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Generation of non-integer high-order harmonics and significant enhancement of harmonic intensity*

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High-order harmonics from helium atom in the orthogonally two-color (OTC) laser field are investigated by solving the two-dimensional time-dependent Schrödinger equation. Non-integer high-order harmonics are obtained in some ratio of frequencies of two components. Pure odd and even harmonics from atoms could be separated in two components by adjusting the ratio of frequencies in OTC scheme, and the resolution of harmonics is improved at the same time. The physical mechanism is explained by the periodicity of dipole. With the same intensity of the incident laser, the intensity of the high-order harmonics from the OTC field scheme is improved by three orders of magnitude compared to the monochromatic laser field scheme. A theoretical scheme is provided for experimentally achieving improving energy resolution and separation of pure odd and even harmonics in atoms. Also, we provide a means for improving harmonic intensity.

Keywords: non-integer high-order harmonic generation, the orthogonally two-color laser field, the enhancement of harmonic intensity

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1. Introduction

Interaction between strong laser fields and atomic or molecular systems has become a hot topic with the development of high intensity ultrashort laser pulsed lasers. The nonlinear coupling of a strong laser field with a target can produce many new phenomena such as high-order harmonic generation (HHG),[1–7] above-threshold ionization (ATI),[8] nonsequential double ionization,[9] and Coulomb explosion.[10] Exploration of these new phenomena has promoted research progress in strong field physics.

HHG has received widespread attention because it is an effective way to generate coherent extreme ultraviolet, x-ray and attosecond pulses.[11–15] Since the late 1980s, many laboratories over the world have carried out a large number of experiments on high-order harmonic spectra and successfully observed the HHG in gas targets.[16–18] From the few orders at the beginning to more than one hundred orders later,[19] the high-order harmonic spectra generated by the intense laser field acting on atoms and molecules has some common features: a monotonic decline, followed by a plateau, a cutoff at the end of the plateau.[20–22] The semi-classical three-step model[23] is used to well explain the appearance of the plateau region, which cannot be explained by the traditional perturbation nonlinear optics theory. The energy at the cutoff position is calculated by the equation $E_{\text{cutoff}} = I_{\text{p}} + 3.17U_{\text{p}}$, where I_{p} is the ionization energy of the atom, U_{p} is the ponderomotive energy calculated by $U_{\text{p}} = \frac{E^2}{4\omega^2}$.[24] In general, only odd harmonics are generated from rare gases in monochromatic laser field.[25–28] Ben-Tal *et al.* introduced the concept of inversion symmetry and proved that the selection rules for HHG satisfy $2k \pm 1$ ($k = 1, 2, 3, \ldots$) when the system is inversely symmetric.[29]

Both odd and even harmonics could be obtained by breaking the symmetry of the system, such as using two-color field or multi-color field,[30–33] adding chirped field,[34] employing inhomogeneous field[35] to break the symmetry of the laser field, or using complex molecular systems to break the symmetry of the target.[36] The appearance of $3n \pm 1$ order harmonic pairs is achieved by the action of targets and two-color counter-rotating circularly polarized laser fields.[37–39] Selection rules for atomic and molecular systems under the N-fold symmetries field radiation were also explored.[40] Recently, only even order harmonics were discussed in the polar molecule CO,[41] nonpolar molecular N_2[42] and symmetrical ion system H_2^+ by adding a static filed.[43] The common point of pure even harmonic research is that in the case of asymmetric electronic motion of the molecular axis, adding linearly polarized laser field in its vertical direction will cause pure even harmonics in the molecular axis direction and pure odd harmonics in the laser direction. In addition, Neufield *et al.* used the Floquet group theory to deduce the selection rules for HHG and its polarization in targets for driving laser field with various temporal and spatial symmetries in 2019.[44]

In either odd or even harmonics, the high-order harmon-

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ics are radiated with the integer order of the incident pulse. However, non-integer order high-order harmonics or pure even harmonics from rare gases have not been reported to our best knowledge. In this work, we propose a solution to achieve the simultaneous generation of non-integer pure-even-order harmonics by numerical simulation in the helium atom. The harmonic intensity in the orthogonally two-color (OTC) laser field is improved by three orders of magnitude compared to the monochromatic field with the same intensity. The physical mechanism for the generation of pure even order harmonics is explained from the characteristics of the dipole acceleration, which is a method different from the Floquet group theory in Ref. [44]. Compared with molecular systems, we provide a simple and easy method for experimentally achieving the separation of odd and even harmonics. It should be noticed that the generation of non-integer orders provides theoretical guidance for experimentally filtering out harmonics of finer frequency multiples, which can achieve the purpose of improving energy resolution.

2. Theory and method

Time-dependent wave function can be obtained by numerically solving the Schrödinger equation under the length gauge (atomic units are used unless stated otherwise)

$$\mathrm{i}\frac{\partial \Psi(x,y,t)}{\partial t} = \left[-\frac{\nabla^2}{2} + V(x,y) + x\cdot E_x(t) + y\cdot E_y(t)\right]\Psi(x,y,t), \quad (1)$$

where the two-dimensional model potential of the helium atom is set to

$$V(x,y) = -\frac{1.5}{\sqrt{x^2+y^2+0.6}}, \quad (2)$$

which corresponds to the binding energy of 0.9 a.u. for the ground state of a helium atom. The OTC laser pulses scheme is used in the simulation, which combines a linearly fundamental field with frequency ω_1 in the x component by combining a weak field with frequency ω_2 in the y component. The laser field in two components can be written as

$$E_x(t) = E_1 f(t)\cos(\omega_1 t), \quad (3)$$

$$E_y(t) = E_2 f(t)\cos(\omega_2 t), \quad (4)$$

where $f(t)$ represents a trapezoidal envelope with a eighteen optical cycle flat top and six optical cycle ramps of the pulse, and one optical cycle is $T_1 = 2\pi/\omega_1$. The results shown in this paper are simulated by keeping $E_2 = 0.2E_1$. We employ the following second-order splitting-operator algorithm to obtained wave function at the next moment by converting the electron wave function between the momentum representation and the coordinate representation:

$$\Psi(x,y,t_0+\Delta t) = \mathrm{e}^{-\mathrm{i}(\frac{p_x^2+p_y^2}{4})\Delta t}\,\mathrm{e}^{-\mathrm{i}V\Delta t}\,\mathrm{e}^{-\mathrm{i}(\frac{p_x^2+p_y^2}{4})\Delta t} \times \Psi(x,y,t_0) + o(\Delta t)^3. \quad (5)$$

The dipole acceleration of the x and y components is derived from the Ehrenfest theorem

$$a_x(t) = \left\langle \Psi(x,y,t)\left| -\frac{\partial V(x,y)}{\partial x} - E_x(t)\right|\Psi(x,y,t)\right\rangle, \quad (6)$$

$$a_y(t) = \left\langle \Psi(x,y,t)\left| -\frac{\partial V(x,y)}{\partial y} - E_y(t)\right|\Psi(x,y,t)\right\rangle. \quad (7)$$

The harmonic spectrum on the two components is obtained from the absolute square of the Fourier transform of the associated dipole acceleration, respectively,

$$S_x(\omega) \sim |a_x(\omega)|^2 = \left|\int a_x(t)\,\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}t\right|^2, \quad (8)$$

$$S_y(\omega) \sim |a_y(\omega)|^2 = \left|\int a_y(t)\,\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}t\right|^2. \quad (9)$$

3. Results and discussion

Figure 1 shows the high-order harmonic spectra of the two components. The intensity of fundamental field is set to $I_1 = 3\times10^{14}$ W/cm^2, $\omega_1 = 0.057$ and $\omega_2 = \frac{2}{3}\omega_1$ at the beginning. Under such combined laser fields, it is surprising that harmonic orders are neither multiples of ω_1 nor multiples of ω_2, but non-integer high-order harmonics in the x direction like $52\frac{1}{3}$, $53\frac{2}{3}$ and in the y direction like $52\frac{2}{3}$, $53\frac{1}{3}$. To explore this phenomenon, the synthesized total laser field is analyzed. Although the periods are $T_1 = 2\pi/\omega_1$ and $T_2 = 2\pi/\omega_2$ in two components, the period of the synthesized total laser field is the least common multiple of T_1 and T_2, define as $T_0 = 2\pi/\omega_0$. Thus, the harmonic order can be defined in the order of ω_0, integer orders are obtained, see the label in Fig. 1. Pure even orders of harmonic are generated in y component, while only odd orders harmonics are obtained in x component. We also calculate the case of changing the relative phase difference of the incident pulse to $\pi/2$, $\pi/3$, $\pi/4$, etc. and then find that the radiation rules of pure odd or pure even harmonics do not depend on the relative phase of the incident pulse, which solves the problem that it is difficult to strictly control the relative phase in the experiment. From the above discussion, pure even harmonics can be generated from rare gas systems, not just molecular systems.[41–43] Different from the molecular system,[40] there is no need to consider the influence of the orientation angle on the selection rules of harmonic order due to the spherical symmetry of rare gas systems. It is easier to control the generation of harmonics in atomic systems in experiments.

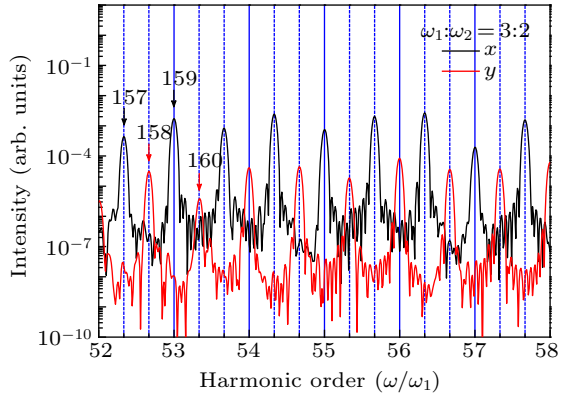


Fig. 1. High-order harmonic spectra from the OTC laser field. In the calculation, $\omega_1 = 0.057$ and $\omega_2 = \frac{2}{3}\omega_1$ are used, which are corresponding to $\lambda_1 = 800$ nm and $\lambda_2 = 1200$ nm. The thin black line and the thick red line represent the high-order harmonics radiated in the x and y directions, respectively.

The emission of pure odd or even high-order harmonics is related to the symmetry of the laser field. The Lissajous figure corresponding to the laser field in Fig. 2(a) is shown in Fig. 2(d). At time $t = 4T_0$, it corresponds to point A in the first quadrant on the Lissajous figure. The time evolution of laser field in the period of $4T_0$–$4.5T_0$ from point A passes point B in the direction of the black solid arrow to point C in the second quadrant. In the period of $4.5T_0$–$5.0T_0$, the laser field returns to the first quadrant along the $C'B'A'$ in the direction of the red hollow arrow. If a specific order harmonic generates at time t_g, and its next radiation time will be at half an optical cycle later, i.e., $t_g + T_0/2$. In order to illustrate the symmetry of the laser field, the laser field directions at t_g and $t_g + T_0/2$ are marked in Fig. 2(d). From the beginning of the integer period, the electric field intensity vector reaches C from the point A along the solid black arrow in the first half-cycle, and returns along the red hollow arrow in the second half-cycle. It causes the laser field directions to be axisymmetric about the y-axis, and the absolute values of the electric field strength at $t_g + T_0/2$ and t_g are equal to each other, while the signs of the laser field are opposite for the x direction. The sign of corresponding dipole acceleration is also reversed due to the change in the sign of the laser field, as shown in Fig. 2(b). The phase reverse results in destructive interference of harmonic frequencies that are exactly even multiples of the driving laser frequency. If the harmonic radiated at time t_g is set as $e^{iN\omega_0 t_g}$, thence the harmonic spectrum emitted by its adjacent half cycle is expressed as $e^{iN\omega_0(t_g+T_0/2)}$, where N is the harmonic order, $T_0 = 2\pi/\omega_0$. Consequently, the interference between the two harmonic emissions in this direction can be written as

$$H_x = e^{iN\omega_0 t_g} - e^{iN\omega_0(t_g+T_0/2)} = \begin{cases} 2e^{iN\omega_0 t_g} & \text{for } N = \text{odd}, \\ 0 & \text{for } N = \text{even}, \end{cases} \tag{10}$$

which is exactly the reason why the atomic system generates only odd order harmonics in a monochromatic field. However, it is very different for the y direction. As shown in Fig. 2(c), the signs of dipole acceleration at t_g and $t_g + T_0/2$ are the same, leading to the same sign of the high-order harmonics radiated in consecutive half cycles. Therefore, the interference in y direction is

$$H_y = e^{iN\omega_0 t_g} + e^{iN\omega_0(t_g+T_0/2)} = \begin{cases} 0 & \text{for } N = \text{odd}, \\ 2e^{iN\omega_0 t_g} & \text{for } N = \text{even}. \end{cases} \tag{11}$$

In a word, in the OTC scheme, the harmonic emission order is neither an integer multiple of ω_1 nor an integer multiple of ω_2, but an integer multiple of the frequency of the synthesized total electric field. In the case of $\omega_1 : \omega_2 = 3 : 2$, only odd and even orders harmonics of ω_0 are radiated in the x and y directions, respectively, which can be explained by Eqs. (10) and (11). Those discussions give an idea to separate pure odd and pure even order harmonics in experiment. Other parameters of the laser field are simulated for further investigation.

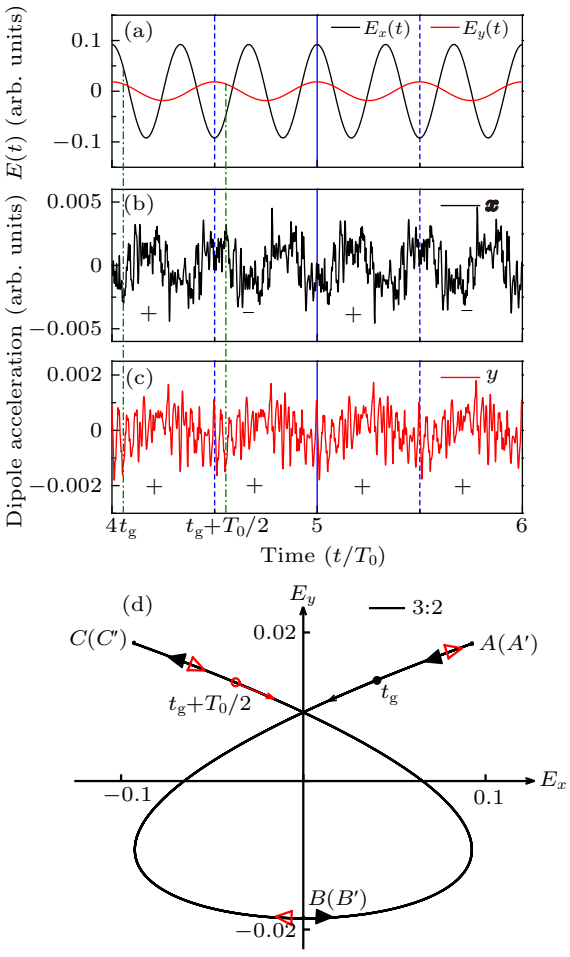


Fig. 2. (a) The evolution of E_x, E_y with period of T_0 over time. (b) and (c) The time-dependent dipole acceleration in x and y directions, respectively. The plus sign + indicates that harmonics have positive phase and the minus sign − indicates that harmonics have negative phase. Two dot-dashed olive lines represent two arbitrary moments that differ by half an optical cycle of T_0. (d) The Lissajous figure of OTC combined laser field. Black solid arrow and red hollow arrow indicate the direction of evolution of the laser field.

At different ratios of frequencies, the local high-order harmonic spectra of the plateau in two components is displayed in Figs. 3(a)–3(d) to show the phenomenon more clearly. The frequency of two incident pulses in Fig. 3(a) is $\omega_1 = 3\omega_0$, $\omega_2 = 4\omega_0$, and the harmonic order is similar to that in Fig. 1. The harmonics generated in the x direction are odd multiple of ω_0, and in the y direction are even multiple of ω_0. Figures 3(b) and 3(c) repeatedly show us the harmonic spectra with frequency ratio $\omega_1 = 3\omega_0$, $\omega_2 = 6\omega_0$ and $\omega_1 = 3\omega_0$, $\omega_2 = 9\omega_0$, and the harmonic orders in both figures are found to be integer multiples of ω_1. This is because the period of the synthesized pulse is the same as the period of the laser field pulse in the x direction, i.e., $\omega_0 = \omega_1$. In Fig. 3(b), there are only even orders in the y direction, while in Fig. 3(c), there are only odd orders. This is due to the symmetry of the Lissajous figure of the synthesized total laser pulse.

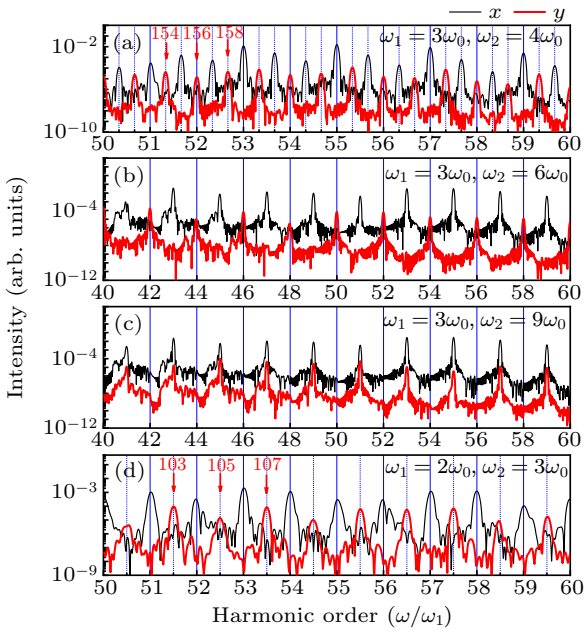


Fig. 3. (a)–(d) Harmonic spectra of helium atom at different frequency ratios of the incident laser field in x and y directions, which are represented by thin black and thick red lines, respectively.

We also simulate the case of $\omega_1 = 2\omega_0$, $\omega_2 = 3\omega_0$, as shown in Fig. 3(d). The results show that pure even order harmonics of ω_0 are generated in the x direction, and their strength is stronger than pure odd order harmonics of ω_0 in the y direction. Compare with Fig. 1, the parameters in Fig. 3 are equivalent to exchanging the intensities of two lasers, and there still exists the phenomenon that pure odd and pure even order harmonics are separated in two directions. We also simulate other intensity ratios and find that the intensity ratio in the OTC scheme has an influence on the harmonic intensity in two directions but the selection of harmonic orders are impervious.

Through the above analysis, a scheme to separate pure odd and pure even harmonics is obtained. If the frequency of two laser pulses is not a integer multiple, the obtained harmonic order is related to the frequency ω_0 of the synthesized total laser field. This shows that although the frequencies ω_1, ω_2 of the incident pulses are relative high, harmonics of the order ω_0 with better resolution can be generated. This provides a good idea for obtaining harmonics with finer orders.

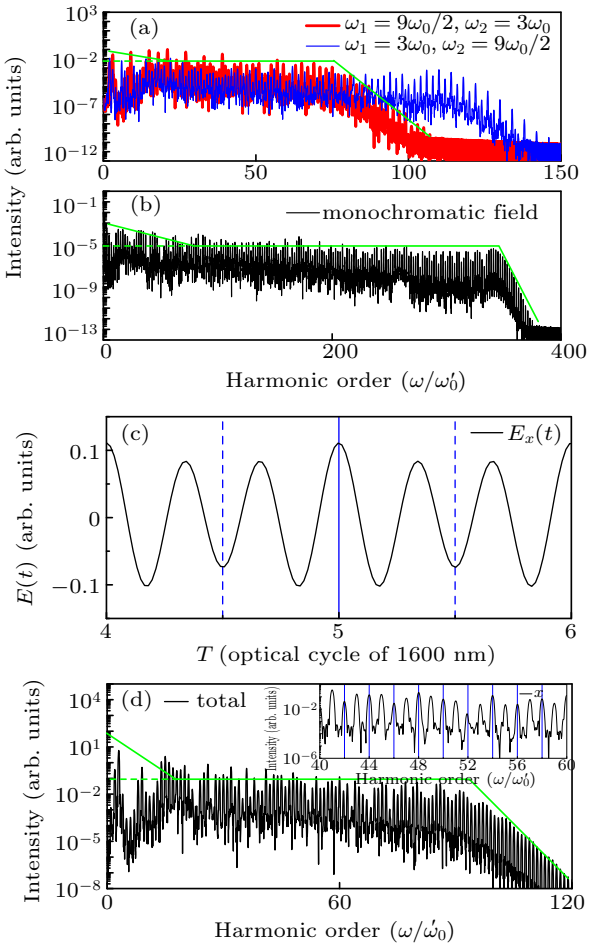


Fig. 4. (a) The total harmonic spectra when the frequency of two incident pulses are $\omega_1 = \frac{9}{2}\omega_0$, $\omega_2 = 3\omega_0$ and $\omega_1 = 3\omega_0$, $\omega_2 = \frac{9}{2}\omega_0$, which are represented by thin red and thick blue lines, respectively. (b) The total harmonic spectra generated by a monochromatic field with a intensity of 3.12×10^{14} W/cm^2 and a wavelength of 1600 nm. (c) The laser field evolution of two-color laser field from parallel polarization over time. (d) High-order harmonic spectra radiating from parallel polarization two-color pulse. The inset is an enlarged view of the harmonic spectra of order 40th to 60th.

Then we change the frequency of the two laser pulses to $\omega_1 = \frac{9}{2}\omega_0$, $\omega_2 = 3\omega_0$ and $\omega_1 = 3\omega_0$, $\omega_2 = \frac{9}{2}\omega_0$ to simulate the high-order harmonic spectra, at this time the corresponding frequency of the synthesized total laser field is no longer ω_0, but becomes $\omega_0' = \frac{3}{2}\omega_0$. In order to compare the intensity of the harmonic emission, we simulate the complete picture of the total harmonics spectra at two frequency ratios of the OTC scheme in Fig. 4(a) and the monochromatic laser field with frequency ω_0' in Fig. 4(b). In the case of $\omega_1 = \frac{9}{2}\omega_0$, $\omega_2 = 3\omega_0$, harmonics with an intensity of about 10^{-2} can be obtained. The harmonic spectrum structure in monochromatic field from helium is a well-known harmonic with only odd order in the

plateau area, the cut-off position is 340th orders, which is wider than the plateau region from the OTC scheme. However, its harmonic intensity is only 10^{-5}, which is about three orders of magnitude lower than the harmonic with an intensity of 10^{-2} in Fig. 4(a). In the case of $\omega_1 = 3\omega_0$, $\omega_2 = \frac{9}{2}\omega_0$, the harmonic intensity is also higher than the monochromatic field. However, compared with the laser field with $\omega_1 = \frac{9}{2}\omega_0$, $\omega_2 = 3\omega_0$, it is found that under the same intensity ratio, the laser field with a long fundamental frequency field wavelength can broaden the harmonic plateau.

In addition, the widely discussed two-color field scheme that two incident lasers are superimposed on the same polarization direction is also simulated, while keeping other laser parameters consistent with the thin red line in Fig. 4(a). It can be seen from Fig. 4(d) that the harmonic intensity of the total harmonic is higher than the monochromatic laser field intensity in Fig. 4(b), which is close to the harmonic intensity of red line in Fig. 4(a). From the evolution of the laser field over time, the direction of the laser field is reversed every half cycle, but the amplitude is different, thus both even and odd harmonics appear in the inset of Fig. 4(d). It can be seen that the two-color field superposition of the same polarization direction cannot separate pure odd and pure even harmonics. To sum up, if you want to have a scheme that can get high intensity harmonics with fine resolution and can separate pure odd and pure even harmonics, then the OTC scheme is the best choice.

4. Conclusion

In summary, we have investigated the interaction of helium atom with orthogonally two-color (OTC) laser field by solving the time-dependent Schrödinger equation. When a combined field of $\omega_1 : \omega_2 = 3 : 2$ is used, the generated harmonic orders are neither integer multiples of ω_1 nor integer multiples of ω_2, but are generated at integer multiples of the synthesized total laser field frequency ω_0. The harmonic spectra present the rules of the pure odd orders in the x direction and the pure even orders in the y direction due to the symmetrical features of Lissajous figures. Then the radiation mechanism of pure even order harmonics is explained using the evolution of laser field and the time-dependent change of dipole acceleration. Compared with the monochromatic laser field of the same total intensity, the OTC laser field has the advantage of significantly improving the harmonic intensity. The conclusion is verified by changing the frequency ratio of the laser field. We propose not only a scheme to achieve the separation of pure odd and pure even order harmonics in atoms, but also a scheme to improve energy resolution, and at the same time provide a simple and effective scheme for significantly improving harmonic intensity. It provides theoretical guidance for the development of atom high-order harmonics in experiments.

References

[1] Han S, Ortmann L, Kim H, Kim Y W, Oka T, Chacon A, Doran B, Ciappina M, Lewenstein M, Kim S W, Kim S and Landsman A S 2019 *Nat. Commun.* **10** 3272
[2] Yu C, Hansen K K and Madsen L B 2019 *Phys. Rev. A* **99** 063408
[3] Zhang C P, Xia C L, Jia X F and Miao X Y 2016 *Opt. Express* **24** 20297
[4] Gemsheim S and Rost J M 2019 *Phys. Rev. A* **100** 043408
[5] Chappuis C, Bresteau D, Auguste T, Gobert O and Ruchon T 2019 *Phys. Rev. A* **99** 033806
[6] Ghimire S and Reis D A 2019 *Nat. Phys.* **15** 10
[7] Bian X B and Bandrauk A D 2014 *Phys. Rev. Lett.* **113** 193901
[8] Agostini P, Fabre F, Mainfray G and Petite G 1979 *Phys. Rev. Lett.* **42** 1127
[9] Fittinghoff D N, Bolton P R, Chang B and Kulander K C 1992 *Phys. Rev. Lett.* **69** 2642
[10] Frasinski L J, Codling K, Hatherly P, Barr J, Ross I N and Toner W T 1987 *Phys. Rev. Lett.* **58** 2424
[11] Xia C L and Liu X S 2013 *Phys. Rev. A* **87** 043406
[12] Zhong C L, Qiao B, Xu X R, Zhang Y X, Li X B, Zhang Y, Zhou C T, Zhu S P and He X T 2020 *Phys. Rev. A* **101** 053814
[13] Luo J, Hong W, Zhang Q, Li Yang and Lu P 2012 *Opt. Express* **20** 21346
[14] Sarantseva T S, Frolov M V, Manakov N L, Silaev A A, Romanov A A, Vvedenskii N V and Starace A F 2020 *Phys. Rev. A* **101** 013402
[15] Yuan K J and Bandrauk A D 2011 *Phys. Rev. A* **84** 023410
[16] Sarukura N, Hata K, Adachi T, Nodomi R, Watanabe M and Watanabe S 1991 *Phys. Rev. A* **43** 1669
[17] Kondo K, Sarukura N, Sajiki K and Watanabe S 1993 *Phys. Rev. A* **47** R2480
[18] L'Huillier A and Balcou Ph 1993 *Phys. Rev. Lett.* **70** 774
[19] Macklin J J, Kmetec J D and Gordon C L 1993 *Phys. Rev. Lett.* **70** 766
[20] Krause J L, Schafer K J and Kulander K C 1992 *Phys. Rev. Lett.* **68** 3535
[21] L'Huillier A, Lewenstein M, Salières P, Balcou Ph, Ivanov M Yu, Larsson J and Wahlström 1993 *Phys. Rev. A* **48** R3433
[22] Li Y, Sato T and Ishikawa K L 2019 *Phys. Rev. A* **99** 043401
[23] Corkum P B 1993 *Phys. Rev. Lett.* **71** 1994
[24] Lewenstein M, Balcou Ph, Ivanov M Yu, L'Huillier A and Corkum P B 1994 *Phys. Rev. A* **49** 2117
[25] Ferray M, L'Huillier A, Li X F, Lompré L A, Mainfray G and Manus C 1988 *J. Phys. B: At. Mol. Opt. Phys.* **21** L31
[26] Li X F, L'Huillier A, Ferray M, Lompré L A and Mainfray G 1989 *Phys. Rev. A* **39** 5751
[27] Le A T, Morishita T and Lin C D 2008 *Phys. Rev. A* **78** 023814
[28] Pfeifer T, Spielmann C and Gerber G 2006 *Rep. Prog. Phys.* **69** 443
[29] Ben-Tal N, Moiseyev N and Beswick A 1993 *J. Phys. B: At. Mol. Opt. Phys.* **26** 3017
[30] Kim I J, Kim C M, Kim H T, Lee G H, Lee Y S, Park J Y, Cho D J and Nam C H 2005 *Phys. Rev. Lett.* **94** 243901
[31] Frolov M V, Manakov N L, Sarantseva T S, Silaev A A, Vvedenskii N V and Starace A F 2016 *Phys. Rev. A* **93** 023430
[32] Frolov M V, Manakov N L, Silaev A A and Vvedenskii N V 2010 *Phys. Rev. A* **81** 063407
[33] Zhai C Y, Shao R Z, Lan P F, Wang B, Zhang Y, Yuan H, Njoroge S M, He L and Lu P 2020 *Phys. Rev. A* **101** 053407
[34] Lara-Astiaso M, Silva R E F, Gubaydullin A, Rivière P, Meier C and Martín F 2016 *Phys. Rev. Lett.* **117** 093003
[35] Guo Y, Liu A, Wang J and Liu X 2019 *Chin. Phys. B* **28** 094212
[36] Zhu X, Zhang C, Gu M and Yao J 2014 *Opt. Express* **22** 7947
[37] Yuan K J and Bandrauk A D 2019 *Phys. Rev. A* **100** 033420
[38] Li M, Jiang W C, Xie H, Luo S, Zhou Y and Lu P 2018 *Phys. Rev. A* **97** 023415
[39] Heslar J, Telnov D A and Chu S I 2018 *Phys. Rev. A* **97** 043419
[40] Liu X, Zhu X, Li L, Li Y, Zhang Q, Lan P and Lu P 2016 *Phys. Rev. A* **94** 033410
[41] Hu H, Li N, Liu P, Li R and Xu Z 2017 *Phys. Rev. Lett.* **119** 173201
[42] Yang Y, Liu L and Zhao Z 2018 *The 9th International Symposium on Ultrafast Phenomena and Terahertz Waves* January 2018, OSA
[43] Xu L and He F 2019 *J. Opt. Soc. Am. B* **36** 840
[44] Neufeld O, Podolsky D and Cohen O 2019 *Nat. Commun.* **10** 405