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Analysis of dark soliton generation in the microcavity with mode-interaction*

Xin Xu(徐昕)†, Xueying Jin(金雪莹), Jie Cheng(程杰), Haoran Gao(高浩然), Yang Lu(陆洋), and Liandong Yu(于连栋)

School of Instrument Science and Opto-electronics Engineering, Hefei University of Technology, Hefei 230009, *China*

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Mode-interaction plays an important role in the dark soliton generation in the microcavity. It is beneficial to the excitation of dark solitons, but also facilitates a variety of dark soliton states. Based on the non-normalized Lugiato–Lefever equation, the evolution of dark soliton in the microcavity with mode-interaction is investigated. By means of mode-interaction, the initial continuous wave (CW) field evolves into a dark soliton gradually, and the spectrum expands from a single mode to a broadband comb. After changing the mode-interaction parameters, the original modes which result in dual circular dark solitons inside the microcavity, are separated from the resonant mode by 2 free spectral ranges (FSR). When the initial field is another feasible pattern of weak white Gaussian noise, the large frequency detuning leads to the amplification of the optical power in the microcavity, and the mode-interaction becomes stronger. Then, multiple dark solitons, which correspond to the spectra with multi-FSR, can be excited by selecting appropriate mode-interaction parameters. In addition, by turning the mode-interaction parameters, the dark soliton number can be regulated, and the comb tooth interval in the spectrum also changes accordingly. Theoretical analysis results are significant for studying the dark soliton in the microcavity with mode-interaction.

Keywords: dark soliton, microcavity, mode-interaction

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1. Introduction

Recently, optical frequency comb technology based on microcavity is widely used in numerous precision measurement fields,[1–4] due to its advantages such as small volume, high integration, low loss, and especially large frequency interval between comb teeth.[5] Owing to the restriction of its structure to the optical field, high power exists inside the microcavity, which leads to a cascaded four-wave mixing effect. Accordingly, the pump mode is transferred to the adjacent modes and a series of comb spectra are generated in the microcavity.[6] A variety of different materials are used to fabricate microcavity, such as Si_3N_4,[7] MgF_2, [8] CaF_2,[9] and so on. Because the materials, dispersion, frequency detuning, and pumping conditions of the microcavities are different, the microcavities can operate in various states. Generally speaking, microcavity with abnormal dispersion is propitious for the bright soliton, which consists of a broadband comb spectrum.[10] When the phase synchronization or mode locking of all the comb-tooth mode is satisfied, the bright soliton can be excited, which is similar to the mode-locked pulses in the mode-locked laser. On the contrary, normally dispersive microcavity is beneficial for the dark soliton, the distribution of which is contrary to bright soliton.[10] The dark soliton is significant for the comb lines in the normally dispersive microcavity, which is separated by only 1 free spectral range (FSR). Furthermore, the generation of dark soliton is momentous to study its physical properties and the practical application, for example, the nonlinear effects and optical communication based on the dark soliton. Therefore, the research on dark soliton is very essential.

Most of the research on dark solitons excited inside the microcavity is theoretical so far. Based on the Lugiato–Lefever equation (LLE), the physical mechanism of dark solitons is studied. A dark soliton can be generated either from the nonlinear evolution of an optical shock wave or the narrowing of a locally broad dark pulse with smoother fronts.[11] Relying on numerically solving the spatiotemporal evolution of dark solitons, the effects of operating parameters on the dark soliton field are also studied. The increment of the microcavity second-order dispersion causes the dark soliton pulse to broaden. And the change of detuning parameter and pump power can lead to the various field distribution other than the dark soliton in the microcavity.[10] Nevertheless, the experiment on the dark soliton in the microcavity is rarely reported. In a recent experiment, it was reported that the mode-locked dark solitons are formed in the normally dispersive microcavity with the aid of the mode-interaction assisted excitation. It is proved that the mode-interaction is beneficial to the excitation of dark solitons, increases the freedom in the microcavity design, and extends the comb spectrum into the

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†Corresponding author. E-mail: xuxin@hfut.edu.cn

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visible.[12] The time-domain waveform of the dark soliton is reconstructed by using the spectral phase information, which is retrieved via line-by-line pulse shaping and the measured power spectrum. Thus, the phase variation of each mode in the microcavity, which is caused by mode-interaction, can be measured in experiments.[13] It provides support for the theoretical research on frequency combs in the microcavity with the mode-interaction.

In this article, based on the non-normalized LLE, the dark soliton production process with mode-interaction is analyzed. And the effect of mode-interaction on the field and spectrum evolution is discussed. Moreover, the multiple dark solitons are excited by selecting appropriate mode-interaction parameters. Taking advantage of turning the mode-interaction parameters, the dark soliton number also can be regulated.

2. Theory model

The field inside the microcavity can be described by the following non-normalized LLE:[12]

$$t_{\mathrm{R}}\frac{\partial E(t,\tau)}{\partial t} = \left[-\alpha - \mathrm{i}\delta_0 - \mathrm{i}L\frac{\beta_2}{2}\frac{\partial^2}{\partial\tau^2} + \mathrm{i}\gamma L|E(t,\tau)|^2\right]E(t,\tau) + \sqrt{\theta}E_{\mathrm{in}}, \tag{1}$$

where $E(t,\tau)$ is the complex field distribution, $t(\tau)$ is the slow (fast) time of the optical field circulation, t_{R} is the round-trip time, the free spectral range is defined as $\mathrm{FSR} = 1/t_{\mathrm{R}}$, α is the loss of the optical field intensity, δ_0 is the frequency detuning between the resonant frequency and the pump frequency, L is the microcavity circumference, β_2 is the second-order dispersion, γ is the nonlinear coefficient, θ is the coupling ratio between pump laser and microcavity, P_{in} is the pump power, and $E_{\mathrm{in}} = \sqrt{P_{\mathrm{in}}}$.

Interaction between comb modes has been encountered in microcavities. It is directly observed as a disruption of the continuity of dispersion in a smaller microcavity.[14] Furthermore, due to the effect of energy conversion, the comb spectrum in the microcavity is symmetric without considering the mode-interaction. Hence, the mode-interaction is indirectly indicated by the asymmetry of the comb spectrum.[15] The spectrum in the microcavity evidently shows broadband, comprising spurious modes and fundamental modes. The spurious modes are not observed in the spectrum, but they cannot be ignored only for the microcavity which supports a single mode.[16] Owing to the unavoidable imperfections of the microcavity structure, the spurious modes and the fundamental modes can interact with each other, namely mode-interaction. Thus, the mode-interaction is essential for studying the field evolution in the microcavity. It leads to the splitting of the higher frequency harmonic modes and causes the frequency shift of these modes. The frequency shift is expressed as

$$\omega_i - \widetilde{\omega}_i = \kappa^2/\Delta, \tag{2}$$

where ω_i is the intrinsic frequency of the arbitrary mode, $\widetilde{\omega}_i$ is the frequency with a shift, κ is the mode-interaction constant reflecting the strength of mode-interaction, and $\Delta = \omega_j - \omega_i$, ω_j is the frequency of any mode which interacts with the mode. On account of $|\Delta| \gg \kappa$, the frequency shift is a relatively small amount.

Based on the above discussion, the mode-interaction is taken into account by adding a phase shift to the modes, which are adjacent to the resonant mode per round trip.[17,18] The accessional phase shift is given by $\Delta\phi = -2\pi\Delta f/\mathrm{FSR}$.[12] And Δf is the resonance shift in Eq. (2). It can be regulated in a certain range by a microheater, which leads to the thermal tuning of the comb modes.[19] The phase shift is processed during the cycle of the split-step Fourier method that is used to solve the non-normalized LLE numerically.

3. Dark soliton generation with mode-interaction

Microcavity with normal dispersion facilitates the generation of the dark soliton. Mode-interaction is significant to regulate produce process and stable pattern of the dark soliton. By optimizing mode-interaction strength and microcavity parameters, various fields of the dark soliton can be obtained. For the aim of discussing the generation of the dark soliton with mode-interaction, we take the normal-dispersion microcavity of Si_3N_4 as an example. The second-order dispersion coefficient β_2 of which is 186.9 $\mathrm{ps}^2/\mathrm{km}$. Other parameters of the microcavity are given as follows: $\alpha = 3.1\times10^{-3}$, $\delta_0 = 1\ \mathrm{m}^{-1}$, $L = 628\ \mu\mathrm{m}$, $\gamma = 0.89\ (\mathrm{m\cdot W})^{-1}$, and $\theta = 1.93\times10^{-3}$. Moreover, the initial field in the microcavity is stable continuous wave (CW) distribution.[20] Numbers of the comb modes are represented by N, where $N = 0$ indicates the resonant mode of the microcavity.

First of all, it is assumed that the resonant mode ($N = 0$) is pumped by the CW laser, and $P_{\mathrm{in}} = 0.1$ W. Owing to mode-interaction, there is resonant shift generating for the neighboring modes. Because the intensity of mode-interaction diminishes as the number of models increases, we just consider the effect of mode-interaction on the first and the second modes ($N = 1$ and $N = 2$) for simplifying the model. And the resonance shifts of mode 1 and mode 2 are represented by Δf_1 and Δf_2. When $\Delta f_1 = 130$ MHz and $\Delta f_2 = -230$ MHz, the field evolution of dark soliton is illustrated in Fig. 1(a). The fields and spectra corresponding to the typical slow time in Fig. 1(a) are demonstrated in Fig. 1(b). In the initial period (for example, slow time is 5 ns), the optical power in microcavity is relatively weak, the fluctuation of which is approximately

10^{-15} W. Consequently, it can be considered as a direct current (DC) field. The corresponding spectrum contains a single resonant mode. There is no mode-interaction inside the microcavity right now. With the accumulation of energy in the microcavity, the nonlinear and dispersion effects are increasing, and the modulation instability results in a sinusoidal field (slow time is 12 ns). New modes located at $N = \pm 1$ arise around the resonant mode. Accordingly, such modes begin to interact with each other, and a dark soliton is excited during the cycle of the optical field. The original dark soliton, the distribution of which comprises some noticeable sharps, is irregular (slow time is 20 ns). At this moment, an unstable comb spectrum generates, and the power of each comb varies with time. After a period of evolution, a stable dark soliton circulates in the microcavity (slow time is 40 ns). It means that nonlinearity, dispersion, and mode interactions in the microcavity are in equilibrium at present. However, due to mode-interaction, the location of the dark soliton changes periodically. The comb spectrum, the frequency interval of which is 1-FSR, is also in a steady state.

The above is the evolution of the dark soliton with mode-interaction. By adjusting the microheater, the mode-interaction conditions can be changed. Thus, various dark solitons can be excited. Remaining the parameters of the microcavity and pump the same as that in Fig. 1, only the resonance shifts Δf_1 and Δf_2 are changed into -130 MHz and 230 MHz, respectively. The updated field evolution in the microcavity is shown in Fig. 2(a). It can be seen that the field evolution is similar to Fig. 1, and the difference is that the field evolves into two dark solitons. As a result of different mode-interaction,

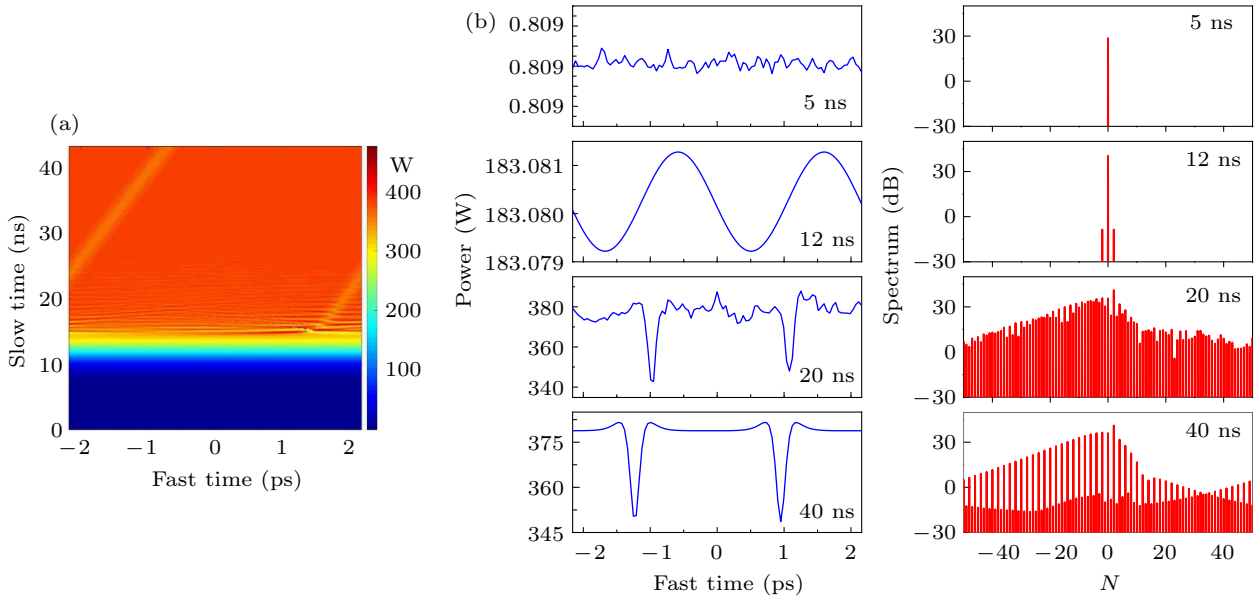


Fig. 1. (a) Generation of a dark soliton ($\Delta f_1 = 130$ MHz, $\Delta f_2 = -230$ MHz), (b) field distribution and spectra corresponding to the slow time in (a).

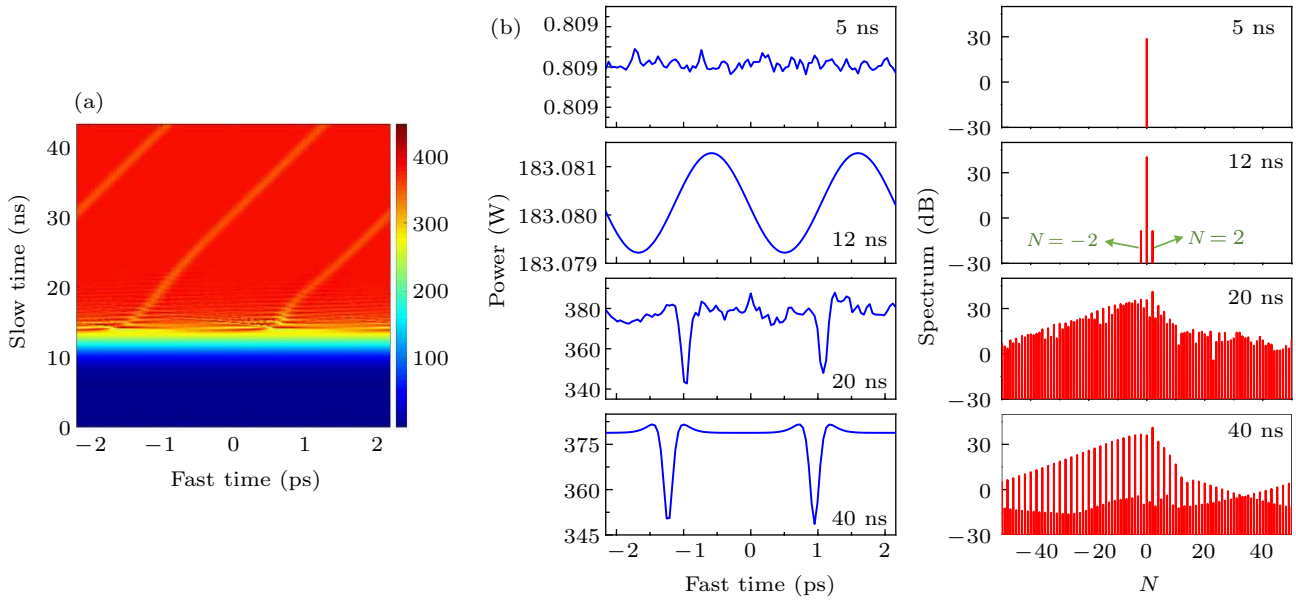


Fig. 2. (a) Generation of dual dark solitons ($\Delta f_1 = -130$ MHz, $\Delta f_2 = 230$ MHz), (b) field distribution and spectra corresponding to the slow time in (a).

in the transition stage between the DC field and the soliton field, the sinusoidal field generated inside the microcavity has two standard oscillation periods (slow time is 12 ns), which results in the new modes located at $N = \pm 2$ in the spectrum. These dual sinusoidal fields ultimately develop into dual circular dark solitons. For the spectrum, the spacing between two adjacent comb teeth is 2-FSR (slow time is 40 ns). It results that different mode-interaction causes distinct forms of dark solitons. Thus, by adjusting the microheater, it can control the mode-interaction, which can further determine the state of dark solitons.

Nevertheless, not all mode-interaction can excite dark solitons in the microcavity. It is found that the resonance shifts Δf_1 and Δf_2 are either positive or negative, and only an irregular distribution can be formed in the microcavity. For example, the pumping and detuning conditions remain constant, but $\Delta f_1 = 130$ MHz and $\Delta f_2 = 230$ MHz. The present field evolution is shown in Fig. 3(a). The field power experiences a gradient from the low level to the high level, which is similar to Fig. 2(a). But the obvious difference is that there is no dark soliton or other regular fields in the microcavity. The field and spectrum at the slow time of 30 ns are plotted in Figs. 3(b) and 3(c), respectively. It can be seen that the field is disordered and there is almost only one comb in the spectrum. Because the modes located at $N = 1$ and $N = 2$ obtain the same phase gain, which results in mode confusion. Except for the resonance mode, no mode can be finally stabilized during the interaction. Thus, only an irregular field exists in the microcavity. Accordingly, the mode-interaction conditions should be controlled within a certain range. Otherwise, the dark solitons cannot generate in the microcavity.

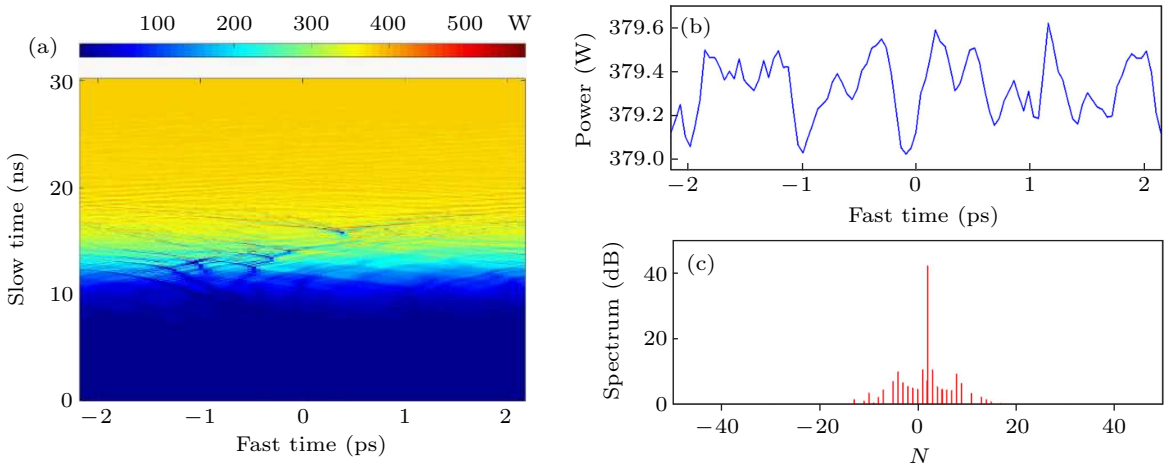


Fig. 3. (a) Field evolution ($\Delta f_1 = 130$ MHz, $\Delta f_2 = 230$ MHz), (b) field distribution and spectrum when the slow time is about 30 ns.

4. Multiple dark solitons generation

When the microcavity is affected by some factors, there are distinct distributions of the initial field inside the microcavity, and the phenomenon of multiple solitons can be motivated. And weak white Gaussian noise is feasible for the initial condition.[11] When the frequency detuning of the microcavity is large, combining with the initial field of the noise, multiple dark solitons can generate. When the frequency detuning δ_0 is set to 4, $\Delta f_1 = -230$ MHz, and $\Delta f_2 = 230$ MHz, the field evolves into four dark solitons, which is illustrated in Fig. 4(a). Due to the large range of optical power variation in this figure, the soliton light field cannot be observed. The final stable soliton field is drawn in Fig. 4(b), which has four dark solitons. It can be found that the increase of the detuning parameter leads to the enhancement of the power in the cavity after stabilization. Moreover, during the evolution of the field, the time experienced by the DC field is significantly shortened. It means that the detuning increase leads to amplification of the optical power in the microcavity, and the mode-interaction becomes stronger. Therefore, the field evolves into the soliton shape much earlier. The optical field when the solitons are just excited is shown in Fig. 4(c). The optical field and spectra at a particular moment are shown in Figs. 4(d) and 4(e), respectively. The solitons are excited from the irregular field, and four dark solitons are produced simultaneously. However, the intensity of the four newly generated solitons is different, and the field at this time is unstable. The modes which can stably exist are selected by the mode-interaction, and they gradually evolve into four identical solitons during the cycle. Corresponding to the field distribution of the four dark solitons, the distance between the two adjacent peak comb teeth is 4-FSR in the spectrograph. In addition, it is apparent that the peak position of the spectrum is significantly shifted relative to the resonant mode. This suggests that the mode-interaction not only results in the creation of multiple solitons, but also effectively transfers the energy of the pumping mode to other modes.

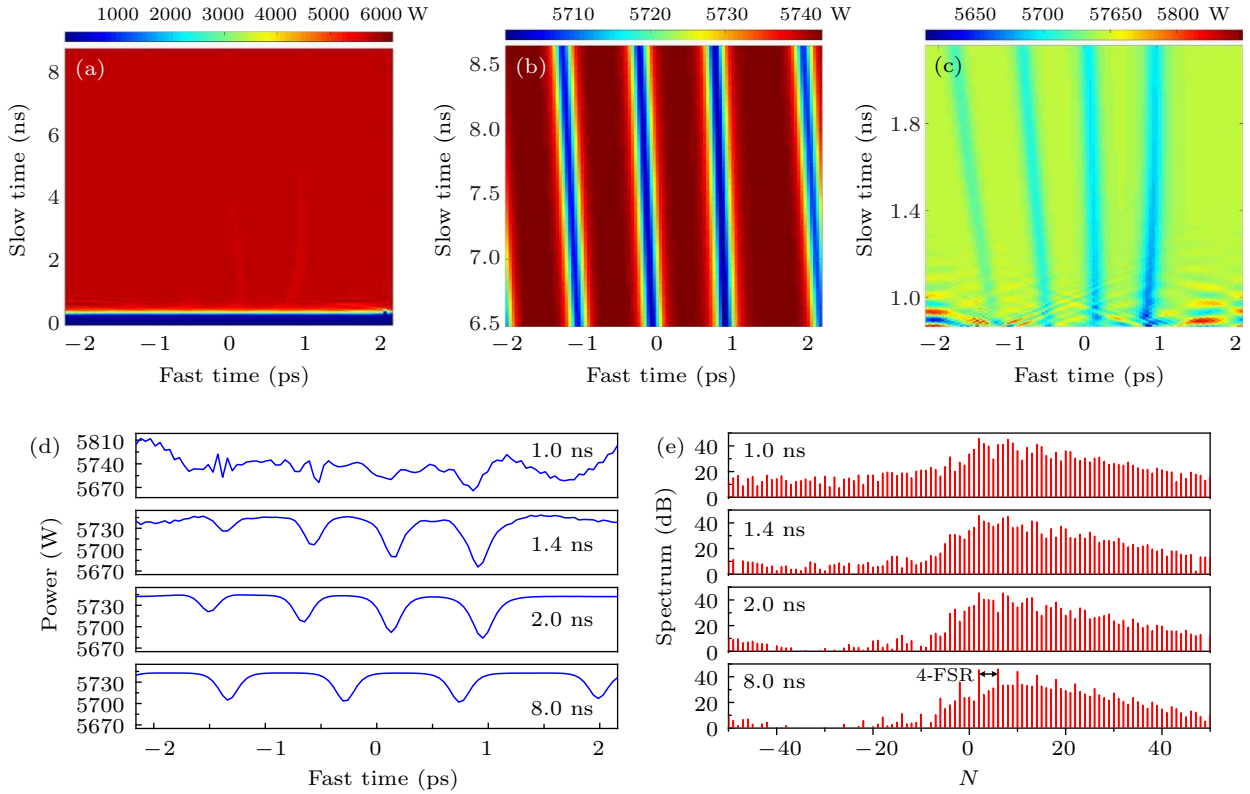


Fig. 4. (a) Field evolution of the serve frequency detuning ($\Delta f_1 = -230$ MHz, $\Delta f_2 = 230$ MHz), (b) field evolution after 6.5 ns in (a), (c) field evolution at the initial stage of soliton formation in (a), (d) optical field at the particular moment, (e) spectra at a particular moment.

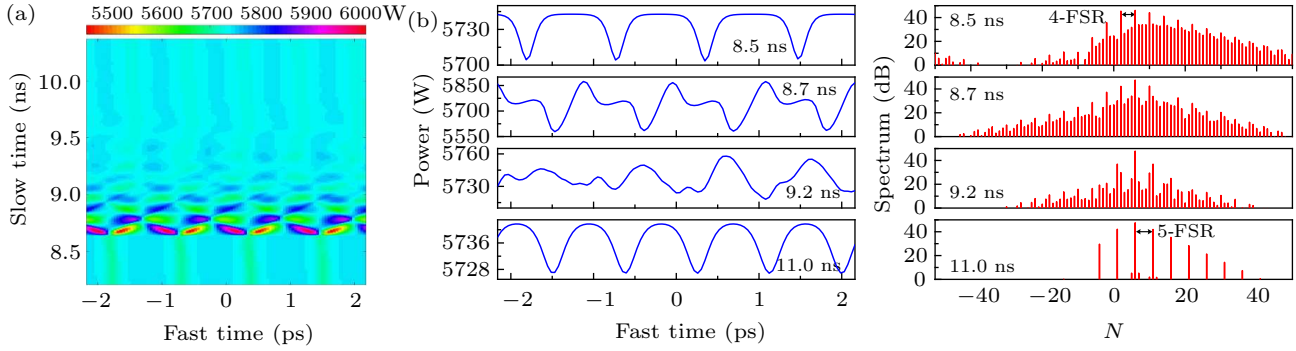


Fig. 5. (a) Field evolution during regulation of dark soliton number, (b) optical field at the particular moment, (c) spectra at a particular moment.

5. Regulation of dark soliton number

Taking advantage of mode-interaction, not only the multi-soliton field can be generated, but also the number of solitons can be controlled. Assuming that the frequencies of dual modes are ω_a and ω_b, the resonance frequencies affected by the mode-interaction are ω'_a and ω'_b can be expressed as[18]

$$\omega'_{a,b} = \frac{\omega_a + \omega_b}{2} \pm \sqrt{\frac{(\omega_a - \omega_b)^2}{4} + |g_0|^2}, \tag{3}$$

where g_0 is the coupling coefficient between the dual modes. According to the equation, the mode-interaction strength can be regulated by changing the difference between ω_a and ω_b, which is determined by the microheater. Therefore, the mode-interaction strength may be engineered for controlling the comb spectrum, which further affects the number of dark solitons number in the microcavity.

Based on the field which consists of four dark solitons in Fig. 4(a), the mode-interaction parameters are changed. The variation of the optical field is shown in Fig. 5(a). When slow time is approximately 8.7 ns, the resonance shifts Δf_1 and Δf_2 are changed into 230 MHz and -130 MHz, respectively. There is an obvious dividing line in the field. The abrupt field distribution, which is no longer dark solitons, is shown in Fig. 5(b) (slow time is 8.7 ns). At this point, because of the novel mode-interaction, the field is in an unstable state, and the power of which fluctuates greatly. It is precise because the peak-to-peak value of the field is large so that the spectrum is broadened. But the comb spectrum of the 4-FSR interval

still remains. Then, the field becomes irregular after mode-interaction reselects the modes in the microcavity (slow time is 9.2 ns). Near the center comb, there are two strong comb teeth at the positions of 4-FSR and 5-FSR spacing. Furthermore, the comb teeth with 5-FSR intervals are much stronger. This means that energy is transferring between different modes. Experiencing the new mode-interaction, the field eventually changes from four solitons to five solitons (slow time is 11 ns), and the peak-to-peak power of which is reduced. It is worth noting that the actual spectrum has a 5-FSR comb tooth interval. It is concluded that by changing the mode interaction parameters, not only the number of solitons in the cavity is controlled, but also the ideal comb spectrum is obtained.

6. Conclusion

In summary, by solving the non-normalized LLE, the field evolution of the dark soliton with mode-interaction is demonstrated. And the mode-interaction is considered by adding a phase shift to the modes which are adjacent to the resonant mode per round trip. With the mode-interaction, the initial CW field experiences a sinusoidal pattern, and ultimately a stable dark soliton is formed in the microcavity. In the spectrogram, because of the mode-interaction, the original mode evolves novel modes at adjacent locations and eventually forms a broadband comb spectrum, the frequency interval of which is 1-FSR. It is concluded that mode-interaction is beneficial for the dark soliton generation. When the mode-interaction parameters are changed, the different modes are excited from the original single model, and they are located at $N = \pm 2$ in the spectrum. Thus, the spectrum changes into the combs and the spacing between two adjacent comb teeth is 2-FSR, which corresponding to dual dark solitons in the spatial distribution.

In addition, when the initial field is weak white Gaussian noise and the frequency detuning is increased, the high optical power in the microcavity leads to stronger mode-interaction. The time to stabilize the field is shortened. Finally, multiple solitons can be excited. When the microcavity operates in a multi-soliton state, the number of dark solitons can be controlled by turning the mode-interaction parameters. In the article, the evolution of four solitons to five solitons is shown. Theoretical analysis results are significant for studying the dark soliton generation in the microcavity with mode-interaction, and they are essential for regulating the mode-interaction parameters for the aim of exciting the dark soliton inside the microcavity.

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