq. (3) during the time evolution process. It shows that the three-qubit W state can be obtained with high-fidelity after the system evolution, i.e., the state remains stable after $t/t_{\rm f}$ is greater than 0.4. The intermediate states $|\phi_5\rangle$–$|\phi_7\rangle$ emerge in a very short time during the time intervals $0.15t/t_{\rm f}$ to $0.4t/t_{\rm f}$, and the population of state $|\phi_4\rangle$ is almost zero during the whole evolution. Hence, the decoherence induced by the resonator decay can be neglected in the proposed superadiabatic shortcut.

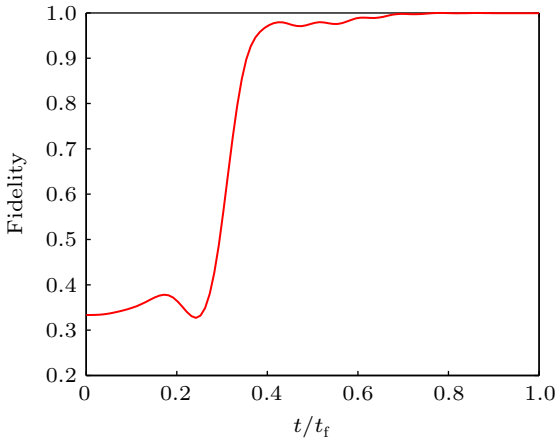



**Fig. 4.** Fidelity of the state during the time evolution with the application of pulses $\Omega^{0}_{\rm p,s}(t)$.

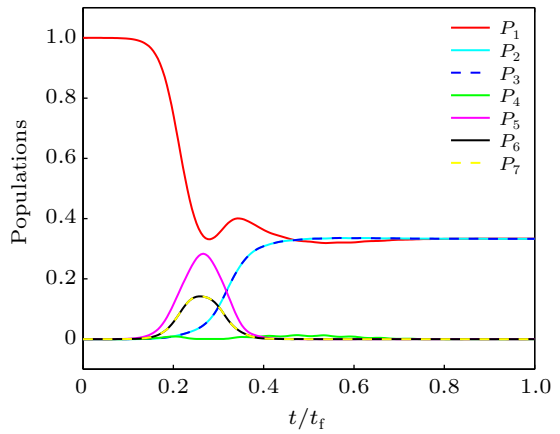



**Fig. 5.** The populations $P_1$–$P_7$ of the states $|\phi_1\rangle$–$|\phi_7\rangle$ during the time evolution with the parameters $\Omega_0 = 6.75t_{\rm f}^{-1}$ and $g = 36t_{\rm f}^{-1}$.

Next, we compare the performance of superadiabatic (STA) and adiabatic cases in Fig. 6. For the adiabatic protocol, the Rabi frequencies of the pulses are selected as the analytic functions of $\Omega_1(t) = -\sqrt{2}\Omega_{\rm p}(t)$ and $\Omega_2(t) = \Omega_3(t) = \sqrt{2}\Omega_{\rm s}(t)$, where $\Omega_{\rm p,s}(t)$ is described by Eq. (16). The results show that when the parameters $\Omega_0 = 6.75t_{\rm f}^{-1}$ and $g = 36t_{\rm f}^{-1}$ are the same as that used in the superadiabatic-based shortcut protocol, the fidelity of the final state is 0.4506 for the adiabatic path (the green curve), while it increases to 1 for the superadiabatic shortcut (the red curve). Otherwise, the parameters for $\Omega_0$ and $g$ should be increased so that the fidelity of the final state for the adiabatic path can approach to 1, for example, $\Omega_0 = 50t_{\rm f}^{-1}$ and $g = 250t_{\rm f}^{-1}$. The smaller the product of the pulse amplitude $\Omega_0$ and the total operation time $t_{\rm f}$ is, the faster the evolution speed can be obtained. The pulse amplitude $\Omega_0 = 6.75t_{\rm f}^{-1}$ of this protocol is much smaller than $\Omega_0 = 50t_{\rm f}^{-1}$ in the adiabatic protocol. In this protocol, we choose parameters $\Omega_0/2\pi = 135$ MHz and $g/2\pi = 720$ MHz,[53] then the system evolution time is 50 ns. In addition, in Ref. [33], the time to obtain the W state is about 0.6 μs. The proposed superadiabatic-based scheme takes $1/12$ of the evolution time

in Ref. [33]. Therefore, the results also indicate that the adiabatic scheme for the generation of high-fidelity three-qubit W state requires a long time to satisfy the adiabatic criteria, while, the proposed superadiabatic one is speeded up with the merits of the adiabatic evolution.

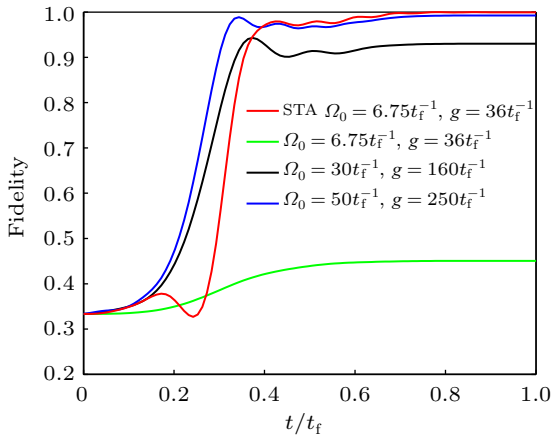



**Fig. 6.** Fidelities versus time for the superadiabatic-based shortcut (with $\Omega_0 = 6.75t_f^{-1}$ and $g = 36t_f^{-1}$ denoted by the red curve) and adiabatic protocols with $\Omega_0 = 6.75t_f^{-1}$ and $g = 36t_f^{-1}$ (green), $\Omega_0 = 30t_f^{-1}$ and $g = 160t_f^{-1}$ (black), and $\Omega_0 = 50t_f^{-1}$ and $g = 250t_f^{-1}$ (blue).

In the following, we take the resonator decay and qubit relaxation into account and analyze their influence on the generation of the W state. The decoherence process can be described by the master equation, which can be written as

$$\dot{\rho} = -\mathrm{i}\,[H_{\text{total}}, \rho] + \sum_k \left[ b_k \rho b_k^\dagger - \frac{1}{2}(b_k^\dagger \rho b_k + \rho b_k^\dagger b_k) \right], \quad (20)$$

where $b_k$ is the dissipative term, and expressed as

$$\begin{aligned}
&b_1 = \sqrt{\gamma_1}|0\rangle_1\langle e|, \ \ b_2 = \sqrt{\gamma_2}|1\rangle_1\langle e|, \ \ b_3 = \sqrt{\gamma_3}|0\rangle_2\langle e|,\\
&b_4 = \sqrt{\gamma_4}|1\rangle_2\langle e|, \ \ b_5 = \sqrt{\gamma_5}|0\rangle_3\langle e|, \ \ b_6 = \sqrt{\gamma_6}|1\rangle_3\langle e|,\\
&b_7 = \sqrt{\kappa}a, \ \ b_8 = \sqrt{\gamma_{\phi_{11}}}(|e\rangle_1\langle e| - |1\rangle_1\langle 1|),\\
&b_9 = \sqrt{\gamma_{\phi_{01}}}(|e\rangle_1\langle e| - |0\rangle_1\langle 0|),\\
&b_{10} = \sqrt{\gamma_{\phi_{12}}}(|e\rangle_2\langle e| - |1\rangle_2\langle 1|),\\
&b_{11} = \sqrt{\gamma_{\phi_{02}}}(|e\rangle_2\langle e| - |0\rangle_2\langle 0|),\\
&b_{12} = \sqrt{\gamma_{\phi_{13}}}(|e\rangle_3\langle e| - |1\rangle_3\langle 1|),\\
&b_{13} = \sqrt{\gamma_{\phi_{03}}}(|e\rangle_3\langle e| - |0\rangle_3\langle 0|). \qquad (21)
\end{aligned}$$

Here, $\gamma_n$ $(n = 1, 2, \ldots, 6)$ is the spontaneous emission rate of the qubit-$l$ from the high-level state $|1\rangle_l$ ($|1\rangle_l$) to the lower state $|e\rangle_l$ ($|0\rangle_l$). The $\kappa$ is the decay rate of the resonator and $\gamma_{\phi_{il}}(i = 1, e; l = 1, 2, 3)$ is the dephasing rate of the qubit $l$ with the levels $|1\rangle$ and $|e\rangle$. For simplicity, we choose $\gamma_n = \gamma$ and $\gamma_{\phi_{il}} = \gamma_\phi$.

In order to quantify the final fidelity caused by the resonator decay $\kappa$ and the spontaneous rate of qubit $\gamma_n$, where the dephasing rate is assumed too small to ignore, we plot the final fidelity $F(t_f)$ versus the resonator decay $\kappa$ and the spontaneous rate of qubit $\gamma_n$ in Fig. 7(a), where $\kappa$ varies from 0 to $0.01/t_f^{-1}$, $\gamma$ is set from 0 to $0.01/t_f^{-1}$. Simultaneously, we plot the final fidelity $F(t_f)$ versus $\gamma_\phi$ and $\gamma$ in Fig. 7(b), and the final fidelity $F(t_f)$ versus $\gamma_\phi$ and $\kappa$ in Fig. 7(c). Figures 7(a) and 7(c) show that the final fidelity remains over 0.988 under different $\kappa$. It is also hinted that the proposed scheme is robust to the resonator decay. As shown in Fig. 7(b), the effect from the qubit decoherence on the final fidelity is also little. The final fidelity is over 0.98, even though $\gamma = 0.01/t_f^{-1}$ and $\gamma_\phi = 0.001/t_f^{-1}$, which is greater than the results in Ref. [45] by using the LR invariant (the final fidelity was 0.94 when the decoherence factors $\gamma = 0.01/t_f^{-1}$, $\gamma_\phi = 0.001/t_f^{-1}$, the final fidelity was 0.982 when $\kappa = 0.01/t_f^{-1}$, $\gamma_\phi = 0.001/t_f^{-1}$). Furthermore, there is not any additional parameter in the proposed scheme in comparison with the method using the LR invariant. Hence, the proposed scheme for the fast generation of three-qubit W state is robust to the resonator decay and qubit decoherence without any additional parameters.

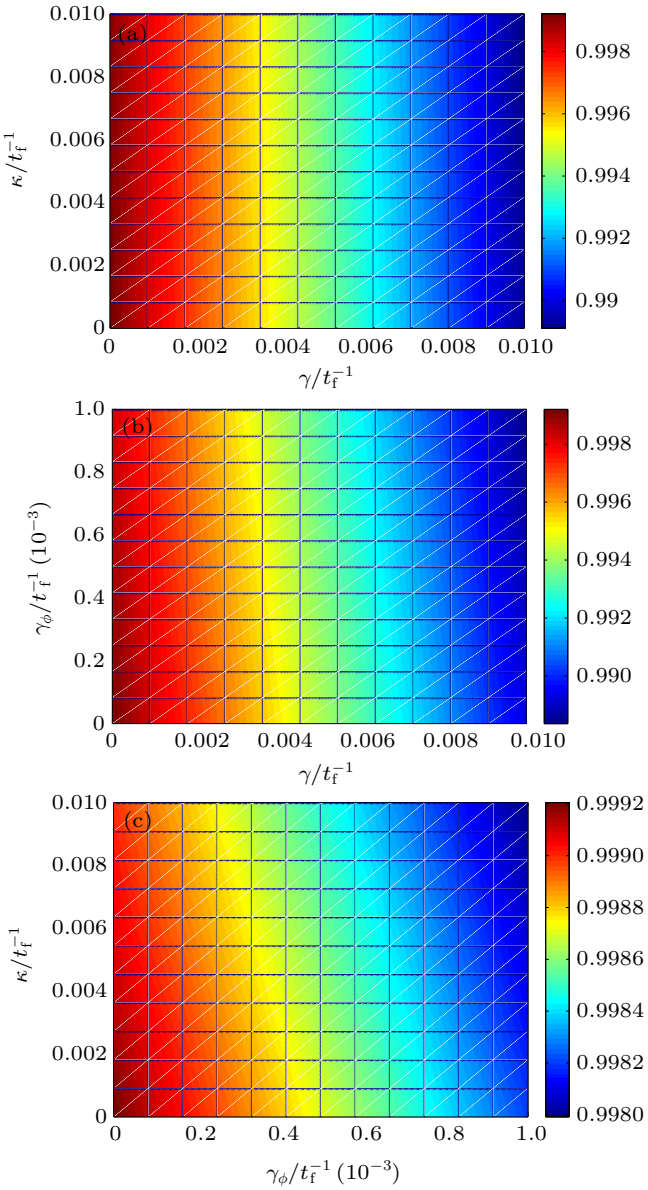



**Fig. 7.** Final fidelity versus the different decoherence factors $\kappa$, $\gamma$, and $\gamma_\phi$. (a) $F(t_f)$ versus different values of $\gamma$ and $\kappa$; (b) is the case of $\gamma$ and $\gamma_\phi$, and (c) shows the effect of decoherence without $\gamma$.

## 5. Conclusion

We have proposed a scheme to prepare the high-fidelity three-qubit W state via superadiabatic-based shortcut. In order to suppress the transitions between different instantaneous eigenstates, the counterdiabatic Hamiltonian with the same form as the effective Hamiltonian has been obtained for the easily implementation in experiment. Additionally, the intricately driven pulses have been replaced by the Gaussian pulses which is feasible experimentally. The numerical results show that the time evolution for the proposed scheme has been significantly speeded up, and the proposed scheme is robust against the decoherence caused by the resonator decay and the spontaneous emission and dephasing of qubit. Compared with the adiabatic case, the proposed protocol could prepare the entanglement state within a very short time. Meantime, it is found that our result can be extended to the case of multiple-qubit W state, especially the odd-number qubit W state.

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