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SPECIAL TOPIC — Quantum computation and quantum simulation

A proposal for preparation of cluster states with linear optics*

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Measurement-based quantum computation in an optical setup shows great promise towards the implementation of large-scale quantum computation. The difficulty of measurement-based quantum computation lies in the preparation of cluster state. In this paper, we propose the method of generating the large-scale cluster state, which is a platform for measurement-based quantum computation. In order to achieve more complex quantum circuits, the preparation protocol of N-photon cluster state will be proposed as a generalization of the preparation of four- and five-photon cluster states. Furthermore, our proposal is experimentally feasible.

Keywords: measurement-based quantum computing, cluster states, linear optical elements

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1. Introduction

Quantum computing is to employ the principles of quantum mechanics to solve the computational issue and improve the efficiency of computing. Quantum computing uses the fundamental properties of quantum mechanics to store and operate information, which is aimed to handle some difficult or even impossible tasks for classical computers. Since quantum computing was proposed, scientists have been exploring the fundamentals and applications of quantum computing. Quantum computing has made significant progress in theoretical research[1–4] and physical implementation,[5–10] greatly improving the computational efficiency on some specific issues. Additionally, different computational models have been proposed, such as quantum circuit model,[11–15] measurement-based quantum computing,[16,17] topological quantum computing,[18,19] adiabatic quantum computing,[20] and dual quantum computing,[21,22] which provide a variety of ideas and methods for constructing quantum computers.[23]

An important advance for appreciating the role of measurement and comprehension is introducing a measurement-based quantum computing model. This quantum computing model requires a series of projection measurements among the subsystems of a unique multi-partite entangled state (cluster state) and classical feedforward of the measurement results. Due to measurement leading to the collapse of a quantum state, the measurement of cluster state is irreversible in time; therefore, it is called measurement-based quantum computing. Another crucial feature of measurement-based quantum computation is universality.[16,24] Put it in another way, any quantum circuit can be realized on a suitable cluster state.

The cluster state is a unique multi-partite entangled state. It was frequently used as a quantum channel to accomplish quantum information processing tasks. It was also used in quantum computing,[17,25,26] quantum error correction,[27] quantum teleportation and dense coding.[28,29] The cluster state was first raised by Briegel and Raussendorf[30] in 2001. Subsequently, they proposed that the single-qubit measurement for two-dimensional (or higher dimensional) cluster state can be used for universal quantum computing.[16] The cluster state possesses the properties of GHZ state and W state, and also maximum connectivity and persistence of entanglement.[30] When particles are more than 3, the cluster state reflects the unique entanglement characteristics. It can be expressed as follows:

$$|C_N\rangle = \frac{1}{2^{N/2}} \bigotimes_{a=1}^{N} (|0\rangle_a \sigma_z^{(a+1)} + |1\rangle_a), \tag{1}$$

where N is the number of particles, $\sigma_z^{(a+1)} = |0\rangle_a\langle 0| - |1\rangle_a\langle 1|$, $(a = 1, 2, \ldots, N)$, $\sigma_z^{(N+1)} = 1$.

For $N = 2$, $|C_2\rangle = \frac{1}{\sqrt{2}}(|0+\rangle + |1-\rangle)$, cluster state can be converted to Bell state by performing local unitary operations. When $N = 3$, $|C_3\rangle = \frac{1}{\sqrt{2}}(|+0+\rangle + |-1-\rangle)$, cluster state is locally equivalent to GHZ state. When $N = 4$, cluster state can be expressed as follows:

$$|C_4\rangle = \frac{1}{2}(|0000\rangle + |0011\rangle + |1100\rangle - |1111\rangle). \tag{2}$$

It can be seen that the four-qubit cluster state is nonequivalent to the four-qubit GHZ state, and it will not become a product state by a single-qubit measurement on a particle. If we measure any one or two particles, the remaining particles will

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remain entangled.

A one-dimensional cluster state can be described by Fig. 1. Each node represents a qubit, and the connections between nodes represent the interaction. A cluster state can be established as follows: all qubits are prepared in the state $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. For generating entanglement, the CPhase gates (control-Z operation) are performed between adjacent connected nodes. We will show how to generate a cluster state later.

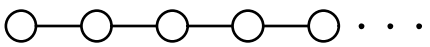


Fig. 1. Diagram of the one-dimensional cluster state.

Many schemes have been proposed to achieve the preparation of multipartite entanglement, and most of them are based on linear optical systems. In 2007, Peng *et al.* experimentally realized the preparation of continuous-variable GHZ states and cluster states in electromagnetic mode.[31] In 2008, Pfister *et al.* prepared continuous-variable cluster states using optical parametric amplifiers under laser pumping conditions.[32] In 2010, Menicucci skillfully used a single quantum nondemolition gate to generate cluster states.[33] In 2012, Su *et al.* proposed a method to prepare the eight-partite cluster state for photonic qumodes through rotation and local Fourier transform.[34] In 2014, Raphael *et al.*[35] proposed a scheme to prepare a continuous variable cluster state by optical spatial mode comb. In 2015, Oussama *et al.* proposed a scheme for generating cluster state in an optomechanical system.[36] Most of the above schemes are based on linear optical systems, the parameters of the key element of these optical multipartite entanglement generation schemes, i.e., PBS, cannot be modified after being fabricated, which may limit the coupling mechanisms for photons. Therefore, it is fundamentally interesting to design regulable coupling mechanisms for photons. A regulable optical circuit is designed for preparing the multi-photon entangled state, which will greatly enrich the coupling mechanism for photons.

The important element in this scheme is the beam displacer (BD). Passing through a BD, the photons with orthogonal polarization states will walk to different positions. Because of this property, the role of the beam splitter from the standard coupling scheme is replaced by the trajectory-exchanging process plus post-selection. A benefit of using beam displacers, as shown in figures, is phase stability between all the different paths. Using a BS or PBS approach would lack this stability. Although this kind of trajectory-exchanging based photonic coupling mechanism was first proposed and utilized in entanglement swapping protocol,[37] it can be used to generate entangled states too.[37] In the previous work, we have proposed the preparation of GHZ state and W state in Ref. [31] in a similar way. Therefore, this method is valuable.

Given a cluster state, two types of basic single-qubit measurements are sufficient to operate a measurement-based quantum computer. In order to describe quantum circuits, encoded qubits are used to distinguish from physical qubits (polarization), which make up the cluster state. Our measurements are carried out on the physical qubits, while the logic gates are implemented on the encoded qubits. The measurements to perform the actual quantum information processing are acting on the basis $B_i(\alpha) = |+\alpha\rangle_i, |-\alpha\rangle_i$, where $|\pm\alpha\rangle_i = (|0\rangle_i \pm \mathrm{e}^{\mathrm{i}\alpha}|1\rangle_i)/\sqrt{2}$ (α is a real number and i is the physical qubit). The selection of measurement basis determines the one-qubit rotations $R(z) = \exp(-\mathrm{i}\alpha\sigma_z/2)$ and $R(x) = \exp(-\mathrm{i}\alpha\sigma_x/2)$ in the cluster state (σ_x, σ_y, σ_z are the Pauli matrix). The universal quantum logic operations can be realized in a sufficiently large cluster state by selecting the correct basis $B_i(\alpha)$. If the measurement result on the physical qubit is $|+\alpha\rangle$, we define it as 0. It is defined as 1 if the result is $|-\alpha\rangle$. For the realization of one-qubit and two-qubit logic gates, we choose the cases in which the result is 0.

Even a small cluster state is sufficient to demonstrate all the basic characteristics of measurement-based quantum computation. Each of the four-qubit cluster state shown in Fig. 2 can implement the quantum circuit below. The quantum circuits consist of a series of one-qubit and two-qubit logic gates. The entire process is realized by sequential single-qubit measurements on cluster state, and the result is stored on the right end qubits. Therefore, these small circuits, which constitute a set of universal logic gates, can be used as the sub-element of full-featured quantum computers.

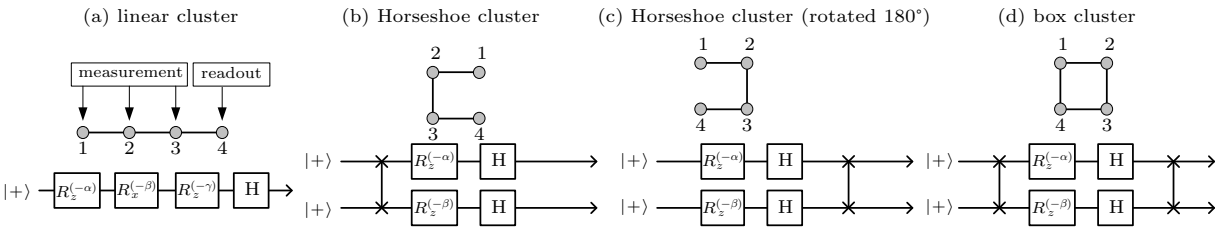


Fig. 2. Cluster states and the realization of quantum circuits. It is given that quantum states of four-qubit cluster state and the computation in the model of measurement-based computer. For the linear cluster state $|\psi_{l4}\rangle$ (a), the sequential measurements of qubits 1, 2, and 3 will be equivalent to a series of one-qubit rotating gates. The encoded input state is performed in an angle controlled single-qubit rotation, and the output is stored on physical qubit 4. In contrast, horseshoe cluster states $|\psi_{\subset 4}\rangle$ (b) and $|\psi_{\supset 4}\rangle$ (c) and box cluster state $|\psi_{\square 4}\rangle$ (d) form more complex circuits including one-qubit and two-qubit gates, both of which are necessary to form a set of universal logic gates for quantum computing. The measurements of two physical qubits will be implemented in the circuit defined by the specific cluster and transmit the logic output to the other two physical qubits.

2. Generation of cluster states

The difficulty of measurement-based quantum computation lies in the preparation of cluster state. For demonstrating the quantum circuits shown in Figs. 2(a)–2(d), it is sufficient to prepare a four-qubit linear cluster state. The implemented circuit is determined by the sequence of subsequent measurements. In order to achieve more complex quantum circuits, we have designed the extensible scheme to prepare an N-photon cluster state. We will display this method in two parts, corresponding to even-photon and odd-photon cluster states, respectively. For an even-photon cluster state, we take the four-photon cluster state as an example, which can be expressed as

$$\begin{aligned}\left|\psi_{\mathrm{C}}^{(4)}\right\rangle &= \frac{1}{2}(|H\rangle_1|H\rangle_2|H\rangle_3|H\rangle_4+|H\rangle_1|H\rangle_2|V\rangle_3|V\rangle_4\\ &\quad +|V\rangle_1|V\rangle_2|H\rangle_3|H\rangle_4-|V\rangle_1|V\rangle_2|V\rangle_3|V\rangle_4), \end{aligned}\tag{3}$$

where $|H\rangle$ and $|V\rangle$ denote horizontally and vertically polarized photon states, respectively, and subscripts denote physical qubits. $|\psi_{\mathrm{cluster}}^{(4)}\rangle$ is equivalent to the four-qubit linear cluster state $|\psi_{l4}\rangle$ and horseshoe cluster states $|\psi_{\subset 4}\rangle$ and $|\psi_{\supset 4}\rangle$ under the local unitary operation $H_1\bigotimes I_2\bigotimes I_3\bigotimes H_4$, where H_i (I_i) is the Hadamard (identity) operation on qubit i. The $|\psi_{\mathrm{cluster}}^{(4)}\rangle$ can be transformed into box cluster state $|\psi_{\square 4}\rangle$ by local unitary operation $H_1\bigotimes H_2\bigotimes H_3\bigotimes H_4$ and exchanging (or relabeling) qubits 2 and 3.

To entangle multiple independent polarized photons, we design an optical scheme with n paths, where $n=1,2,3,\ldots,N$. The key operation in our scheme is the trajectory-exchanging operation, which exchanges the path and position information without changing the polarization state of two photons. The quantum state will evolve to the proper state by the correct trajectory-exchanging operation. The functions can be described as follows:

$$a_{\varphi_1,x_1}^{\dagger}|0\rangle\rightarrow a_{\varphi_1,y_2}^{\dagger}|0\rangle\ ,\quad a_{\varphi_2,y_2}^{\dagger}|0\rangle\rightarrow a_{\varphi_2,x_1}^{\dagger}|0\rangle\ ,\tag{4}$$

where $|0\rangle$ is a vacuum state, φ in $a_{\varphi,x_n}^{\dagger}$ labels the polarization state, x and n label the current position and the path, respectively.

For the generation of four-photon cluster state, the positions 0 of four paths 1, 2, 3 and 4 are set as input-ports, in which four independent photons in the horizontal polarization state are inputted. The four-photon system can be expressed as follows:

$$\left|\psi_{\mathrm{in}}^{(4)}\right\rangle=a_{H,0_1}^{\dagger}a_{H,0_2}^{\dagger}a_{H,0_3}^{\dagger}a_{H,0_4}^{\dagger}|0\rangle.\tag{5}$$

As shown in Fig. 3, Hi ($i=1,2$) is a half-wave plate, which can achieve different polarization rotating operations by setting at different angles θ. The function can be expressed in matrix

$$\mathrm{HWP}_{\theta}=\begin{pmatrix}\cos 2\theta & \sin 2\theta\\ \sin 2\theta & -\cos 2\theta\end{pmatrix},\tag{6}$$

where $\theta\in(0,\pi/2)$, which represents the angle between the optical axis of the half-wave plate and the horizontal polarization. Corresponding to the different HWPs, the polarization rotation operations U are expressed as

$$U_1=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix},\quad U_2=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}.\tag{7}$$

The function of a BD, which displaces the horizontal polarization component (H) away from the vertical polarization component (V) can be expressed as

$$S=\sum_x|H,x+1\rangle\langle H,x|+|V,x-1\rangle\langle V,x|.\tag{8}$$

Let us move to a detailed description of this scheme. As depicted in Fig. 3(a), the input photons at positions 0 of paths 1, 2 and 3 pass through H1, and the photon of path 4 pass through no element for the time being. After HWPs, three independent photons enter the corresponding first beam displacer BD1. The output modes of BD1 in positions $+1$ of paths 1 2 and positions -1 of paths 2 3 are exchanged via mirrors, and enter the $+1$ and -1 positions of corresponding BD2, after passing through H2. Other output positions (position -1 of paths 1 and position $+1$ of paths 3) successively pass through phase retarder and H2, and then enter the corresponding positions of BD2. The photon in output position 0 of BD2 in path 3 and the photon of path 4 pass through H1 and BD3. The output modes of BD3 in position $+1$ of paths 3 and 4 are exchanged via four mirrors, and enter the position $+1$ of corresponding BD4, after passing through H2. H2 is added at the position 0 of paths 3 and 4. After all the operations shown in Fig. 3(a), we take positions 0 of paths 1, 2, 3 and 4 as output-ports. At the output-ports, the four photons are in the following state:

$$\begin{aligned}\left|\psi_{\mathrm{out}}^{(4)}\right\rangle &= \frac{1}{4}\left(a_{V,0_2}^{\dagger}+a_{H,0_1}^{\dagger}\right)\left[a_{V,0_1}^{\dagger}+\frac{1}{\sqrt{2}}\left(a_{H,0_4}^{\dagger}+a_{V,0_3}^{\dagger}\right)\right]\\ &\quad\times\left[\frac{1}{\sqrt{2}}\left(a_{H,0_4}^{\dagger}-a_{V,0_3}^{\dagger}\right)+a_{H,0_2}^{\dagger}\right](a_{H,0_3}^{\dagger}+a_{V,0_4}^{\dagger})|0\rangle\\ &= \frac{1}{4\sqrt{2}}\left(a_{H,0_1}^{\dagger}a_{H,0_2}^{\dagger}a_{H,0_3}^{\dagger}a_{H,0_4}^{\dagger}+a_{H,0_1}^{\dagger}a_{H,0_2}^{\dagger}a_{V,0_3}^{\dagger}a_{V,0_4}^{\dagger}\right.\\ &\quad\left.+a_{V,0_1}^{\dagger}a_{V,0_2}^{\dagger}a_{H,0_3}^{\dagger}a_{H,0_4}^{\dagger}-a_{V,0_1}^{\dagger}a_{V,0_2}^{\dagger}a_{V,0_3}^{\dagger}a_{V,0_4}^{\dagger}\right)|0\rangle\\ &\quad+\left|\phi_{\mathrm{other}}^{(4)}\right\rangle\\ &= \frac{1}{2\sqrt{2}}|\mathrm{Cluster}_4\rangle+\left|\phi_{\mathrm{other}}^{(4)}\right\rangle. \end{aligned}\tag{9}$$

All the terms of the output state are divided into two parts: the contribution part and the non-contribution part. In the contribution part, the four photons occupy the four output-ports, respectively. That is to say, there is only one photon in each output-port, and the four photons are in the exact four-photon cluster state if we can post-select it from the output state. Other terms (abbreviated as $|\phi_{\mathrm{other}}^{(4)}\rangle$) of the output state are categorized as the non-contribution part. By analyzing the output state, it is straightforward to see that the four-photon cluster state can be generated by post-selecting the event where there is only one photon in each of the four output-ports with probability $P_4=1/8$.

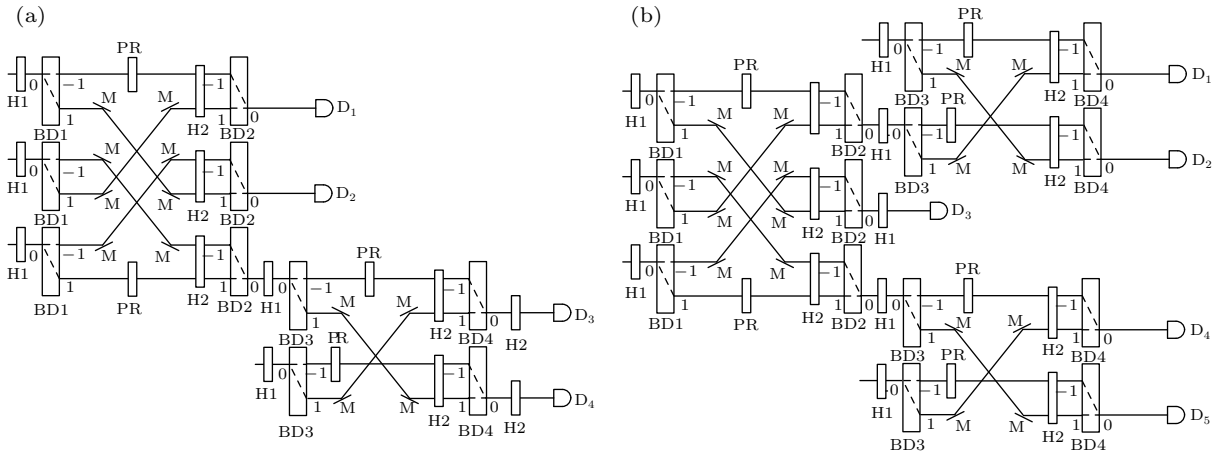


Fig. 3. The optical circuit for preparing four- (a) and five-photon (b) polarization-entangled cluster states. Hi ($i = 1, 2$) and BD respectively indicate half-wave plate and beam displacer, which I will explain later. M is the mirror, and PR denotes phase retarder, which is used for compensating the optical path difference.

The generation of the odd-photon cluster state is slightly different from that of the even-photon cluster state. We will show this method by the generation of the five-photon cluster state. To generate the five-photon cluster state via a similar mechanism, we need to modify the optical circuit in Fig. 3(a) appropriately. The setup for generating the five-photon state is depicted in Fig. 3(b). The five-photon cluster state can be expressed as

$$\begin{aligned}\left|\psi_{\mathrm{C}}^{(5)}\right\rangle = \frac{1}{2}\left[|H\rangle_1(|H\rangle_2|H\rangle_3|H\rangle_4|H\rangle_5+|V\rangle_2|V\rangle_3|V\rangle_4|V\rangle_5)\right.\\ \left.+|V\rangle_1(|H\rangle_2|H\rangle_3|V\rangle_4|V\rangle_5+|V\rangle_2|V\rangle_3|H\rangle_4|H\rangle_5)\right],\end{aligned} \tag{10}$$

The positions 0 of five paths 1, 2, 3, 4, and 5 are set as input-ports, in which five independent photons in horizontal polarization state are inputted, respectively. The five-photon system can be expressed as follows:

$$\left|\psi_{\mathrm{in}}^{(5)}\right\rangle = a^{\dagger}_{H,0_1}a^{\dagger}_{H,0_2}a^{\dagger}_{H,0_3}a^{\dagger}_{H,0_4}a^{\dagger}_{H,0_5}|0\rangle. \tag{11}$$

Similarly, after all the operations shown in Fig. 3(b) we take positions 0 of paths 1, 2, 3, 4, and 5 as output-ports. The input state becomes

$$\begin{aligned}&\left|\psi_{\mathrm{out}}^{(5)}\right\rangle\\ &= \frac{1}{16}(a^{\dagger}_{V,0_1}+a^{\dagger}_{H,0_2})(a^{\dagger}_{H,0_3}-a^{\dagger}_{V,0_3}+a^{\dagger}_{V,0_2}+a^{\dagger}_{H,0_1})\\ &\quad\bigotimes(a^{\dagger}_{V,0_2}-a^{\dagger}_{H,0_1}+a^{\dagger}_{V,0_4}+a^{\dagger}_{H,0_5})\\ &\quad\bigotimes(a^{\dagger}_{V,0_4}-a^{\dagger}_{H,0_5}+a^{\dagger}_{H,0_3}+a^{\dagger}_{V,0_3})(a^{\dagger}_{V,0_5}+a^{\dagger}_{H,0_4})|0\rangle\\ &= \frac{1}{8}\left[a^{\dagger}_{H,0_3}(a^{\dagger}_{H,0_1}a^{\dagger}_{H,0_2}a^{\dagger}_{H,0_4}a^{\dagger}_{H,0_5}+a^{\dagger}_{V,0_1}a^{\dagger}_{V,0_2}a^{\dagger}_{V,0_4}a^{\dagger}_{V,0_5})\right.\\ &\quad\left.+a^{\dagger}_{V,0_3}(a^{\dagger}_{H,0_1}a^{\dagger}_{H,0_2}a^{\dagger}_{V,0_4}a^{\dagger}_{V,0_5}+a^{\dagger}_{V,0_1}a^{\dagger}_{V,0_2}a^{\dagger}_{H,0_4}a^{\dagger}_{H,0_5})\right]|0\rangle\\ &\quad+\left|\phi_{\mathrm{other}}^{(5)}\right\rangle\\ &= \frac{1}{4}|\mathrm{Cluster}_5\rangle+\left|\phi_{\mathrm{other}}^{(5)}\right\rangle.\end{aligned} \tag{12}$$

As the N-photon expansion for the preparation of cluster state (see Figs. 4(a) and 4(b)), we need respectively add one path above and below the optical circuit and input one horizontally polarized photon in position 0 of each new path, which is based on the existing scheme. The new photons pass through the optical elements as shown in Fig. 4 and exchange the trajectory of position $+1$ (or -1) with the neighbors (adjacent paths). N-photon cluster state can be obtained by the post-selection operation with the probability $P_N = \frac{1}{2^{N-1}}$.

In the previous work, we have realized the preparation of GHZ state and W state in Ref. [38] in a similar way. The key to generation is to design proper trajectory-exchanging operations. Photons will evolve to the target entangled state by appropriate trajectory-exchanging operations. The entangled state can be easily obtained after performing the post-selection operation. The indiscernibility of identical particles is an important theoretical basis for this scheme. Entanglement occurs when identical photons overlap in space. After some predetermined operations, the quantum state will evolve into a final state containing useful entanglement, which can be separated by the post-selection operation.

In addition, one may argue that BDs are a specific type of PBS and that normal PBS also performs a trajectory-exchanging operation, so how can the trajectory-exchanging operation used here lead to a concrete advantage over the traditional PBS based schemes. In our scheme, four BDs with trajectory exchanging can mimic the action of a traditional PBS, but a benefit of using beam displacers, as shown in figures, is phase stability between all the different paths. Using a BS or PBS approach would lack this stability. And there is a concrete difference between a PBS and a BD, which makes BDs more tunable and may induce more rich actions on the input photons. That is to say, the function of the BDs-plus-trajectory-exchanging case can be tuned if we can insert some wave plates in between the BDs. But the function of a PBS is definite and cannot be tuned once it is manufactured.

Up to now, we were studying the ideal case where after quantum walks, the spatial wave-functions of the photons overlap perfectly on some positions of the beam displacers.

What will happen if the spatial wave-functions of the photons do not overlap after trajectory-exchanging (in general, this is what may happen in the lab)? If the spatial wave-functions of the photons do not overlap on some positions of the beam displacers, the photons are distinguishable in time, which will cause decoherence in the polarization degree of freedom. Therefore, the temporal synchronicity problem will affect the schemes considerably, and the experimental setup must be designed in such a way that spatial wave-functions of different photons do overlap when they reach the same position of a beam displacer.

In 2016, Pan *et al.*[39] used a laser to excite an artificial atom in a semiconductor crystal. By using finely tuned laser pulses, they produced individual photons with near-perfect uniformity. The device emitted 3.7 million high-quality photons per second – a rate that makes it good enough for practical applications. This advance remains to allow our schemes to be implemented practically.

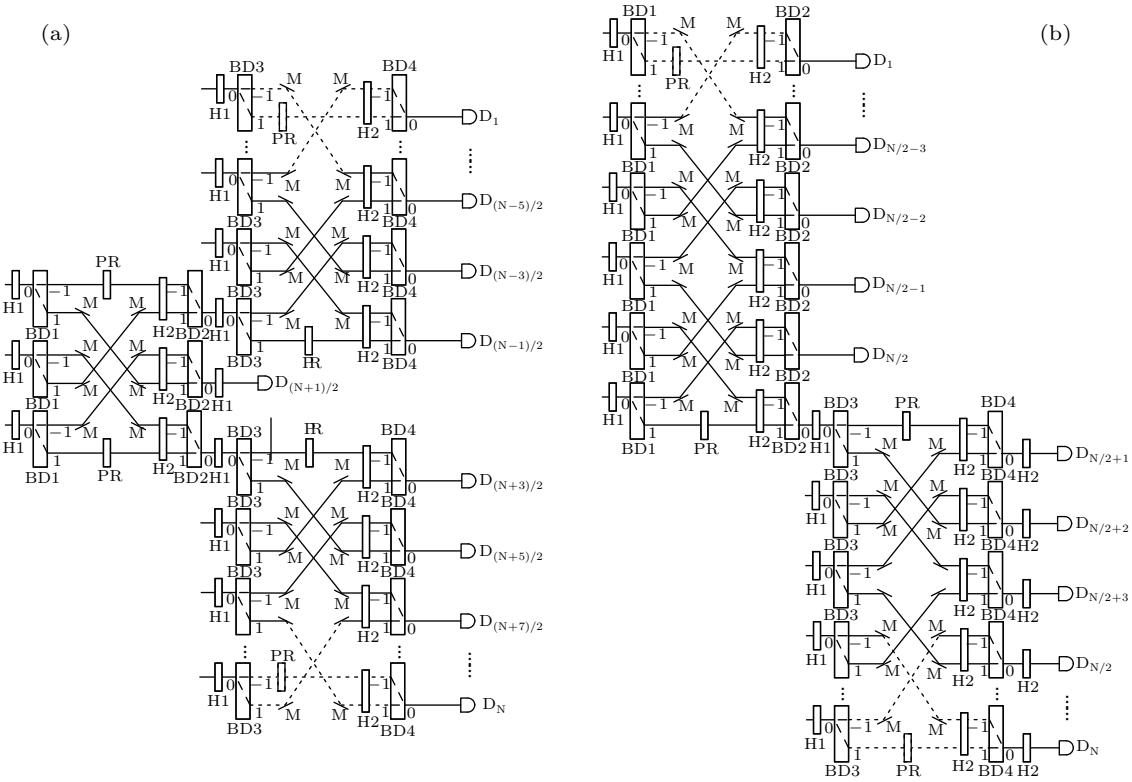


Fig. 4. The optical circuit for preparing odd- (a) and even-photon (b) polarization-entangled cluster states.

Finally, we would like to point out that, with the current technology, it is impossible to perform "non-demolition" measurements of photons, i.e., it is impossible to detect a photon's presence without immediately annihilating it. Therefore, in order to make use of the extracted cluster states, one has to, upon collapsing the three photons onto the right output positions, perform a suitable (say, Bell-like, in case of testing local realism) measurement. We also consider the event-ready schemes. This part needs further research, which will be the focus of our next study.

3. Discussion

In conclusion, optical schemes for generating photonic polarization-entangled cluster states are proposed based on the indistinguishability of identical photons via quantum walk in this paper. The most distinctive aspect of the schemes is that the coupling between photons is realized by trajectory-exchanging operation plus post-selection, the simplicity of which makes its realization promising with current technology. Our schemes demonstrate that trajectory-exchanging operation plus post-selection can single out entangled states from product states, which induces a non-dynamical coupling between two photons. The intrinsic physics underlying this new coupling mechanism is the quantum statistics between different photons entering the device simultaneously.

In addition, the post-selection operation and the trajectory-exchanging operation used here are not the only ones that can be realized. There are many different choices of them, which can induce more new multi-photon non-dynamical coupling mechanisms for quantum computation and quantum communication. Moreover, our new coupling mechanism for photons is more phase stable and tunable than the PBS-based one, which may greatly enrich the coupling mechanism for photons.

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