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Discrete wavelet structure and discrete energy of classical plane light waves

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We find by the wavelet transform that the classical plane light wave of linear polarization can be decomposed into a series of discrete Morlet wavelets. In the theoretical frame, the energy of the classical light wave becomes discrete; interestingly, the discretization is consistent with the energy division of P portions in Planck radiation theory, where P is an integer. It is shown that the changeable energy of a basic plane light wave packet or wave train is $H_{0k} = np_{0k}\omega$ ($n = 1,2,3,\ldots; k = |\boldsymbol{k}|$), with discrete wavelet structure parameter n, wave vector $\boldsymbol{k}$ and idler frequency ω, and a constant p_{0k}. The wave-particle duality from the Mach–Zehnder interference of single photons is simulated by using random basic plane light wave packets.

Keywords: classic plane light wave, discrete wavelet structure, discrete energy

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1. Introduction

One of the apparitions in physics is the wave–particle duality of particles.[1,2] The property of light quantum (photon) is a typical example.[1–4] Although there have been Dirac,[5] Gupta,[6] and Fermi's[7] approaches towards quantizing the radiation field and various models presented for photon,[8–13] it is still a challenge to understand the property of photon thoroughly due to its multi-faceted and elusive nature.[14,15] The difficulty comes mainly from the depiction of its wave feature. Unlike an electron, whose wave state can be described by a coordinates function (the probability amplitude of spatial localization), the single photon with energy $\hbar\omega$ has no analogous probability amplitude available[16] despite a light wave involving the photon. It is known that the interaction between a light wave and a matter will result in discrete energy exchange in the basic unit of $\hbar\omega$. However, it is still a question on what change will happen about the wave state of light due to the absorption of photons. The following thought experiment may be enlightened for giving an insight into the property of photon(s). In the thought experiment, a coherent plane light wave with frequency ω is incident onto an absolute blackbody, and then the state change of the light wave is monitored during the process of light absorption. The question then arises: what happens to the plane light wave when the photons are absorbed by the absolute blackbody? Does the amplitude of the whole plane wave change constantly, or does the wave disappear segment by segment? In our experiment, a chopped beam of He–Ne laser is incident onto a photomultiplier tube in a black cavity, which shows that the absorption of photons in the front of the light wave is impossible to affect the amplitude of the succeeding part of the light wave. This suggests that the plane light wave would disappear segment by segment during the process of light absorption. And it also indicates that, by the time inversion, the plane light wave could be reconstructed by the translation and superposition of a segment by segment wave packet or wave train. This leads to another question: what method can realize the reconstruction? The wavelet transform[17] should be a good candidate. In fact, the wavelet transform has been adopted to investigate physical problems for many years.[17–19] And Morlet wavelet function has been widely used owing to its advantage in simplicity and time-frequency analysis.[20] It has been found that the wavelet is more suitable for present physics. In this paper, the wavelet transform is used to decompose the classical linearly polarized plane light wave into a series of discrete Morlet wavelets (or basic wavelets). And we find that the energy of the light wave can be related to its discrete wavelet structure and can be discrete as well. Finally, the random light wave packets are used to simulate the Mach–Zehnder interference of single photons, showing the wave-particle duality of light.

2. Discrete wavelet structure and discrete energy of classical plane light waves

Here, we start to investigate the discrete wavelet structure and discrete energy of classical plane light waves. We first consider the radiation field in free space. As well known, the classical radiation fields in free space are governed by Maxwell's equations. The light wave equations for the elec-

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tric field can be derived from Maxwell's equations as follows:

$$\nabla^2 \boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = 0, \tag{1}$$

$$\nabla \cdot \boldsymbol{E} = 0, \tag{2}$$

where $\boldsymbol{E}$ is the electric field, and c is the speed of light in vacuum. By solving Eq. (1), a plane wave solution of wavelength λ and wave vector $\boldsymbol{k}$ can be found and expressed as

$$\boldsymbol{E}_k(\boldsymbol{r},t) = q_k(t)\boldsymbol{E}_k(\boldsymbol{r}) + q_k^*(t)\boldsymbol{E}_k^*(\boldsymbol{r}), \tag{3}$$

where $q_k(t) \propto \mathrm{e}^{-\mathrm{i}\omega t}$, $\boldsymbol{E}_k(\boldsymbol{r}) = \boldsymbol{E}_{k0}\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}$, and $\boldsymbol{E}_{k0}$ is a constant vector with two independent polarized components and perpendicular to wave vector $\boldsymbol{k}$ due to Eq. (2); $q_k^*(t)$ and $\boldsymbol{E}_k^*(\boldsymbol{r})$ are conjugate complexes of $q_k(t)$ and $\boldsymbol{E}_k(\boldsymbol{r})$, respectively. The general radiation field can be expressed as the linear superposition of all possible plane waves. Now we try to decompose the electric field of the plane light wave into a series of wavelets. For the sake of simplicity, we consider a linearly polarized monochromatic plane wave propagating along the z-direction, with a spatial function $E_k(\boldsymbol{r}) = E_k(z) = E_{k0}\mathrm{e}^{\mathrm{i}k\cdot z}$, $\lambda = 2\pi/k = 500$ nm. And the simplified Morlet wavelet function is used for the wavelet transform, which is[21]

$$\psi_K(z) = \mathrm{e}^{\mathrm{i}Kz}\mathrm{e}^{-z^2/(2s)}, \tag{4}$$

where K is the central wave number of the Morlet wavelet, and s is a parameter. For the function $E_k(z)$ with $k = K$, the wavelet transform can be expressed as[17]

$$\begin{aligned} Wf(a,b) &= \frac{E_{k0}}{\sqrt{a}}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}kz}\mathrm{e}^{-\mathrm{i}k(\frac{z-b}{a})}\mathrm{e}^{-\left(\frac{z-b}{a\sqrt{2s}}\right)^2}\mathrm{d}z \\ &= E_{k0}\sqrt{as}\,\mathrm{e}^{\mathrm{i}bk}\mathrm{e}^{-s(1-a)^2k^2/2}. \end{aligned} \tag{5}$$

We notice that the inverse wavelet transform[17] can be performed only by translation of wavelets, i.e., with $a = 1$, the function $E_k(z)$ can be reconstructed as

$$\begin{aligned} E_k(z) &= \frac{E_{k0}}{C_\psi}\int_{-\infty}^{\infty}\sqrt{s}\,\mathrm{e}^{\mathrm{i}bk}\,\mathrm{e}^{\mathrm{i}k(z-b)}\,\mathrm{e}^{-(\frac{z-b}{\sqrt{2}s})^2}\,\mathrm{d}b \\ &= E_{k0}\,\mathrm{e}^{\mathrm{i}kz}\frac{1}{C_\psi}\int_{-\infty}^{\infty}\sqrt{s}\,\mathrm{e}^{-\frac{(z-b)^2}{2s}}\,\mathrm{d}b, \end{aligned} \tag{6}$$

where C_ψ is a constant, obviously $C_\psi = \int_{-\infty}^{\infty}\sqrt{s}\,\mathrm{e}^{-(z-b)^2/2s}\,\mathrm{d}b$. It means that a plane light wave can be reconstructed by a series of wavelets through translation and superposition. This is the case of the continuous wavelet transform. However, we are interested in the discrete case. The theory of discrete wavelet transform is rather complicated. Here we simplify the way according to our experience. The key is to choose a suitable parameter s. We find that when $s = (\lambda/c_1)^2/2$, $c_1 = 0.886231921$, Eq. (6) can be discretized as

$$E_k(z) = \frac{E_{k0}}{2}\mathrm{e}^{\mathrm{i}kz}\lim_{n\to\infty}\sum_{r=-n}^{n+1}\mathrm{e}^{\frac{-(z-r\lambda+\lambda/2)^2}{2s}}, \tag{7}$$

where n is the discrete wavelet structure parameter. Equation (7) shows the discrete wavelet structure of the plane light wave, which is the translation and superposition of infinite wavelets. In order to see clearly the way to reconstruct the plane light wave, we consider the cases of a finite n. Figure 1 shows the results of wavelet reconstruction, where Figs. 1(a)–1(d) show the cases for $n = 0$, 1, 2, and 6, respectively. The dotted line in the figure is the waveform of the plane light wave, while the solid lines denote the reconstructed light wave packets or wave trains with discrete wavelet structure. One can see that there is not a complete period for the case of $n < 2$.

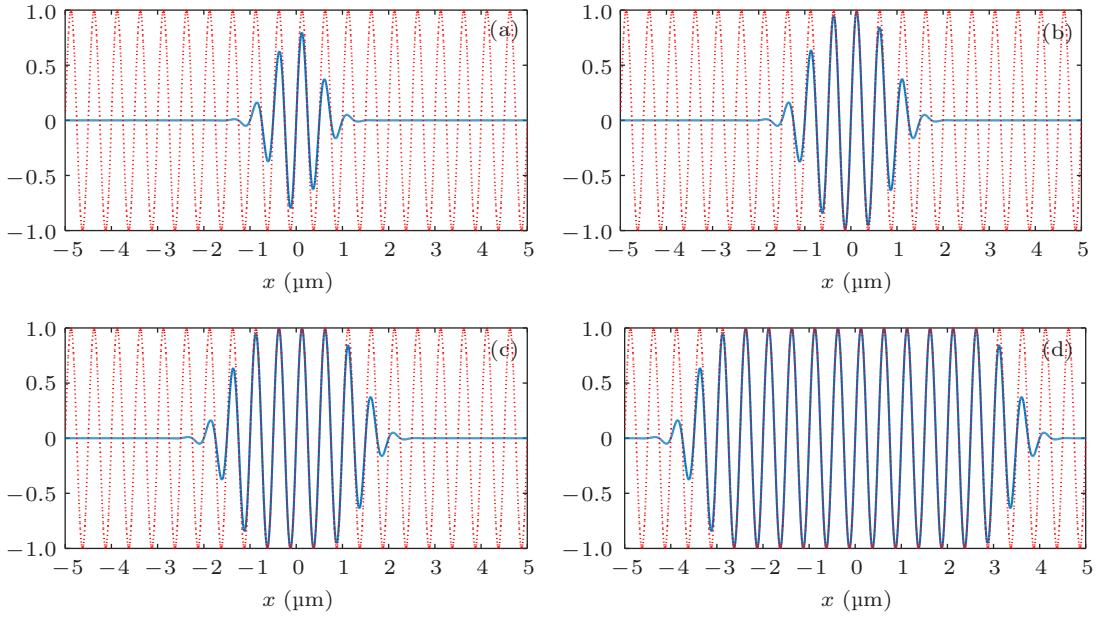


Fig. 1. Morlet wavelet reconstruction of the classical plane wave with $E_{k0} = 1$ V/m. The dotted line in the figure is the waveform of the plane light wave. Solid lines are the reconstructed light wave packets or wave trains with discrete wavelet structure. (a)–(d) show the cases for $n = 0$, 1, 2, and 6 in Eq. (7), respectively.

But when $n=2$, there are two complete periods. When n increases by 1, two complete periods will be added. When $n=6$, there are 10 complete periods. The larger n is, the more complete periods appear. When $n\to\infty$, the whole plane wave is reconstructed finally. These results will be used below. It is easy to check that, if n is finite in Eq. (7), by making the transformation $z\to z-ct$, then

$$E_k(z-ct)=\frac{E_{k0}}{2}\,\mathrm{e}^{\mathrm{i}k(z-ct)}\sum_{r=-n}^{n+1}\mathrm{e}^{\frac{-[(z-ct)-r\lambda+\lambda/2]^2}{2s}} \tag{8}$$

is also the solution to Eqs. (1) and (2), representing a plane light wave packet or wave train with $\lambda=2\pi/k$ (the idler frequency is $kc=\omega$). Figure 1 can be taken as the waveforms of light wave packets or wave trains reconstructed by wavelets at $t=0$.

By far, we have obtained the discrete wavelet structure of plane light waves. Now we study the discrete energy of these light waves associated with their discrete wavelet structure.

For the plane electromagnetic waves in free space, the energy density is $\varepsilon_0/2\cdot|\boldsymbol{E}|^2+1/2\mu_0\cdot|\boldsymbol{B}|^2=\varepsilon_0|\boldsymbol{E}|^2$, where $\boldsymbol{E}=\sum_k\boldsymbol{E}_k(\boldsymbol{r},t)$ denotes the total electric field of all possible plane electromagnetic waves. According to Eq. (3), in a finite volume $\tau=L_xL_yL_z$, the total energy of the plane electromagnetic waves is

$$\begin{aligned}\varepsilon_0\int|\boldsymbol{E}|^2\mathrm{d}\tau\;=\;&\varepsilon_0\sum_{k,k'}\omega_k\omega_{k'}\int\mathrm{d}\tau(q_k\boldsymbol{E}_{1k}+q_k^*\boldsymbol{E}_{1k}^*)\\&\times(q_{k'}\boldsymbol{E}_{1k'}+q_{k'}^*\boldsymbol{E}_{1k'}^*),\end{aligned} \tag{9}$$

where $\omega_k\boldsymbol{E}_{1k}=\boldsymbol{E}_k(\boldsymbol{r})$ (or $\omega_k\boldsymbol{E}_{1k0}=\boldsymbol{E}_{k0}$). Besides, it is known that the plane electromagnetic waves should meet the following periodic conditions:

$$\begin{aligned}&k_x=\frac{2\pi}{L_x}l,\;\;k_y=\frac{2\pi}{L_y}m,\;\;k_z=\frac{2\pi}{L_z}n,\\&(l,n,m=0,\pm1,\pm2,\pm3,\ldots;\text{ not all zero}),\end{aligned} \tag{10}$$

where each set of (l,m,n) corresponding to a wave vector. Then we can get the following expression:

$$\begin{aligned}&\int\mathrm{d}\tau(\boldsymbol{E}_{1k}\cdot\boldsymbol{E}_{1k'}^*)\\&=\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{e}^{\mathrm{i}[(k_x-k_x')x+(k_y-k_y')y+(k_z-k_z')z]}\\&\quad\times(\boldsymbol{E}_{1k0}\cdot\boldsymbol{E}_{1k'0})\\&=(\boldsymbol{E}_{1k0}\cdot\boldsymbol{E}_{1k'0})L_xL_yL_z\delta_{k_xk_x'}\delta_{k_yk_y'}\delta_{k_zk_z'},\end{aligned} \tag{11}$$

$$\begin{aligned}&\varepsilon_0\int|\boldsymbol{E}|^2\mathrm{d}\tau\\&=\sum_\lambda\varepsilon_0L_xL_yL_z\left[\left|\boldsymbol{E}_{1k0\parallel}\right|^2+\left|\boldsymbol{E}_{1k0\perp}\right|^2\right]\omega_k^2\left[q_kq_k^*+q_k^*q_k\right],\end{aligned} \tag{12}$$

where $|\boldsymbol{E}_{1k0\parallel}|^2$ and $|\boldsymbol{E}_{1k0\perp}|^2$ correspond to two independent polarized components, respectively. By considering only one of the components and make the following transformations: $Q_k=\varepsilon_0\tau|\boldsymbol{E}_{1k0}|^2(q_k+q_k^*)$, $\dot{Q}_k=\dot{q}_k+\dot{q}_k^*=-\mathrm{i}\omega_k\varepsilon_0\tau|\boldsymbol{E}_{1k0}|^2(q_k-q_k^*)$, Eq. (12) can be expressed as

$$\varepsilon_0\int|\boldsymbol{E}|^2\mathrm{d}\tau=\sum_k\left[\frac{1}{2}\dot{Q}_k^2+\frac{1}{2}\omega_k^2Q_k^2\right]=\sum_kH_k. \tag{13}$$

According to the classical canonical transformation,[22] each Hamiltonian H_k can get such a form of $H_k=\dot{Q}_k^2/2+\omega_k^2Q_k^2/2=P_k\omega_k$, where P_k is a constant independent of time. For a given λ, we choose $L_x<\simeq\lambda$, $L_y<\simeq\lambda$ and $L_z=m\lambda,(m=1,2,3,\ldots)$. According to the periodic conditions above, we have $k_x=k_y=0$, $k_z\neq0$, which means a plane light wave is propagating along the z-direction. By using the relation of $L_z=m\lambda$, P_k for this plane wave can be represented as mP_k' since $\tau=L_xL_yL_z=m(L_xL_y\lambda)$. Then, by dropping the subscript of ω_k, the energy for the plane light wave in volume τ becomes

$$H_k=mP_k'\omega. \tag{14}$$

The result is valid for any value of m. In other words, the energy of a segment of an m-period light wave in an infinitely long one is proportional to the number of periods. This result is consistent with the energy division of Planck radiation theory.[23]

Next, we discuss this in more detail. As is well known, in a cavity of volume V, the number of electromagnetic vibration modes (oscillators) with the frequency between ν and $\nu+\mathrm{d}\nu$ is $N=8\pi V\nu^2/c^3\cdot\mathrm{d}\nu$. And in his radiation theory, Planck supposed that the total energy belonging to these N oscillators could be divided into P portions of ε (i.e., the total energy is $P\varepsilon$), where P is an integer. The energy $P\varepsilon$ is then assigned to N oscillators statistically and the average energy per oscillator is

$$U=\varepsilon/(\mathrm{e}^{\varepsilon/\kappa T}-1)=h\nu/(\mathrm{e}^{h\nu/\kappa T}-1),$$

where h, κ, and T are the Planck constant, Boltzmann constant, and absolute temperature, respectively. So it is that

$$NU=\frac{8\pi V}{c^3}\cdot\frac{h\nu^3}{(\mathrm{e}^{h\nu/\kappa T}-1)}\cdot\mathrm{d}\nu=Ph\nu,$$

i.e.,

$$P=\frac{8\pi V}{c^3}\cdot\frac{\nu^2}{(\mathrm{e}^{h\nu/\kappa T}-1)}\cdot\mathrm{d}\nu.$$

Also $P/N=1/(\mathrm{e}^{h\nu/\kappa T}-1)$, from which we know that: i) When $h\nu/\kappa T\gg1$, $P/N=\mathrm{e}^{-h\nu/\kappa T}\ll1$, which means that even if the energy is distributed evenly, each oscillator cannot get one portion of energy $h\nu$; ii) $h\nu/\kappa T\ll1$, $P/N=\kappa T/h\nu\gg1$, which means that each oscillator can get far more than one of $h\nu$; and further $P=\frac{8\pi V}{c^3}\cdot\frac{\kappa T\nu}{h}\cdot\mathrm{d}\nu$, i.e., $P\propto\nu\propto1/\lambda$, meaning that the number of photons in given volume V is inverse proportional to λ. This supports our above result, where $m\propto1/\lambda$ in a given volume $\tau=m(L_xL_y\lambda)$.

A similar result for the wave train described by Eq. (8) can also be obtained by taking a segment of wave with com-

plete periods. Referring to the above discussion on the discrete wavelet structure of plane light waves, one can find that the wave train described by Eq. (8) has $m = 2(n-1)$ complete periods, and its energy is $2(n-1)P'_k\omega$ in a volume τ. And for the whole wave train in a volume τ' with cross-section $L_x \times L_y$ (where $L_x <\simeq \lambda$, $L_y <\simeq \lambda$), the numerical results show that it carries energy $0.59(2P'_k\omega)$ for $n = 0$ or $(n+0.58)2P'_k\omega$ for $n \geq 1$. By denoting $2P'_k = p_k$, the energy in τ' carried by the light wave of Eq. (8) can then be expressed as

$$H'_k = \begin{cases} 0.59 p_k \omega, & (n=0), \\ (n+0.58) p_k \omega, & (n = 1,2,3,\ldots < \infty). \end{cases} \tag{15}$$

A similar discussion can be made on a general cross-section for a plane light wave packet or wave train. With $n \to \infty$, the result turns to that for the infinite plane wave of Eq. (7).

Let us continue our discussion on the light wave described by Eq. (8). For a general value of n, if we regard it as a state of an electromagnetic wave mode (degree of freedom) with idler frequency $\omega = kc$ rather than simply take it as a wave packet or wave train not indispensable, then E_{k0} in Eq. (8) could not be zero. Since it is well known that electromagnetic vibration in free space does not depend on a medium. And the electromagnetic wave modes only depend on the electromagnetic vibrations. If we assume that each admissive electromagnetic wave mode is a physical existence and always nonempty, then the electromagnetic vibration for its mode should always exist. If $E_{k0} = 0$, the mode with it will disappear. Therefore, E_{k0} would have a minimum value of nonzero, $E_{k0\min}$, which cannot be further divided. In other words, p_k in Eq. (15) has a minimum value of nonzero, denoted by p_{0k}, being indivisible either, which suggests that a general p_k would be composed of some p_{0k}. For the case of $p_k = p_{0k}$, Eq. (15) reads

$$H'_{0k} = \begin{cases} 0.59 p_{0k} \omega, & (n=0), \\ (n+0.58) p_{0k} \omega, & (n = 1,2,3,\ldots < \infty). \end{cases} \tag{16}$$

Therefore, the plane light wave packet or wave train of Eq. (8) with $E_{k0\min}$ is the basic one. It can be seen that, for such kind of basic light wave packet with $n = 1$, the minimum changeable energy is nearly a portion of $p_{0k}\omega$ in an absorption process. The remaining part of $0.59 p_{0k}\omega$ could not be further absorbed for ensuring the existence of its electromagnetic wave vibration mode. Therefore, for a basic plane light wave packet or wave train described by Eq. (8), practically, its changeable energy can only take the form of $H_{0k} = n p_{0k}\omega$ $(n = 1,2,3,\ldots)$. Now there is a question: how to determine the minimum value of p_{0k}? The answer is experiment, for example, the experiment of photoelectric effect.[3,24] And the value of p_{0k} is expected to be $h/2\pi$. One would also ask whether the wave packets of Eq. (8) can show the wavelength λ $(= 2\pi/k_\lambda)$ in an experiment? We will investigate this by the simulation on Mach–Zehnder interference (MZI) of single photons.[4]

3. Simulation on Mach–Zehnder interference of single photons

We use the interference field of random wave packets with minimum amplitude $E_{k0\min}$ to perform the simulation. The interference field of each pair of wave packets can be expressed as

$$F(t,x) = \frac{E_{k0\min}}{2} \mathrm{e}^{\mathrm{i}\omega t} \sum_{j=-n}^{n+1} \mathrm{e}^{-\frac{(t-jT+T/2)^2}{2s/c^2}} + \frac{E_{k0\min}}{2} \mathrm{e}^{\mathrm{i}\omega\left(t+\frac{x}{c}\right)} \sum_{j=-n}^{n+1} \mathrm{e}^{-\frac{\left(t+\frac{x}{c}-jT+T/2\right)^2}{2s/c^2}}, \tag{17}$$

where $T = \lambda/c$ with $\lambda = 500$ nm; x is the optical path difference between two arms of Mach–Zehnder interferometer, ranging from -3λ to 3λ; and n is set to 10. Two terms on the right side of Eq. (17) represent two wave packets passing through two arms of the interferometer, respectively, simulating single photons passing through two different arms of the interferometer. For simplicity, we take $E_{k0\min}$ as two units. And the intensity distribution of the interference after counting the pairs of wave packets m times per round of measurement is

$$I(x) = \sum_{r=1}^{m} \frac{u_r \cdot \mathrm{e}^{-u_r}}{30T} \int_{-18T}^{18T} |F(t,x)|^2 \mathrm{d}t, \tag{18}$$

where $u_r$is a random number taken from the sequence [0, 1, 2, 3, 4, 5], meaning that the pairs of wave packets in each time of count would be zero, one, or more than one; and e^{-u_r} is a weight factor, meaning that the probability for more pairs of wave packets is less than that for fewer pairs, in each time of count. Figure 2 shows the numerical results for $m = 3, 30, 300$, respectively. Figure 2(a) is the case for $m = 3$, where none of the interference fringes can be observed. When $m = 30$, see Fig. 2(b), the interference fringes appear but not so clear. For a large enough number of m, for example, $m = 300$, the interference fringes become very distinct, as shown in Fig. 2(c). For comparison, a classical interference figure of MZI is presented in Fig. 2(d), whose amplitude and period are like those in Fig. 2(c). One can see that the space of the interference fringes (corresponding to the optical path difference) in Fig. 2(c) is 500 nm, being the wavelength λ $(= 2\pi/k_\lambda)$. The result here looks like the experiment of MZI of single photons,[4] and like that of the electron two-hole interference[25] as well. The light wave packets here behave like photons with wave–particle duality. In the simulation, the wavelength of light can be revealed by numerous wave packets through interference but cannot be identified by a few wave packets, which show no wave information about light; instead, they behave as if they are "particles" like electrons.[25]

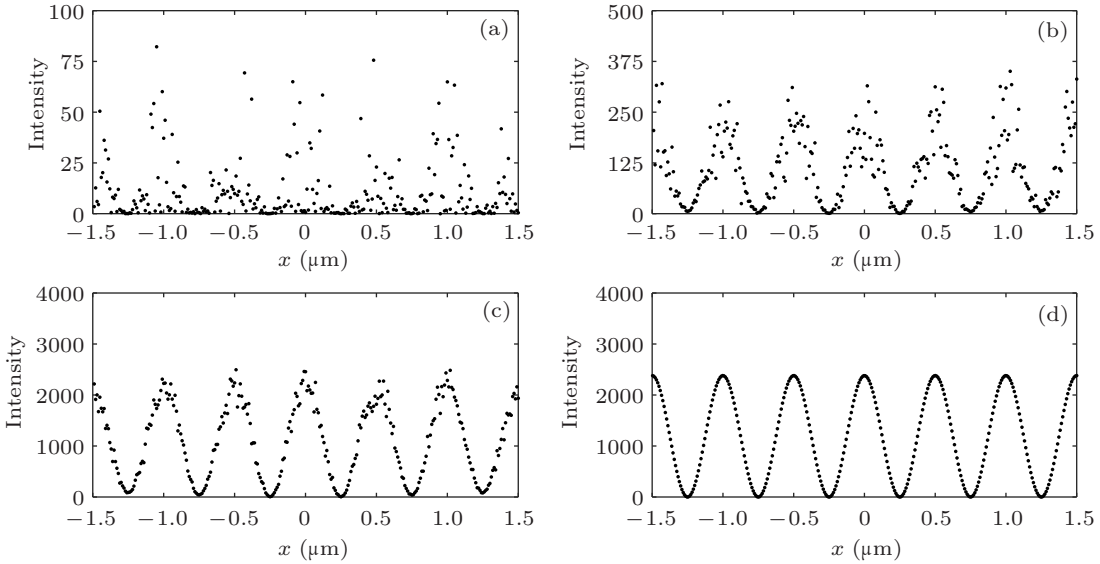


Fig. 2. Simulation on Mach–Zehnder interference of single photons by using the random light wave packets with discrete wavelet structure of $n = 10$, where m is the number of wave packet pairs for each optical path difference x; (a) $m = 3$, (b) $m = 30$, (c) $m = 300$, and (d) a classical interference figure of Mach–Zehnder interference for comparison.

4. Conclusion

By wavelet transform, a classical plane light wave of linear polarization can be viewed as a string of discrete Morlet wavelets and is found to be with discrete energies. And the changeable energy of basic plane light wave packet or wave train with discrete wavelet structure is shown to be that of $H_{0k} = np_{0k}\omega$ $(n = 1, 2, 3, \ldots)$, practically. Finally, the wave-particle duality from the Mach–Zehnder interference of single photons is simulated by using random basic plane light wave packets.

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