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Dynamical stability of dipolar condensate in a parametrically modulated one-dimensional optical lattice*

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We study the stabilization properties of dipolar Bose–Einstein condensate in a deep one-dimensional optical lattice with an additional external parametrically modulated harmonic trap potential. Through both analytical and numerical methods, we solve a dimensionless nonlocal nonlinear discrete Gross–Pitaevskii equation with both the short-range contact interaction and the long-range dipole–dipole interaction. It is shown that, the stability of dipolar condensate in modulated deep optical lattice can be controled by coupled effects of the contact interaction, the dipolar interaction and the external modulation. The system can be stabilized when the dipolar interaction, the contact interaction, the average strength of potential and the ratio of amplitude to frequency of the modulation satisfy a critical condition. In addition, the breather state, the diffused state and the attractive-interaction-induced-trapped state are predicted. The dipolar interaction and the external modulation of the lattice play important roles in stabilizing the condensate.

Keywords: Bose–Einstein condensate, optical lattice, dipole–dipole interaction, periodic modulation

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1. Introduction

Optical lattices are ideal test platform for condensed matter theories, putting Bose–Einstein condensate (BEC) into optical lattices immediately leads to much richer physics as a nonlinearity is introduced into the problem. Typical effects associated with periodic lattices, such as Bloch oscillations and Landau–Zener tunneling, are caused by atom-atom interactions.[1] The atomic interactions play a crucial role in research of strongly correlated systems realized with ultra-cold atoms, and greatly enrich the physical properties of ultra-cold quantum gases.[2] Particularly, ultra-cold quantum gases have occurred a major development when the long-range dipole–dipole interaction terms are introduced into the system.[3,4] The dipole–dipole interaction has attracted great interests for two reasons. On the one hand, significant experimental progress was made in recent years in the cooling and trapping of polar molecules,[5] for polar molecules, a very effective technique can be associated with ultra-cold atoms by means of Feshbach resonances,[6] the ^{52}Cr condensate was achieved in 2004.[7] In the next place, the properties of the dipole–dipole interaction are radically different from the ones of contact interactions. The dipole–dipole interaction is long-range and anisotropic, and its strength and sign depend on the polarization direction and the angle between the two particle directions,[8] hence, a number of unexpected phenomena have been discovered such as novel quantum phases including supersolid and checkerboard phases,[9,10] unusual equilibrium shapes, roton-maxon character of the excitation spectrum[11] and the dependence of stability on the trap geometry.[12]

Recent studies with ultra-cold atomic quantum gases in the context of optical lattices demonstrate that periodic driving can also be an effective tool for the dynamics of quantum many-body systems and their coherent manipulation.[13,14] Those studies include the manipulate of superfluid-Mott-insulator transition,[15,16] the magnetic magnetism parameters,[17] the kinetics of phase transitions,[18,19] and the dressed matter waves.[20,21] In general, periodic modulation is primarily an external periodic potential and the modulated interatomic interactions[22] or scattering length[23] using the Feshbach resonance. The stabilization of trapless dipolar condensate by temporal modulation of the contact interaction[24] and the s-wave scattering length is realized.[25] It was found that there is an enhancement of the condensate stability due to the inclusion of long-range dipolar interaction in addition to the short-range contact interaction. A global parametric modulation of the trapping potential to control the stability of the interacting quantum gas by tuning the atomic short-range contact interaction strength is given.[26] Thus, the natural question arises, how do dipolar interactions affect the stability of a globally parametrically driven quantum many-body system? Can long-range dipolar interaction stabilize a quantum gas in an external periodic potential which would otherwise be unstable?

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The objective of the present work is to study the stability of the dipolar condensate in a one-dimensional optical lattice with an additional external parametrically modulated harmonic trap potential. A dimensionless nonlocal discrete nonlinear Gross–Pitaevskii (GP) equation and the kinematic equations are obtained using the tight-binding approximation[20,27,28] and the variational method. We analyze the effects of the long-range dipolar interactions on the stability of the system in detail, and the critical conditions for maintaining the stability of the system are obtained analytically. The results show that the dipolar condensate can be stabilized when the dipolar interactions and the contact interaction satisfy an analytical condition. The analytical results are confirmed by direct numerical simulation.

The organization of the paper is as follows. In Section 2, we present the physical model for the dipolar condensate in a modulated deep one-dimensional optical lattice potential. The stability of the system and the critical conditions for the occurrence of stable state, attractive-interaction-induced-trapped state and diffused state are studied both analytically and numerically in Section 3. Finally, Section 4 contains our conclusion.

2. The model and variational analysis

We consider dipolar condensate in a deep one-dimensional optical lattice with an additional external parametrically modulated harmonic trap potential (see Fig. 1). Using the tight-binding approximation, the system can be described by the dimensionless nonlocal discrete nonlinear Gross–Pitaevskii equation[29]

$$\begin{aligned} \mathrm{i}\dot{\phi}_l &= -(\phi_{l+1}+\phi_{l-1})+V(t)(l-l_0)^2\phi_l \\ &\quad +[U_{\rm cd}|\phi_l|^2+U_{\rm dd1}(|\phi_{l+1}|^2+|\phi_{l-1}|^2) \\ &\quad +U_{\rm dd2}(|\phi_{l+2}|^2+|\phi_{l-2}|^2)]\phi_l, \end{aligned} \tag{1}$$

where ϕ_l is the complex amplitude of the condensate in the lth lattice site, $U_{\rm cd}=U+U_{\rm dd0}$ (U represents the on-site contact interaction strength and $U_{\rm dd0}$ represents the on-site dipolar-dipolar interaction) is the total on-site interaction strength, $U_{\rm ddj}$ $(j=1,2)$ ($U_{\rm dd1}$ is the nearest-neighbor dipolar interaction, $U_{\rm dd2}$ is the next-nearest-neighbor interaction) represent the inter-site dipolar interaction. $V(t)=V_0+\delta V\sin(\Omega t)$ represents parametrically modulated harmonic trap potential, the potential has a average strength V_0, which is parametrically modulated with the strength δV and the frequency Ω.[26] Consider a lattice with M sites, l_0 is the lattice center.

Note that Eq. (1) is time-dependent, in the high-frequency region $\Omega \gg \max\{U_{\rm cd},\ U_{\rm ddj}\}$, the wave function can be approximately described by a slowly varying and rapidly varying function. Thus, we can make a substitution[30] as

$$\phi_l(t)=\psi_l(t)\exp\left\{-\mathrm{i}(l-l_0)^2\left[\int \delta V\sin(\Omega t')\mathrm{d}t'-\frac{\delta V}{\Omega}\right]\right\}, \tag{2}$$

where ψ_l represents slowly varying wave function. Substituting Eq. (2) into Eq. (1) and time averaging over a period (just integrating for the exponential part, and regarding ψ_l as a constant), we can obtain the following equation for the slowly varying amplitude:

$$\begin{aligned} \mathrm{i}\dot{\psi}_l &= -\{J_0[(2l'+1)\eta]\psi_{l+1} \\ &\quad +J_0[(2(l'-1)+1)\eta]\psi_{l-1}\}+V_0 l'^2\psi_l \\ &\quad +U_{\rm cd}|\psi_l|^2\psi_l+U_{\rm dd1}(|\psi_{l+1}|^2+|\psi_{l-1}|^2)\psi_l \\ &\quad +U_{\rm dd2}(|\psi_{l+2}|^2+|\psi_{l-2}|^2)\psi_l, \end{aligned} \tag{3}$$

where $J_0[(2l'+1)\eta]$ and $J_0[(2(l'-1)+1)\eta]$ are the zero-order Bessel function of the first kind, $l'=l-l_0$, $\eta=\delta V/\Omega$. The first term on the right-hand side of Eq. (3) is the kinetic term, the second term represents the potential energy, the last three terms represent the on-site interaction term, the nearest-neighbor dipolar interaction term and the next-nearest-neighbor dipolar interaction term, respectively. The effects of the dipolar interaction $U_{\rm ddj}$, the contact interaction U and the average strength of potential V_0 on the dynamic stability of dipolar condensate that trapped in optical lattice with an external parametrically modulated harmonic trap are researched by the variational approach. The Hamiltonian function corresponding to Eq. (3) is

$$\begin{aligned} H &= -\sum_l J_0[(2(l-l_0)+1)\eta][\psi_l^*\psi_{l+1}+\psi_{l+1}^*\psi_l] \\ &\quad +V_0\sum_l(l-l_0)^2|\psi_l|^2+\frac{U_{\rm cd}}{2}\sum_l|\psi_l|^4 \\ &\quad +\frac{U_{\rm dd1}}{2}\sum_l(|\psi_{l+1}|^2+|\psi_{l-1}|^2)|\psi_l|^2 \\ &\quad +\frac{U_{\rm dd2}}{2}\sum_l(|\psi_{l+2}|^2+|\psi_{l-2}|^2)|\psi_l|^2, \end{aligned} \tag{4}$$

and the Lagrangian density is

$$L=\sum_l\frac{\mathrm{i}}{2}(\psi_l^*\partial_t\psi_l-\psi_l\partial_t\psi_l^*)-H. \tag{5}$$

To obtain an analytic approximation for the time evolution of the condensate, we use a Gaussian trial wave function

$$\psi_l(t)=\left(\frac{1}{\pi\alpha^2}\right)^{1/4}\times\exp\left[-\frac{(l-l_0)^2}{2\alpha^2}+\mathrm{i}\beta(l-l_0)^2\right], \tag{6}$$

with the time-dependent width of the wave packet $\alpha(t)$, $\beta(t)$ is its rate of change. The wave function Eq. (6) is substituted in the Lagrangian density and the effective Lagrangian is calculated by integrating the Lagrangian density as

$$\begin{aligned} L_{\rm eff} &= 2\mathrm{e}^{-\gamma}(1+\frac{1}{2}\alpha^2\eta^2)-\frac{1}{2}(V_0+\dot{\beta})\alpha^2 \\ &\quad -\frac{U_{\rm cd}}{2\sqrt{2\pi}\alpha}-\frac{U_{\rm dd1}}{\sqrt{2\pi}\alpha}\mathrm{e}^{-\frac{1}{2\alpha^2}}-\frac{U_{\rm dd2}}{\sqrt{2\pi}\alpha}\mathrm{e}^{-\frac{2}{\alpha^2}}, \end{aligned} \tag{7}$$

where $\gamma=\frac{1}{4\alpha^2}+\alpha^2\beta^2$. Here, because of the high-frequency approximation is used, $J_0[(2(l-l_0)+1)\eta]\approx 1+[(2(l-l_0)+$

$1)\eta]^2/2$, and $\eta \ll 1$. The wave-packet dynamical evolution can be obtained by a variational principle from the Lagrangian $L_{\rm eff}$, with the equations of motion for the variational parameters $x(t)=\alpha(t), \beta(t)$ by the Euler–Lagrange equations $\partial_x L = \frac{d}{dt}\partial_{\dot{x}} L$,

$$\dot{\alpha} = 4\mathrm{e}^{-\gamma}\alpha\beta(1+\frac{3}{2}\alpha^2\eta^2),$$
$$\dot{\beta} = 2\mathrm{e}^{-\gamma}\left[\eta^2+\left(\frac{1}{2\alpha^4}-2\beta^2\right)\left(1+\frac{1}{2}\alpha^2\eta^2\right)\right]-V_0+g, \quad (8)$$

where

$$g = U_{\rm cd}/(2\sqrt{2\pi}\alpha^3)+U_{\rm dd1}/(\sqrt{2\pi}\alpha^3)\mathrm{e}^{-1/2\alpha^2}(1-1/\alpha^2) + U_{\rm dd2}/(\sqrt{2\pi}\alpha^3)\mathrm{e}^{-2/\alpha^2}(1-4/\alpha^2).$$

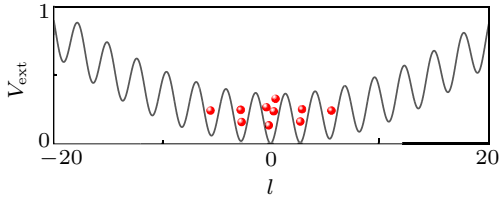


Fig. 1. Schematic diagram of the deep one-dimensional optical lattice with an additional external parametrically modulated harmonic trap potential.

3. Stability analysis of the system

First, we consider the equilibrium state. From Eq. (8), the equilibrium wave packet width α_0 satisfies the following equation when $\dot{\alpha}=0, \dot{\beta}=0$:

$$V_0\alpha_0^4 = \mathrm{e}^{-\frac{1}{4\alpha_0^2}}(1+\frac{\eta^2\alpha_0^2}{2}+2\eta^2\alpha_0^4)+\alpha_0^4 g_0, \quad (9)$$

where

$$g_0 = \frac{U_{\rm cd}}{2\sqrt{2\pi}\alpha_0^3}+\frac{U_{\rm dd1}}{\sqrt{2\pi}\alpha_0^3}\mathrm{e}^{-1/2\alpha_0^2}(1-1/\alpha_0^2) + \frac{U_{\rm dd2}}{\sqrt{2\pi}\alpha_0^3}\mathrm{e}^{-2/\alpha_0^2}(1-4/\alpha_0^2).$$

In Fig. 2(a), variation of the equilibrium condensate width α_0 against the periodic driving force $\delta V/\Omega$ is shown for different V_0 and $U_{\rm dd2}$. Dotted lines represent the next-nearest-neighbor dipolar interaction $U_{\rm dd2}=0$ with different V_0, and solid lines represent $U_{\rm dd2}=-0.5$ with different V_0. We can find that, for $\delta V/\Omega < V_0$, the equilibrium condensate width α_0 is almost constant, for $\delta V/\Omega > V_0$, the $\delta V/\Omega$ term becomes comparable in size to the V_0 term which leads to a rapid growth of α_0. Moreover, the effect of the next-nearest-neighbor dipolar interaction $U_{\rm dd2}$ on α_0 is significant for weak harmonic trap V_0. However, with the increase of V_0, α_0 is less and less affected by $U_{\rm dd2}$, or even almost no influence. In Fig. 2(b), the equilibrium condensate width α_0 as a function of the periodic driving force $\delta V/\Omega$ for different nearest-neighbor dipolar interaction $U_{\rm dd1}$ is shown. The equilibrium condensate width α_0 decreases when the attractive dipolar interaction ($U_{\rm dd1}=-1$) is considered and increases when the repulsive dipolar interaction ($U_{\rm dd1}=1$) is considered. As shown in Fig. 2, because the physical phenomena of the system are mainly determined by the on-site interaction and the nearest-neighbor dipolar interaction, and the qualitative characteristics of the system remain the same if the next-nearest-neighbor dipolar interaction is considered.[31–33] Generally, the next-nearest-neighbor dipolar interaction is weak, and the nearest-neighbor and the next-nearest-neighbor dipolar interaction satisfy $U_{\rm dd2}=U_{\rm dd1}/\chi$, $\chi\sim(4-8)$.[34] Here we set $\chi=8$ in the following results.

Next, we address the dipolar condensate stability in parametrically modulated harmonic trap potential. For a parametrically modulated trap, we can gain an equation of motion for α from Eq. (8)

$$\ddot{\alpha}+2\alpha^2\beta\nu\dot{\alpha}+4\mathrm{e}^{-\gamma}\left[V_0(1+\frac{3}{2}\alpha^2\eta^2)-2\mathrm{e}^{-\gamma}\eta^2\right]\alpha = 4\mathrm{e}^{-\gamma}\left(1+\frac{3}{2}\alpha^2\eta^2\right)\left(\frac{\mathrm{e}^{-\gamma}}{\alpha^3}+\alpha g\right), \quad (10)$$

where $\nu=2\mathrm{e}^{-\gamma}\eta^2-V_0+g$. Equation (10) displays an effective unstable oscillation of the dipolar condensate width when $\nu\neq 0$, which depends on the sign of ν. When $\nu<0$, the second term on the left-hand side of Eq. (10) has a damping effect, and the damped oscillation of α occurs, and attractive-interaction-induced-trapped state will take place. On the contrary, when $\nu>0$, the second term on the left-hand side of Eq. (10) has an excitation effect, and α increases with time, and the diffused state will occur. Note that, in the diffused state, because of the existence of the external harmonic trap V_0, the condensate will finally oscillates in the harmonic trap with a large α. Only when $\nu=0$, the condensate width stably oscillates around the initial equilibrium state α_0. We note that α oscillates around a mean value (approximately the initial wake packet width α_0). Generally, the oscillating amplitude is weak.

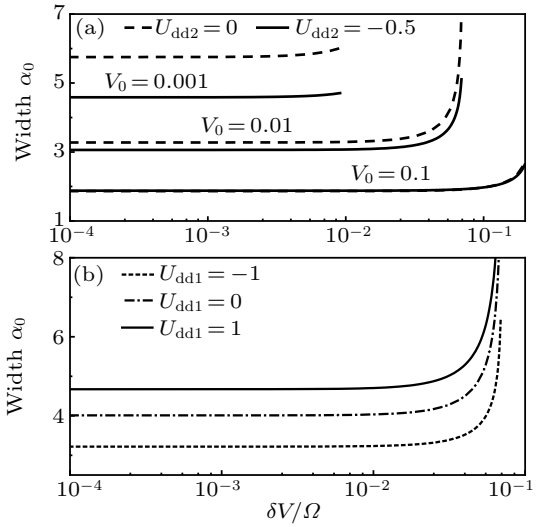


Fig. 2. The stationary condensate width α_0 as a function of the periodic driving force $\delta V/\Omega$ for different (a) next-nearest-neighbor dipolar interaction $U_{\rm dd2}$ and average strength of potential V_0 ($U_{\rm cd}=2$, $U_{\rm dd1}=-1$), and (b) nearest-neighbor dipolar interaction $U_{\rm dd1}$ ($U_{\rm cd}=2$ and $V_0=0.01$).

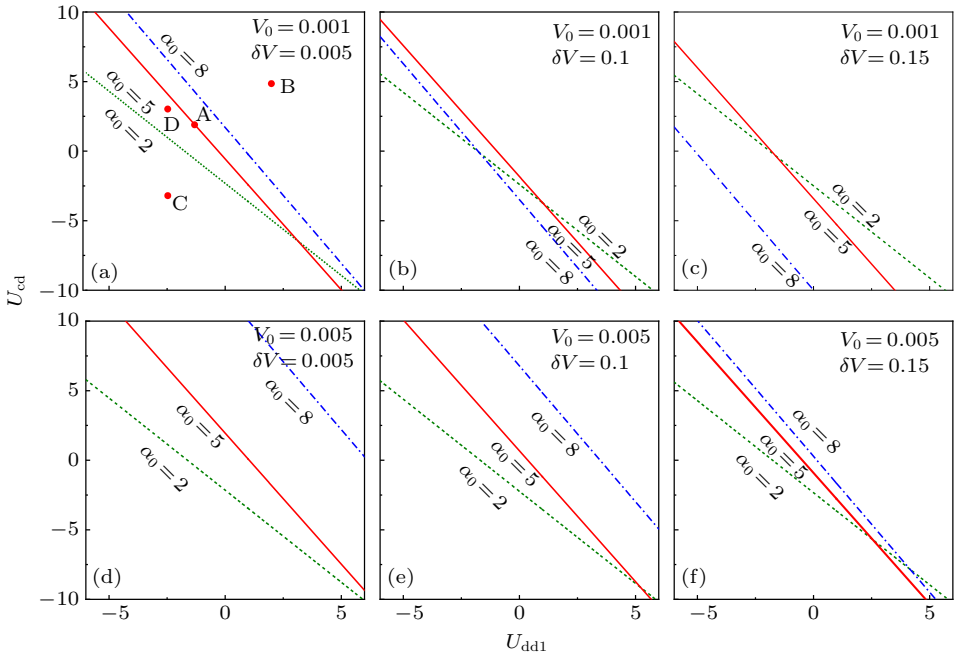


Fig. 3. The critical value U_{cd} against U_{dd1} for achieving stable condensate for different average strengths of potential V_0, driving strength δV and condensate width α_0. Here $\Omega = \pi$.

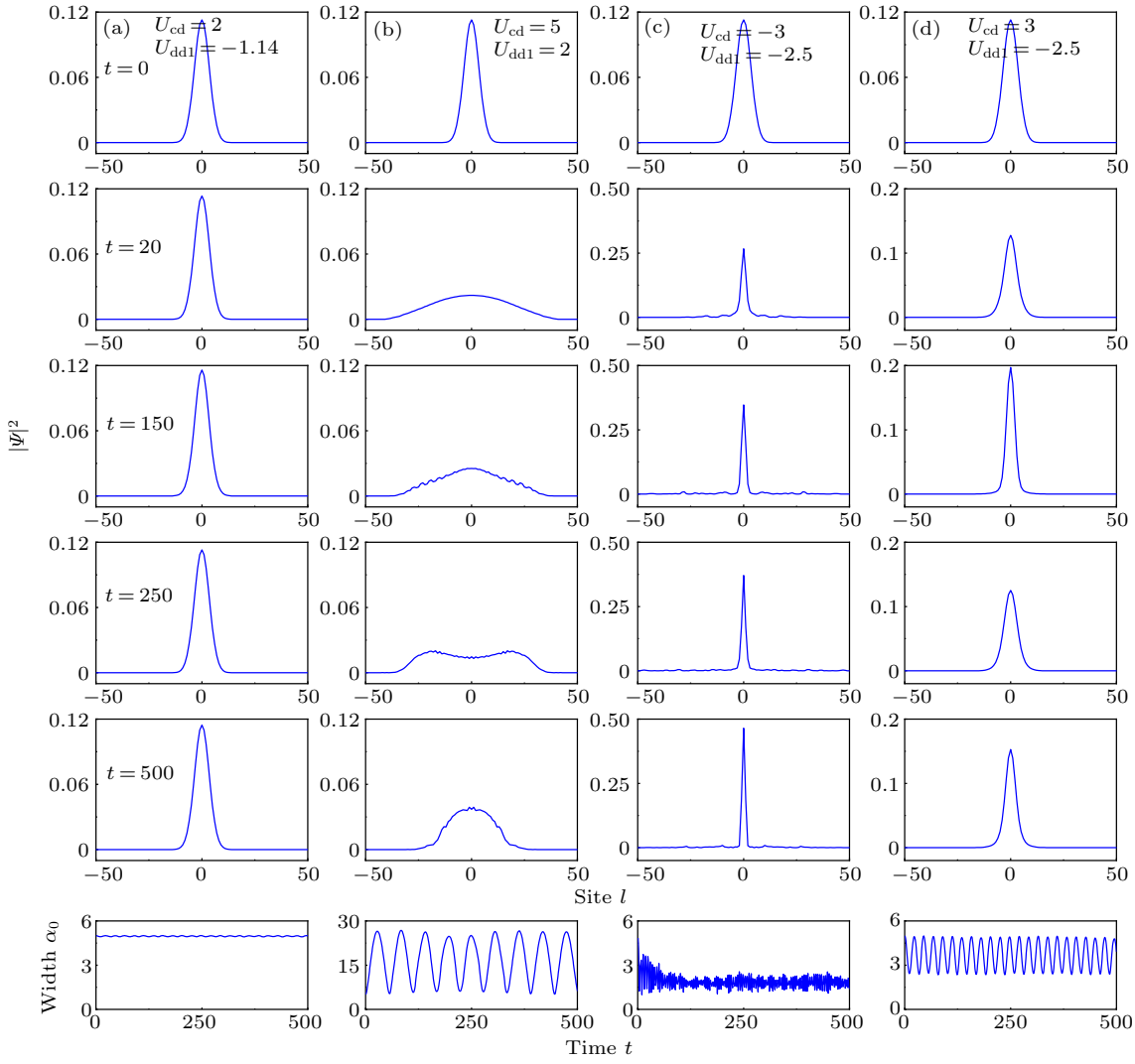


Fig. 4. The time evolution of the dipole condensate wave packet for different states, the stable state (the first column, as marked by A in Fig. 3(a)), diffused state (the second column, as marked by B in Fig. 3(a)), attractive-interaction-induced-trapped state (the third column, as marked by C in Fig. 3(a)) and breathing state (the fourth column, as marked by D in Fig. 3(a)), and the corresponding time evolution of α (the last row). With $\alpha_0 = 5$, $V_0 = 0.001$, $\delta V = 0.005$, and $\Omega = \pi$.

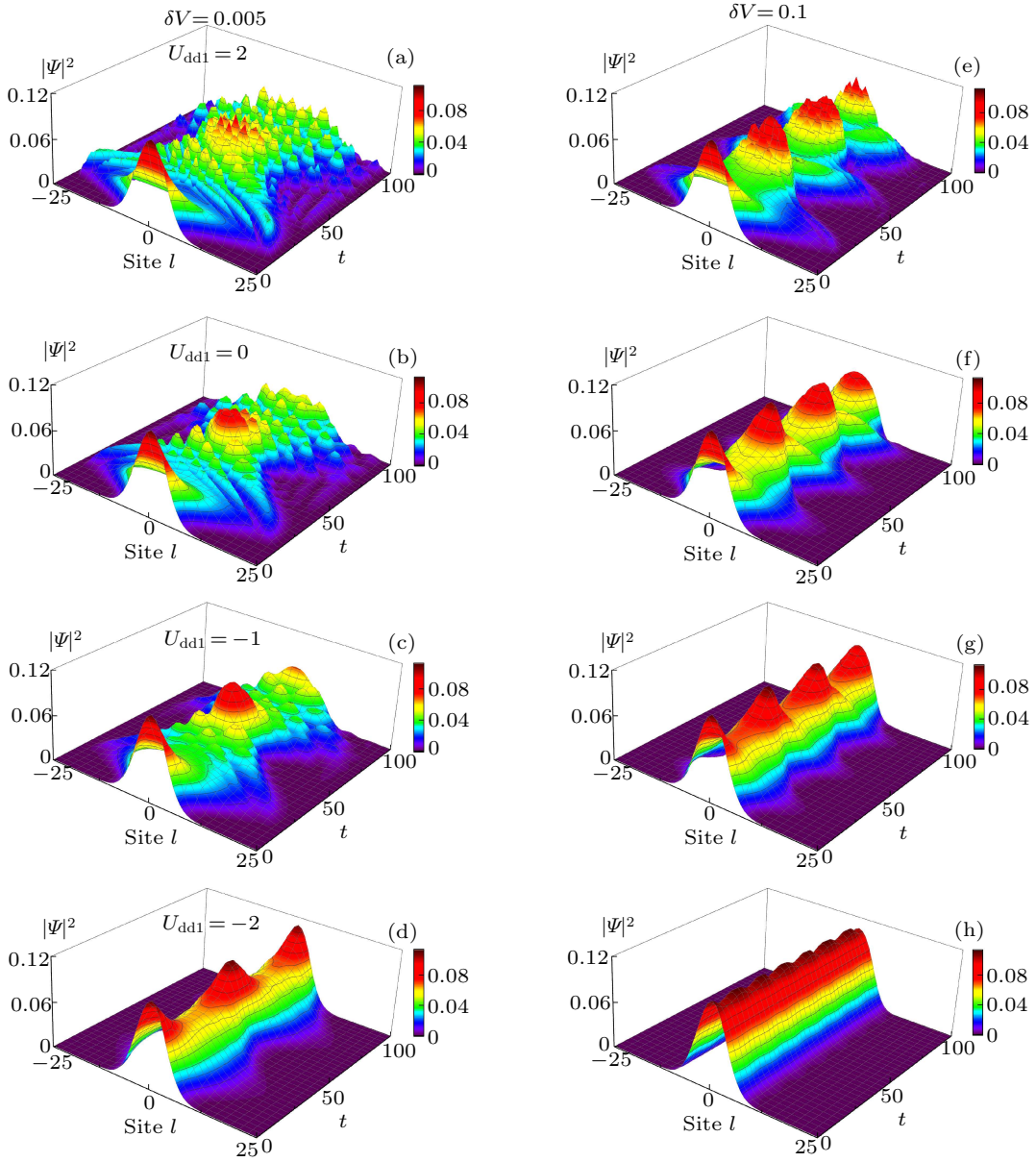


Fig. 5. Time evolution of wave packet of the dipole condensate for different intersite dipole interaction (from the first row to the fourth row, $U_{\rm dd1} = 2$, 0, -1, -2, respectively) and the driving strength (the first column with $\delta V = 0.005$ and the second column with $\delta V = 0.1$). $V_0 = 0.001$, $\alpha_0 = 5$, $U_{\rm cd} = 5$ and $\Omega = \pi$.

Hence, we can assume that α tends to a constant of α_0, and $\beta^2 \simeq 0$, then we can gain $\nu \approx 2\eta^2 \mathrm{e}^{-1/4\alpha_0^2} - V_0 + g_0$. The unstable oscillation rate is proportional to ν, depending on V_0, $\delta V/\Omega$, $U_{\rm cd}$, $U_{\rm dd1}$ and α_0. Then setting $\nu = 0$, we obtain a critical condition

$$U_{\rm cd} = 2\sqrt{2\pi}\alpha_0^3(V_0 - 2\eta^2 \mathrm{e}^{-\gamma}) - 2U_{\rm dd1}\,\mathrm{e}^{-1/2\alpha_0^2}(1 - 1/\alpha_0^2) - 2U_{\rm dd2}\,\mathrm{e}^{-2/\alpha_0^2}(1 - 4/\alpha_0^2), \tag{11}$$

when the condition (11) is satisfied, the wave packet width α is stable, the stability of the dipole system can be achieved. The condition (11) is one of the key conclusion of this article. In Fig. 3, the critical value of $U_{\rm dd1}$ against $U_{\rm cd}$ for stable oscillation of α is given by condition (11), which is strongly dependent on α_0, δV and V_0. Attractive-interaction-induced-trapped state can occur in the regions below each curve ($\nu < 0$) and diffused state can occur in the regions above each curve ($\nu > 0$). Interestingly, if we choose the parameters close to the condition (11), i.e., $\nu \to 0$ ($\nu \neq 0$), the breathing dynamics will occur.

To confirm the analytical prediction, we perform a direct numerical simulation for the GP Eq. (3) with parameters as marked in Fig. 3(a) by A, B, C, D, the corresponding results are shown in Figs. 4(a)–4(d). We show the stability properties of the dipole condensate that satisfy or does not satisfy condition (11). When we choose A $(U_{\rm dd1}, U_{\rm cd}) = (-1.14, 2)$, which satisfies condition (11), as expected, in this case the stable state occurs (see Fig. 4(a), the first column), the stability of dipolar condensate is achieved (also see the last row of the first column of Fig. 4, α keeps as a constant). When we choose B $(U_{\rm dd1}, U_{\rm cd}) = (2, 5)$, which does not satisfy condition (11), due to the strong repulsive total interaction prevents the system from stabilizing, the wave packet will be

diffused (see Fig. 4(b), the second column). However, because of existence of the external harmonic trap potential V_0, the condensate finally oscillates in the harmonic trap with a large α (also see the last row of the second column of Fig. 4). When we choose C $(U_{\rm dd1}, U_{\rm cd}) = (-2.5, -3)$, which still does not satisfy condition (11), but the instability of the system is mainly caused by the strong attractive intersite dipole interaction and on-site interaction, and condensate is in a attractive-interaction-induced-trapped state (see Fig. 4(c), the third column), the condensate is trapped approximatively in one lattice site (also see the last row of the third column of Fig. 4). Interestingly, if we choose the parameters close to condition (11) (as marked by D in Fig. 3(a)), the breathing dynamics occurs, i.e., α oscillates around α_0 periodically (see the last column of Fig. 4). That is, the dynamic stability of the dipole system is well predicted by numerical simulation and variational analysis. In summary, the dipole system is stable only when inter-site dipole interaction and on-site interaction satisfy the critical condition (11).

We further study the effects of interplay of both the periodic modulation and the inter-site dipole interaction on stability of dipolar condensate. Here, we set the on-site interaction $U_{\rm cd} = 5$, and use Eq. (6) as an initial wave function for dynamical evaluation of the system with $\alpha_0 = 5$. The time dynamics of the condensate for various $U_{\rm dd1}$ are depicted in Figs. 5(a)–5(d) with $\delta V = 0.005$. As can be seen, the instability of the condensate is gradually decreased with increasing the attractive nearest-neighbor dipole interaction $U_{\rm dd1}$. It is clear that attractive nearest-neighbor dipole interaction can help the stabilizing of condensate. The results depicted in Figs. 5(e)–5(h) for $\delta V = 0.1$ show that the increase of the driving strength of external modulation can enhance the stability of the system.

4. Conclusion

In summary, we have theoretically investigated the stability of dipolar condensate in a deep optical lattice with an additional external modulated harmonic potential. It is shown that when the dipolar interaction, the contact interaction, the average strength of potential and the ratio of amplitude to frequency of the modulation satisfy a critical condition, the stabilization of the dipolar condensate in deep optical lattice can be achieved. The dipolar interaction can balance the instability caused by the contact interaction. In addition, the breather dynamics, the diffusion dynamics and the attractive-interaction-induced-trapped dynamics are predicted. To further prove this, we perform a direct numerical simulation for GP equation, as expected, the results of numerical simulation are in good agreement with the analytical result. The dipolar interaction and the external modulation of the lattice play important roles in stabilizing condensates.

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