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Citation: Chin. Phys. B, 2021, 30 (**3**): 030308. DOI: 10.1088/1674-1056/abcf4a
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Nonlocal advantage of quantum coherence in a dephasing channel with memory*

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(Received 10 September 2020; revised manuscript received 2 October 2020; accepted manuscript online 1 December 2020)

We investigate nonlocal advantage of quantum coherence (NAQC) in a correlated dephasing channel modeled by the multimode bosonic reservoir. We obtain analytically the dephasing and memory factors of this channel for the reservoir having a Lorentzian spectral density, and analyze how they affect the NAQC defined by the l_1 norm and relative entropy. It is shown that the memory effects of this channel on NAQC are state-dependent, and they suppress noticeably the rapid decay of NAQC for the family of input Bell-like states with one excitation. For the given transmission time of each qubit, we also obtain the regions of the dephasing and memory factors during which there is NAQC in the output states.

Keywords: quantum coherence, correlated quantum channel, memory effects

PACS: 03.67.Mn, 03.65.Ta, 03.65.Yz **DOI:** 10.1088/1674-1056/abcf4a

1. Introduction

Quantum coherence is an embodiment of the superposition principle of states. It characterizes quantumness of a state that marks the departure of quantum theory from classical physics.[1] Recent studies further reveal that quantum coherence underpins different forms of quantum correlations[2] and is also responsible for the power of emerging quantum communication and computation tasks.[3] Hence, it is crucial to characterize it quantitatively. Baumgratz *et al.*[4] made a first step toward this problem. They introduced the defining conditions for a faithful coherence measure, together with two coherence measures defined by the l_1 norm and the relative entropy. Such a progress stimulates a number of subsequent studies focused on quantifying coherence, such as the entanglement-based coherence measures,[5] the robustness of coherence,[6,7] the skew information measure of coherence,[8,9] and those convex roof measures of coherence.[10–12]

Starting from these coherence measures, the roles of coherent states in the quantum information processing tasks such as quantum state merging,[13] deterministic quantum computation with one qubit,[14] quantum heat engine,[15] quantum metrology,[6,9,16] and several quantum algorithms,[17,18] have been explored. Moreover, these coherence measures can also be used to interpret the wave–particle duality[19,20] and to detect quantum phase transitions in various spin-chain systems.[21–25]

The above coherence measures are also useful in interpreting quantum correlations.[2] Besides the entanglement-based coherence measures,[5] the progresses in this respect include the interpretations of quantum entanglement[12,26] and various discord-like quantum correlations[26–29] by considering distribution of quantum coherence among the subsystems. Another route for connecting quantum coherence to quantum correlations can be achieved by considering the steered coherence of a state.[30–33] In particular, by virtue of the mutually unbiased bases, Mondal *et al.*[31] introduced the nonlocal advantage of quantum coherence (NAQC) for a two-qubit state, which was subsequently generalized to the $(d\times d)$-dimensional states (d is a prime power),[32] and it is shown that it characterizes a type of quantum correlation stronger than entanglement. For the two-qubit states, the connection between NAQC and Bell nonlocality was also established.[33]

Due to its practical role in quantum information processing, it is significant to seek ways to protect the NAQC in the noisy environments. For the case of the surrounding noise acting independently on each qubit of a system, the freezing of coherence[34–37] and the decay of coherence[38–40] have already been studied. But if the time windows for a sequence of qubits being inside a channel are partially overlapped, the actions of this channel may

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11675129, 11774406, and 11934018), the National Key R&D Program of China (Grant Nos. 2016YFA0302104 and 2016YFA0300600), the Strategic Priority Research Program of Chinese Academy of Sciences (Grant No. XDB28000000), and the Research Program of Beijing Academy of Quantum Information Sciences (Grant No. Y18G07).

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be correlated. In fact, for the correlated Pauli channel introduced phenomenologically,[41–43] its influence on entropic uncertainty,[44] quantum coherence,[45,46] and quantum Fisher information[47] have been explored. In this paper, we consider a Hamiltonian model of correlated dephasing channel.[48–50] We will show that the memory effects arising from correlated actions of such a Hamiltonian dephasing channel on NAQC are state dependent, and the decay rate of NAQC for the input Bell-like states with only one excitation can be suppressed evidently.

2. Measures of the NAQC

The NAQC was introduced based on the resource theory of quantum coherence.[2] Specifically, for a two-qubit state ρ_{AB} shared between Alice and Bob who agree on three mutually unbiased observables $\{\sigma_{x,y,z}\}$ ($\sigma_{x,y,z}$ are the Pauli operators) in advance, the NAQC can be formulated in an operational way.[31] First, Alice chooses randomly one of $\sigma_{x,y,z}$ (e.g., σ_i with $i=x$, y, or z) and performs the local measurements $\Pi_i=\{\Pi_i^{\pm}\}$ with $\Pi_i^{\pm}=(\mathbb{1}_2\pm\sigma_i)/2$ on qubit A, then she communicates to Bob her choice σ_i and outcomes $a\in\{+,-\}$, based on which Bob first calculates the average coherence of his conditional states with respect to the eigenbasis of σ_j ($j\neq i$) as

$$\bar{C}_q^{\sigma_j}(\{p_{B|\Pi_i^a},\rho_{B|\Pi_i^a}\})=\sum_a p_{B|\Pi_i^a}C_q^{\sigma_j}(\rho_{B|\Pi_i^a}), \tag{1}$$

with $p_{B|\Pi_i^a}=\mathrm{tr}(\Pi_i^a\rho_{AB})$, $\rho_{B|\Pi_i^a}=\mathrm{tr}_A(\Pi_i^a\rho_{AB})/p_{B|\Pi_i^a}$, $C_q^{\mathcal{B}}(\cdot)$ is the coherence defined in the eigenbasis of $\mathcal{B}$, and the subscript q is introduced for distinguishing different coherence measures. Then, Bob further averages $\bar{C}_q^{\sigma_j}(\{p_{B|\Pi_i^a},\rho_{B|\Pi_i^a}\})$ over the two sets of the eigenbases of the remaining σ_j (e.g., $j=\{y,z\}$ for $i=x$, and likewise for other i), and after Alice's three measurements $\{\Pi_{x,y,z}\}$ with equal probability, the steered coherence at Bob's hand can be obtained as

$$C_q^{na}(\rho_{AB})=\frac{1}{2}\sum_{i,j,a}p_{B|\Pi_i^a}C_q^{\sigma_{j\neq i}}(\rho_{B|\Pi_i^a}). \tag{2}$$

By considering different coherence measures, one could obtain different $C_q^{na}(\rho_{AB})$. In this paper, we consider two faithful coherence measures introduced by Baumgratz *et al.*,[4] i.e., the l_1 norm of coherence and the relative entropy of coherence which are given, respectively, by

$$C_{l_1}^{\mathcal{B}}(\rho)=\sum_{i\neq j}|\rho_{ij}|,\quad C_{\mathrm{re}}^{\mathcal{B}}(\rho)=S(\rho_{\mathrm{diag}})-S(\rho), \tag{3}$$

where $\rho_{ij}=\langle i|\rho|j\rangle$, with $\{|i\rangle\}$ being eigenvectors of the Hermitian operator $\mathcal{B}$, ρ_{diag} is a density operator obtained by deleting the off-diagonal elements of ρ, $S(\rho)=-\mathrm{tr}(\rho\log_2\rho)$ is the von Neumann entropy of ρ, and likewise for $S(\rho_{\mathrm{diag}})$.

Mondal *et al.*[31] showed that whenever $C_q^{na}(\rho_{AB})$ exceeds a lower bound C_q^m, there is NAQC in ρ_{AB}, that is, the steered coherence at Bob's hand is not achievable by any single-qubit state. For the l_1 norm of NAQC, $C_{l_1}^m=\sqrt{6}$, and for the relative entropy of NAQC, $C_{\mathrm{re}}^m\simeq 2.232$.

3. Solution of the memory dephasing channel

We consider the case of a sequence of qubits transmitting a dephasing channel modeled by the overall Hamiltonian

$$\begin{aligned}H(t)=&\sum_\alpha\omega_\alpha b_\alpha^\dagger b_\alpha-\frac{1}{2}\sum_{\alpha,n}[\lambda f_n(t)b_\alpha^\dagger\sigma_z^{(n)}+\mathrm{H.c.}]\\&+\sum_{\alpha,n}\frac{\lambda^2}{4\omega_\alpha}\sigma_z^{(n)},\end{aligned} \tag{4}$$

where the first term on the right-hand side denotes the Hamiltonian of the bosonic reservoir, the second term characterizes the switchable coupling of the qubits to the reservoir, and the last term is a counter term.[51] Moreover, α labels the bosonic field mode with frequency ω_α, $b_\alpha^\dagger$ (b_α) is the bosonic creation (annihilation) operator, and $\sigma_z^{(n)}$ is the third Pauli operator for qubit n. The function $f_n(t)$, which equals 1 (0) when qubit n is inside (outside) the channel, is introduced for controlling the coupling between qubit n and the channel, with the strength being denoted by λ.

If the initial state $\rho^S(0)$ of the N qubits and that of the multimode bosonic field ρ^E are in a product form and ρ^E is initially in the thermal equilibrium state $\rho^E=\mathrm{e}^{-\beta H_E}/Z$ (H_E is the first term on the right-hand side of Eq. (4), Z is the partition function, $\beta=1/k_{\mathrm{B}}T$, with k_{B} being the Boltzmann constant and T the temperature), the state of the system at time t will be given by $\rho^{SE}(t)=U(t)[\rho^S(0)\otimes\rho^E]U^\dagger(t)$, with $U(t)=\mathrm{T}\mathrm{e}^{-\mathrm{i}\int_0^t H(t')\,\mathrm{d}t'}$ and T is the time-ordering operator. The output state of the N qubits can then be obtained by partial trace over E as[48]

$$\rho^S(t)=\sum_{jl}\mathrm{e}^{-\lambda^2\gamma_{jl}}\rho_{jl}^S(0)|j\rangle\langle l|, \tag{5}$$

where $|j\rangle=|j_1,j_2,\ldots,j_N\rangle$ ($j_k\in\{0,1\}$, $k=1,2,\ldots,N$), and likewise for $|l\rangle$. By denoting $S(\omega)$ for the spectral density of the multimode field, τ_{p} for the time duration of each qubit to cross the channel, and τ the time interval between two consecutive actions of the channel, the decoherence rate γ_{jl} in Eq. (5) can be obtained as[48]

$$\gamma_{jl}=\int_0^\infty\mathrm{d}\omega S(\omega)\frac{1-\cos(\omega\tau_{\mathrm{p}})}{\pi\omega^2}\left|\sum_{n=1}^N(j_n-l_n)\mathrm{e}^{\mathrm{i}\omega(n-1)\tau}\right|^2, \tag{6}$$

then it is clear to see that the diagonal elements of $\rho^S(t)$ remain unchanged as $\gamma_{jj}=0$ ($\forall j$).

For the special case of one qubit traversing the channel, one can obtain from Eq. (6) that $\gamma_{01}=\gamma_{10}=\gamma_0$, with

$$\gamma_0=\int_0^\infty\mathrm{d}\omega S(\omega)\frac{1-\cos(\omega\tau_{\mathrm{p}})}{\pi\omega^2}, \tag{7}$$

while for two qubits traversing the channel subsequently, one has $\gamma_{03,30}=\gamma_+$ and $\gamma_{12,21}=\gamma_-$, with

$$\gamma_\pm = 2\int_0^\infty \mathrm{d}\omega S(\omega)\frac{1-\cos(\omega\tau_\mathrm{p})}{\pi\omega^2}[1\pm\cos(\omega\tau)], \tag{8}$$

and the remaining γ_{jl} with $j\neq l$ equal γ_0. For $N\geqslant 3$, one can also obtain γ_{jl} from Eq. (6), but as we consider the NAQC of two qubits, we do not list them here.

The decoherence rates γ_0 and $\gamma_\pm$ are different for the channels with different spectral densities. In the following, we consider the Lorentzian spectrum

$$S(\omega)=\frac{2\tau_\mathrm{c}}{1+\omega^2\tau_\mathrm{c}^2}, \tag{9}$$

for which γ_0 and $\gamma_\pm$ can be derived analytically as

$$\begin{aligned}
\gamma_0 &= \tau_\mathrm{c}\Big(\tau_\mathrm{p}-\tau_\mathrm{c}+\tau_\mathrm{c}\mathrm{e}^{-\frac{\tau_\mathrm{p}}{\tau_\mathrm{c}}}\Big),\\
\gamma_+ &= \tau_\mathrm{c}\Big[3\tau_\mathrm{p}-\tau+|\tau-\tau_\mathrm{p}|+\tau_\mathrm{c}\Big(\mathrm{e}^{-\frac{\tau+\tau_\mathrm{p}}{\tau_\mathrm{c}}}+\mathrm{e}^{-\frac{|\tau-\tau_\mathrm{p}|}{\tau_\mathrm{c}}}\Big)\\
&\quad +2\tau_\mathrm{c}\Big(\mathrm{e}^{-\frac{\tau_\mathrm{p}}{\tau_\mathrm{c}}}-\mathrm{e}^{-\frac{\tau}{\tau_\mathrm{c}}}-1\Big)\Big],\\
\gamma_- &= \tau_\mathrm{c}\Big[\tau_\mathrm{p}+\tau-|\tau-\tau_\mathrm{p}|-\tau_\mathrm{c}\Big(\mathrm{e}^{-\frac{\tau+\tau_\mathrm{p}}{\tau_\mathrm{c}}}+\mathrm{e}^{-\frac{|\tau-\tau_\mathrm{p}|}{\tau_\mathrm{c}}}\Big)\\
&\quad +2\tau_\mathrm{c}\Big(\mathrm{e}^{-\frac{\tau_\mathrm{p}}{\tau_\mathrm{c}}}+\mathrm{e}^{-\frac{\tau}{\tau_\mathrm{c}}}-1\Big)\Big],
\end{aligned} \tag{10}$$

with τ_c being the correlation time scale of the channel.

From Eqs. (7) and (8), one can note that the memory effect of the channel originates from $\gamma_\pm\neq 2\gamma_0$, that is, the actions of the channel on two consecutive qubits cannot be factorized as those on each one. Then, it is natural to introduce the following factor:[49]

$$\mu=\frac{\gamma_+}{2\gamma_0}-1=1-\frac{\gamma_-}{2\gamma_0}, \tag{11}$$

to characterize such a memory effect, where the second equality is due to $\gamma_++\gamma_-=4\gamma_0$, see Eqs. (7) and (8). Moreover, we would like to remark here that the origin of this memory effect is different from that of the non-Markovian memory effect due to the temporal correlations in the evolution process of each qubit. It characterizes the memory related to the history of the multiple uses of a channel, whereas the non-Markovian memory effect characterizes the back-flow of information from the environment to the central system.[41]

From Eqs. (10) and (11), one can see that the memory factor μ is determined by τ/τ_c and $\tau_\mathrm{p}/\tau_\mathrm{c}$, so we display in Fig. 1 the τ dependence of μ with different τ_p (in units of τ_c). From this figure, one can see that for the given transition time τ_p of each qubit, μ decreases with the increasing time interval τ that separates two consecutive qubits entering the dephasing channel. Such an observation is understandable because with the increase of τ, the channel is reset to its initial state gradually, and in the limit of $\tau\gg\tau_\mathrm{p,c}$, one has $\gamma_\pm=2\gamma_0$ and $\mu=0$. Thus its actions on the two qubits are uncorrelated. In the limit of $\tau,\tau_\mathrm{p}\ll\tau_\mathrm{c}$, however, one has $\gamma_+=4\gamma_0$ and $\gamma_-=0$, which corresponds to $\mu=1$. So the consecutive actions of the channel are fully correlated. For the general case of $\mu\in(0,1)$, the channel has partial memory about its actions on two consecutive qubits passing through it.

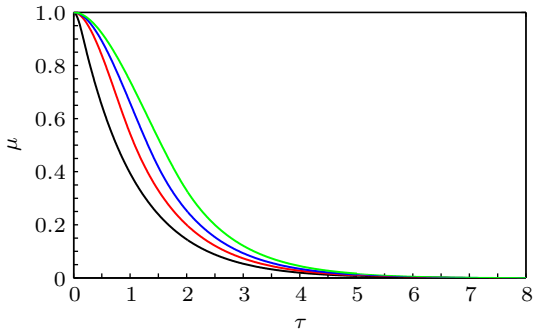


Fig. 1. The memory factor μ of the memory dephasing channel versus τ (in units of τ_c), where the lines from bottom to top correspond to $\tau_\mathrm{p}=0.5\tau_\mathrm{c}$, τ_c, $1.5\tau_\mathrm{c}$, and $2\tau_\mathrm{c}$, respectively.

4. NAQC in the memory dephasing channel

Based on the above preliminaries, we explore effects of the Hamiltonian dephasing channel on NAQC in this section. We consider the case of two qubits being prepared initially in one of the following Bell-like states:

$$\begin{aligned}
|\Psi\rangle &= \cos(\theta/2)|00\rangle+\mathrm{e}^{\mathrm{i}\phi}\sin(\theta/2)|11\rangle,\\
|\Phi\rangle &= \cos(\theta/2)|01\rangle+\mathrm{e}^{\mathrm{i}\phi}\sin(\theta/2)|10\rangle,
\end{aligned} \tag{12}$$

where $\theta\in[0,\pi]$ and $\phi\in[0,2\pi]$. By using Eq. (5), the corresponding output state ρ_+ for the initial state $|\Psi\rangle$ and ρ_- for the initial state $|\Phi\rangle$ can be obtained as

$$\begin{aligned}
\rho_+ &= \cos^2(\theta/2)|00\rangle\langle 00|+\sin^2(\theta/2)|11\rangle\langle 11|\\
&\quad +\frac{1}{2}h_+\sin\theta(\mathrm{e}^{-\mathrm{i}\phi}|00\rangle\langle 11|+\mathrm{e}^{\mathrm{i}\phi}|11\rangle\langle 00|),\\
\rho_- &= \cos^2(\theta/2)|01\rangle\langle 01|+\sin^2(\theta/2)|10\rangle\langle 10|\\
&\quad +\frac{1}{2}h_-\sin\theta(\mathrm{e}^{-\mathrm{i}\phi}|01\rangle\langle 10|+\mathrm{e}^{\mathrm{i}\phi}|10\rangle\langle 01|),
\end{aligned} \tag{13}$$

where the dephasing factors $h_\pm$ are given by

$$h_\pm=\mathrm{e}^{-\lambda^2\gamma_\pm}. \tag{14}$$

Then by the definition of NAQC in Eq. (2), one can obtain

$$\begin{aligned}
C_{l_1}^{na}(\rho_\pm) &= 1+h_\pm\sin\theta+\sqrt{\cos^2\theta+h_\pm^2\sin^2\theta\cos^2\phi},\\
C_\mathrm{re}^{na}(\rho_\pm) &= 1+H_2\left(\cos^2\frac{\theta}{2}\right)+H_2\left(\frac{1+h_\pm\sin\theta\sin\phi}{2}\right)\\
&\quad -2H_2\left(\frac{1+\sqrt{\cos^2\theta+h_\pm^2\sin^2\theta}}{2}\right),
\end{aligned} \tag{15}$$

where $H_2(x)=-x\log_2 x-(1-x)\log_2(1-x)$ denotes the binary Shannon entropy function. Then one can see that $C_{l_1}^{na}(\rho_\pm)$ are monotonic increasing functions of $h_\pm$, irrespective of θ and ϕ. Moreover, one can prove that $C_\mathrm{re}^{na}(\rho_\pm)$ are

also monotonic increasing functions of $h_\pm$ (see Appendix A). But their strengths depend strongly on θ and ϕ in the initial states, and for given $h_\pm$, they take their maxima for $\theta = \pi/2$ and $\phi = \{0, \pi\}$, which correspond to the maximally entangled input states.

As for the initial state with fixed θ and ϕ, $C_q^{na}(\rho_\pm)$ are monotonic increasing functions of $h_\pm$, we display in Fig. 2 the τ dependence of $h_\pm$ with different τ_p. By comparing the lines from top to bottom, one can see that they decrease with the increase of τ_p, which can be understood by showing that $\partial\gamma_\pm/\partial\tau_p > 0$. For the fixed τ_p, however, h_+ and h_- exhibit completely different dependences on τ. While h_+ increases with the increase of τ, h_- decreases with the increase of τ. By combining this with the τ dependence of μ as was shown in Fig. 1, one can obtain that with the increasing strength of the memory factor μ, h_+ is decreased and h_- is increased. This, together with Eq. (15), indicates that the decay rate of $C_q^{na}(\rho_+)$ for the initial state $|\Psi\rangle$ will be accelerated due to the correlated actions of this channel. Therefore, in order to protect the NAQC, one could tune the time interval τ so that there is no superposition between the time windows for the two qubits being inside the channel simultaneously. If the two qubits are prepared initially in $|\Phi\rangle$, however, the decay rate of $C_q^{na}(\rho_-)$ will be greatly suppressed, and for such a situation, the memory effect arising from the superposition of the time windows for two consecutive qubits being inside the channel simultaneously is beneficial for postponing the rapid decay of NAQC. This shows that the memory effects of this channel on NAQC are state dependent. They play a positive role for certain input states and a negative role for the other input states.

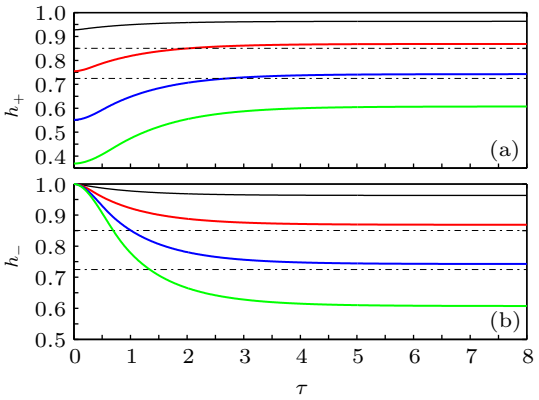


Fig. 2. The dephasing factors (a) h_+ and (b) h_- of the memory dephasing channel versus τ (in units of λ and τ_c), where the solid lines from top to bottom correspond to $\tau_p = 0.2\tau_c$, $0.4\tau_c$, $0.6\tau_c$, and $0.8\tau_c$, respectively, while the horizontal dash-dotted lines at $h_\pm = 0.7247$ (0.8502) show the critical $h_{\pm,cr}$ larger than which there is l_1 norm (relative entropy) of NAQC in $\rho_\pm$.

The different dependence of $C_q^{na}(\rho_\pm)$ on the memory factor μ could also be understood by combining Eqs. (11) and (14), from which one can obtain

$$h_\pm = \mathrm{e}^{-2\lambda^2\gamma_0(1\pm\mu)}. \tag{16}$$

Hence, h_+ (h_-) decreases (increases) with the increasing memory factor μ, which further induces the decrease (increase) of $C_q^{na}(\rho_+)$ [$C_q^{na}(\rho_-)$] with an increase in μ.

Moreover, when $C_{l_1}^{na}(\rho_\pm) > \sqrt{6}$ or $C_{re}^{na}(\rho_\pm) \gtrsim 2.232$, there is NAQC in $\rho_\pm$. By fixing $\theta = \pi/2$ and $\phi = 0$, which corresponds to the initial Bell states, equation (15) reduces to

$$C_{l_1}^{na}(\rho_\pm) = 1 + 2h_\pm,\ C_{re}^{na}(\rho_\pm) = 3 - 2H_2\left(\frac{1+h_\pm}{2}\right), \tag{17}$$

and an exemplified plot of $C_{l_1}^{na}(\rho_\pm)$ versus τ is shown in Fig. 3 (the dependence of $C_{re}^{na}(\rho_\pm)$ on τ is qualitatively the same). From Eq. (17), one can see that there exist critical $h_{\pm,cr}$ smaller than which the NAQC will vanish. When considering the l_1 norm of NAQC, one has $h_{\pm,cr} = (\sqrt{6}-1)/2$. For state ρ_-, as $C_{l_1}^{na}(\rho_-)$ decreases with the increase of both τ and τ_p, one can obtain that if τ_p is shorter than a critical value, there is l_1 norm of NAQC in the whole region of τ (see Fig. 3). Such a critical τ_p is determined by the following equation:

$$1 + 2\mathrm{e}^{-2\lambda^2\tau_c(t_p-\tau_c+\tau_c\mathrm{e}^{-\tau_p/\tau_c})} > \sqrt{6}, \tag{18}$$

which corresponds to $\tau \to \infty$, and can be solved numerically as $\tau_p \lesssim 0.6265$ (in units of λ and τ_c). When $\tau_p \gtrsim 0.6265$, however, there exists a critical time interval τ_{cr} longer than which there will be no l_1 norm of NAQC in ρ_-. As is shown by the blue circles in the top panel of Fig. 4, τ_{cr} decreases monotonically with the increase of τ_p and approaches asymptotically to a steady value when $\tau_p \to \infty$. Such a steady value is about 0.6265 which can be obtained numerically by using Eqs. (10), (14), and (17). As μ always decreases with the increase of τ (see Fig. 1), the above observation also shows that for fixed transition time τ_p of each qubit, the memory factor μ enabling NAQC in ρ_- should be stronger than a critical value, and such a critical value increases with the increasing τ_p.

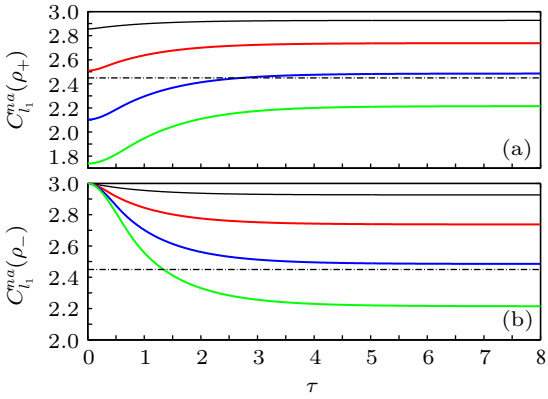


Fig. 3. (a) $C_{l_1}^{na}(\rho_+)$ and (b) $C_{l_1}^{na}(\rho_-)$ versus τ (in units of λ and τ_c) in the memory dephasing channel, where the parameters $\theta = \pi/2$ and $\phi = 0$. The solid lines from top to bottom correspond to $\tau_p = 0.2\tau_c$, $0.4\tau_c$, $0.6\tau_c$, and $0.8\tau_c$, respectively, while the horizontal dash-dotted lines represent the bound $C_{l_1}^m$.

For state ρ_+, as $C_{l_1}^{na}(\rho_+)$ increases with an increase in τ (Fig. 3) and $h_+ = h_-$ when $\tau \gg \tau_{p,c}$, there is no l_1 norm of NAQC in the whole region of τ if $\tau_p \gtrsim 0.6265$. If $\tau_p \lesssim 0.4299$,

which is obtained by solving the equation (in units of λ and τ_c)

$$1+2\,\mathrm{e}^{-4\lambda^2\tau_c(\tau_p-\tau_c+\tau_c\,\mathrm{e}^{-\tau_p/\tau_c})}>\sqrt{6}, \tag{19}$$

there will be NAQC in the whole region of τ. In the region of $\tau_p\in(0.4299,0.6265)$, there exists a critical time interval τ_{cr} shorter than which there will be no l_1 norm of NAQC in ρ_+. As is shown by the red squares in the top panel of Fig. 4, such a τ_{cr} increases with the increase of τ_p, which is contrary to that of τ_{cr} for ρ_-. It indicates that for given transition time τ_p, the memory factor μ enabling NAQC in ρ_+ should be weaker than a critical value.

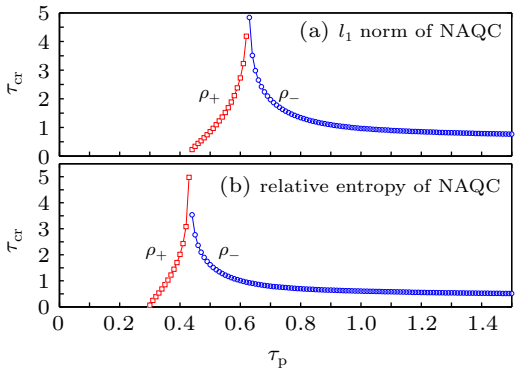


Fig. 4. The critical time interval τ_{cr} longer (shorter) than which there will be no NAQC in ρ_- (ρ_+) versus τ_p (in units of λ and τ_c). Note that when $\tau_p\lesssim 0.6265$ (0.4318), one always has the l_1 norm (relative entropy) of NAQC in ρ_-, when $\tau_p\lesssim 0.4299$ (0.2991), one always has the l_1 norm (relative entropy) of NAQC in ρ_+, and when $\tau_p\gtrsim 0.6265$ (0.4299), there will be no l_1 norm (relative entropy) of NAQC in ρ_+ in the whole region of τ.

Similarly, when considering the relative entropy of NAQC, the critical $h_{\pm,cr}$ can be obtained approximately as 0.8502, and one can see from the bottom panel of Fig. 4 that τ_{cr} exhibits qualitatively the same τ_p dependence as that for the l_1 norm of NAQC. The only difference is that the corresponding critical τ_p is shifted to the short-time region, and τ_{cr} for $C_{re}^{na}(\rho_-)$ approaches asymptotically to the steady value of about 0.4318 in the infinite limit of τ_p.

One may further explore NAQC in the correlated dephasing channel defined phenomenologically,[43] for which decoherence of the maximally coherent input states has been studied.[46] Similarly, for the input Bell states, one can obtain

$$\begin{aligned} C_{l_1}^{na}(\rho)&=3-8(1-\mu)(p-p^2),\\ C_{re}^{na}(\rho)&=3-2H_2[2(1-\mu)(p-p^2)], \end{aligned} \tag{20}$$

where $p\in[0,1]$ is a decoherence factor determined by the decoherence rate of this channel and the transition time of each qubit.[43] One can see immediately that the two NAQC measures are increasing functions of the memory factor μ, that is, the correlated actions of this channel are beneficial for enhancing the NAQC for all the four input Bell states. Such a behavior is clearly different from that of the NAQC in the Hamiltonian dephasing channel considered above.

Finally, we make a comparison of the NAQC with entanglement and Bell nonlocality. For the output states $\rho_\pm$, one can obtain the concurrence (a measure of entanglement)[52] and the maximally Bell-inequality violation[53] as

$$C(\rho_\pm)=h_\pm|\sin\theta|,\quad B_{\max}(\rho_\pm)=1+h_\pm^2\sin^2\theta, \tag{21}$$

which are independent of the phase factor ϕ. In particular, the two qubits are maximally entangled and Bell nonlocal for $\theta=\pi/2$ and $h_\pm=1$, whereas the two measures of NAQC are given by $C_{l_1}^{na}(\rho_\pm)=2+|\cos\phi|$ and $C_{re}^{na}(\rho_\pm)=2+H_2(1/2+\sin\phi/2)$, both of which take their maxima of 3 only when $\phi=0$ and π. Hence, the maximally entangled and Bell nonlocal states may not always be the maximally NAQC states. There is even no NAQC for certain maximally entangled two-qubit states, e.g., $C_{l_1}^{na}(\rho_\pm)>\sqrt{6}$ only when $|\cos\phi|>\sqrt{6}-2$. This confirms a previous observation that the NAQC captures a kind of quantum correlation which is stronger than entanglement[31] and Bell nonlocality.[33]

5. Summary

We have investigated the two-qubit NAQC in a dephasing channel modeled by the multimode bosonic field. We first obtained analytically the dephasing and memory factors of this channel for the bosonic field having a Lorentzian power spectrum, and analyzed in detail their dependence on the transmission time of each qubit, the time interval of two consecutive qubits entering the channel, and the characteristic correlation time of the field. Then, by considering the NAQC defined by virtue of the l_1 norm and the relative entropy, and choosing two types of Bell-like input states, we showed explicitly that the memory effects of this dephasing channel on NAQC are state dependent. For certain input states, the memory effects are beneficial for maintaining the NAQC, whereas for the other input states, they are detrimental to the NAQC. We have also analyzed the critical dephasing factors $h_\pm$ smaller than which there will be no NAQC in the output states $\rho_\pm$, as well as the corresponding critical memory factor μ. Moreover, we have presented an analysis of the decay behaviors of the NAQC with that of the entanglement and Bell nonlocality for the same two qubits, and showed that they exhibit qualitatively the same dependence on the dephasing factors, but the maximally entangled states may not always be the maximal NAQC states.

Appendix A: Proof of the monotonicity of the relative entropy of SQC

For $C_{re}^{na}(\rho_\pm)$, its first-order derivative with respect to $h_\pm$ can be obtained as

$$\begin{aligned} \frac{\partial C_{re}^{na}(\rho_\pm)}{\partial h_\pm}&=\frac{\sin\theta\sin\phi}{2}\log_2\frac{1-h_\pm\sin\theta\sin\phi}{1+h_\pm\sin\theta\sin\phi}\\ &\quad-\frac{h_\pm\sin^2\theta}{\sqrt{\alpha}}\log_2\frac{1-\sqrt{\alpha}}{1+\sqrt{\alpha}}, \end{aligned} \tag{A1}$$

where we have defined $\alpha = \cos^2\theta + h_\pm^2 \sin^2\theta$. One can see that $\partial C_{\rm re}^{na}(\rho_\pm)/\partial h_\pm$ are independent of the sign of $\sin\phi$, so we only need to consider the case of $\phi \in [0,\pi]$.

By using the fact that $\log_2[(1-x)/(1+x)]$ is a monotonically decreasing function of x for $x \in [0,1]$ and $\sin\phi \in [0,1]$ (note that $\phi \in [0,\pi]$), one can obtain

$$\frac{\sin\phi}{2}\log_2\frac{1-h_\pm\sin\theta\sin\phi}{1+h_\pm\sin\theta\sin\phi} \geqslant \log_2\frac{1-h_\pm\sin\theta}{1+h_\pm\sin\theta}, \tag{A2}$$

and because $(1/x)\log_2[(1-x)/(1+x)]$ is also a monotonically decreasing function of x for $x \in [0,1]$ and $h_\pm \sin\theta \leqslant \sqrt{\alpha}$ (note that $\theta \in [0,\pi]$), one has

$$\frac{1}{h_\pm\sin\theta}\log_2\frac{1-h_\pm\sin\theta}{1+h_\pm\sin\theta} \geqslant \frac{1}{\sqrt{\alpha}}\log_2\frac{1-\sqrt{\alpha}}{1+\sqrt{\alpha}}. \tag{A3}$$

Then by using the above three equations, one can show directly that $\partial C_{\rm re}^{na}(\rho_\pm)/\partial h_\pm \geqslant 0$, thus $C_{\rm re}^{na}(\rho_\pm)$ are monotonic increasing functions of $h_\pm$.

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