

Ground state cooling of an optomechanical resonator with double quantum interference processes

Shuo Zhang(张硕), Tan Li(李坦), Qian-Hen Duan(段乾恒), Jian-Qi Zhang(张建奇), and Wan-Su Bao(鲍皖苏)
Citation: Chin. Phys. B, 2021, 30 (**2**): 023701. DOI: 10.1088/1674-1056/abc2c0
Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Tuning the velocity and flux of a low-velocity intense source of cold atomic beam

Shu Chen(陈姝), Ying-Ying Li(李营营), Xue-Shu Yan(颜学术), Hong-Bo Xue(薛洪波), Yan-Ying Feng(冯焱颖)
Chin. Phys. B, 2017, 26 (**11**): 113703. DOI: 10.1088/1674-1056/26/11/113703

Development of adjustable permanent magnet Zeeman slowers for optical lattice clocks

Xiao-Hang Zhang(张晓航), Xin-Ye Xu(徐信业)
Chin. Phys. B, 2017, 26 (**5**): 053701. DOI: 10.1088/1674-1056/26/5/053701

BaF radical: A promising candidate for laser cooling and magneto-optical trapping

Liang Xu(许亮), Bin Wei(魏斌), Yong Xia(夏勇), Lian-Zhong Deng(邓联忠), Jian-Ping Yin(印建平)
Chin. Phys. B, 2017, 26 (**3**): 033702. DOI: 10.1088/1674-1056/26/3/033702

Microwave-mediated magneto-optical trap for polar molecules

Dizhou Xie(谢笛舟), Wenhao Bu(卜文浩), Bo Yan(颜波)
Chin. Phys. B, 2016, 25 (**5**): 053701. DOI: 10.1088/1674-1056/25/5/053701

Temperature and number evolution of cold cesium atoms inside a wall-coated glass cell

Huang Jia-Qiang, Zhang Jian-Wei, Wang Shi-Guang, Wang Zheng-Bo, Wang Li-Jun
Chin. Phys. B, 2015, 24 (**11**): 113701. DOI: 10.1088/1674-1056/24/11/113701

Ground state cooling of an optomechanical resonator with double quantum interference processes*

Shuo Zhang(张硕)[1], Tan Li(李坦)[1,†], Qian-Hen Duan(段乾恒)[1], Jian-Qi Zhang(张建奇)[2,‡], and Wan-Su Bao(鲍皖苏)[1,3,§]

[1] *Henan Key Laboratory of Quantum Information and Cryptography, SSF IEU, Zhengzhou, Henan* 450001, *China*
[2] *Innovation Academy for Precision Measurement Science and Technology, Wuhan* 430071, *China*
[3] *Synergetic Innovation Center of Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei* 230026, *China*

(Received 17 July 2020; revised manuscript received 23 September 2020; accepted manuscript online 20 October 2020)

We present a cooling scheme with a tripod configuration atomic ensemble trapped in an optomechanical cavity. With the employment of two different quantum interference processes, our scheme illustrates that it is possible to cool a resonator to its ground state in the strong cavity–atom coupling regime. Moreover, with the assistance of one additional energy level, our scheme takes a larger cooling rate to realize the ground state cooling. In addition, this scheme is a feasible candidate for experimental applications.

Keywords: laser cooling, coherent optical effects, multiphoton processes, optomechanics

PACS: 37.10.Rs, 42.50.−p **DOI:** 10.1088/1674-1056/abc2c0

1. Introduction

Over the last ten years, the study of macro- or nano-mechanical resonators (MRs) have attracted considerable attention from both scientific and industrial communities. It results from the fact that the MRs hold a great potential for creating mesoscopic quantum states, developing novel quantum devices, and constructing highly sensitive sensors for metrology.[1–3] However, most of these applications are limited by the thermal phonons on the MRs for their low frequencies. Hence, cooling a MR to its ground state is a fundamental step towards its various applications.

To achieve the ground state cooling of the MRs, great efforts have been paid for searching different cooling methods, such as sideband cooling,[4–7] feedback cooling,[8] measurement cooling,[9] squeezed-light-based cooling,[10–12] and so on.[13]

In experiments, the well developed technology is based on the sideband cooling,[6,7] which can be realized by coupling a MR to a high quality optical cavity via radiation pressure coupling. With the assistance of the cavity dissipation and setting the system in the regime of the first-order red sideband, the MR can be cooled to its ground state in the resolved sideband regime, where the linewidth of the optical cavity is much smaller than the vibrational frequency of the MR. Moreover, to further reduce the thermal noise of the environment, the squeezed field is introduced to the sideband cooling, and the ground state cooling of the MR can be beyond the quantum backaction limit.[11] Nevertheless, limited by the manufacturing technology, the sideband cooling can only be realized by a few research groups.

On the other hand, to reduce the difficulty in the ground state cooling of the MR in the experiments, a variety of new cooling approaches based on quantum interference have been proposed in non-resolved sideband regime in which the linewidth of the optical cavity is larger than the vibrational frequency of the MR. These works for quantum inference cooling can be achieved with the assistance of coupled cavities,[14–16] superconductor systems,[17] quantum dots,[18–20] nitrogen-vacancy (NV) centers,[21–24] and cavity quantum electrodynamics systems.[25–36] In these schemes, the internal degrees of freedom (d.o.f) works as a hybrid-multilevel configuration, and ground-state cooling can be achieved by the quantum interference effects, such as electromagnetically induced transparency (EIT)[37–39] and EIT-like effects.[40–42] Under the action of the quantum interference, the first red-sideband transition can be enhanced, while the one for blue-sideband will be suppressed, and the ground state cooling of the MR can be achieved in this way. As a result, it can be more efficient than the sideband cooling.

In this work, we present a scheme to cool a MR to its ground-state with two quantum interference processes. We assume an optomechanical cavity strongly couples to a tripod configuration atomic ensemble, and there is a double-quantum interference structure in the hybrid systems to improve the cooling performance. Then, we use the first quantum inter-

*Project supported by the National Key Research and Development Program of China (Grant No. 2017YFA0304503), Key Research and Development Project of Guangdong Province, China (Grant No. 2020B030300001), and the National Natural Science Foundation of China (Grant Nos. 828330256, 11636220, 11805279, 1173401, and 11504430).

†Corresponding author. E-mail: lt@qiclab.cn
‡Corresponding author. E-mail: changjianqi@gmail.com
§Corresponding author. E-mail: bws@qiclab.cn

© 2021 Chinese Physical Society and IOP Publishing Ltd

ference (EIT-like resonance) to enhance the narrow red sideband transition, and the mean phonon number of the MR can be cooled to the order of $1/C_N$, where C_N is the cooperativity between cavity and atomic ensemble. Therefore, ground-state cooling can be achieved in strong atoms-cavity coupling regime. Moreover, to avoid the photon absorption by the atoms and provide a larger optomechanical coupling than previous atom-assisted cooling schemes,[25,26,28,31,32] the second quantum interference (cavity-EIT resonance) is introduced to cancel this process. As a result, the cooling rate of our scheme can be larger than relevant schemes, and it is more robust against heating caused by the thermal noise.

2. Model

2.1. Hamiltonian

The experimental setup is shown in Fig. 1. An ensemble of N atoms is confined in an optomechanical cavity.[2] The vibrational frequency of the movable cavity mirror is $\omega_{\rm m}$, and the cavity is pumped by an external laser field $\Omega_{\rm p}$ with frequency $\omega_{\rm p}$. Each atom has one excited state $|e\rangle$ and three ground states $|g\rangle$, $|r\rangle$, and $|s\rangle$. The transition $|e\rangle \leftrightarrow |g\rangle$ couples to a cavity field with single atom–cavity coupling strength g_0[26] and frequency $\omega_{\rm c}$, and the one between states $|e\rangle$ and $|r\rangle$ ($|s\rangle$) is driven by the laser field Ω_r (Ω_s) at frequency $\omega_{\rm L1}$ ($\omega_{\rm L2}$).

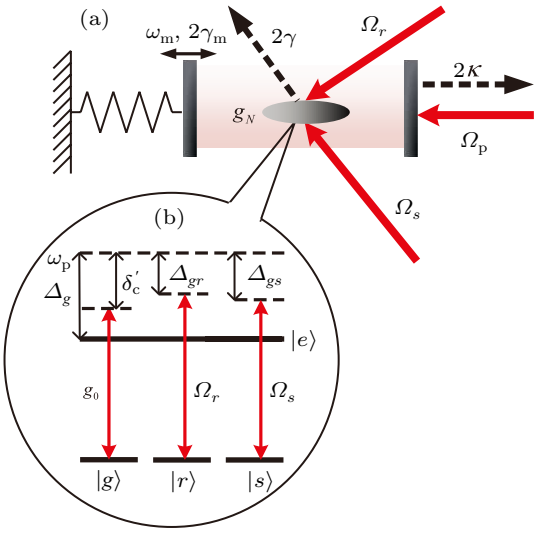


Fig. 1. (a) Experimental setup. An atomic ensemble is confined in an optomechanical cavity. The cavity is driven by a pump field. Each atom in ensemble simultaneously couples to two laser fields and the cavity field, as it is shown in panel (b).

In the rotating frame of frequencies $\omega_{\rm p}$, $\omega_{\rm L1}$, and $\omega_{\rm L2}$, the Hamiltonian of the system is ($\hbar = 1$)

$$H = H_{\rm m} + H_{\rm c} + H_{\rm a} + H_{\rm I}. \tag{1}$$

Here,

$$H_{\rm m} = \omega_{\rm m} b^\dagger b, \tag{2}$$

$$H_{\rm c} = -\delta_{\rm c}' a^\dagger a, \tag{3}$$

$$H_{\rm a} = -\Delta_g \sum_{j=1}^{N} |e\rangle_{jj}\langle e| - \Delta_{gr} \sum_{j=1}^{N} |r\rangle_{jj}\langle r| - \Delta_{gs} \sum_{j=1}^{N} |s\rangle_{jj}\langle s|, \tag{4}$$

describe the effective energy of the MR, cavity mode and atoms, respectively. a (b) denotes the annihilation operator of the cavity (phonon) mode.

The corresponding detunings in Fig. 1 are defined by

$$\begin{aligned} \delta_{\rm c}' &= \omega_{\rm p} - \omega_{\rm c}, \\ \Delta_g &= \omega_{\rm p} - \omega_{eg}, \\ \Delta_{gr} &= \Delta_g - (\omega_{\rm L1} - \omega_{er}), \\ \Delta_{gs} &= \Delta_g - (\omega_{\rm L2} - \omega_{es}), \end{aligned} \tag{5}$$

with $\omega_{e\alpha}$ being the transition frequency between states $|e\rangle$ and $|\alpha\rangle$ for $\alpha = g, r, s$.

The interaction Hamiltonian

$$H_{\rm I} = H_{\rm at-cav} + H_{\rm at-L} + H_{\rm p} + H_{\rm m-cav} \tag{6}$$

is composed of

$$\begin{aligned} H_{\rm at-cav} &= g_0 \sum_{j=1}^{N} \left(a^\dagger \sigma_{ge,j}^- + a \sigma_{ge,j}^+ \right), \\ H_{\rm at-L} &= \Omega_r \sum_{j=1}^{N} \left(\sigma_{re,j}^- + \sigma_{re,j}^+ \right) + \Omega_s \sum_{j=1}^{N} \left(\sigma_{se,j}^- + \sigma_{se,j}^+ \right), \\ H_{\rm p} &= \Omega_{\rm p} \left(a^\dagger + a \right), \\ H_{\rm m-cav} &= \lambda_0 a^\dagger a \left(b^\dagger + b \right), \end{aligned} \tag{7}$$

where $H_{\rm p}$ describes that the optomechanical cavity is driven by the pump field, $H_{\rm m-cav}$ shows the radiation pressure coupling between the optomechanical cavity and the MR, and $H_{\rm at-cav}$ ($H_{\rm at-L}$) represents the interaction between atoms and cavity (optical) field. λ_0 is the single-photon radiation pressure coupling strength, and operator $\sigma_{\alpha\beta}^-$ is defined by $\sigma_{\alpha\beta}^- = |\alpha\rangle_{jj}\langle\beta|$ for $\alpha, \beta = g, r, s, e$.

Assume the effective atom-cavity coupling $g\langle a\rangle$ is much weaker than the coupling strengths Ω_r and Ω_s ($g|\langle a\rangle| \ll \{\Omega_r, \Omega_s\}$), so that most of the atoms would stay in their ground state $|g\rangle$. Then, we can employ the bosonization approximations,[26,32] $\frac{1}{\sqrt{N}} \sum_{j=1}^{N} \sigma_{ge,j}^- \to E$, $\frac{1}{\sqrt{N}} \sum_{j=1}^{N} \sigma_{gr,j}^- \to R$, and $\frac{1}{\sqrt{N}} \sum_{j=1}^{N} \sigma_{gs,j}^- \to S$ with $\left[F, F^\dagger\right] = 1$ $(F = E, R, S)$, to rewrite Hamiltonian (1) as

$$\begin{aligned} H =\ & \omega_{\rm m} b^\dagger b - \delta_{\rm c}' a^\dagger a - \Delta_g E^\dagger E - \Delta_{gr} R^\dagger R - \Delta_{gs} S^\dagger S \\ & + g_N \left(a E^\dagger + a^\dagger E \right) + \Omega_r \left(R E^\dagger + R^\dagger E \right) + \Omega_s \left(S E^\dagger + S^\dagger E \right) \\ & + \Omega_{\rm p} \left(a^\dagger + a \right) + \lambda_0 a^\dagger a \left(b^\dagger + b \right). \end{aligned} \tag{8}$$

Here $g_N = g_0\sqrt{N}$ means that only the effective atom-cavity coupling between states $|g\rangle$ and $|e\rangle$ is enhanced by a factor $\sqrt{N}$ due to the collective effects.

2.2. Canonical transformation

The equations of motion based on the Hamiltonian (8) can be written as

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}a &= \left(\mathrm{i}\delta_{\mathrm{c}}'-\kappa\right)a-\mathrm{i}g_N E-\mathrm{i}\lambda_0 a\left(b^{\dagger}+b\right)-\mathrm{i}\Omega_{\mathrm{p}}+a_{\mathrm{in}},\\
\frac{\mathrm{d}}{\mathrm{d}t}b &= -\left(\mathrm{i}\omega_{\mathrm{m}}+\gamma_{\mathrm{m}}\right)b-\mathrm{i}\lambda_0 a^{\dagger}a+b_{\mathrm{in}},\\
\frac{\mathrm{d}}{\mathrm{d}t}E &= \left(\mathrm{i}\Delta_g-\gamma\right)E-\mathrm{i}g_N a-\mathrm{i}\Omega_r R-\mathrm{i}\Omega_s S+E_{\mathrm{in}},\\
\frac{\mathrm{d}}{\mathrm{d}t}R &= \mathrm{i}\left(\Delta_{gr}R-\Omega_r E\right)+R_{\mathrm{in}},\\
\frac{\mathrm{d}}{\mathrm{d}t}S &= \mathrm{i}\left(\Delta_{gs}S-\Omega_s E\right)+S_{\mathrm{in}},
\end{aligned}\tag{9}$$

where 2γ and 2κ are the atom and cavity decay rates, respectively. The noise terms fulfill the time correlation functions $\left\langle a_{\mathrm{in}}(t)\,a_{\mathrm{in}}^{\dagger}(t')\right\rangle=\kappa\delta(t-t')$, $\left\langle E_{\mathrm{in}}(t)\,E_{\mathrm{in}}^{\dagger}(t')\right\rangle=\gamma\delta(t-t')$, $\left\langle R_{\mathrm{in}}(t)\,R_{\mathrm{in}}^{\dagger}(t')\right\rangle=\left\langle S_{\mathrm{in}}(t)\,S_{\mathrm{in}}^{\dagger}(t')\right\rangle=0$, $\left\langle b_{\mathrm{in}}^{\dagger}(t)\,b_{\mathrm{in}}(t')\right\rangle=\gamma_{\mathrm{m}}n_{\mathrm{th}}\delta(t-t')$, and $\left\langle b_{\mathrm{in}}(t)\,b_{\mathrm{in}}^{\dagger}(t')\right\rangle=\gamma_{\mathrm{m}}(n_{\mathrm{th}}+1)\,\delta(t-t')$ with $2\gamma_{\mathrm{m}}$ being the MR dissipation rate and n_{th} being the temperature dependent thermal phonon number. Note that, the decays from the excited state $|e\rangle$ to $|r\rangle$ and $|s\rangle$ can be ignored since we assume all the atoms are in its ground state $|g\rangle$.[41]

Then the corresponding steady solutions of Eq. (9) can be given as

$$\begin{aligned}
\langle a\rangle &= \frac{\Omega_{\mathrm{p}}\left[\Delta_{gr}\Omega_s^2+\Delta_{gs}\Omega_r^2+\left(-\Delta_g-\mathrm{i}\gamma\right)\Delta_{gs}\Delta_{gr}\right]}{\mathcal{D}},\\
\langle b\rangle &= -\frac{\lambda_0}{\omega_{\mathrm{m}}-\mathrm{i}\gamma_{\mathrm{m}}}|\langle a\rangle|^2,\\
\langle E\rangle &= -\frac{g_N\Omega_{\mathrm{p}}\Delta_{gs}\Delta_{gr}}{\mathcal{D}},\\
\langle R\rangle &= -\frac{g_N\Omega_{\mathrm{p}}\Delta_{gs}\Omega_r}{\mathcal{D}},\\
\langle S\rangle &= -\frac{g_N\Omega_{\mathrm{p}}\Delta_{gr}\Omega_s}{\mathcal{D}},
\end{aligned}\tag{10}$$

with

$$\begin{aligned}
\mathcal{D} &= \left(\delta_{\mathrm{c}}+\mathrm{i}\kappa\right)\left(\Delta_{gr}\Omega_s^2+\Delta_{gs}\Omega_r^2\right)\\
&\quad+\left[g_N^2-\left(\delta_{\mathrm{c}}+\mathrm{i}\kappa\right)\left(\Delta_g+\mathrm{i}\gamma\right)\right]\Delta_{gs}\Delta_{gr}
\end{aligned}\tag{11}$$

and the effective detuning $\delta_{\mathrm{c}}=\delta_{\mathrm{c}}'+2\lambda_0^2|\langle a\rangle|^2/\omega_{\mathrm{m}}$.

With the application of the linear transformation $O\to\langle O\rangle+\delta O$, for $O=a,b,E,R,S$, and δO being the fluctuation of the operator O, the linear Hamiltonian of fluctuations for Eq. (8) can be written as

$$\begin{aligned}
H_{\mathrm{L}} &= \omega_{\mathrm{m}}b^{\dagger}b-\delta_{\mathrm{c}}a^{\dagger}a-\Delta_g E^{\dagger}E-\Delta_{gr}R^{\dagger}R-\Delta_{gs}S^{\dagger}S\\
&\quad+g_N\left(aE^{\dagger}+a^{\dagger}E\right)+\Omega_r\left(RE^{\dagger}+R^{\dagger}E\right)+\Omega_s\left(SE^{\dagger}+S^{\dagger}E\right)\\
&\quad+\lambda\left(a^{\dagger}+a\right)\left(b^{\dagger}+b\right),
\end{aligned}\tag{12}$$

where the optomechanical coupling strength is $\lambda=\lambda_0|\langle a\rangle|$, and the notation δ of the fluctuation operator δO has been dropped in Eq. (12) for simplicity.

Then the master equation of the whole system can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho=-\mathrm{i}[H_{\mathrm{L}},\rho]+\mathcal{L}_{\gamma}\rho+\mathcal{L}_{\kappa}\rho+\mathcal{L}_{\gamma_{\mathrm{m}}}\rho,\tag{13}$$

where

$$\begin{aligned}
\mathcal{L}_{\gamma}\rho &= \gamma\sum_j\left(2\sigma_{g,j}^{-}\rho\sigma_{g,j}^{+}-\sigma_{g,j}^{+}\sigma_{g,j}^{-}\rho-\rho\sigma_{g,j}^{+}\sigma_{g,j}^{-}\right)\\
&= \gamma\left(2E\rho E^{\dagger}-E^{\dagger}E\rho-\rho E^{\dagger}E\right),\\
\mathcal{L}_{\kappa}\rho &= \kappa\left(2a\rho a^{\dagger}-a^{\dagger}a\rho-\rho a^{\dagger}a\right),\\
\mathcal{L}_{\gamma_{\mathrm{m}}}\rho &= (n_{\mathrm{th}}+1)\gamma_{\mathrm{m}}(2b\rho b^{\dagger}-b^{\dagger}b\rho-\rho b^{\dagger}b)\\
&\quad+n_{\mathrm{th}}\gamma_{\mathrm{m}}(2b^{\dagger}\rho b-bb^{\dagger}\rho-\rho bb^{\dagger})
\end{aligned}\tag{14}$$

are the dissipation terms of atoms, cavity, and the MR, respectively.

2.3. Heating and cooling coefficients

After adiabatically eliminating the internal d.o.f[4,43] of the master Eq. (13), one can obtain the rate equation

$$\frac{\mathrm{d}}{\mathrm{d}t}p_n=p_{n+1}A_-+np_{n-1}A_+-p_n\left[(n+1)A_++nA_-\right],\tag{15}$$

where p_n is the phonon occupation probability on Fock state $|n\rangle$, and A_+ (A_-) is the heating (cooling) coefficient. Then the dynamics of the mean phonon number $\langle n\rangle=\sum_n np_n$ can be described by

$$\langle n(t)\rangle=n_{ss}+(n_0-n_{ss})\,\mathrm{e}^{-Wt}\tag{16}$$

with an initial mean phonon number n_0, cooling rate $W=A_--A_+$, and final mean phonon number $n_{ss}=A_+/W$.

The heating and cooling coefficients $A_{\pm}$, which can be calculated with the scattering amplitude method[44,45] (see Appendix A), are in the form of

$$A_{\pm}=A_{\pm}^{\gamma}+A_{\pm}^{\kappa}+A_{\pm}^{\gamma_{\mathrm{m}}},\tag{17}$$

where

$$A_{\pm}^{\gamma}=2\lambda^2\gamma\left|\frac{g_N\left(\Delta_{gr}\mp\omega_{\mathrm{m}}\right)\left(\Delta_{gs}\mp\omega_{\mathrm{m}}\right)}{f(\mp\omega_{\mathrm{m}})}\right|^2,\tag{18}$$

$$\begin{aligned}
A_{\pm}^{\kappa}=2\lambda^2\kappa\Bigg|\frac{1}{f(\mp\omega_{\mathrm{m}})}\big[&\left(\Delta_g\mp\omega_{\mathrm{m}}+\mathrm{i}\gamma\right)\left(\Delta_{gr}\mp\omega_{\mathrm{m}}\right)\left(\Delta_{gs}\mp\omega_{\mathrm{m}}\right)\\
&-\left(\Delta_{gr}\mp\omega_{\mathrm{m}}\right)\Omega_s^2-\left(\Delta_{gs}\mp\omega_{\mathrm{m}}\right)\Omega_r^2\big]\Bigg|^2
\end{aligned}\tag{19}$$

with

$$\begin{aligned}
f(\omega) &= \left(\omega+\Delta_{gr}\right)\left(\omega+\Delta_{gs}\right)\left[-g_N^2-\kappa\gamma+\left(\omega+\delta_{\mathrm{c}}\right)\left(\omega+\Delta_g\right)\right]\\
&\quad-\left(\omega+\delta_{\mathrm{c}}\right)\left(\omega+\Delta_{gr}\right)\Omega_s^2-\left(\omega+\delta_{\mathrm{c}}\right)\left(\omega+\Delta_{gs}\right)\Omega_r^2\\
&\quad+\mathrm{i}\Big\{\gamma\left(\omega+\Delta_{gr}\right)\left(\omega+\Delta_{gs}\right)\left(\omega+\delta_{\mathrm{c}}\right)\\
&\quad+\kappa\left[\left(\omega+\Delta_{gr}\right)\left(\omega+\Delta_{gs}\right)\left(\omega+\Delta_g\right)-\left(\omega+\Delta_{gr}\right)\Omega_s^2\right.\\
&\quad\left.-\left(\omega+\Delta_{gs}\right)\Omega_r^2\right]\Big\},\\
A_+^{\gamma_{\mathrm{m}}} &= 2\gamma_{\mathrm{m}}n_{\mathrm{th}},\quad A_-^{\gamma_{\mathrm{m}}}=2\gamma_{\mathrm{m}}(n_{\mathrm{th}}+1).
\end{aligned}$$

3. Results

3.1. Displaced frame and cooling processes

In the following, we would like to analyze the cooling dynamics with the displaced frame in Fig. 2(a).[16] Assume that the internal state is initially in the state $|1\rangle = |N_G, N_E, N_R, N_S\rangle |n_c\rangle$, where N_j indicates the N_j-th Fock state of the atomic ensemble excitation ($j = G, E, R, S$) and $|n_c\rangle$ ($|n_b\rangle$) is the Fock states of cavity mode (the MR). The transition processes between the different states can be realized as the follows: the state $|1\rangle|n_b\rangle$ can be coupled to states $|2\rangle|n_b \pm 1\rangle = |N_G, N_E, N_R, N_S\rangle |n_c + 1\rangle |n_b \pm 1\rangle$ via the radiation pressure coupling λ, the transition from state $|2\rangle$ to state $|3\rangle = |N_G, N_E + 1, N_R, N_S\rangle |n_c\rangle$ can be realized by the cavity-ensemble interaction g_N, the state $|3\rangle$ can be transferred to states $|4\rangle = |N_G, N_E, N_R + 1, N_S\rangle |n_c\rangle$ and $|5\rangle = |N_G, N_E, N_R, N_S + 1\rangle |n_c\rangle$ with laser fields Ω_r and Ω_s, respectively, and states $|2\rangle$ and $|3\rangle$ can dissipate to internal steady state $|1\rangle$ with the atomic decay γ and cavity decay κ, respectively.

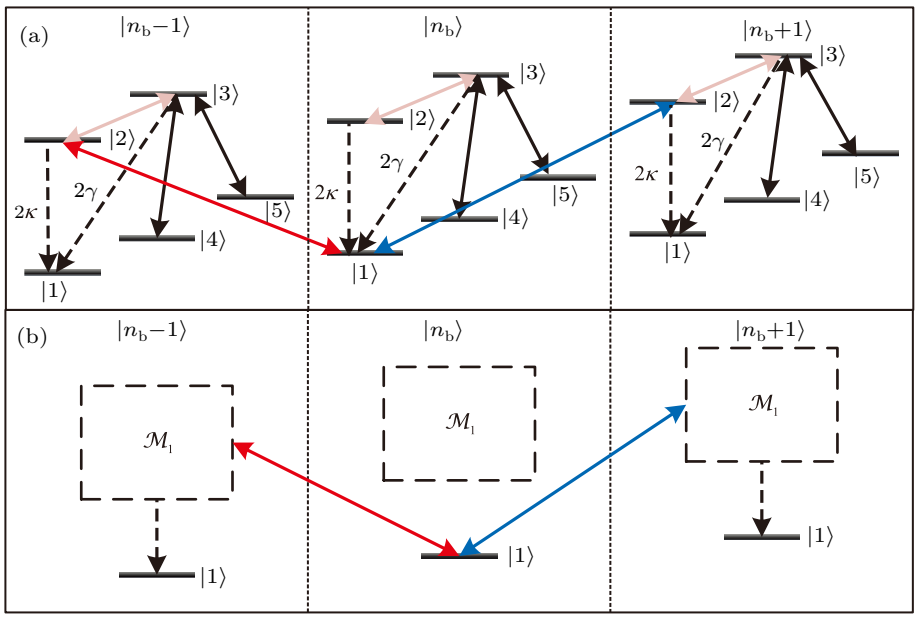


Fig. 2. (a) The cooling dynamics in displaced frame. (b) The cooling dynamics can be reduced to transitions between the steady state and the first excited subspace.

Therefore, the internal d.o.f can be divided into the steady state $|1\rangle$ and the first excited subspace $\mathcal{M}_1$ spanned by states $\{|2\rangle, |3\rangle, |4\rangle, |5\rangle\}$. In this view, the Hamiltonian (12) should be rewritten as

$$H'_L = H'_{L0} + V', \tag{20}$$

with

$$\begin{aligned} H'_{L0} &= \omega_m b^\dagger b - \delta_c |2\rangle\langle 2| - \Delta_g |3\rangle\langle 3| - \Delta_{gr} |4\rangle\langle 4| - \Delta_{gs} |5\rangle\langle 5| \\ &\quad + g_N (|2\rangle\langle 3| + |3\rangle\langle 2|) + \Omega_r (|3\rangle\langle 4| + |4\rangle\langle 3|) \\ &\quad + \Omega_s (|3\rangle\langle 5| + |5\rangle\langle 3|), \end{aligned} \tag{21}$$

$$V' = \lambda (|1\rangle\langle 2| + |2\rangle\langle 1|)(b + b^\dagger). \tag{22}$$

Then the cooling processes based on the above Hamiltonian can be understood as the follows. The initial state $|1\rangle|n_b\rangle$ can be excited to the state $|2\rangle|n_b - 1\rangle$ for cooling and $|2\rangle|n_b + 1\rangle$ for heating via sideband interaction V'. However, there are two kinds of quantum interference processes (see Section 3.2 for detail), and the transition from the state $|1\rangle|n_b\rangle$ to state $|2\rangle|n_b - 1\rangle$ for cooling can be enhanced, while the one from the state $|1\rangle|n_b\rangle$ to state $|2\rangle|n_b + 1\rangle$ for heating would be suppressed. As a result, the transition for $|1\rangle|n_b\rangle \leftrightarrow |2\rangle|n_b - 1\rangle$ will dominate the system. After that, the state $|2\rangle|n_b - 1\rangle$ will be kept in the first excited subspace $|m\rangle|n_b - 1\rangle$ with $m = 2, 3, 4, 5$ (see Fig. 4(b)) for H'_{L0} until it decays to the final state $|1\rangle|n_b - 1\rangle$ via the transition $|2\rangle|n_b\rangle \leftrightarrow |1\rangle|n_b - 1\rangle$ ($|3\rangle|n_b\rangle \leftrightarrow |1, n_b - 1\rangle$) for the cavity (atom) decay.

3.2. Cooling conditions

Considering that $A_\pm^{\gamma_m}$ are the constants for the thermal phonon number which depends on the temperature, to find the optimal cooling conditions for $A_- \gg A_+$, we can neglect these two terms in $A_\pm$ in the idea case ($T = 0$ mK). When the number of atoms N is large enough, we can suppose the coupling between the atomic ensemble and cavity is sufficiently strong, which can ensure the cooperativity satisfies $C_N = g_N^2/\kappa\gamma \gg 1$. Then the optimal cooling condition can be derived as the follows.

Firstly, efficient cooling can be achieved by enhancing the cooling rate W, which is proportional to $\lambda^2 = \lambda_0^2 |\langle a \rangle|^2$. According to the steady solution $\langle a \rangle$ in (10), one would get the

minimum of the denominator $|\mathcal{D}|$ when

$$\Delta_{gs} = 0, \tag{23}$$

which is the condition for the cavity-induced EIT resonance. In this situation, atoms are transparent to the cavity field, and the photon absorption vanishes. Therefore, the maximum of the optomechanical coupling strength λ is around $\Delta_{gs} = 0$. Then the mean photon number of the cavity can be reduced to

$$|\langle a\rangle|^2 \approx \left|\frac{\Omega_{\rm p}}{\delta_{\rm c}+{\rm i}\kappa}\right|^2. \tag{24}$$

Secondly, as shown in Refs. [29,44], the cooling coefficient A_- can reach its maximum when $\mathrm{Re}\{f(\omega_{\rm m})\} = 0$. In our scheme, it can be easily achieved by choosing that

$$\delta_{\rm c} = \Delta_{gr} = -\omega_{\rm m}. \tag{25}$$

The above physical picture can be understood in the displaced frame (as plotted in Figs. 3(a) and 3(b)): when the detunings $\delta_{\rm c}$ and Δ_{gr} are tuned to the same ($\delta_{\rm c} = \Delta_{gr}$), an EIT-like resonance takes place among states $|2\rangle$, $|3\rangle$, and $|4\rangle$, and a dark-state-like eigenstate of internal d.o.f $H'_{\rm L0}$ is

$$|d\rangle = \frac{1}{\sqrt{g_N^2+\Omega_r^2}}\left(\Omega_r|2\rangle - g_N|4\rangle\right), \tag{26}$$

$$H'_{\rm L0}|d\rangle = -\delta_{\rm c}|d\rangle. \tag{27}$$

In the case of $g_N \gg \Omega_r$, the linewidth of $|d\rangle$ is $2\gamma_d = 2\frac{\Omega_r^2}{g_N^2}\kappa$, which is much smaller than the natural linewidth of the cavity. Condition (25) ensures the red sideband transition is in resonance to the one for the narrow dressed states $|1\rangle|n_{\rm b}\rangle \to |d\rangle|n_{\rm b}-1\rangle$, as shown in Fig. 3(c).

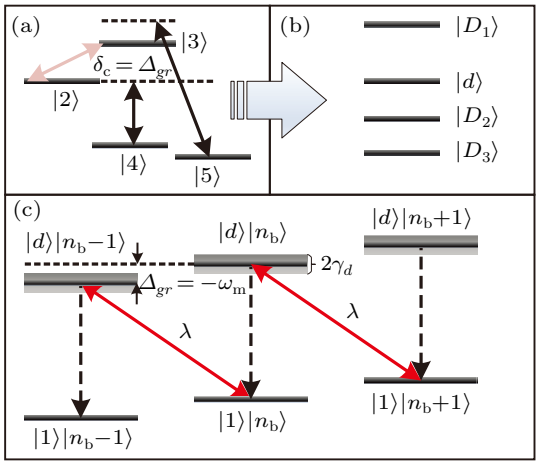


Fig. 3. (a) The interaction between states $|2\rangle$, $|3\rangle$, $|4\rangle$, and $|5\rangle$. Under the condition $\delta_{\rm c} = \Delta_{gr}$, an EIT-like quantum interference between transitions $|2\rangle \leftrightarrow |3\rangle$ and $|5\rangle \leftrightarrow |3\rangle$. (b) The internal d.o.f $H'_{\rm L0}$ can be diagonalized in dressed-state representation, including a narrow linewidth state $|d\rangle$. (c) By tuning $\Delta_{gs} = -\omega_{\rm m}$, the red sideband transition is resonant with the narrow resonance, and the cooling dynamics is analogous to resolved-sideband cooling.

In this situation, the cooling coefficient A_- in Eq. (17) can be reduced to

$$A_- = 2\lambda^2\frac{1}{\kappa}, \tag{28}$$

with $\lambda^2 = \lambda_0^2\left|\frac{\Omega_{\rm p}}{\delta_{\rm c}+{\rm i}\kappa}\right|^2$.

Thirdly, one can find that the minimum value of A_+ is at

$$2\omega_{\rm m}(-\omega_{\rm m}+\Delta_g) + 2\Omega_s^2 + \Omega_r^2 = 0. \tag{29}$$

Condition (29) ensures a quantum destructive interference involving transitions $|1\rangle|n_{\rm b}\rangle \to |2\rangle|n_{\rm b}+1\rangle \to |3\rangle|n_{\rm b}+1\rangle$, $|4\rangle|n_{\rm b}+1\rangle \to |3\rangle|n_{\rm b}+1\rangle$, and $|5\rangle|n_{\rm b}+1\rangle \to |3\rangle|n_{\rm b}+1\rangle$, and A_+ is highly suppressed, which is similar to the optimal condition in double-EIT cooling scheme of a trapped ion.[39]

The combination of the optimal parameter conditions (23), (25), and (29) leads to

$$A_+ \simeq 2\lambda^2\frac{4\gamma g_N^2}{4g_N^4+16\gamma^2\omega_{\rm m}^2}. \tag{30}$$

Then we can get a reduced cooling rate

$$W \simeq A_- = 2\lambda^2\frac{1}{\kappa}, \tag{31}$$

and its corresponding final mean phonon number

$$n_{ss} \simeq \frac{1}{C_N}. \tag{32}$$

Equation (32) shows the final phonon number is limited by the cooperativity C_N. When the cooperativity is much larger than 1 ($C_N \gg 1$), the final phonon number will be approaching to the phononic ground state as $n_{ss} \ll 1$. After the item $A_\pm^{\gamma_{\rm m}}$ is included in Eq. (17), the cooling rate and final mean phonon number will respectively be

$$\begin{aligned} W' &= 2\lambda^2\frac{1}{\kappa} + 2\gamma_{\rm m} \\ n'_{ss} &= \frac{A_+ + 2\gamma_{\rm m}n_{\rm th}}{W'} = \frac{1}{C_N} + \frac{\kappa\gamma_{\rm m}n_{\rm th}}{\lambda^2}. \end{aligned} \tag{33}$$

4. Numerical simulations and discussion

To verify the validity of the optimal cooling conditions in the experiments, we choose the experimental parameters as follows: $\omega_{\rm m} = 2\pi\times 1$ MHz, $\lambda_0 = 2\pi\times 2.5$ kHz or $2\pi\times 5$ kHz, $2\kappa = 2\pi\times 10$ MHz, $N = 10^6$, $2\gamma = 2\pi\times 10$ MHz, $g_0 = 2\pi\times 200$ kHz, $\Omega_{\rm p} = 2\pi\times 300$ MHz $\Omega_r = \Omega_s = 2\pi\times 50$ MHz, and present some numerical calculations below.

The final phonon numbers as functions of detunings Δ_{gr}, $\delta_{\rm c}$, and Δ_g are plotted in Fig. 4. According to the optimal parameter conditions (23), (25), and (29), the minimal final mean phonon number is at $\Delta_{gr} = \delta_{\rm c} = -\omega_{\rm m}$ and $\Delta_g = -3750\omega_{\rm m}$. To find the optimal parameters for cooling with Fig. 4, we choose the two optimal detunings in theory, and numerically simulate the final mean phonon number as a function of the third detuning. One can find that the optimal values of the third detunings coincide with analytical ones.

To demonstrate our scheme can cool the MR to its ground state more efficiently, we would like to compare the our scheme with three-level atomic ensemble assisted cooling schemes[25,26,28,31–33] via the cooling rate W. As is shown in Eq. (33), the increase of coupling strength $\lambda = \lambda_0|\langle a\rangle|$

leads to a larger cooling rate and a lower final temperature. In the three-level atomic ensemble assisted cooling methods, the steady value of the cavity mode is $|\langle a\rangle| \simeq \left|\frac{\Omega_{\rm p}}{\delta_{\rm c}+{\rm i}\kappa+\chi_{\rm EIT}}\right|$, with an EIT medium susceptibility $\chi_{\rm EIT} = -\frac{g_N^2}{\left(\Delta_g+\frac{\Omega_r^2}{\Delta_{gr}}\right)+{\rm i}\gamma}$.[26,46] In the strong coupling regime and optimal cooling conditions,[26,33] the EIT medium susceptibility would be much larger than the detuning and cavity decay rate ($|\chi_{\rm EIT}| \gg \delta_{\rm c}, \kappa$), which means that the intracavity photons are highly absorbed by the atomic ensemble. On the other hand, in our scheme, we introduce the fourth atomic level to provide an additional interference channel to avoid this absorption. For this reason, in the case of $\Delta_{gs} = 0$, there is a cavity-EIT resonance for transitions $|g\rangle \to |e\rangle$ and $|s\rangle \to |e\rangle$, and this quantum interference can ensure that atoms are transparent to the cavity (a zero absorption $|\chi_{\rm EIT}| = 0$) and larger photons in cavity mode for $|\langle a\rangle| = \left|\frac{\Omega_{\rm p}}{\delta_{\rm c}+{\rm i}\kappa}\right|$) can be realized. Therefore, under the action of the same drive strength $\Omega_{\rm p}$, our scheme can provide a larger optomechanical coupling strength than previous schemes, which yields a better cooling performance.

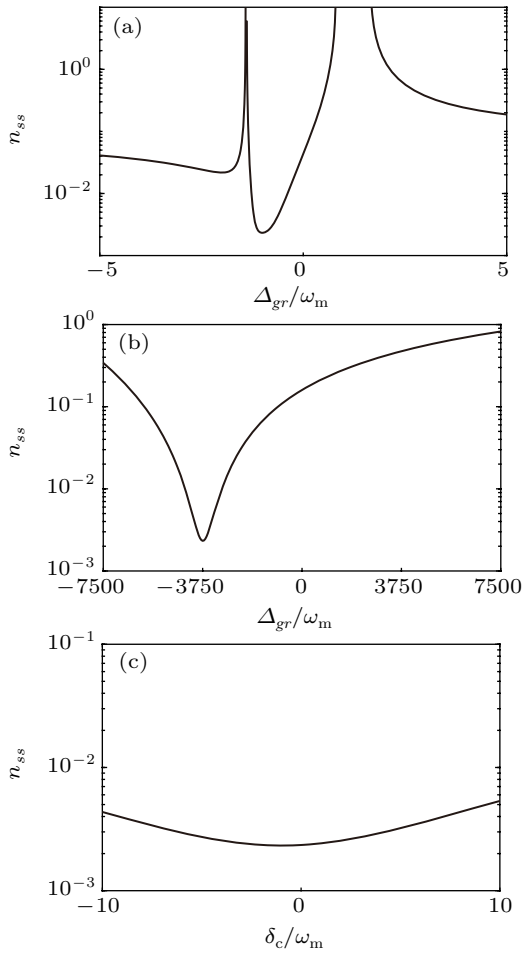


Fig. 4. Numerical simulations of the final mean phonon number as functions of (a) Δ_{gr}, (b) $\delta_{\rm c}$, and (c) Δ_g. The parameters are $\omega_{\rm m} = 2\pi \times 1$ MHz, $\lambda_0 = 2\pi \times 2.5$ kHz, $2\kappa = 2\pi \times 10$ MHz, $N = 10^6$, $2\gamma = 2\pi \times 10$ MHz, $g_0 = 2\pi \times 200$ kHz, $\Omega_{\rm p} = 2\pi \times 300$ MHz, $\Omega_r = \Omega_s = 2\pi \times 50$ MHz, $\delta_{\rm c} = -\omega_{\rm m}$ and $\Delta_g = -3750\omega_{\rm m}$ for (a), $\Delta_{gr} = -\omega_{\rm m}$ and $\Delta_g = -3750\omega_{\rm m}$ for (b), and $\Delta_{gr} = \delta_{\rm c} = -\omega_{\rm m}$ for (c).

In Fig. 5, we compare three-level atoms assisted cooling[26] (red dashed lines) with four-level atoms assisted cooling (black solid line). The intracavity photon number $|\langle a\rangle|^2$, cooling rate W, and final mean phonon number n_{ss} as functions of Δ_{gr} are plotted in Figs. 5(a)–5(c), respectively. From Figs. 5(a)–5(c), we can find that intracavity photon number of four-level atoms assisted cooling is independent of Δ_{gr}, and the maximal cooling rate and minimal final mean phonon number are at the detuning $\Delta_{gr} = -\omega_{\rm m}$, as the above discussions. That is very different from the three-level atoms assisted cooling, where the maximum intracavity photon number and cooling rate are at $\Delta_{gr} = 0$, while the minimal final mean phonon number is at $\Delta_{gr} = -\omega_{\rm m}$. The simulations also show the cooling rate in our scheme is about one order of magnitude larger than three-level atoms assisted cooling.

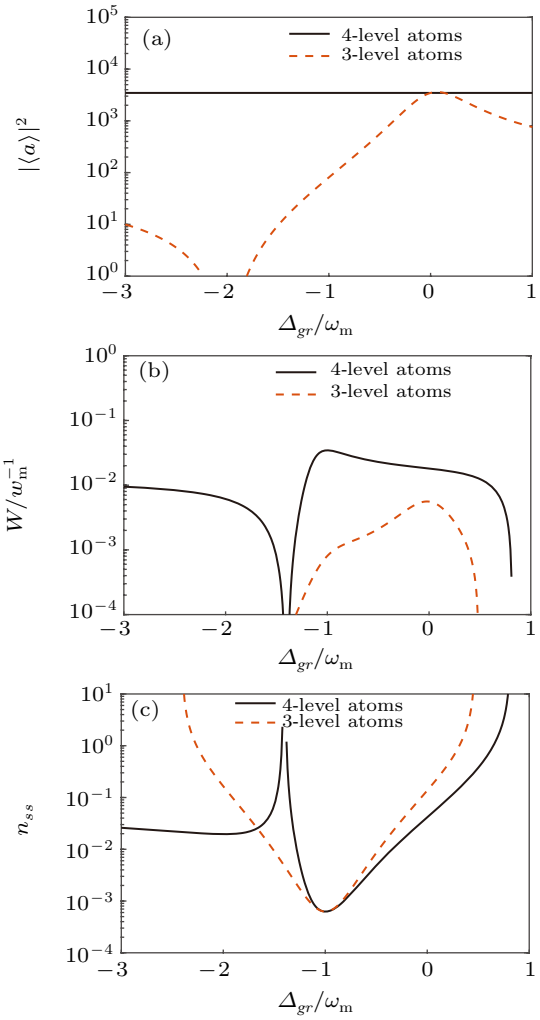


Fig. 5. The intracavity photon number $|\langle a\rangle|^2$ (a), cooling rate W (b) and final mean phonon number n_{ss} (c) as functions of Δ_{gr} for four-level atoms (black solid line) and three-level atoms (red dashed line). The parameters are $\omega_{\rm m} = 2\pi \times 1$ MHz, $\lambda_0 = 2\pi \times 5$ kHz, $2\kappa = 2\pi \times 10$ MHz, $2\gamma = 2\pi \times 10$ MHz, $g_N = 2\pi \times 200$ MHz, $\Omega_{\rm p} = 2\pi \times 300$ MHz, and $\delta_{\rm c} = -\omega_{\rm m}$. For four-level atoms, $\Delta_{gs} = 0$, $\Omega_r = \Omega_s = 2\pi \times 50$ MHz, and the corresponding optimal detunings are $\Delta_g = -3750\omega_{\rm m}$ and $\Delta_{gr} = -\omega_{\rm m}$; for three-level atoms, $\Omega_r = 2\pi \times 50$ MHz, and the corresponding optimal detunings are $\Delta_g = -1250\omega_{\rm m}$ and $\Delta_{gr} = -\omega_{\rm m}$.

Moreover, we would like to compare our scheme with the

double-EIT cooling scheme.[39] Both of the cooling schemes are achieved by a double-EIT structure in four-level atom(s). In Ref. [39], the scheme is proposed to cool the motion of a trapped ion, and it use two EIT structure to eliminate the carrier and blue sideband heating transitions, respectively; while our scheme is ground state cooling of the MR, and it employ the first cavity-EIT structure to avoid photon absorption by atoms and the second EIT-like structure to generate a narrow-linewidth dressed state. Therefore, the cooling mechanism of our scheme is significantly different from Ref. [39].

Since the system always interacts with the environment, it is necessary to study the ground state cooling of the MR with different temperature. We plot the cooling result affected by the thermal noise in Fig. 6. This simulation shows our scheme can cool the MR to its ground state $n_{ss} < 1$ at the temperature $T \sim 50$ mK and $\gamma_{\rm m} = 10^{-5}$. Moreover, similar to Refs. [4,47],

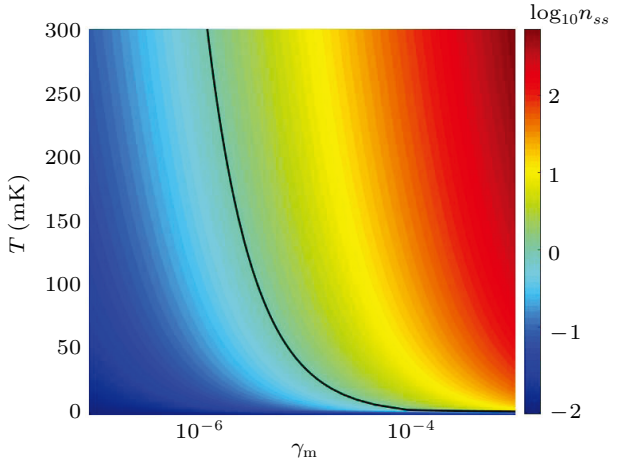


Fig. 6. Numerical simulation of the final mean phonon number $\log_{10} n_{ss}$ as a function of T and $\gamma_{\rm m}$. The parameters are $\omega_{\rm m} = 2\pi \times 1$ MHz, $\lambda_0 = 2\pi \times 5$ kHz, $2\kappa = 2\pi \times 10$ Hz, $2\gamma = 2\pi \times 10$ MHz, $g_N = 2\pi \times 200$ MHz, $\Omega_{\rm p} = 2\pi \times 300$ MHz, and $\Omega_r = \Omega_s = 2\pi \times 50$ MHz. The black line corresponds to $n_{ss} = 1$.

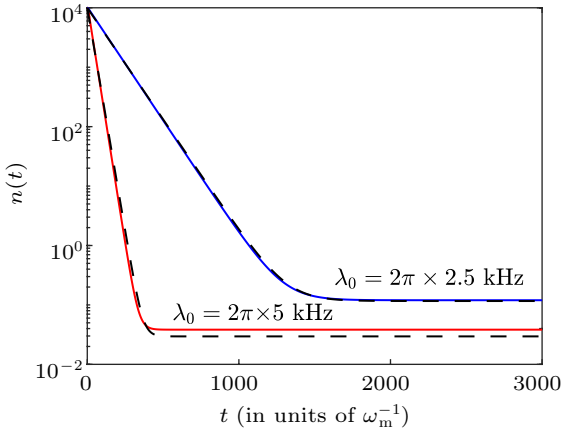


Fig. 7. Numerical simulations of the mean phonon number $n(t)$ for $\lambda_0 = 2\pi \times 5$ kHz (red solid line) and $\lambda_0 = 2\pi \times 2.5$ kHz (blue solid line). The dashed curves are their corresponding analytical prediction, respectively. The other parameters are $\Omega_{\rm p} = 2\pi \times 300$ MHz, $\Delta_{gs} = 0$, $\Delta_{gr} = \delta_{\rm c} = -\omega_{\rm m}$, $\Delta_g = -3750\omega_{\rm m}$, $\omega_{\rm m} = 2\pi \times 1$ MHz, $g_N = 2\pi \times 200$ MHz, $\Omega_r = \Omega_s = 2\pi \times 50$ MHz, $\gamma = \kappa = 2\pi \times 10$ MHz, $\gamma_{\rm m} = 5 \times 10^{-7}\omega_{\rm m}$, and $T = 24$ mK.

we also simulation the time evolution of mean phonon number $\langle n \rangle = \langle b^\dagger b \rangle$ by numerically calculating the equation of motion of the arbitrary second-order moment in Fig. 7. Figure 7 shows that the numerical simulation is in good agreement with the analytical solutions (16).

5. Conclusion

In conclusion, we have presented a ground state cooling scheme of a MR with the assistance of a four-level atomic ensemble. A better cooling result can be achieved by employing a double-quantum interference structures with different quantum interference processes. With the EIT-like quantum interference in atoms, the effective linewidth of the internal dressed state can be much smaller than the cavity linewidth, so that the cooling transition rate can be largely enhanced, while the heating transition is almost eliminated even in the non-resolved sideband regime. In addition, another quantum interference for the cavity induced transparency can be used to avoid photon absorption from atoms, a larger optomechanical coupling strength can be obtained, and a larger cooling rate can be achieved. As a result, this scheme can realize a faster cooling scheme than previous ones with the experimental parameters.

Appendix A: The calculation of $A_\pm^\kappa$, $A_\pm^\gamma$

The dynamics of the rate equation for phonon occupation probability p_n is in the form of

$$\frac{\mathrm{d}}{\mathrm{d}t} p_n = p_{n+1}\Gamma_{n+1\to n} + n\Gamma_{n-1\to n} - p_n\left(\Gamma_{n\to n+1} + \Gamma_{n\to n-1}\right), \tag{A1}$$

with $\Gamma_{i\to j}$ being the transition rate from p_i to p_j. Then the transition rates $\Gamma_{n\to n\pm1}$ can be obtained using scattering amplitude method in displaced frame. According to Refs. [44,45], we can define

$$\Gamma_{n\to n\pm1} = \Gamma^\kappa_{n\to n\pm1} + \Gamma^\gamma_{n\to n\pm1}, \tag{A2}$$

where $\Gamma^\kappa_{n\to n\pm1}$ and $\Gamma^\gamma_{n\to n\pm1}$ are scattering rates from $|\psi_{n_{\rm b}}\rangle = |1\rangle|n_{\rm b}\rangle$ to $|\psi_{n\pm1}\rangle = |1\rangle|n_{\rm b}\pm1\rangle$ via the cavity decay and atomic spontaneous decay, respectively. The transition rates can be calculated with

$$\begin{aligned}\Gamma^\kappa_{n\to n\pm1} &= 2\kappa\left|\langle\psi_{n\pm1}|W_\kappa\frac{1}{E_n - H_{\rm eff}}V'|\psi_n\rangle\right|^2\\ \Gamma^\gamma_{n\to n\pm1} &= 2\gamma\left|\langle\psi_{n\pm1}|W_\gamma\frac{1}{E_n - H_{\rm eff}}V'|\psi_n\rangle\right|^2,\end{aligned} \tag{A3}$$

where operators $W_\kappa = |1\rangle\langle 2|$ and $W_\gamma = |1\rangle\langle 3|$ describe cavity decay and atomic spontaneous decay, respectively, E_n denotes the energy of $|\psi_n\rangle$, and the effective Hamiltonian is $H_{\rm eff} = H'_{\rm L0} - {\rm i}\kappa|2\rangle\langle 2| - {\rm i}\gamma|3\rangle\langle 3|$.

With the assistance of Eq. (A3), one can get

$$\Gamma^\kappa_{n\to n+1} = (n+1)A^\kappa_+, \quad \Gamma^\kappa_{n\to n-1} = nA^\kappa_-$$

$$\Gamma^{\gamma}_{n\to n+1} = (n+1)A^{\gamma}_{+}, \quad \Gamma^{\gamma}_{n\to n-1} = nA^{\gamma}_{-}. \tag{A4}$$

As a result, the explicit form of $A^{\kappa}_{\pm}$ and $A^{\gamma}_{\pm}$ in Eqs. (18) and (19) can be obtained in this way.

References

[1] Kippenberg T J and Vahala K J 2008 *Science* **321** 1172
[2] Aspelmeyer M, Kippenberg T J and Marquardt F 2014 *Rev. Mod. Phys.* **86** 1391
[3] Marquardt F and Girvin S M 2009 *Physics* **2** 40
[4] Wilso-Rae I, Nooshi N, Zwerger W and Kippenberg T J 2007 *Phys. Rev. Lett.* **99** 093901
[5] Marquardt F, Chen J P, Clerk A and Girvin S M 2007 *Phys. Rev. Lett.* **99** 093902
[6] Teufel J D, Donner T, Li D, Harlow J W, Allman M S, Cicak K, Sirois A J, Whittaker J D, Lehnert K W and Simmonds R W 2011 *Nature* **475** 359
[7] Chan J, Mayer Alegre T P, Safavi-Naeini A H, Hill J T, Krause A, Groblacher S, Aspelmeyer M and Painter O 2011 *Nature* **478** 89
[8] Poggio M, Degen C L, Mamin H J and Rugar D 2007 *Phys. Rev. Lett.* **99** 017201
[9] Vanner M R, Hofer J, Cole J D and Aspelmeyer M 2013 *Nat. Commun.* **4** 2295
[10] Saxena G M and Agarwal A 2006 *Opt. Commun.* **267** 124
[11] Clark J B, Lecocq F, Simmonds R W, Aumentado J and Teufel J D 2017 *Nature* **541** 191
[12] Gan J H, Liu Y C, Lu C, Wang X, Tey M K and You L 2019 *Laser Photonic Rev.* **13** 1900120
[13] Wang D Y, Bai C H, Liu S T, Zhang S and Wang H F 2018 *Phys. Rev. A* **98** 023816
[14] Gu W J and Li G X 2013 *Phys. Rev. A* **87** 025804
[15] Guo Y, Li K, Nie W and Li Y 2014 *Phys. Rev. A* **90** 053841
[16] Liu Y C, Xiao Y F, Luan X S, Gong Q H and Wong C W 2015 *Phy. Rev. A* **91** 033818
[17] Xia K and Evers J 2009 *Phys. Rev. Lett.* **103** 227203
[18] Wilson-Rae I, Zoller P and Imamoğlu A 2004 *Phys. Rev. Lett.* **92** 075507
[19] Zhu J P, Li G X and Ficek Z 2012 *Phys. Rev. A* **85** 033835
[20] Zhu J P and Li G X 2012 *Phys. Rev. A* **86** 053828
[21] Rabl P, Cappellaro P, Gurudev Dutt M V, Jiang L, Maze J R and Lukin M D 2009 *Phys. Rev. B* **79** 041302
[22] Zhang J Q, Zhang S, Zou J H, Chen L, Yang W, Li Y and Feng M 2013 *Opt. Express* **21** 29695
[23] Yan L L, Zhang J Q, Zhang S and Feng M 2015 *Sci. Rep.* **5** 14977
[24] Yan L L, Zhang J Q, Zhang S and Feng M 2016 *Phys. Rev. A* **94** 063419
[25] Genes C, Ritsch H and Vitali D 2009 *Phys. Rev. A* **80** 061803
[26] Genes C, Ritsch H, Drewsen M and Dantan A 2011 *Phys. Rev. A* **84** 051801
[27] Breyer D and Bienert M 2012 *Phys. Rev. A* **86** 053819
[28] Vogel B, Stannige K and Zoller P 2013 *Phys. Rev. A* **87** 023816
[29] Zhang S, Zhang J Q, Zhang J, Wu C W, Wu W and Chen P X 2014 *Opt. Express* **22** 28118
[30] Yi Z, Li G X, Wu S P and Yang W P 2014 *Opt. Express* **22** 20060
[31] Chen X, Liu Y C, Peng P, Zhi Y Y and Xiao Y F 2015 *Phys. Rev. A* **92** 033841
[32] Zeng R P, Zhang S, Wu C W, Wu W and Chen P X 2015 *JOSAB* **32** 2314
[33] Li T, Zhang S, Huang H L, Li F G, Fu X Q, Wang X and Bao W S 2018 *J. Phys. B: At. Mol. Opt. Phys.* **51** 045503
[34] Liu Y M, Bai C H, Wang D Y, Wang T, Zheng M H, Wang H F, Zhu A D and Zhang S 2018 *Opt. Express* **26** 6143
[35] Liao Q H, Dai Y Z, Nie W J, Liu X and Liu Y C 2020 *J. Phys. B: At. Mol. Opt. Phys.* **53** 085402
[36] Li L C, Luo R H, Liu L J, Zhang S and Zhang J Q 2018 *Sci. Rep.* **8** 14276
[37] Fleischhauer M, Imamoğlu A and Marangos J P 2005 *Rev. Mod. Phys.* **77** 633
[38] Morigi G, Eschner J and Keitel C H 2000 *Phys. Rev. Lett.* **85** 4458
[39] Evers J and Keitel C H 2004 *Europhys. Lett.* **68** 370
[40] Rice P R and Brecha R J 1996 *Opt. Comm.* **126** 230
[41] Bienert M and Morigi G 2012 *New J. Phys.* **14** 023002
[42] Zhang S, Duan Q H, Guo C, Wu C W, Wu W and Chen P X 2014 *Phys. Rev. A* **89** 013402
[43] Cirac J I, Blatt R and Zoller P 1992 *Phys. Rev. A* **46** 2668
[44] Zippilli S and Morigi G 2005 *Phys. Rev. A* **72** 053408
[45] Cohen-Tannoudji C, Dupont-Roc J and Grynberg G 1998 *Atom–Photon Interactions: Basic Processes and Applications* (New York: John Wiley and Sons)
[46] Dantan A, Albert M and Drewsen M 2012 *Phys. Rev. A* **85** 013840
[47] Liu Y C, Shen Y F, Gong Q H and Xiao Y F 2014 *Phys. Rev. A* **89** 053821