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Citation: Chin. Phys. B, 2021, 30 (**1**): 015202. DOI: 10.1088/1674-1056/abc53b
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A fitting formula for electron–ion energy partition fraction of 3.54-MeV fusion alpha particles in hot dense deuterium–tritium plasmas*

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(Received 1 September 2020; revised manuscript received 19 October 2020; accepted manuscript online 28 October 2020)

Based on our previous work (*Phys. Plasmas* **25** 012704 (2018)), a fitting formula is given for electron–ion energy partition fraction of 3.54-MeV fusion alpha particles in deuterium–tritium (DT) plasmas as a function of plasma mass density ρ, electron temperature T_e, and ion temperature T_i. The formula can be used in a huge range of the plasma state, where ρ varies between 1.0 g/cc $\sim 10.0^3$ g/cc and both T_e and T_i change from 0.1 keV to 100.0 keV. Relativistic effect for electrons is investigated including the effect of the projectile recoil in the plasmas at $T_e \geq 50.0$ keV. The partition fraction for $T_e > T_i$ is found to be close to that for $T_e = T_i$. The comparisons with other fitting results are made at some plasma densities when $T_e = T_i$, and the difference is explained. The fitting result is very close to the calculated one in most cases, which is convenient for the simulation of alpha heating in hot dense DT plasmas for inertial confined fusion.

Keywords: inertial confinement fusion, deuterium–tritium plasma, alpha heating, energy loss

PACS: 52.40.Mj, 34.50.Bw, 52.20.Hv, 52.58.Hm **DOI:** 10.1088/1674-1056/abc53b

1. Introduction

Heating of deuterium (D) and tritium (T) ions by 3.54-MeV alpha particles in plasmas is a basic problem in controlled fusion, which is called as alpha heating. The relevant investigation has been made over half a century.[1,2] An important parameter to describe the heating is the electron–ion energy partition fraction η, defined as the ratio of the energy obtained by both D and T ions to 3.54-MeV projectile energy. In order to simulate the pellet gain and the burn processes in inertial confinement fusion (ICF),[3] the values of the partition fraction should be known for wide ranges of the electron (T_e) and ion (T_i) temperatures from tens of eV to 200.0 keV, and electron number density (n_e) from $\sim 10^{21}$ cm^{-3} to $\sim 10^{26}$ cm^{-3}.[4,5]

As the energy partition fraction is important for the simulation of alpha heating in DT plasmas, some fitting formulas have been given at some plasma densities when $T_e = T_i$. As far as we know, the earliest formula was presented by Fraley *et al.*,[6] by means of the binary collision (BC) theory.[7] They found $\eta = T_e/(T_e + 32)$ (T_e is measured in unit keV) when $\rho = 0.213$ g/cc, and $\eta = 0.5$ when $\rho = 10^4$ g/cc and $T_e = 24$ keV. Later by the same BC model, Atzeni and Meyer-ter-Vehn (AM)[8] gave $\eta = T_e/(T_e + 25)$ (T_e is in unit of keV) when $\rho = 10.0$ g/cc–100.0 g/cc for typical ICF fuel density. However, they all ignored the important cases when $T_e \neq T_i$, which usually happen in the heating process. Thus it is necessary to carry out a fitting formula as a function of ρ, T_e, and T_i for convenient application in related simulation. This is the main motivation of the present work.

So far there are several models to calculate η from different physical pictures, including the classical binary collision model,[7] the collective effect or plasma polarization model[9] by means of the classical dielectric function,[10] Li and Petrasso (LP) model[11] which combined the binary collision and collective effect, and the model[12] proposed by Brown, Preston, and Singleton Jr (BPS) on the basis of a simplified Fokker–Plank equation.[13] In recent time we suggested a model[14] that was able to bring reliable values of η in wide ranges of the plasma densities and temperatures. In the present work the fitting formula is given according to the data by our model.

Totally speaking, previous research about the energy fraction partition was obtained in the non-relativistic framework and the possible relativistic effect was not investigated but it might appear when T_e becomes very high such as 100 keV–200 keV. Therefore, it is necessary to check whether the relativistic motion of the electrons in the plasmas affects the energy partition fraction. This is the second purpose of the present work. For this goal, the relativistic model suggested by Prentice[15] is revised to gain more reasonable result of stopping power.

This paper is organized as follows. In Section 2, a

*Project supported by the National Key Research and Development Program of China (Grant Nos. 2017YFA0402300 and 2017YFA0403200), the National Natural Science Foundation of China (Grant No. 11574034), the Innovation Development Foundation of China Academy of Engineering Physics (CAEP) (Grant No. CX20200029), and the Science Challenge Project (Grant Nos. JCKY201612A501).

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brief description about our model is presented, followed by some typical features of the energy partition fraction. In Section 3, an investigation of relativistic motion of electrons for $T_e \geq 50.0$ keV is made for the possible influence upon η. In the last section, a fitting formula is given together with the fitting error. Comparisons with other works are made in this section, and the difference is explained. Finally, some conclusions are drawn. In the following the word plasma means the dense ICF DT plasma. In the current work, the plasma mass density varies from 1.0 g/cc to 10.0^3 g/cc, and temperature changes from 100 eV to 100 keV. The projectile is always alpha particle, and the maximum of kinetic energy E is 3.54 MeV. Deuteron and triton in the plasmas are called as DT ion for simplicity. The atomic units ($e = m_e = \hbar = 1$) are used in this work unless otherwise explicitly indicated.

2. Brief introduction of our model and some features of η

Although some models have been suggested to estimate the energy partition fraction, further revisions are still necessary to get more reliable data for relevant research. In the BC model the projectile slows down due to the Coulomb collision with the charged particles in the plasmas. Here the Coulomb logarithm $\log\Lambda$ related to Debye length λ_D is used, but it is invalid at high plasma density and low temperature. Moreover, the results from this model depend on the choice of Coulomb logarithm. In the picture of collective effect, the projectile polarizes the plasma and an electro-static field is excited. Therewith the projectile feels a retarding force that retards its motion. However, the projectile is usually assumed to move in a straight line so that the energy loss always happens even the projectile energy E_p is far below the plasma temperature. Obviously it is unreasonable since the projectile must be thermalized at the ambient plasma temperature. In LP model the collective effect was represented by a logarithmic term, which is valid only when the projectile velocity V_p is much larger than the thermal velocity of the electrons or ions in the plasmas, as pointed in Ref. [10]. However, the collective effect at a lower V_p is also necessary for the accurate estimation of the energy transferred to the plasmas. In BPS model, the basic dimensionless parameter g ($= 1/(\lambda_D T_e)$) should be much smaller than 1. This is not satisfied in the case of high plasma density and low temperature. For example, g is higher than 0.34 when T_e is around 0.1 keV while n_e is around 10^{26} cm^{-3}.

In order to better explore the slowing down of alpha particles in ICF dense DT plasmas, the mechanism of the plasma polarization described by the quantum dielectric function $\varepsilon(k,\omega)$ and the effect of the projectile recoil[16–18] were combined in our previous work.[12] Here the rate of projectile energy change due to plasma electron or ion is obtained as

$$-\frac{\mathrm{d}E_s}{\mathrm{d}t} = \frac{2Z_p^2}{\pi V_p}\int_0^\infty \mathrm{d}k \int_{k^2/2M_p - kV_p}^{k^2/2M_p + kV_p} \mathrm{d}\omega \frac{\omega \cdot n_s(\omega)}{k} \times \mathrm{Im}\frac{\varepsilon(\boldsymbol{k},\omega)}{|\varepsilon(\boldsymbol{k},\omega)|}, \quad (1)$$

where s denotes electron or ion component and

$$\varepsilon(\boldsymbol{k},\omega) = 1 + \frac{4\pi}{k^2}\sum_s \int \mathrm{d}\boldsymbol{v}\frac{f_s(\boldsymbol{v}) - f_s(\boldsymbol{v}-\boldsymbol{k})}{\omega - \boldsymbol{k}\cdot\boldsymbol{v} + k^2/2m_s + \mathrm{i}\delta}$$

with $\delta \to 0$ being the quantum dielectric function. T_s and $f_s(\boldsymbol{v})$ are the temperature and the Fermi–Dirac distribution of the plasma species, respectively. $n_s(\omega)$ is the Bose statistics $[\exp(\omega/T_s) - 1]^{-1}$. The corresponding equation in electron plasmas has been given in Eq. (12) of Ref. [17] or Eq. (7) of Ref. [18]. Stopping power $SP_s(E_p)$ is defined as $-(1/V_p)(\mathrm{d}E_s/\mathrm{d}t)$ for simplicity. In the following $SP_i(E_p)$ and $SP_e(E_p)$ denote the contribution from ions and electrons, respectively.

According to Eq. (1), the energy transferred to the ion is determined as $E_i = \int_{E_m}^{E_0} \mathrm{d}E_p \tau(E_p)$, where $\tau(E_p) = SP_i(E_p)/[SP_e(E_p) + SP_i(E_p)]$. A minimum of the projectile energy E_m is chosen to ensure both $SP_i(E_p)$ and $SP_e(E_p)$ are positive. E_m is found to be close to the maximum of T_e and T_i. Then η is obtained by E_i/E_0.

It is worth noting three features of η. The first one is that when both T_e and T_i are fixed, η decreased very slowly with rising ρ. This has also been observed in other models such as in BC model[6,8] and BPS model[11] though the reason is still unknown to us. The second feature is that when both T_e and ρ are fixed, η decreased significantly with rising T_i when $T_e < T_i$. This is easy to understand since $SP_i(E_p)$ declines with increasingT_i. The final one is that when both T_e and ρ are fixed, η for $T_e \geq T_i$ is close to that for $T_e = T_i$. For example, at $T_e = 5$ keV, η varies from 0.1326 to 0.1314 as T_i changes from 1.0 keV to 4.2 keV and is 0.1313 at $T_i = 5.0$ keV. This can be understood according to Fig. 1.

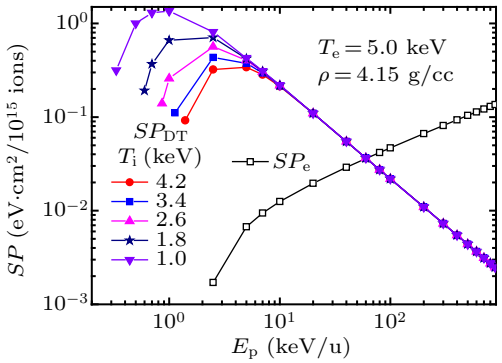


Fig. 1. Ionic component of stopping power SP_i in DT plasmas as a function of E_p at different T_i when $\rho = 4.15$ g/cc and $T_e = 5.0$ keV. The line with empty circles denotes SP_e.

Figure 2 shows the SP_i and SP_e in DT plasmas as a function of E_p at different T_i when $\rho = 4.15$ g/cc and $T_e = 5.0$ keV.

Here SP_e is almost independent of T_i since the mass of electron is much smaller than that of DT ions. This figure indicates that the SP_i at different T_i is close to each other for $E_p = 10$ keV/u. This is also valid when E_p is higher. This is the asymptotic behavior of stopping power[19] since the corresponding V_p is much higher than the thermal velocity of the DT ions. Hence, for E_p between 10 keV/u and 885 keV/u the corresponding $\tau(E_p)$ is irrelevant with T_i. Moreover, $\tau(E_p)$ in this E_p range contributes more than 90% of the energy partition fractions. These make η in this case weak dependence upon T_i. Therefore, the asymptotic behavior of SP_i and weak dependence of SP_e upon T_i lead to the last feature of η.

3. The η in relativistic plasmas

As the plasma temperature may be quite high, such as $T_e = 100$ keV, the relativistic effect of electrons in the plasmas should be considered although the effect for ions is still very weak. The reason is that the ion is much heavier than the electron so that the ion thermal velocity is much smaller than the light speed c. More than 50 years ago, Prentice[15] derived some analytical expressions for stopping power of ions in very hot plasmas, where relativistic effect was completely included on the base of classical dielectric function. However, in that work the projectile was assumed to move in a straight line in the plasmas, which made it still lose energy even if its E_p is much smaller than the plasma T_e. This is not suitable according to the description in the last section. For this reason the projectile recoil is introduced in the same way as Eq. (5) in Ref. [20], and the energy loss rate comes to be

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \left(\frac{Z_p}{\pi}\right)^2 \int \mathrm{d}\boldsymbol{q} \frac{\omega N(\omega)}{q^2} \mathrm{Im}\left[\frac{-1}{\varepsilon(\boldsymbol{q},\omega)}\right]. \tag{2}$$

Since T_e is quite high and both alpha particle and DT ion are non-relativistic, the corresponding dielectric function $\varepsilon(\boldsymbol{q},\omega)$ should be introduced in the consideration of the plasma static-electro field caused by the energetic alpha particle. For this reason let us see the relativistic Vlasov equation

$$\frac{\partial f}{\partial t} + \boldsymbol{V}\cdot\frac{\partial f}{\partial \boldsymbol{x}} - (\boldsymbol{E} + \boldsymbol{V}\times\boldsymbol{B})\cdot\frac{\partial f}{\partial \boldsymbol{p}} = 0. \tag{3}$$

Then the classical dielectric response function for electronic component in the plasmas is obtained by the similar method in Ichimaru's book[10]

$$\varepsilon(\boldsymbol{q},\omega) = 1 - \frac{\omega_{pe}^2}{q^2}\int \frac{1}{\boldsymbol{q}\cdot\boldsymbol{v} - \omega - \mathrm{i}\nu}\boldsymbol{q}\cdot\frac{\partial f_0}{\partial \boldsymbol{p}}\mathrm{d}\boldsymbol{p}, \tag{4}$$

where p is the relativistic momentum,

$$f_0 = \exp(-\gamma/\alpha^2)/4\pi c^3\alpha^2 \mathrm{K}_2(\alpha^{-2})$$

is the electron momentum distribution with K_2 the modified Bessel function of the second-order, $\alpha = \sqrt{T_e}/c^2$ and $\gamma = \sqrt{1+p^2/c^2}$. According to Prentice' result[15]

$$\varepsilon(\boldsymbol{q},\omega) = 1 + s^{-2}\left[\phi(u,\alpha) - \mathrm{i}\,\psi(u,\alpha)\cdot S(u\alpha)\right] \tag{5}$$

with

$$\begin{aligned}\phi(u,\alpha) &= -2\pi c^2\alpha^2\int_0^\infty p\gamma\frac{\partial f_0}{\partial p} \\ &\quad\times\left\{2 - g\cdot\log\left|\frac{1+g}{1-g}\right|\right\}\mathrm{d}p, \end{aligned}\tag{6}$$

$$\begin{aligned}\psi(u,\alpha) &= \frac{\pi u\alpha}{2K_2(\alpha^{-2})}\left(1+2\alpha^2\xi^{1/2}+4\alpha^4\xi\right) \\ &\quad\times\exp\left(-\alpha^{-2}\xi^{-1/2}\right)\xi^{-1}. \end{aligned}\tag{7}$$

Here $S(x) = 1$ at $|x| \le 1$, or else is zero. $\xi = 1 - u^2\alpha^2 = \omega/kc\alpha$, $\beta = V_p/c$, $s = k/k_D$, and $g = cu\alpha\gamma/p$. Then the stopping power SP_e due to electron in the plasmas can be obtained. If $N(\omega)$ is chosen as 1, the result for longitudinal energy loss rate, just as Eq. (6.13) in Ref. [15], is recovered. The following figure shows the related result at $T_i = T_e = 100$ keV and $n_e = 10^{26}$/cc.

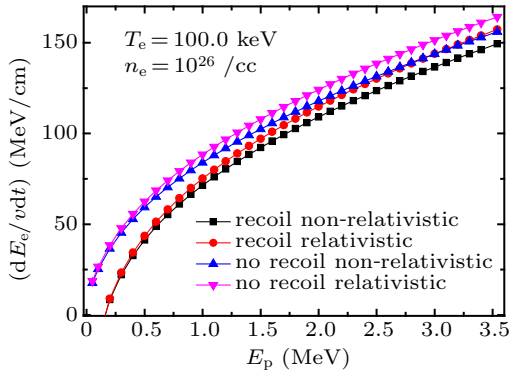


Fig. 2. SP_e in DT plasmas as a function of E_p by different models at $T_i = T_e = 100$ keV and $n_e = 10^{26}$/cc. The line with squares and circles denote the result considering the recoil of the projectile with non-relativistic and relativistic momentum distribution of the electrons, respectively. The line with up and down triangles denote the result without the recoil of the projectile when non-relativistic and relativistic momentum distribution of the electrons are considered, respectively.

The above figure indicates clearly that the results with the recoil of the projectile is quite different from that without the recoil. Meanwhile the result by relativistic momentum distribution is close to that by non-relativistic distribution when $E_p < 1.0$ MeV, which covers most range for SP_i to play its role to affect η. With E_p increasing, SP_e by relativistic momentum distribution becomes a few higher than that by non-relativistic distribution. The reason may be related to the fact that the corresponding velocity distribution under the non-relativistic momentum distribution is a little flatter than that under the relativistic momentum distribution.[21] However, in the considered range of E_p the SP_e under non-relativistic distribution is at most 6.5% lower than that under relativistic distribution. Similar results are found for many other plasma cases when T_e is high enough. This makes η close to each other in the

two kinds of momentum distribution. In other words, our previous result of η in Ref. [14] is appropriate even if the non-relativistic distribution was used for the electron when T_e is very high.

4. Fitting formula of η and comparison with other works

According to Eq. (1), lots of calculations have been made to get η in the ranges of both T_i and T_e from 0.1 keV to 100 keV, and ρ between 1. and 10^3 g/cc. By fitting these data a formula of η is found as

$$\begin{aligned}\eta_f = & \frac{T_e[1+2.155\cdot\exp(-1.33\cdot T_e^{0.499})]}{T_e+36.31\cdot\rho^{-0.0248}} \\ & \times\exp[-0.00116[W(T_i-T_e)]^{0.781}]+3.01\cdot10^{-7} \\ & \times\exp(-0.141\cdot T_e)[W(T_i-T_e)]^{2.6} \\ & -0.00033\cdot T_e^{-0.0267}[W(T_i-T_e)]^{1.21} \\ & -2.83\cdot10^{-6}\cdot\rho^{0.763}[W(T_i-T_e)]^{0.793} \end{aligned} \quad (8)$$

when 100 keV $\geq T_e \geq$ 2.0 keV, and

$$\begin{aligned}\eta_f = & -1.08+3.51T_e+0.91\exp[(0.3691\rho^{0.22}-0.08T_e) \\ & +0.129T_e\cdot W(T_i-T_e)-0.106\cdot W(T_i-T_e)], \end{aligned} \quad (9)$$

when $T_e < 2.0$ keV. Here $W(x) = 0$ if $x < 0$, otherwise it is equal to x.

For the fitting results, there are some errors defined as $1-\eta_f/\eta$, where η_f is the fitting value of η. Figure 3 plots the error as a function of T_e and ρ when $T_i = T_e$. Generally speaking, the error decreases with the increase of T_e when ρ is fixed. The less absolute value of error when $T_e > 30$ keV is not plotted in the figure. It is easy to see that in most cases the size of the error is less than 5%, and it may exceed 7% when $T_e < 5$ keV and $\rho > 350$ g/cc. Its maximum may be 15% at $T_e = 2.0$ keV and $\rho > 350$ g/cc, where $\eta(\approx 0.05)$ is small.

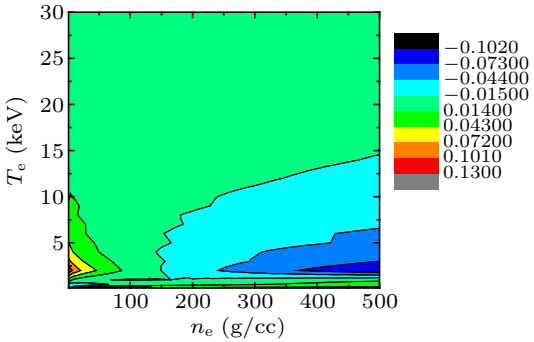


Fig. 3. Error as a function of T_e and ρ when $T_i = T_e$.

Figure 4 shows the error as a function of T_e, T_i, and ρ when $T_i > T_e$. This figure indicates that the size of the error is less than 3% in most cases. The error may become 10%, even 15% when T_e is around 2.0 keV while both T_i and ρ are in a very narrow range of theirs. However, in this case η is small and less than 6.0%. Hence, totally speaking, the fitting formula is reliable in most cases where the size of the error is less than 5%, and the error may become 10%–15% only when T_e is around 2.0 keV and both T_i and ρ are in very narrow ranges and η is less than 6.0% in this situation.

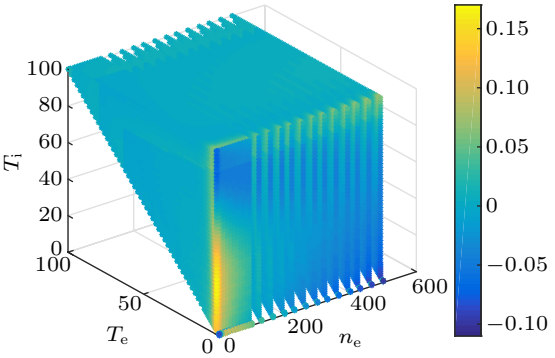


Fig. 4. Error as a function of T_e, T_i, and ρ when $T_i > T_e$.

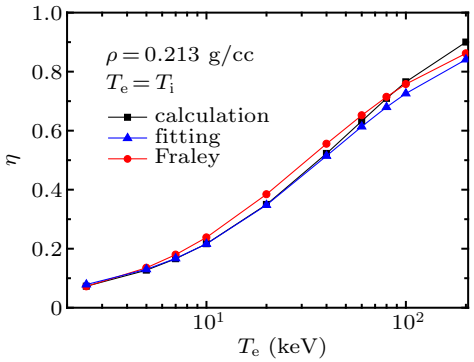


Fig. 5. Electron–ion energy partition fraction comparison among our calculation, fitting and Fraley's result when $T_e = T_i$ and $\rho = 0.213$ g/cc.

It is necessary to compare the fitting formula with those obtained by others. In the case of $T_e = T_i$ and $\rho = 0.213$ g/cc, figure 5 shows the energy partition fraction comparisons among our calculation, fitting and Fraley's result.[6] Our calculation is according to Eq. (1) and the fitting is based on the fitting formulas (8) and (9). The fitting result is very close to the calculation when T_e is small but the difference increases gradually with the increasing of T_e. The fitting expression is close to Farley's[6] $\eta = T_e/(T_e+32)$ and their results are close to each other, especially at the low temperature range. Fraley's expression gives $\eta = 0.238$ at $T_e = T_i = 10$ keV while $\eta = 0.217$ and $\eta_f = 0.210$ are estimated by our calculation and fitting formula. When DT ions obtain half of the incident energy, namely $\eta = 0.5$, T_e is 37.7 keV according to our result, that is a few higher than 32 keV by Fraley's work. The fitting formula gives $\eta_f = T_e/(T_e+36.31\cdot\rho^{-0.0248})$ at $T_e \geq 2.0$ keV and $T_e = T_i$. Thus $\eta_f = T_e/(T_e+34.3)$ at $\rho = 10.0$ g/cc and $\eta_f = T_e/(T_e+32.4)$ at $\rho = 100.0$ g/cc. Such tendency of η_f with ρ and T_e agrees with those in Fig. 1(b) in Ref. [6] and Fig. 11.14 in Ref. [8] but the formula is apparently different from Eq. (11.90) in Ref. [8], where $\eta = T_e/(T_e+25)$ when $\rho = 10.0$ g/cc $\sim$ 100.0 g/cc. Our results show $\eta = 0.231$ and $\eta_f = 0.225$ at $T_e = T_i = 10.0$ keV and $\rho = 10.0$ g/cc. For the

same T_e and T_i but $\rho = 100.0$ g/cc, $\eta = 0.245$, and $\eta_f = 0.236$. In both cases AM gave $\eta = 0.285$.[8] So in general, our result is closer to Fraley's than to AM's.

The difference among ours, Fraley and AM is mainly due to the choice of the Coulomb logarithm $\log\Lambda$ in respective calculation. The choice is a little arbitrary in their calculations while no such arbitrariness exists in our model. In Fraley's work[6] the following expression was used:

$$\frac{\mathrm{d}U}{\mathrm{d}x} = -23.2\left(\frac{\rho}{\rho_0}\right)\frac{U^{1/2}}{T_e^{3/2}}1+0.17\log\left[T_e\left(\frac{\rho_0}{\rho}\right)^{1/2}\right]\Bigg\} - 0.047\left(\frac{\rho}{\rho_0}\right)\frac{1}{U}\left\{1+0.075\log\left[T_e\left(\frac{\rho_0}{\rho}\right)U\right]\right\}, \quad (10)$$

where $U = E/E_0$, $\rho_0 = 0.213$ g/cc, T_e is measured in unit keV. In the above expression the first term and the second term of the right side represent the energy deposition in electron and DT ion, respectively. AM[8] used the following formula to get η:

$$\frac{(\mathrm{d}V_p/\mathrm{d}t)_e}{(\mathrm{d}V_p/\mathrm{d}t)_i} = \frac{\log(\Lambda_e)}{\log(\Lambda_i)}\left(\frac{78}{T_e}U\right)^{3/2}, \quad (11)$$

where $\log\Lambda$ due to electron and DT ion are expressed as

$$\log(\Lambda_e) = 7.1 - 0.5\log[n_e] + \log[T_e] \quad \text{for} \quad T_e \leq 10\ \text{eV}, \quad (12)$$

$$\log(\Lambda_i) = 9.2 - 0.5\log[n_e] + 1.5\log[T_i] \quad \text{for} \quad T_i \leq 40\ \text{keV}. \quad (13)$$

According to Eq. (10) a similar expression with Eq. (11) can be obtained only with different Λ_e and Λ_i Therefore, all the related Coulomb logarithms used in the two works are quite different. This leads to the difference of their results.

In summary, on the basis of our previous work, a fitting formula is obtained for electron–ion energy partition fraction for 3.54-MeV fusion alpha particles in DT plasmas as a function of plasma mass density ρ, electron temperature T_e, and ion temperature T_i. In the formula, ρ varies between 1.0 g/cc and 10.0^3 g/cc, and both T_e and T_i change from 0.1 keV–100.0 keV. It covers a wide range of the plasma state. The formula is reliable in the above-mentioned range, and the error is less than 5% in most cases while the maximum error may reach 10%–15% only when T_e is around 2.0 keV with ρ and T_i varying in a small range. Relativistic effect for electrons is investigated with the consideration of the projectile recoil in the plasmas at $T_e \geq 50.0$ keV. Little influence of the effect is found upon the results of partition fraction. The fraction at $T_e > T_i$ is close to that at $T_e = T_i$. The comparisons with other fitting results are made at some plasma density at $T_e = T_i$, and the difference is explained. Our formula is convenient for the simulation of alpha heating in inertial confined fusion hot dense DT plasmas.

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