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Citation: Chin. Phys. B, 2021, 30 (**1**): 017504. DOI: 10.1088/1674-1056/abca26
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Exact soliton solutions in anisotropic ferromagnetic wires with Dzyaloshinskii–Moriya interaction*

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(Received 23 October 2020; revised manuscript received 10 November 2020; accepted manuscript online 13 November 2020)

We theoretically investigate the exact soliton solutions of anisotropic ferromagnetic wires with Dzyaloshinskii–Moriya interaction. For example, we give the bright and black soliton solutions. From these results we find that the Dzyaloshinskii–Moriya interaction affects the existence region of soliton, spin-wave transport, and soliton dynamic properties. As the Dzyaloshinskii–Moriya interaction grows, the soliton width is widened, which provides a way to control the soliton dynamics.

Keywords: bright soliton, black soliton, Dzyaloshinskii–Moriya interaction

PACS: 75.60.Ch, 75.70.Jj, 75.78.Fg **DOI:** 10.1088/1674-1056/abca26

1. Introduction

In ferromagnetic wires, there are many nonlinear magnetic excitations, such as spin waves, domain wall, and dynamic solitons. The magnetic domain wall describes the localized states, in which the magnetization gradually inverses its direction and it can be used to store and exchange two bits in race track memory.[1–4] In the last thirty years, physical scientists have mainly used magnetic fields and spin-polarized currents to drive domain wall motion,[5–8] in which the main theoretical method is the Walker method.[9–14] On the other hand, two inverted domain walls can form a dynamic soliton. By continuous deformation it can be reduced to a uniform magnetization. This magnetic soliton can define the typical scale for magnetic texture and enable high density magnetic storage in nanomagnetism.[15–17]

In the inversion symmetry broken system, the research of Dzyaloshinskii–Moriya interaction (DMI)[18,19] has attracted great attention for the effect of spin–orbit scattering of electrons, such as the specific metallic alloys with B20 structure[20–24] and the surface or interface of magnetic multi-layers,[25–29] while it vanishes when the system is centrosymmetric.[30] For this reason, the presence of DMI favors a certain chirality of the magnetization such as skyrmions,[31,32] vortex structures,[33–35] and spin spirals.[36–44]

It has been reported that the DMI can give rise to very interesting phenomena for domain wall dynamics driven by the magnetic fields[45–51] or charge currents.[52–60] Also, various novel property has been clarified in the magnon Hall effect,[61] molecular magnetism,[62] and multiferroicity.[63] Besides the above studies for nonlinear excited states, however the exact soliton solutions have not well explored in ferromagnetic wires with the presence of DMI. In this paper, we obtain three exact soliton solutions in anisotropic ferromagnetic wires with DMI. We find that the DMI affects the velocity, processional frequency, and the internal structure distortion of the solitons.

2. Exact Dzyaloshinskii–Moriya soliton solutions

As a model, we consider a uniaxial anisotropic ferromagnetic wire with DMI. In the continuous limit, the dimensionless Hamiltonian can be written as[57,58]

$$\mathcal{H} = \int \mathrm{d}z \left\{ \frac{1}{2} \left(\frac{\partial \boldsymbol{m}}{\partial z} \right)^2 + \lambda_z m_z^2 + D_0 \boldsymbol{m} \cdot (\boldsymbol{e}_z \times \frac{\partial \boldsymbol{m}}{\partial z}) \right\}, \quad (1)$$

where $\boldsymbol{m} = M/M_\mathrm{s}$, $\boldsymbol{M}$ is the magnetization with the saturation magnetization M_s, λ_z is the magnetic anisotropy constant, and D_0 is the DMI constant assuming that the wire is cut or grown along the DMI vector. The parameter z represents dimensionless space, and $\boldsymbol{e}_z$ is the unit vector in the z direction. It is well known that the magnetization dynamics can be described by the dimensionless Landau–Lifshitz equation[57]

$$\frac{\partial \boldsymbol{m}}{\partial t} = \boldsymbol{m} \times \boldsymbol{h}_\mathrm{eff}, \quad (2)$$

where the dimensionless effective magnetic field $\boldsymbol{h}_\mathrm{eff} = \partial^2 \boldsymbol{m}/\partial z^2 - 2D_0 (\boldsymbol{e}_z \times \partial \boldsymbol{m}/\partial z) - \lambda_z m_3 \boldsymbol{e}_z$.

*Project supported by the Science and Technology Project of Hebei Education Department, China (Grant No. ZD2015133), the National Natural Science Foundation of China (Grant Nos. 61774001 and 12075102), the Natural Science Foundation of Hebei Province of China (Grant No. F2019202141), and the Program of State Key Laboratory of Quantum Optics and Quantum Optics Devices (Grant No. KF201906).

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Equation (2) has a norm form of Landau–Lifshitz type for the magnetization dynamics in a uniaxial anisotropic ferromagnetic wire with DMI. The exact solution of Eq. (2) may be very complex, and the ansatz method is an effective straightforward technique. Firstly, it applies a direct transformation to the nonlinear equation. With some reasonable assumptions, the exact solutions can be constructed effectively by the symbolic computation. In the following, we obtain three exact soliton solutions for Eq. (2).

2.1. Bright Dzyaloshinskii–Moriya soliton

The first type of bright Dzyaloshinskii–Moriya soliton takes the form

$$\begin{aligned} m_1 &= \tanh\theta_1 \cos\varphi_1, \\ m_2 &= \tanh\theta_1 \sin\varphi_1, \\ m_3 &= \frac{1}{\cosh\theta_1}, \end{aligned} \tag{3}$$

where $\theta_1 = (z - z_1)/\Delta$ with $\Delta = 1/\sqrt{\lambda_z + D_0^2}$, and $\varphi_1 = k_1 z$. Here z_1 and k_1 are the only free parameters in the solution, respectively. The solution in Eq. (3) denotes the magnetic soliton on the spin wave background as shown in Fig. 1. From Fig. 1 we find that equation (3) represents the static solution as $z \to \pm\infty$, i.e., $m_3 \to 0$ and $m_+ \equiv m_1 + \mathrm{i}m_2 = \mathrm{e}^{\mathrm{i}\varphi_1}$ with wave number k_1. If applying the external field or spin current field, the magnetization can take the dynamic precession. From Eq. (3) we see that the soliton exists under the condition $D_0^2 > |\lambda_z|$, while it only exists in the case of easy-plane ($\lambda_z > 0$) without DMI. It implies that the DMI widens the existence region of soliton. Also, the soliton width Δ increases with the increasing DMI term $|D_0|$.

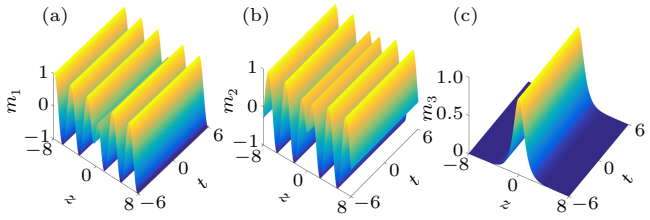


Fig. 1. Evolution of bright Dzyaloshinskii–Moriya soliton solution in Eq. (3). As $z \to \pm\infty$, it reprents a periodic solution, i.e., $m_3 \to 0$ and $m_+ \equiv m_1 + \mathrm{i}m_2 = \mathrm{e}^{\mathrm{i}\varphi_1}$. Parameters are $k_1 = 2$, $\lambda_z = 0.3$, and $D_0 = 0.5$.

By a tedious calculation, we obtain the second type of bright Dzyaloshinskii–Moriya soliton

$$\begin{aligned} m_1 &= \frac{2c\tanh\theta_2}{1 + (c\tanh\theta_2)^2}\cos\varphi_2, \\ m_2 &= \frac{2c\tanh\theta_2}{1 + (c\tanh\theta_2)^2}\sin\varphi_2, \\ m_3 &= \frac{1 - (c\tanh\theta_2)^2}{1 + (c\tanh\theta_2)^2}, \end{aligned} \tag{4}$$

where

$$\begin{aligned} \theta_2 &= \frac{2G}{1+4G^2}(z - z_2)/\Delta, \\ \varphi_2 &= D_0 z + \frac{1-4G^2}{1+4G^2}t/\Delta. \end{aligned} \tag{5}$$

Here c, z_2, and G are the only free parameters in the solution. Similar to the first type soliton, the solution in Eqs. (3) and (5) also has the zero moving velocity of magnetic soliton, while takes a dynamic precession, as shown in Fig. 2. When $z \to \pm\infty$, we have $m_3 \to (1-c^2)/(1+c^2)$, $m_+ \equiv m_1 + \mathrm{i}m_2 = 2c/(1+c^2)\,\mathrm{e}^{\mathrm{i}\varphi_2}$. This result shows that the spin wave has the amplitude $2c/(1+c^2)$, wave number being D_0, and the precession velocity being $(1-4G^2)/(1+4G^2)/\Delta$. Without DMI, the spatial phase difference vanishes for the spin wave solution. Therefore the DMI affects the spin-wave transport in the magnetic structures. This solution also exists under the condition $D_0^2 > |\lambda_z|$, while it only exists in the case of easy-plane($\lambda_z > 0$) without DMI. The soliton width increases with the increasing DMI term $|D_0|$, which can be easy found in Eqs. (4) and (5).

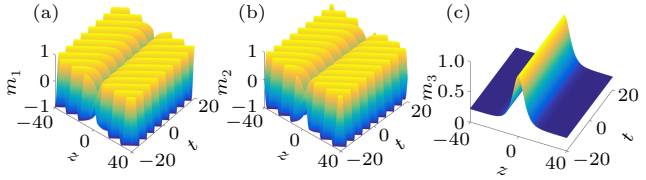


Fig. 2. Graphical illustration of exact bright Dzyaloshinskii–Moriya soliton solution in Eq. (4). The magnetization excitation forms a a dynamic precession, and as $z \to \pm\infty$, $m_3 \to (1-c^2)/(1+c^2)$, $m_+ \equiv m_1 + \mathrm{i}m_2 = 2c/(1+c^2)\,\mathrm{e}^{\mathrm{i}\varphi_2}$. The DMI favors the spational phase difference. Parameters are given as follows: $c = 0.8$, $G = 5$, $\lambda_z = 0.3$, and $D_0 = 0.5$.

2.2. Black Dzyaloshinskii–Moriya soliton

The black type of Dzyaloshinskii–Moriya soliton takes the form

$$\begin{aligned} m_1 &= -\frac{2\sqrt{b}\sinh\theta_3}{1 + b\sinh^2\theta_3}\cos\varphi, \\ m_2 &= -\frac{2\sqrt{b}\sinh\theta_3}{1 + b\sinh^2\theta_3}\sin\varphi, \\ m_3 &= 1 - \frac{2}{1 + b\sinh^2\theta_3}, \end{aligned} \tag{6}$$

where

$$\begin{aligned} \theta_3 &= a(z - z_3), \quad \varphi_3 = D_0 z + \zeta t, \\ \zeta &= a^2 + 1/\Delta, \quad b = \zeta/(4a^2). \end{aligned}$$

Here a and z_3 are the only free parameters in the solution. The solution in Eq. (6) represents a black soliton solution of magnetization on a ground state background as shown in Fig. 3. From Fig. 3, we find that equation (6) reduces to the ground state as $z \to \pm\infty$, i.e., $m_3 \to 1$ and $m_+ \equiv m_1 + \mathrm{i}m_2 \to 0$. In the region of soliton, the components m_1 and m_2 precesses around with m_3. The precession wave number is D_0, and the frequency is ζ. These results show that the DMI favors the

spatial phase for the magnetization state, and determines the precession velocity.

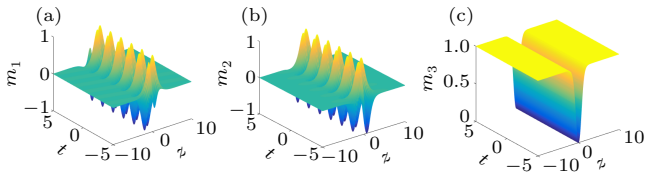


Fig. 3. Evolution of exact black Dzyaloshinskii–Moriya soliton solution in Eq. (6). The solution of magnetization reduces to the ground state, $m_3 \to 1$ and $m_+ \equiv m_1 + \mathrm{i}m_2 \to 0$ as $z \to \pm\infty$. The DMI favors the spatial phase for the magnetizaion state. Parameters are given as follows: $a = 1.5$, $\lambda_z = 0.3$, and $D_0 = 0.5$.

3. Conclusion

In this paper, we have constructed the exact soliton solutions of anisotropic ferromagnetic wires with the Dzyaloshinskii–Moriya interaction. The effect of DMI on magnetization motion was studied in detail by using these analytical solutions. These results show that the DMI can affect the existence region of soliton, spin-wave transport, and soliton dynamic properties in the magnetic structures.

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