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Impact of counter-rotating-wave term on quantum heat transfer and phonon statistics in nonequilibrium qubit–phonon hybrid system*

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Counter-rotating-wave terms (CRWTs) are traditionally viewed to be crucial in open small quantum systems with strong system–bath dissipation. Here by exemplifying in a nonequilibrium qubit–phonon hybrid model, we show that CRWTs can play the significant role in quantum heat transfer even with weak system–bath dissipation. By using extended coherent phonon states, we obtain the quantum master equation with heat exchange rates contributed by rotating-wave-terms (RWTs) and CRWTs, respectively. We find that including only RWTs, the steady state heat current and current fluctuations will be significantly suppressed at large temperature bias, whereas they are strongly enhanced by considering CRWTs in addition. Furthermore, for the phonon statistics, the average phonon number and two-phonon correlation are nearly insensitive to strong qubit–phonon hybridization with only RWTs, whereas they will be dramatically cooled down via the cooperative transitions based on CRWTs in addition. Therefore, CRWTs in quantum heat transfer system should be treated carefully.

Keywords: quantum transport, open systems, nonequilibrium and irreversible thermodynamics, phonons or vibrational states in low-dimensional structures and nanoscale materials

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1. Introduction

Understanding and managing nonequilibrium energy transfer at nanoscale is a long-standing problem, which has attracted great attention with scientific interest and practical importance.[1–3] Theoretically, the microscopic description of the system–bath coupling in open small quantum systems is crucial to model the quantum heat flow, as the system–bath coupling describes the dissipation of the small quantum system to the external environment.[4,5] In particular, the strong couplings between the small quantum system and surrounding baths significantly contribute to the transient and steady state heat transport properties, including energy exchange dynamics,[6–11] quantum thermodynamics and thermal machines,[12–18] and quantum heat transfer.[19–27]

One of the most representative paradigms of the system–bath dissipation is the spin–boson (qubit–bath) coupling[28]

$$\hat{V}_{\rm SB} = \hat{\sigma}_x \sum_k (g_k \hat{b}_k^\dagger + g_k^* \hat{b}_k) = \hat{V}_{\rm RWT} + \hat{V}_{\rm CRWT},$$

where

$$\hat{V}_{\rm RWT} = \sum_k (g_k \hat{b}_k^\dagger \hat{\sigma}_- + g_k^* \hat{b}_k \hat{\sigma}_+),$$

$$\hat{V}_{\rm CRWT} = \sum_k (g_k \hat{b}_k^\dagger \hat{\sigma}_+ + g_k^* \hat{b}_k \hat{\sigma}_-),$$

with $\hat{\sigma}_x$ the Pauli operator of the central qubit, or say, a two-level system, g_k the qubit–bath coupling strength, $\hat{b}_k^\dagger$ ($\hat{b}_k$) the bosonic creator (annihilator) in the thermal bath. This system–bath dissipation form has been widely used for the heat transport in quantum phononics. The rotating-wave-terms (RWTs) $\hat{V}_{\rm RWT}$ describe energy conserved processes that can occur sequentially in an incoherent picture, while the counter-rotating-wave-terms (CRWTs) $\hat{V}_{\rm CRWT}$ depict two energy non-conserved processes that can only occur coherently.[29]

It is well believed that in the weak qubit–bath dissipation limit, $\hat{V}_{\rm CRWT}$ is negligible and $\hat{V}_{\rm RWT}$ dominates the dissipation process. Only when the dissipation strength increases, $\hat{V}_{\rm CRWT}$ is necessarily included to properly characterize the heat exchange.[5,7] In quantum dissipative dynamics, $\hat{V}_{\rm CRWT}$ is found to significantly enhance anti-Zeno signal,[30–33] quantum correlation,[34–36] and spontaneous emission[37–39] at strong system–bath coupling. At steady state, CRWTs result in the non-canonical thermal equilibrium distribution[29,40–42] and optimal enhancement of the heat current.[19,23,25] Recently, due to the fast development of circuit-QED with high quality microwave resonators, the spin–boson coupling is reduced to the qubit–photon hybridization $\hat{V}_{\rm SB} = \hat{\sigma}_x(g\hat{b}^\dagger + g^*\hat{b})$ with a single mode of bosons (photons).[43,44] The importance

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of CRWTs $\hat{V}_{\rm CRWT} = (g\hat{b}^\dagger\hat{\sigma}_+ + g^*\hat{b}\hat{\sigma}_-)$ in photonic hybrid quantum systems on energy level structure has been experimental demonstrated.[45,46] The investigation of steady state coherence and heat transport has been conducted in the QED based hybrid quantum systems.[47–49] Moreover, from the aspect of photon statistics, the qubit–photon interaction ($\hat{V}_{\rm SB}$) will affect the two-photon correlation function in dressed picture at equilibrium, which exhibits novel features.[50–52]

For steady state heat transfer with two baths at nonequilibrium setup, the nonequilibrium spin–boson (qubit–bath) dissipation $\hat{V}_{\rm NSB} = \hat{\sigma}_x\sum_{k;v=L,R}(g_{k,v}\hat{b}^\dagger_{k,v} + g^*_{k,v}\hat{b}_{k,v})$ is considered generic to establish the thermodynamic bias between phononic baths L and R.[53,54] Specifically, in the weak qubit–bath interaction limit, the heat flow shows a sequential process, which is dominated by RWTs. In sharp contrast, the heat current and fluctuation exhibit cooperative energy exchange processes between two phononic baths at strong qubit–bath coupling,[19–21,23] where CRWTs play the leading role. While under the adiabatic modulation of two bath temperatures, the finite heat pump based on the Redfield approximation is exhibited with weak qubit–bath dissipation,[55] which is irrelevant with the dissipation strength. However, in strong dissipation limit, the heat pump can be observed only for the biased qubit.[24,56] Moreover, the exploration of such distinction originating from RWTs and CRWTs has been extended to the quantum thermal transistor from weak to strong spin–boson couplings.[57–64]

Here, we propose that at strong qubit–phonon hybridization inside the central system, CRWTs of the (even weak) qubit–bath dissipation may novelly contribute to the transport behaviors of hybrid quantum systems. The main points of this work have three manifolds: (i) Based on the quantum master equation encoded with extended coherent phonon states, we obtain microscopic pictures of heat exchange processes, which are separately contributed by RWTs and CRWTs. (ii) By including full counting statistics (FCS) with weak system–bath interactions, the heat current and fluctuations are strongly suppressed at large temperature bias with the qubit–bath dissipation considering only RWTs, whereas the heat current fluctuations are greatly enhanced by considering CRWTs in addition. This clearly demonstrates the novel role of CRWTs on steady state heat transfer. (iii) The steady state phonon number and two-phonon correlation function with RWTs are nearly insensitive to the strong qubit–phonon hybridization, whereas they are optimally cooled down via the cooperative transition path contributed by both RWTs and CRWTs.

This paper is organized as follows. We describe the nonequilibrium qubit–phonon hybrid model in Subsection 2.1, derive the quantum master equation in Subsection 2.2, and obtain the expression of heat current fluctuations by including full counting statistics in Subsection 2.3. In Section 3, we investigate the steady state heat transfer, which is characterized by heat current, noise power, and skewness. In Section 4, we analyze the steady state phonon number and two-phonon correlation function of phonon statistics. Finally, we give a conclusion in Section 5.

2. Model and method

2.1. Qubit–phonon hybrid system

The nonequilibrium qubit–phonon hybrid system, which is composed by two-level qubits interacting with a single vibrational mode, each individually coupled to a bosonic thermal bath, is described as ($\hbar = 1$)

$$\hat{H} = \hat{H}_s + \sum_{u=qu,ph}(\hat{H}_u + \hat{V}_u). \tag{1}$$

Specifically, the hybrid system is expressed as

$$\hat{H}_s = \varepsilon\hat{J}_z + \omega_0\hat{a}^\dagger\hat{a} + \lambda(\hat{a}^\dagger + \hat{a})\hat{J}_z, \tag{2}$$

where the collective qubit operators are $\hat{J}_\alpha = \frac{1}{2}\sum_{i=1}^N\hat{\sigma}^i_\alpha$ ($\alpha = x,y,z$) with N the qubit number and $\hat{\sigma}^i_\alpha$ is the Pauli operator of the i-th qubit, ε is the energy splitting of the qubits, $\hat{a}^\dagger$ ($\hat{a}$) creates (annihilates) one phonon with frequency ω_0, and λ is the hybridization strength between the qubits and the phononic field. The u-th thermal bath is described as $\hat{H}_u = \sum_k\omega_k\hat{b}^\dagger_{k,u}\hat{b}_{k,u}$, where $\hat{b}^\dagger_{k,u}$ ($\hat{b}_{k,u}$) creates (annihilates) one boson in the u-th thermal bath with the momentum k and frequency $\omega_{k,u}$. The interaction between the phononic mode and the ph-th bath is given by

$$\hat{V}_{ph} = \sum_k(g_{k,ph}\hat{b}^\dagger_{k,c} + g^*_{k,ph}\hat{b}_{k,c})(\hat{a}^\dagger + \hat{a}), \tag{3}$$

with $g_{k,ph}$ the coupling strength. While for the interaction between the qubits and the qu-th thermal bath, the interaction can be generally expressed as

$$\hat{V}_{qu} = \sum_k(g_{k,qu}\hat{b}^\dagger_{k,qu}\hat{S} + g^*_{k,q}\hat{b}_{k,q}\hat{S}^\dagger), \tag{4}$$

with $g_{k,qu}$ the coupling strength between the qubits and the qu-th thermal bath. For the qubit operator $\hat{S}$, if we analyze $\hat{V}_{qu}$ within the rotating wave approximation (RWA), it is specified as $\hat{S} = \hat{J}_-$. Hence, under the influence of rotating-wave terms (RWTs) [i.e., $\hat{V}^{\rm RWT}_{qu} = \sum_k(g_{k,qu}\hat{b}^\dagger_{k,qu}\hat{J}_- + g^*_{k,q}\hat{b}_{k,qu}\hat{J}_+)$], the particle number of the whole system is conserved as the exchange processes occur between the qubits and the qu-th bath. However, when we consider the full interaction between the qubits and the qu-th bath $\hat{V}^{\rm full}_{qu}$, the operator becomes $\hat{S} = 2\hat{J}_x$, which includes both the RWTs and the counter-rotating-wave terms (CRWTs) [i.e., $\hat{V}^{\rm CRT}_{qu} = \sum_k(g_{k,qu}\hat{b}^\dagger_{k,qu}\hat{J}_+ + g^*_{k,q}\hat{b}_{k,qu}\hat{J}_-)$]. Under the effect of CRWTs, both the system and corresponding thermal bath will be excited (annihilated) simultaneously, which apparently breaks the particle number (energy) conservation. In the single qubit limit, i.e., $N = 1$, the model of the qubit–phonon hybrid system in Eq. (1) is identical with the

counterpart in Ref. [49], where we investigated the steady state heat current in a nonequilibrium qubit–phonon hybrid model at weak qubit–phonon hybridization regime. We would like to point out that although the angular momentum conservation is not explicitly considered at present, it is implicitly contained in the spin–boson dissipation, where the raising and lowering of spin angular momentum are compensated by the coupled bosons due to their intrinsic spin.[65,66] Moreover, it needs to stress that the spin–boson and qubit–bath dissipations (couplings) denote the spin coupled to the bosonic thermal bath, which is composed by continuous boson modes, whereas the qubit–phonon hybridization describes the spin interacting with the single mode phononic resonator.

In this work, we mainly analyze the novel role of CRWTs on the steady state behaviors of the hybrid quantum system in Eq. (1). For the the qubit–phonon hybrid system $\hat{H}_s$ in Eq. (2), the eigensolution can be exactly solved as $\hat{H}_s|\phi_m^k\rangle = E_m^k|\phi_m^k\rangle$. The eigenvalue is

$$E_m^k = \varepsilon m + \omega_0 k - \lambda^2 m^2/\omega_0, \tag{5}$$

and the extended coherent phonon state is[67–69]

$$|\phi_m^k\rangle = |j,m\rangle \otimes \left[\frac{(\hat{a}^\dagger + g_m)^k}{\sqrt{k!}}|0\rangle_m\right], \tag{6}$$

where the phonon excitation number is $k = 0,1,2,\ldots$, the angular momentum state is $\hat{J}_z|j,m\rangle = m|j,m\rangle$ $(m = -j,-j+1,\ldots,j)$ with $j = N/2$, the displacement coefficient is $g_m = \lambda m/\omega_0$, $|0\rangle_m = \mathrm{e}^{-g_m\hat{a}^\dagger - g_m^2/2}|0\rangle_a$, and the bare vacuum state is $\hat{a}|0\rangle_a = 0$.

It should be noted that though the nonequilibrium qubit–phonon hybrid system is theoretically investigated in the present paper, it has experimental correspondences. Specifically, the hybrid quantum system can be specified by the system composed by the nanomechanical resonator and single quantum dot,[70] where the resonator and quantum dot interact with the bosonic thermal and magnon reservoir,[71,72] respectively. The CRWTs in Eq. (4) can be realized by the interfacial interaction with non-spin-conservation.[73] Moreover, it could also be realized by the circuit-QED setup,[48] where one josephson junction could be longitudinally coupled to a LC resonator.[74–77] While for the spin–boson model, under the reaction coordinate mapping approach, it can be mapped to another type of qubit–phonon hybrid model.[6,10,78–82] From the aspect of inverse design, the analysis of quantum heat transfer in nonequilibrium qubit–phonon hybrid systems could fertilize theoretical interact and practical application of the spin–boson model.

2.2. Quantum master equation

Considering weak system–bath interactions, we separately perturb $\hat{V}_{ph}$ and $\hat{V}_{qu}$ to obtain the quantum master equation. Under the Born–Markov approximation, the total density operator is decomposed as $\hat{\rho}_{\rm tot}(t) \approx \hat{\rho}_s(t) \otimes \hat{\rho}_b$, where $\hat{\rho}_s(t)$ is the reduced density operator of the qubit–phonon hybrid system, and $\hat{\rho}_b = [\Pi_{u=ph,qu}\exp(-\hat{H}_b^u/k_{\rm B}T_u)]/\mathcal{Z}$ is the equilibrium density operator of thermal baths, with $k_{\rm B}$ the Boltzmann constant, T_u the temperature of the u-th thermal bath, and $\mathcal{Z} = {\rm Tr}\{\Pi_{u=ph,qu}\exp(-\hat{H}_b^u/k_{\rm B}T_u)\}$ the partition function. In this paper, we set $k_{\rm B} = 1$ for convenience. Then, by tracing over the degrees of freedom of thermal baths, the generalized master equation is obtained at Eq. (A1) in Appendix A.

From the generalized quantum master equation, it is known that for transient dynamics the populations are generally coupled to the off-diagonal terms.[83] However, after long time evolution, it is numerically checked in a wide parameter regime that the off-diagonal terms become negligible. Hence, the generalized quantum master equation is reduced to the dressed master equation as

$$\begin{aligned}\frac{{\rm d}\hat{\rho}_s(t)}{{\rm d}t} &= \hat{\mathcal{L}}_0\hat{\rho}_s(t) + \sum\{\Gamma_u^+(E_{m'k'}^{mk})\hat{\mathcal{L}}_+(|\phi_m^k\rangle\langle\phi_{m'}^{k'}|)\hat{\rho}_s(t) \\ &\quad + \Gamma_u^-(E_{m'k'}^{mk})\hat{\mathcal{L}}_-(|\phi_{m'}^{k'}\rangle\langle\phi_m^k|)\hat{\rho}_s(t)\},\end{aligned} \tag{7}$$

where the dissipators are given by

$$\begin{aligned}\hat{\mathcal{L}}_0\hat{\rho}_s(t) &= -{\rm i}[\hat{H}_s,\hat{\rho}_s(t)] \\ &\quad - \frac{1}{2}\sum\Gamma_u^+(E_{m'k'}^{mk})(|\phi_{m'}^{k'}\rangle\langle\phi_{m'}^{k'}|\hat{\rho}_s + \hat{\rho}_s|\phi_{m'}^{k'}\rangle\langle\phi_{m'}^{k'}|) \\ &\quad - \frac{1}{2}\sum\Gamma_u^-(E_{m'k'}^{mk})(|\phi_m^k\rangle\langle\phi_m^k|\hat{\rho}_s + \hat{\rho}_s|\phi_m^k\rangle\langle\phi_m^k|),\end{aligned} \tag{8}$$

$$\hat{\mathcal{L}}_+(|\phi_m^k\rangle\langle\phi_{m'}^{k'}|)\hat{\rho}_s = |\phi_m^k\rangle\langle\phi_{m'}^{k'}|\hat{\rho}_s(t)|\phi_{m'}^{k'}\rangle\langle\phi_m^k|, \tag{9}$$

$$\hat{\mathcal{L}}_-(|\phi_m^k\rangle\langle\phi_{m'}^{k'}|)\hat{\rho}_s = |\phi_{m'}^{k'}\rangle\langle\phi_m^k|\hat{\rho}_s(t)|\phi_m^k\rangle\langle\phi_{m'}^{k'}|. \tag{10}$$

The nonzero rates assisted by the ph-th bath are

$$\Gamma_{ph}^{\pm}(\phi_m^{k-1}|\phi_m^k) = \pm k\gamma_{ph}(E_{m,k-1}^{m,k})n_{ph}(\pm E_{m,k-1}^{m,k}), \tag{11}$$

with the energy gap $E_{m,k-1}^{m,k} = E_{m,k} - E_{m,k-1}$. $\Gamma_{ph}^+(\phi_m^{k-1}|\phi_m^k)$ $[\Gamma_{ph}^-(\phi_m^{k-1}|\phi_m^k)]$ describes the phonon excitation (relaxation) process from the extended coherent phonon state $|\phi_m^{k-1}\rangle$ $(|\phi_m^k\rangle)$ to $|\phi_m^k\rangle$ $(|\phi_m^{k-1}\rangle)$ by absorbing (releasing) one boson with the energy $E_p^{m,k}m,k-1$ from (into) the ph-th thermal bath, with the qubits state unchanged. For the qubit–bath interaction under rotating-wave approximation, the rates assisted by the qu-th bath are given by

$$\begin{aligned}\Gamma_{qu,{\rm RWT}}^{\pm}(\phi_{m-1}^l|\phi_m^k) &= \pm\theta(E_{m-1,l}^{m,k})(j_{m-1}^+)^2 D_{k,l}^2\left(\frac{\lambda}{\omega_0}\right) \\ &\quad \times\gamma_{qu}(E_{m-1,l}^{m,k})n_{qu}(\pm E_{m-1,l}^{m,k}),\end{aligned} \tag{12}$$

where the Heaviside step function is $\theta(\omega{\geq}0) = 1$ and $\theta(\omega{<}0) = 0$, the angular momentum factor is $j_m^\pm = \sqrt{j(j+1) - m(m\pm1)}$, the extended coherent phonon state overlap coefficient is[67,68]

$$D_{k,l}(x) = \mathrm{e}^{-x^2/2}\sum_{n=0}^{\min[k,l]}\frac{(-1)^n\sqrt{k!l!}x^{k+l-2n}}{(k-n)!(l-n)!n!}, \tag{13}$$

and the energy gap is $E_{m-1,l}^{m,k} = E_{m,k} - E_{m-1,l}$. The rate $\Gamma_{qu,{\rm RWT}}^+(\phi_{m-1}^l|\phi_m^k)$ $[\Gamma_{qu,{\rm RWT}}^-(\phi_{m-1}^l|\phi_m^k)]$ characterizes the mi-

croscopic transfer process from the extended coherent phonon state $|\phi^l_{m-1}\rangle$ ($|\phi^k_m\rangle$) to $|\phi^k_m\rangle$ ($|\phi^l_{m-1}\rangle$) by exchange $|l-k|$ phonon number and the energy $E^{m,k}_{m-1,l}$ involved with the qu-th thermal bath in Fig. 1(a) [Fig. 1(b)], which is simultaneously bounded by the unidirectional transition of the qubits state from $|j,m-1\rangle$ ($|j,m\rangle$) to $|j,m\rangle$ ($|j,m-1\rangle$). While for the full qubit–bath interaction including both RWTs and CRWTs, the transition rate assisted by the qu-th bath is expressed as $\Gamma^{\pm}_{qu}(\phi^l_n|\phi^k_m)=\Gamma^{\pm}_{qu,\mathrm{RWT}}(\phi^l_{m-1}|\phi^k_m)\delta_{n,m-1}+\Gamma^{\pm}_{qu,\mathrm{CRWT}}(\phi^l_{m+1}|\phi^k_m)\delta_{n,m+1}$, where the rates contributed by CRWTs are given by

$$\Gamma^{\pm}_{qu,\mathrm{CRWT}}(\phi^l_{m+1}|\phi^k_m) = \pm\theta(E^{m,k}_{m+1,l})(j^-_{m+1})^2 D^2_{k,l}\left(\frac{\lambda}{\omega_0}\right) \times\gamma_{qu}(E^{m,k}_{m+1,l})n_{qu}(\mp E^{m,k}_{m+1,l}), \tag{14}$$

with the positive energy gap $E^{m,k}_{m+1,l}=E_{m,k}-E_{m+1,l}$. For $\Gamma^{+}_{qu}(\phi^l_n|\phi^k_m)$ [$\Gamma^{-}_{qu}(\phi^l_n|\phi^k_m)$], it is found that besides the transfer processes mastered by $\Gamma^{+}_{qu,\mathrm{RWT}}(\phi^l_{m-1}|\phi^k_m)$ [$\Gamma^{-}_{qu,\mathrm{RWT}}(\phi^l_{m-1}|\phi^k_m)$], the rate also describes the another distinct transition from state $|\phi^l_{m+1}\rangle$ ($|\phi^k_m\rangle$) to $|\phi^k_m\rangle$ ($|\phi^l_{m+1}\rangle$) in Fig. 1(c) [Fig. 1(d)], which is characterized by the rate component $\Gamma^{+}_{qu,\mathrm{CRWT}}(\phi^l_{m+1}|\phi^k_m)$ [$\Gamma^{-}_{qu,\mathrm{CRWT}}(\phi^l_{m+1}|\phi^k_m)$]. It is noted that this process is accompanied by the angular momentum transition from $|j,m+1\rangle$ ($|j,m\rangle$) to $|j,m\rangle$ ($|j,m+1\rangle$). This additional transfer process contributed by CRWTs will significantly affect the steady state features of the hybrid quantum system, e.g., nonequilibrium heat transfer and phonon statistics, even with weak qubit–bath dissipation.

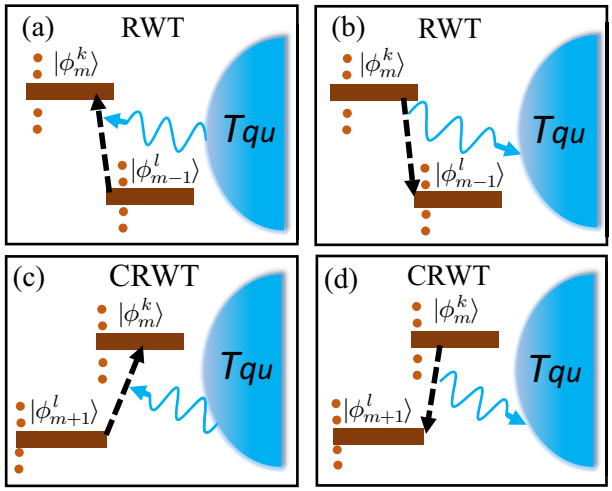


Fig. 1. Microscopic heat transfer processes assisted by the qu-th thermal bath. Under the effect of RWTs: (a) the excitation process from the extended coherent phonon state $|\phi^l_{m-1}\rangle$ to $|\phi^k_m\rangle$ by absorbing one boson from the qu-th bath with the energy $E^{m,k}_{m-1,l}$ characterized by the rate $\Gamma^{+}_{qu,\mathrm{RWT}}(\phi^l_{m-1}|\phi^k_m)$; (b) the relaxation process from the extended coherent phonon state $|\phi^k_m\rangle$ to $|\phi^l_{m-1}\rangle$ by releasing one boson into the qu-th bath with the energy $E^{m,k}_{m-1,l}$ characterized by the rate $\Gamma^{-}_{qu,\mathrm{RWT}}(\phi^l_{m-1}|\phi^k_m)$. Under the effect of CRWTs: (c) the excitation process from the extended coherent phonon state $|\phi^l_{m+1}\rangle$ to $|\phi^k_m\rangle$ by absorbing one boson from the qu-th bath with the energy $E^{m,k}_{m+1,l}$ characterized by the rate $\Gamma^{+}_{qu,\mathrm{CRWT}}(\phi^l_{m+1}|\phi^k_m)$; (d) the relaxation process from $|\phi^k_m\rangle$ to $|\phi^l_{m+1}\rangle$ by releasing one boson into the qu-th bath with the energy $E^{m,k}_{m+1,l}$ characterized by the rate $\Gamma^{-}_{qu,\mathrm{CRWT}}(\phi^l_{m+1}|\phi^k_m)$.

2.3. Quantum master equation combined with FCS

We apply the quantum master equation combined with full counting statistics (FCS) to study the steady state heat transfer. FCS[84] is considered as a powerful approach to measure the energy current and the current fluctuations,[9,19–22,55,85,86] which was initially proposed by L. Levitov *et al.* to investigate the electron current fluctuations.[87,88] It is based on the two-time measurement protocol. Here, we add the counting parameter to count the energy flow into the ph-th thermal bath based on the dressed master equation in Eq. (7), where the off-diagonal elements become negligible. Specifically, we first introduce the generalized density operator $\hat{\rho}(t,Q_t)$, where Q_t is the transferred energy into the ph-th bath during the time interval t. Accordingly, the master equation can be described as

$$\begin{aligned}&\mathrm{d}\hat{\rho}_s(t,Q_t)/\mathrm{d}t\\ &=\hat{\mathcal{L}}_0\hat{\rho}_s(t,Q_t)+\sum\{\Gamma^{+}_u(E^{mk}_{m'k'})\hat{\mathcal{L}}_{+}(|\phi^k_m\rangle\langle\phi^{k'}_{m'}|)\hat{\rho}^{+}_s(t,Q_t)\\ &\quad+\Gamma^{-}_u(E^{mk}_{m'k'})\hat{\mathcal{L}}_{-}(|\phi^{k'}_{m'}\rangle\langle\phi^k_m|)\hat{\rho}^{-}_s(t,Q_t)\},\end{aligned} \tag{15}$$

with $\hat{\rho}^{\pm}_s(t,Q_t)=\hat{\rho}_s(t,Q_t\mp E^{mk}_{m'k'})$. The probability of counting the energy Q_t after the time t is given by $P(t,Q_t)=\mathrm{Tr}\{\hat{\rho}_s(t,Q_t)\}$. Then, we apply a Fourier transformation in the energy space by including the counting parameter χ via $\hat{\rho}_s(t,\chi)=\sum_{Q_t}\hat{\rho}_s(t,Q_t)\mathrm{e}^{\mathrm{i}\chi Q_t}$. This leads to the modified dressed master equation

$$\begin{aligned}&\mathrm{d}\hat{\rho}_s(t,\chi)/\mathrm{d}t\\ &=\hat{\mathcal{L}}_0\hat{\rho}_s(t,\chi)+\sum\{\Gamma^{+}_u(E^{mk}_{m'k'})\hat{\mathcal{L}}^{\chi}_{+}(|\phi^k_m\rangle\langle\phi^{k'}_{m'}|)\hat{\rho}_s(t,\chi)\\ &\quad+\Gamma^{-}_u(E^{mk}_{m'k'})\hat{\mathcal{L}}^{\chi}_{-}(|\phi^{k'}_{m'}\rangle\langle\phi^k_m|)\hat{\rho}_s(t,\chi)\},\end{aligned} \tag{16}$$

where $\hat{\mathcal{L}}^{\chi}_{+}(|\phi^k_m\rangle\langle\phi^{k'}_{m'}|)=\mathrm{e}^{(-\mathrm{i}\chi E^{mk}_{m'k'}\delta_{u,ph})}\hat{\mathcal{L}}_{+}(|\phi^k_m\rangle\langle\phi^{k'}_{m'}|)$, $\hat{\mathcal{L}}^{\chi}_{-}(|\phi^{k'}_{m'}\rangle\langle\phi^k_m|)=\mathrm{e}^{(\mathrm{i}\chi E^{mk}_{m'k'}\delta_{u,ph})}\hat{\mathcal{L}}_{-}(|\phi^{k'}_{m'}\rangle\langle\phi^k_m|)$, $\delta_{u,ph}=1$ for $u=ph$, and $\delta_{u,ph}=0$ for $u=qu$. Then, the generating function is obtained as $\mathcal{G}(t,\chi)=\mathrm{Tr}\{\hat{\rho}_s(t,\chi)\}$. After long time evolution, the steady state cumulant generating function is given by $\mathcal{Z}(\chi)=\lim_{t\to\infty}\frac{1}{t}\ln[\mathcal{G}(\chi,t)]$.[84,86] Consequently, the n-th cumulant of steady state heat flow is given by

$$J^{(n)}=\partial^n\mathcal{Z}(\chi)/\partial(\mathrm{i}\chi)^n\big|_{\chi=0}. \tag{17}$$

In particular, the first cumulant is the heat current $J=\partial\mathcal{Z}(\chi)/\partial(\mathrm{i}\chi)|_{\chi=0}$, the second cumulant is the noise power $J^{(2)}=\partial^2\mathcal{Z}(\chi)/\partial(\mathrm{i}\chi)^2|_{\chi=0}$, and the third cumulant is the skewness $J^{(3)}=\partial^3\mathcal{Z}(\chi)/\partial(\mathrm{i}\chi)^3|_{\chi=0}$, which are the representative quantities to characterize the steady state heat transfer.

We plot the heat current at resonance in Fig. 2(a) as one typical instance to analyze the effect of the finite qubit number N on steady state behaviors. It is found that in the small qubit number limit (e.g., $N=1,2$) the heat current with the full interaction between the qubits and the qu-th bath is dramatically enhanced over a wide qubit–phonon coupling regime, compared to the counterpart affected by only RWTs. This

demonstrates the nontrivial contribution from CRWTs. However, as the qubit number becomes large (e.g., $N = 8$), the currents from two different types of qubit–bath interactions (i.e., RWTs and full interaction) become nearly identical, which implies that RWTs dominate the behavior of the heat current. Moreover, it needs to note that we set the truncation number of phonons N_{ph} to 30, which is sufficient to make the steady state quantities converge based on numerical calculations, e.g., the heat current with full qubit–bath interaction in Fig. 2(b). Therefore, we select $N = 1$ in the following to manifest the novel role of CRWTs.

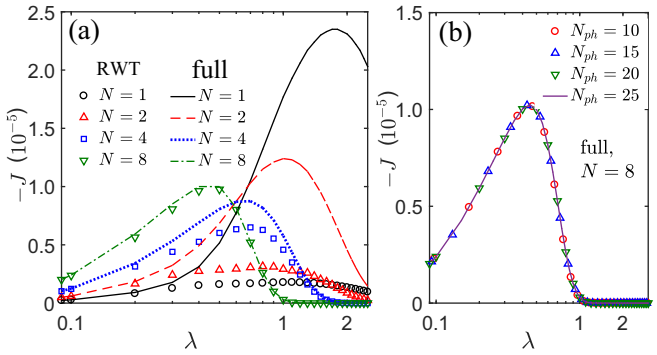


Fig. 2. Steady state heat current out of the *ph*-th thermal bath as a function of the qubit–phonon hybridization strength (a) under the effects of only RWTs and full qubit–bath interaction(including both RWTs and CRWTs) with various qubit numbers, and (b) by tuning the truncation number of phonons N_{ph} with full qubit–bath interaction and $N = 8$. The other system parameters are given by $\varepsilon = 1$, $\omega_0 = 1$, $\alpha_{ph} = \alpha_{qu} = 0.005$, $T_{ph} = 1.5$, $T_{qu} = 0.5$, and $\omega_c = 10$.

3. Quantum heat transfer

In this section, we investigate heat current, noise power, and skewness at steady state, which are the representative characteristics of quantum heat transfer. In one previous work, we mainly investigated the steady state heat current and the effect of negative differential thermal conductance (NDTC) with weak qubit–phonon hybridization,[49] where the role of CRWTs was not explicitly explored. Here, we focus on the comparison of heat current fluctuations owning RWTs to the counterpart with full qubit–bath interaction, i.e., including both RWTs and CRWTs, at strong qubit–phonon hybridization.

We first study the steady state heat current out of the *ph*-th thermal bath ($-J$) in Fig. 3. Under the influence of RWTs, figure 3(a) exhibits a globally optimal peak at finite temperature bias ($\Delta T \approx 0.5$) and strong qubit–phonon hybridization strength ($\lambda \approx 2$). This clearly demonstrates that the effect of NDTC can also be observed with strong qubit–phonon hybridization with only RWTs. In sharp contrast, by including both RWTs and CRWTs we find that the heat current shows monontonic enhancement by increasing the temperature bias in the strong hybridization regime, as shown in Fig. 3(b). Moreover, we unravel such distinction from the aspect of heat current fluctuations. Specifically, for the noise power and skewness with only RWTs in Figs. 3(c) and 3(e), they are significantly suppressed at large temperature bias (e.g., $T_{ph} \approx 2$ and $T_{qu} \approx 0$), Whereas the counterparts show dramatic enhancement by including CRWTs in Figs. 3(d) and 3(f). Hence, we conclude that CRWTs nontrivially enhance the steady state heat transfer with strong qubit–phonon hybridization, particularly in large temperature bias regime.

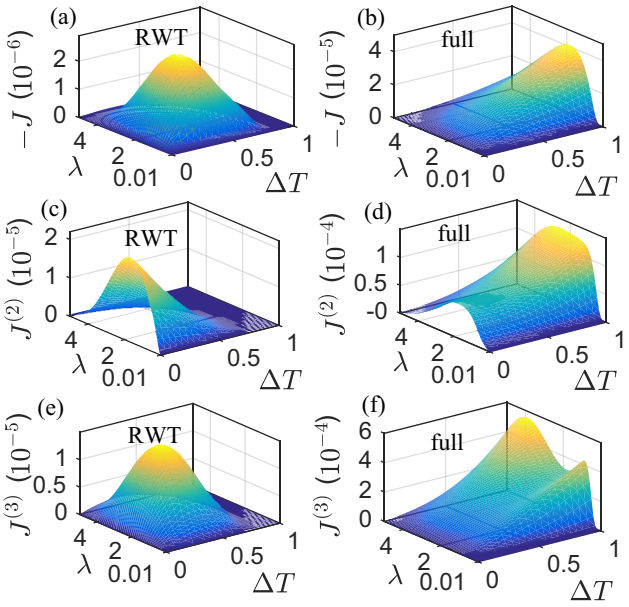


Fig. 3. (a) and (b) Steady state heat current ($-J$), (c) and (d) noise power $[J^{(2)}]$, (e) and (f) skewness $[J^{(3)}]$ by modulating both the temperature bias ΔT and qubit–phonon hybridization strength λ under the effects of RWTs and CRWTs, with $T_{ph} = T_0 + \Delta T$, $T_{qu} = T_0 - \Delta T$, and $T_0 = 1$. The other system parameters are given by $\varepsilon = 1$, $\omega_0 = 1$, $N = 1$, $\alpha_{ph} = \alpha_{qu} = 0.005$, and $\omega_c = 10$.

Next, we try to explore the underlying mechanism for the distinction of heat current fluctuations with two different types of qubit–bath dissipations. Under the effect of only RWTs at large temperature bias limit, the excitation transition quantified by the rate $\Gamma^{+}_{qu,\mathrm{RWT}}(\phi^{l}_{-\frac{1}{2}}|\phi^{k}_{\frac{1}{2}})$ naturally vanishes, due to $n_{qu}(E^{-\frac{1}{2},l}_{\frac{1}{2},k}) = 0$. Consequently, the relaxation transition from the extended coherent phonon state with higher angular momentum state ($|\frac{1}{2},\frac{1}{2}\rangle$) to the extended coherent phonon state with lower counterpart ($|\frac{1}{2},-\frac{1}{2}\rangle$) dominates the heat exchange processes between the dressed qubit and the *qu*-th bath, which is characterized by $\Gamma^{-}_{qu,\mathrm{RWT}}(\phi^{l}_{-\frac{1}{2}}|\phi^{k}_{\frac{1}{2}})$. This leads to the population depletion of the extended coherent phonon states associated with $P_{\frac{1}{2},k}$. And the nonzero steady state population becomes $P_{-\frac{1}{2},k} = \mathrm{e}^{-k\omega_0/(k_{\mathrm{B}}T_{ph})}[1 - \mathrm{e}^{-\omega_0/(k_{\mathrm{B}}T_{ph})}]$, which is fully thermalized by the *ph*-th bath. Finally, such depletion of spin-up branch population prevents the hybrid system from establishing the thermodynamic bias to drive steady state heat current and fluctuations, resulting in the persistent suppression of the current fluctuations.

While for steady state heat transfer including both RWTs and CRWTs at large temperature bias, though the spin flip-up transition accompanied by the energy excitation in Fig. 1(a)

driven by the RWTs is suppressed, such spin-flip transition can be alternatively realized by releasing one boson into the *qu*-th bath under the effect of CRWTs in Fig. 1(d). Hence, under the cooperative contributions of the *qu*-th bath assisted transfer processes in Figs. 1(b) and 1(d) and the processes accompanied by the *ph*-th bath characterized by the rates $\Gamma_{ph}^{\pm}(\phi_m^{k-1}|\phi_m^k)$ in Eq. (11), the steady state populations $P_{\frac{1}{2},k}$ can be dramatically excited by increasing the temperature T_{ph}. Simultaneously, a completely thermodynamic cycle can be restored. This could explain the enhancement of the heat current in Fig. 3(b) and current fluctuations in Figs. 3(d) and 3(f).

4. Two-phonon statistics

In the quantum theory of optical coherence, the zero-time delay two-photon correlation function was initially defined by R. J. Glauber as[89]

$$g_a^{(2)}(0)=\frac{\langle\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}\rangle}{\langle\hat{a}^\dagger\hat{a}\rangle^2}, \tag{18}$$

where $\hat{a}^\dagger$ ($\hat{a}$) creates (annihilates) one photon in the cavity, and $\langle\hat{A}\rangle$ denotes the expectation value of the operator $\hat{A}$. $g_a^{(2)}(0)$ is traditionally applied to study the statistical features of photons. Specifically, the super-Poisson distribution of photon–photon correlation is characterized as $g_a^{(2)}(0)>1$, which implies the bunching effect. While $g_a^{(2)}(0)<1$ as the two-photon statistics becomes sub-Poisson distribution, implying the antibunching effect. Moreover, the correlation function $g_a^{(2)}(0)=2$ for the thermal state.[90]

However, it was later proposed that such definition of $g_a^{(2)}(0)$ may only be properly adopted to study the photon statistics with weak light–matter hybridization.[50–52,91–94] As the light–matter interaction becomes strong, the two-photon correlation function should be modified in the dressed picture of the hybrid quantum system[50]

$$g^{(2)}(0)=\frac{\langle\hat{X}_+^2\hat{X}_-^2\rangle}{\langle\hat{X}_+\hat{X}_-\rangle^2}, \tag{19}$$

where the transition projector is $\hat{X}_-=-\mathrm{i}\sum_{k>j}\Delta_{kj}X_{jk}|\phi_j\rangle\langle\phi_k|$, $\hat{X}_+=(\hat{X}_-)^\dagger$, the energy gap is $\Delta_{kj}=E_k-E_j$, the transition coefficient is $X_{jk}=\langle\phi_j|(\hat{a}^\dagger+\hat{a})|\phi_k\rangle$, and $|\phi_k\rangle$ the eigenstate of the hybrid system. Physically, $\hat{X}_-$ ($\hat{X}_+$) describes the relaxing (exciting) transfer process from the eigenstate $|\phi_k\rangle$ ($|\phi_l\rangle$) to the eigenstate $|\phi_l\rangle$ ($|\phi_k\rangle$), which is bounded by $\Delta_{kj}>0$. For the qubit–phonon hybrid system, the transition projector under the extended coherent phonon state basis is generally expressed as

$$\hat{X}_-=-\mathrm{i}\omega_0\sum_{m,k}\sqrt{k}|\phi_m^{k-1}\rangle\langle\phi_m^k|, \tag{20}$$

and the definitions of $g_a^{(2)}(0)$ and $g^{(2)}(0)$ are different. However, in the weak qubit–phonon hybridization limit (i.e., $\lambda/\omega_0\approx0$), the transition operator is simplified to $\hat{X}_-=-\mathrm{i}\hat{a}$, and the one- and two-phonon correlation terms are specified as

$$\langle\hat{X}_+\hat{X}_-\rangle=\omega_0^2 n_{ph}(\omega_0),\quad \langle\hat{X}_+^2\hat{X}_-^2\rangle=2\omega_0^4 n_{ph}^2(\omega_0), \tag{21}$$

which leads to the zero-time delay two-phonon correlation function obtained as $g^{(2)}(0)=2$.

Here, we adopt the definition of the correlation function in Eq. (19) to investigate the two-phonon statistics at steady state in Fig. 4. We first study the phonon statistics at thermal equilibrium (i.e., $T_{ph}=T_{qu}=T_0$). The populations are given by $P_{m,k}=\mathrm{e}^{(-E_{m,k}/k_\mathrm{B}T_0)}/[\sum_{m,k}\mathrm{e}^{(-E_{m,k}/k_\mathrm{B}T_0)}]$ ($m=\pm1/2;k=0,1,2,\ldots$), which is valid both with and without CRWTs. Then, the average phonon number is $\langle\hat{X}_+\hat{X}_-\rangle=\omega_0^2 n_{ph}(\omega_0)$, the two-phonon correlation term is $\langle\hat{X}_+^2\hat{X}_-^2\rangle=2\omega_0^4 n_{ph}^2(\omega_0)$, and the two-phonon correlation function is $g^{(2)}(0)=2$. It should be noted that this result is analytically obtained for arbitrary qubit–phonon hybridization strength, which is distinct from the counterpart in Eq. (21) at weak qubit–phonon hybridization limit. Hence, the CRWTs show negligible contribution to the phonon statistics at thermal equilibrium.

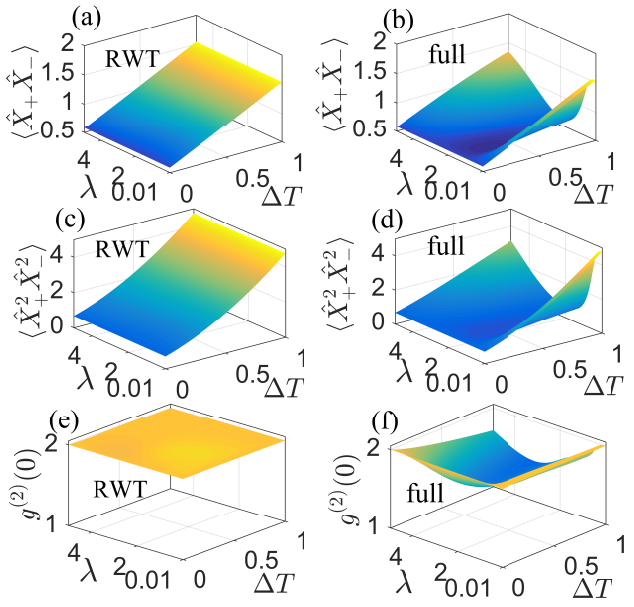


Fig. 4. Representative quantities of phonon statistics at steady state by tuning both the qubit–phonon hybridization strength λ and temperature bias ΔT, which include only RWTs and full interaction, respectively. (a) and (b) The expectation values of the phonon number $\langle\hat{X}_+\hat{X}_-\rangle$; (c) and (d) the two-phonon correlation terms $\langle\hat{X}_+^2\hat{X}_-^2\rangle$; (e) and (f) zero-time delay two-phonon correlation functions $g^{(2)}(0)$. The other system parameters are the same as those in Fig. 3.

Next, we investigate the phonon statistics at the finite thermodynamic bias. For the average phonon number with RWTs in Fig. 4(a), it is generally insensitive to the qubit–phonon hybridization strength. It shows linear increase by increasing ΔT, and becomes most significant in the bias limit $T_{ph}\approx2$ and $T_{qu}\approx0$. The reason is that in the low temperature regime of T_{qu}, the *qu*-th bath mainly assists the unidirectional transition from $|\frac{1}{2},\frac{1}{2}\rangle$ branch of extended coherent phonon states to the $|\frac{1}{2},-\frac{1}{2}\rangle$ branch of extended coherent phonon states. The mechanism is quite similar for the

two-phonon correlation term $\langle\hat{X}_+^2\hat{X}_-^2\rangle$, which is exhibited in Fig. 4(c). On the contrary, for $\langle\hat{X}_+\hat{X}_-\rangle$ with full qubit–bath interaction, it is interesting to find that by increasing the qubit–phonon hybridization strength to strong coupling (e.g., $\lambda = 2$), the average phonon number is dramatically suppressed, as shown in Fig. 4(b). Compared to the transitions only with RWTs, the full qubit–bath interaction includes additional transition channels to significantly cool down the phonon field, i.e., $|\phi^l_{-1/2}\rangle\rightarrow|\phi^k_{1/2}\rangle\rightarrow|\phi^{l'}_{-1/2}\rangle$ with the energy restriction $E^l_{-1/2} > E^k_{1/2} > E^{l'}_{-1/2}$ and phonon excitation number bias $(l-l')\geq 1$. Moreover, the existence of such collective transition paths also greatly decreases the magnitude of the two-phonon term $\langle\hat{X}_+^2\hat{X}_-^2\rangle$. This results in $g^{(2)}(0)$ apparently lower than 2. Therefore, we conclude that CRWTs generate additional novel transfer paths to suppress both the average phonon number and two-phonon correlation function.

5. Conclusion

To summarize, we have studied steady state statistics of the nonequilibrium qubit–phonon hybrid system by applying quantum master equation under the extended coherent phonon states, where the phonon–bath and qubit–bath dissipations are weak. At steady state, the natural disappearance of off-diagonal terms of the reduced hybrid system density matrix simplifies the generalized master equation to the dressed mater equation. For steady state heat transfer, we have adopted the full counting statistics to compare the heat current and current fluctuations under the effects of only RWTs and full qubit–bath interaction including both RWTs and CRWTs.

We first investigated the heat current at strong qubit–phonon hybridization. It has been found that the current always shows the behavior of NDTC under the effect of RWTs. In particular at the large temperature bias limit, the unidirectional transition from the extended coherent phonon states in spin-up branch to the counterparts in spin-down branch prevents the hybrid system from establishing a thermodynamic cycle, as shown in Fig. 1(b). Such suppression mechanism persists also for current fluctuations, e.g., noise power and skewness. On the contrary, the heat current and current fluctuations contributed by both RWTs and CRWTs exhibit monotonic increase by increasing the temperature bias. The novel transition controlled by CRWTs in Fig. 1(d) restores the thermodynamic cycle.

Moreover, we have analyzed the average phonon number and two-phonon correlation function at strong qubit–phonon hybridization and finite temperature bias. It has been shown that under the effect of RWTs the phonon is mainly thermally distributed in the extended coherent phonon states with spin-down branch. The average phonon number is approximately given by $\langle\hat{X}_+\hat{X}_-\rangle = \omega_0^2 n_{ph}(\omega_0)$, and the two-phonon correlation function is $g^{(2)}\approx 2$, which are both nearly insensitive to the qubit–phonon hybridization strength. While including CRWTs, we have discovered that the average phonon number and the two-phonon correlation function are significantly cooled down in the optimally strong hybridization regime, due to the collective energy down transition controlled by the processes in Figs. 1(b) and 1(d).

Appendix A: Generalized quantum master equation

By individually perturbing the phonon–bath and qubit–bath interactions in Eqs. (3) and (4), we obtain the generalized master equation under the Born–Markov approximation as

$$\begin{aligned}\frac{\mathrm{d}\hat{\rho}_s(t)}{\mathrm{d}t} = &-\mathrm{i}[\hat{H}_s,\hat{\rho}_s(t)]\\ &+\frac{1}{2}\sum_{u=ph,qu;\omega'>0}\Big\{\kappa_u^+(\omega')\big[\hat{S}_u^\dagger\hat{\rho}_s(t)\hat{S}_{u,<}(-\omega')+\mathrm{H.c.}\big]\\ &+\kappa_u^-(\omega')\big[\hat{S}_u\hat{\rho}_s(t)\hat{S}_{u,<}^\dagger(-\omega')+\mathrm{H.c.}\big]\\ &-\kappa_u^+(\omega')\big[\hat{S}_u\hat{S}_{u,<}^\dagger(-\omega')\hat{\rho}_s(t)+\mathrm{H.c.}\big]\\ &-\kappa_u^-(\omega')\big[\hat{S}_u^\dagger\hat{S}_{u,<}(-\omega')\hat{\rho}_s(t)+\mathrm{H.c.}\big]\Big\},\end{aligned}\tag{A1}$$

where the operators are $\hat{S}_{ph} = \hat{a}$, $\hat{S}_{qu} = \hat{S}$, the components are obtained as $\hat{S}_u(-\tau) = \sum_{\omega>0}[\hat{S}_{u,>}(\omega)\mathrm{e}^{-\mathrm{i}\omega\tau} + \hat{S}_{u,<}(-\omega)\mathrm{e}^{\mathrm{i}\omega\tau}] + \sum_{\omega=0}\hat{S}_{u,0}$, and the transition rates between two extended coherent phonon states are $\kappa_u^+(\omega) = \gamma_u(\omega)n_u(\omega)$, $\kappa_u^-(\omega) = \gamma_u(\omega)[1+n_u(\omega)]$, with the Bose–Einstein distribution function $n_u(\omega) = 1/[\exp(\omega/k_\mathrm{B}T_u)-1]$.

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