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Coherent-driving-assisted quantum speedup in Markovian channels*

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As is well known, the quantum evolution speed of quantum state can never be accelerated in the Markovian regime without any operators on the system. The Hamiltonian corrections induced by the action of coherent driving forces are often used to fight dissipative and decoherence mechanisms in experiments. For this reason, considering three noisy channels (the phase-flip channel, the amplitude damping channel and the depolarizing channel), we propose a scheme of speedup evolution of an open system by controlling an external unitary coherent driving operator on the system. It is shown that, in the presence of the coherent driving, no-speedup evolution can be transformed into quantum speedup evolution in the Markovian dynamics process. Additionally, under the fixed coherent driving strength in the above three noisy channels, the best way to achieve the most degree of quantum speedup for the system has been acquired by rotating the system with appropriate driving direction angles, respectively. Finally, we conclude that the reason for this acceleration is not the non-Markovian dynamical behavior of the system but due to the oscillation of geometric distance between the initial state and the target final state.

Keywords: quantum dynamics control, quantum speed limit, Markovian dynamics

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1. Introduction

Quantum speed limit time (QSLT) defined as the shortest time of evolution between two distinguishable states,[1–19] is a key method for characterizing the maximal evolution speed of a quantum system. The understanding of QSLT has been applied in quantum communication,[20,21] non-equilibrium thermodynamics,[22] quantum metrology,[23] quantum optimal control[24–27] and other fields. For a closed quantum system, the quantum speed limit usually has two different boundaries, namely Mandelstam–Tamm (M-T) bound[1] and Margolus–Levitin (M-L) type boundary.[2] However, in real situations, any quantum information process is inevitably subject to environmental noise. Hence, how to delimit QSLT in the nonunitary dynamics of the open system is followed up by more and more researchers in quantum information field. In Refs. [13,15], relative purity is used to define the QSLT of arbitrary initial state. Deffner and Lutz[28] unified the M-T and M-L boundaries of QSLT using the Bures angle for the open system. Then, in 2019, Campaioli *et al.* proposed a solution method of the QSLT of any quantum state in the process of unitary dynamics[28] and non-unitary dynamics.[29] Recently, a new bound of QSLT[30] for open systems was proposed using the gauge invariant and geometric natures of quantum mechanics.

Moreover, many valuable efforts have also been taken to exploration of QSLT in open systems under non-unitary evolution. For instance, the effects of entanglement on multipartite systems,[33–36] decoherence speed limit of the spin boson system,[37] the effect of coherent or excited state population on QSLT,[17,18,39,40] research on QSLT in non-equilibrium environment[41] and QSLT in classical and quantum systems.[42,43] In addition, Deffner and Lutz[14] firstly pointed out that the non-Markovian regime of the environment can promote the speedup of quantum process. Therefore, a common way to speedup the quantum evolution process is to transform Markovian dynamics into non-Markovian dynamics,[14–18,44–46] the corresponding scheme is also obtained in the experiment.[47] Even though the non-Markovianity of environment has been currently manipulated in few experiments, they are often limited to certain physical systems. Hence, how to look for an oversimplified way to design an operational and experimental implementation for quantum speedup without considering the non-Markovian dynamical behavior is necessary and extremely significant.

The Hamiltonian corrections through the action of coherent driving forces are often used to fight dissipative and decoherence mechanisms.[48–50] The coherent driving Hamiltonian corresponding to a simple quantum rotation operation on the qubit could be easily realized in a variety of actual physical systems.[51–54] Based on the effects of the coherent driving Hamiltonian, in this paper we calculate and analyze the dynamics of an open system in three typical decoherent noise channels (the phase-flip channel, the amplitude damping channel and the depolarizing channel). By choosing QSLT[29] as a description of the achievable evolution speed of the open sys-

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tem, it is interesting to show that, in the presence of the coherent driving, no-speedup evolution in the Markovian dynamics process can be transformed into quantum speedup evolution. We also analyze the best driving way to achieve the most degree of quantum speedup in the considered three noisy channels, respectively. Furthermore, it is worth mentioning that the dynamics of the system still retains Markovian behavior. That is to say, the coherent driving to the system can not transfer the original Markovian behavior to the non-Markovian behavior. Unlike the previous speedup scheme, this speedup scheme can be achieved in the Markovian regime, and does not have to generate the non-Markovian behavior of the dynamics.

The organization structure of this work is as follows. In Section 2, we introduce the calculation method of the dynamics of the system in detail. In Section 3, we mainly analyze the results of the quantum speedup of a qubit in three different noisy channels. In Section 4, we present the conclusion of this work.

2. Dynamics in Markovian channels

In order to get the density matrix at any time by a given initial state of a quantum system, we mainly discuss the one-qubit system under the classical noisy influence of environment. Now we introduce the Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) master equation[58,59]

$$\dot{\rho}_t = \mathscr{L}[\rho_t] = \gamma \mathscr{D}[\rho_t] - \mathrm{i}\mathscr{H}[\rho_t], \tag{1}$$

with

$$\mathscr{D}[\rho_t] = \sum_{j=1}^{d^2-1} \left(L_j \rho_t L_j^\dagger - \frac{1}{2}[L_j^\dagger L_j, \rho_t]_+ \right), \tag{2}$$

$$\mathscr{H}[\rho_t] = [\hat{H}, \rho_t]_-. \tag{3}$$

Equation (1) consists of two parts: the first is a purely dissipative term, and the second is a coherence-preserving contribution gauged by a Hamiltonian term. Here L_j are the Lindblad operators, γ is the damping parameter, and d is the dimension of the system. The symbols $[\cdots,\cdots]_\pm$ indicate the commutator $(-)$ and anticommutator $(+)$ brackets, respectively. The driving of Hamiltonian term in Eq. (1) can be expressed by

$$\hat{H} = \omega \hat{n} \cdot \hat{\sigma}, \tag{4}$$

$$\hat{n} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta), \tag{5}$$

with $\hat{\sigma} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$. Here $\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z$ are the pauli matrices; ω is the uniform coherent driving strength to control the external unitary coherent driving on the qubit. Two driving direction angles θ and φ can be used to indicate the weights of the driving in three directions $(\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$. In this way, we can simultaneously analyze the influence of the driving strength and direction weights on the dynamics of the qubit. According to these two driving direction angles, the kinds of the coherence-preserving driving acted on the qubit can be given in Table 1 in the case of various special values.

Table 1. Driving direction at different special angles.

θ	φ			
	0	$\pi/2$	π	$3\pi/2$
0	$\hat{\sigma}_z$	$\hat{\sigma}_z$	$\hat{\sigma}_z$	$\hat{\sigma}_z$
$\pi/2$	$\hat{\sigma}_x$	$\hat{\sigma}_y$	$-\hat{\sigma}_x$	$-\hat{\sigma}_y$
π	$-\hat{\sigma}_z$	$-\hat{\sigma}_z$	$-\hat{\sigma}_z$	$-\hat{\sigma}_z$

In the operator basis $(E^{00}, E^{10}, E^{01}, E^{11})$ formed by the external products $E^{ij} = |i\rangle\langle j|$ of the computational basis (here $i, j = 1, 2$), the Lindblad superoperator $\mathscr{L}[\rho_t]$ can be written as a 4×4 matrix. The evolutional density matrix of the qubit $\rho_t = \Phi_t \rho_0$. Since γ in the Markovian channel does not involve time, the dynamic evolution operator Φ_t can be acquired by $\Phi_t = \mathrm{Exp}[t\mathscr{L}]$, which can be expressed as

$$\Phi_t = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{pmatrix}, \tag{6}$$

and then by rearranging the matrix Φ_t in terms of four bases, four 2×2 matrices (M_1, M_2, M_3, M_4) are expressed as

$$M_1 = \begin{pmatrix} a_1 & c_1 \\ b_1 & d_1 \end{pmatrix}, \quad M_2 = \begin{pmatrix} a_2 & c_2 \\ b_2 & d_2 \end{pmatrix},$$
$$M_3 = \begin{pmatrix} a_3 & c_3 \\ b_3 & d_3 \end{pmatrix}, \quad M_4 = \begin{pmatrix} a_4 & c_4 \\ b_4 & d_4 \end{pmatrix}. \tag{7}$$

For a generally initial state, ρ_0 of the one-qubit quantum system can be written in terms of Pauli matrices, whose coefficients define the so-called Bloch vector $\rho_0 = \frac{1}{2}(I + r_x\hat{\sigma}_x + r_y\hat{\sigma}_y + r_z\hat{\sigma}_z)$, where I is the identity operator of the qubit, and $r = (r_x, r_y, r_z) \in B := (r \in \Re^3; \|r\| \le 1)$. It can be written as

$$\rho_0 = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1-r_z & r_x - \mathrm{i}r_y \\ r_x + \mathrm{i}r_y & 1+r_z \end{pmatrix}. \tag{8}$$

Then the reduced evolution density matrix ρ_t at any time can be expressed as

$$\rho_t = M_1 \cdot \rho_{00} + M_2 \cdot \rho_{10} + M_3 \cdot \rho_{01} + M_4 \cdot \rho_{11}. \tag{9}$$

3. Quantum speedup of dynamics process

The coherent driving term acting on qubits can be easily implemented in experiments. As we all know, for the Markovian dynamics process, if no coherent driving term acts on the system ($\omega = 0$), the dynamics of quantum system would not be accelerated.[14,15] In order to illustrate the role of the coherent driving on the quantum speed of the open system, a proper QSLT could be effectually defined by the bound of minimal evolution time for arbitrary initial states ρ_0 to the corresponding target evolution state ρ_τ, facilitating to analyze the maximal evolution speed of an open system. In 2019, Campaioli

et al. derived the expression of QSLT by relying on the Euclidean distance to measure the divisibility between two quantum states in a generalized Bloch sphere,[29] the QSLT has been defined as

$$\tau_{\rm qsl} = \frac{\|\rho_0 - \rho_\tau\|_{\rm hs}}{\overline{\|\dot{\rho}_t\|_{\rm hs}}}, \tag{10}$$

with $\overline{\|\dot{\rho}_t\|_{\rm hs}} = \frac{1}{\tau}\int_0^\tau d\tau \|\dot{\rho}_t\|$ and $\|A\|_{\rm hs} = \sqrt{\sum_i f_i^2}$. Here f_i are the singular values of A, and τ is set as the actual evolution time of the dynamics process. The advantage of this definition is tighter and easier to calculate for almost all quantum evolution processes. Physical explanation of QSLT is as follows: If the ratio of $\tau_{\rm qsl}$ to the actual evolution time τ is equal to 1, the dynamics of the quantum state would not be speedup, that is to say, the speed is already maximum. If the ratio of $\tau_{\rm qsl}$ to actual evolution time τ is less than 1, the dynamics of the quantum state may be speedup. Moreover, the smaller the value of $\tau_{\rm qsl}/\tau$ is, the greater the quantum speedup could be. In this section, by considering three different Markovian channels, we use the bound $\tau_{\rm qsl}$ to demonstrate the control of the quantum dynamical speedup from an initial state ρ_0 to a final state ρ_τ by adjusting the relevant coherent driving parameters, such as the driving strength ω and the driving direction angles θ and φ.

3.1. Phase-flip channel

Phase-flip channel is a noisy process of pure quantum mechanics property. It describes the loss of quantum information without loss of energy, reflected in the decay of the off-diagonal elements of the density matrix with time. In this channel, the Lindblad operator is $L_1 = \hat{\sigma}_z/2$. Hence Eq. (1) can be rewritten as

$$\mathscr{L}[\rho_t] = \frac{\gamma}{2}(\hat{\sigma}_z\rho_t\hat{\sigma}_z - \rho_t) - {\rm i}\omega[\hat{n}\cdot\hat{\sigma}, \rho_t]_-. \tag{11}$$

Substituting operator basis $(E^{00}, E^{10}, E^{01}, E^{11})$ into Eq. (11) yields the matrix for each of the four bases,

$$\begin{aligned}
L(E^{00}) &= \begin{pmatrix} 0 & -{\rm i}\omega(-\alpha - {\rm i}\beta) \\ -{\rm i}\omega(\alpha - {\rm i}\beta) & 0 \end{pmatrix}, \\
L(E^{10}) &= \begin{pmatrix} -{\rm i}\omega(-\alpha + {\rm i}\beta) & 0 \\ -\gamma - 2{\rm i}\omega\cos\varphi & -{\rm i}\omega(\alpha - {\rm i}\beta) \end{pmatrix}, \\
L(E^{01}) &= \begin{pmatrix} -{\rm i}\omega(-\alpha + {\rm i}\beta) & -\gamma + 2{\rm i}\omega\cos\varphi \\ 0 & -{\rm i}\omega(\alpha - {\rm i}\beta) \end{pmatrix}, \\
L(E^{11}) &= \begin{pmatrix} 0 & -{\rm i}\omega(\alpha + {\rm i}\beta) \\ -{\rm i}\omega(-\alpha + {\rm i}\beta) & 0 \end{pmatrix}.
\end{aligned} \tag{12}$$

Accordingly, we can obtain the Lindblad superoperator[49] of the phase-flip channel as follows:

$$\mathscr{L} = \begin{pmatrix} 0 & -{\rm i}\omega(\alpha + {\rm i}\beta) & -{\rm i}\omega(-\alpha + {\rm i}\beta) & 0 \\ -{\rm i}\omega(\alpha - {\rm i}\beta) & -\gamma - 2{\rm i}\omega\cos\varphi & 0 & -{\rm i}\omega(-\alpha + {\rm i}\beta) \\ -{\rm i}\omega(-\alpha - {\rm i}\beta) & 0 & -\gamma + 2{\rm i}\omega\cos\varphi & -{\rm i}\omega(\alpha + {\rm i}\beta) \\ 0 & -{\rm i}\omega(-\alpha - {\rm i}\beta) & -{\rm i}\omega(\alpha - {\rm i}\beta) & 0 \end{pmatrix}, \tag{13}$$

with $\alpha = \sin\theta\cos\varphi$ and $\beta = \sin\theta\sin\varphi$.

First, we analyze the effect of coherent driving term on the dynamics of the qubit in the phase-flip channel. Setting the initial state of the qubit in the maximal coherent state ($r_x = 1$, $r_y = r_z = 0$), the quantum speed of this initial state cannot be accelerated in the Markovian phase-flip channel, as reported in Refs. [15,40]. Therefore, in the following, by inducing the coherent driving on the qubit and choosing certain driving direction angles, we would discuss the relationship between QSLT of the qubit and the driving parameter ω.

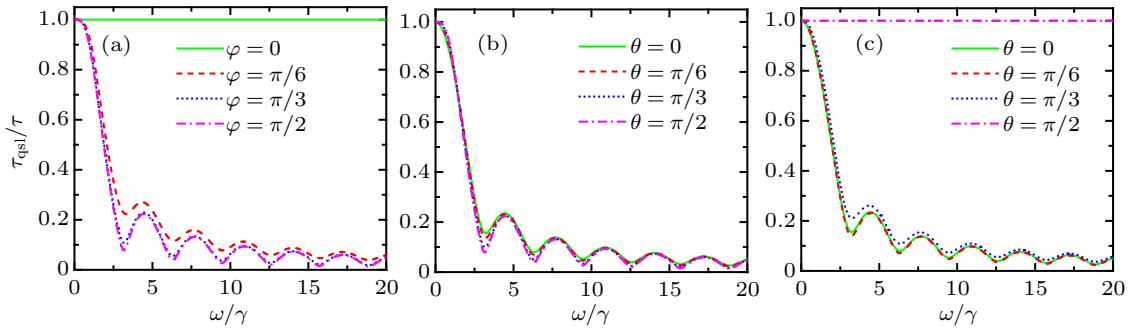


Fig. 1. The relationship between QSLT and the driving strength ω in the phase-flip channel with $\gamma = 1$: (a) driving the $\hat{\sigma}_x$–$\hat{\sigma}_y$ direction, $\theta = \pi/2$; (b) driving the $\hat{\sigma}_z$–$\hat{\sigma}_y$ direction, $\varphi = \pi/2$; (c) driving the $\hat{\sigma}_z$–$\hat{\sigma}_x$ direction, $\varphi = 0$.

As shown in Fig. 1, without coherent driving ($\omega = 0$), the whole quantum process cannot be speedup, i.e., $\tau_{\rm qsl}/\tau = 1$. While for the case $\omega \neq 0$ (adding the external coherent driving on the qubit), the values of $\tau_{\rm qsl}/\tau < 1$ have been obtained. In addition, the oscillation reduction behavior of $\tau_{\rm qsl}/\tau$ will appear with increasing the driving strength ω. This means

that the original no-speedup dynamics process has become the speedup dynamics process by introducing an external coherent driving operating on the qubit in the phase-flip channel. Moreover, from Fig. 1, we also find that, with the increasing ω/γ, QSLT could tend to zero numerically. Here, we must remind the fact that $\tau_{\rm qsl}/\tau \to 0$ is physically meaningless. At this time, $\tau_{\rm qsl}/\tau \to 0$ corresponds to a very loose bound of the quantum speed limit, and it can not mean the infinite acceleration of the evolution of the system.

However, it is crucial to note that, in the case ($\theta = \pi/2$, $\varphi = 0$ means the $\hat{\sigma}_x$ operating acting on the qubit), the no-speedup dynamics $\tau_{\rm qsl}/\tau = 1$ always appears no matter what value of the driving parameter ω is taken in Figs. 1(a) and 1(c). That is to say, the speedup dynamics depends not only on the driving parameter ω, but also on the driving direction angles. Hence, how to choose suitable driving angles would also have a crucial impact on the quantum speedup of the open system. To further investigate the effects of the coherent driving direction angles on the QSLT, Fig. 2 shows the ratio $\tau_{\rm qsl}/\tau$ as a function of two driving angles θ and φ in the phase-flip channel. Under the condition of the weaker driving strength, such as $\omega = 0.2$ in Fig. 2(a), the value of $\tau_{\rm qsl}/\tau$ is not symmetric in the regions $\theta \in (0, \pi)$ and $\varphi \in (0, 2\pi)$. Here, "non symmetric" means that the value of $\tau_{\rm qsl}/\tau$ does not follow axisymmetry with two symmetry axis $\theta = \pi/2$ and $\varphi = \pi$. With the driving strength increasing, it is worth noting that the value of $\tau_{\rm qsl}/\tau$ looks more and more symmetrical about $\theta = \pi/2$ and $\varphi = \pi$, as shown in Figs. 2(c) and 2(d). This means that the effects of angles θ and φ on the QSLT are gradually regular when ω is large enough.

Combining Fig. 2 with Table 1, it can be seen that no matter how strong the driving is, only the driving in the $\hat{\sigma}_x$ direction has no-speedup effect on the whole quantum process, i.e. $\tau_{\rm qsl}/\tau = 1$. Therefore, it can be considered that for this channel, the driving $\hat{\sigma}_x$ direction does not contribute to quantum speedup. For the coherent driving in any other directions, the quantum process can be speedup to different degrees, i.e., $\tau_{\rm qsl}/\tau < 1$. Furthermore, a speedup evolution of the open system would be preferable to deal with the robustness of quantum simulators and computers against decoherence. How to explore the maximum speedup condition of quantum process is meaningful. Therefore, the minimum value of $\tau_{\rm qsl}/\tau$ must be found to characterize the maximum speed of evolution. However, according to the different driving strengths, the driving direction angles (θ and φ) corresponding to the minimum QSLT are also different. For example in Fig. 2(a) in a quarter of one period (0 to $\pi/2$), when the two driving angles $\theta = 0$ and $\varphi = 0$, the QSLT takes the minimum value, that is, at this time the dynamics of the qubit can be speedup most. In Fig. 2(b) the minimum value of QSLT will appear when $\theta = 0.14\pi$ and $\varphi = 0.035\pi$. As ω increases, this condition is going to $\theta = \pi/2$ and $\varphi = \pi/2$ in Figs. 2(c) and 2(d). Therefore, with the increase of driving strength ω, the optimal driving operating changes from $\hat{\sigma}_z$ direction to $\hat{\sigma}_y$ direction. In order to find the driving direction with the best speedup effect of this channel, the value of its driving parameters must be considered. For the smaller driving strength, the optimal driving direction is obtained near the $\hat{\sigma}_z$ direction. While for the larger driving strength, the optimal driving direction is obtained near the $\hat{\sigma}_y$ direction.

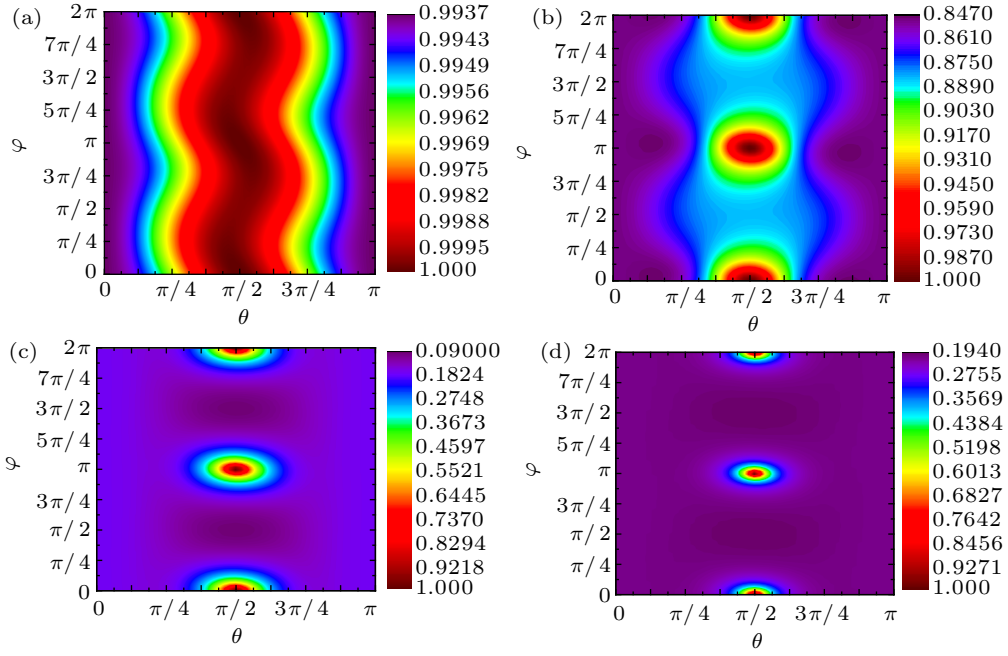


Fig. 2. The relationship between QSLT and driving angles of the coherent driving in the phase-flip channel with $\tau = 1$: (a) $\omega/\gamma = 0.2$, (b) $\omega/\gamma = 1$, (c) $\omega/\gamma = 3$, (d) $\omega/\gamma = 5$.

3.2. Amplitude damping channel

Under the amplitude damping channel, there are a certain probability that energy will be dissipated from the quantum system to the environment and a certain probability that energy will remain unchanged. The Lindblad operators of the amplitude damping channel are $L_1=\sqrt{N+1}\hat{\sigma}_-$, $L_2=\sqrt{N}\hat{\sigma}_+$, with N being a non-negative number that gauges the mean thermal photon number of the environment, and $\hat{\sigma}_\pm=\frac{1}{2}(\hat{\sigma}_x\pm \mathrm{i}\hat{\sigma}_y)$ being the ladder operators. To simplify the calculation without loss of generality, here we set $N=0$, and consider the initial state of the qubit in the excited state $r_x=r_y=0$, and $r_z=1$, as noted in Ref. [14]. Equation (1) hence is in the form

$$\mathscr{L}[\rho_t]=\gamma(L_1\rho_t L_1^\dagger-\frac{L_1^\dagger L_1\rho_t+\rho_t L_1^\dagger L_1}{2})-\mathrm{i}\omega[\hat{n}\cdot\hat{\boldsymbol{\sigma}},\rho_t]_-. \quad (14)$$

Similar to the previous method, by substituting operator basis $(E^{00},E^{10},E^{01},E^{11})$ into Eq. (14) to get the matrix for each of the four bases, the Lindblad superoperator[49] of the amplitude damping channel is as follows:

$$\mathscr{L}=\begin{pmatrix} 0 & -\mathrm{i}\omega(\alpha+\mathrm{i}\beta) & -\mathrm{i}\omega(-\alpha+\mathrm{i}\beta) & \gamma \\ -\mathrm{i}\omega(\alpha-\mathrm{i}\beta) & -\frac{\gamma}{2}-2\mathrm{i}\omega\cos\varphi & 0 & -\mathrm{i}\omega(-\alpha+\mathrm{i}\beta) \\ -\mathrm{i}\omega(-\alpha-\mathrm{i}\beta) & 0 & -\frac{\gamma}{2}+2\mathrm{i}\omega\cos\varphi & -\mathrm{i}\omega(\alpha+\mathrm{i}\beta) \\ 0 & -\mathrm{i}\omega(-\alpha-\mathrm{i}\beta) & -\mathrm{i}\omega(\alpha-\mathrm{i}\beta) & -\gamma \end{pmatrix}. \quad (15)$$

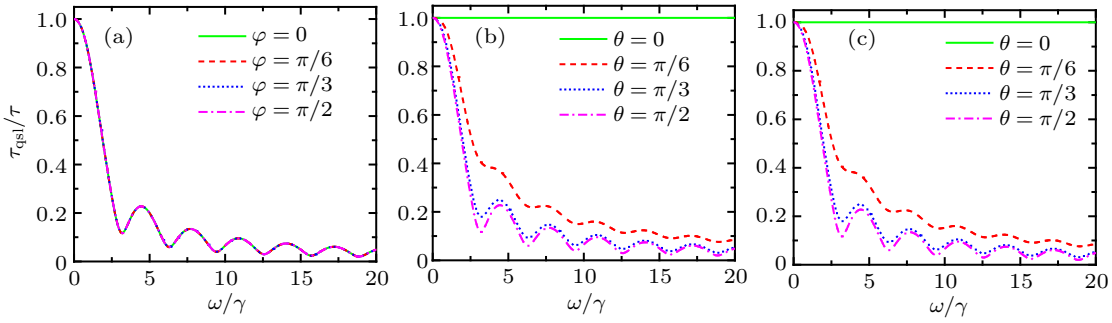


Fig. 3. The relationship between QSLT and driving strength ω for the qubit in the amplitude damping channel with $\tau=1$, $N=0$: (a) driving the $\hat{\sigma}_x$–$\hat{\sigma}_y$ direction, $\theta=\pi/2$, (b) driving the $\hat{\sigma}_z$–$\hat{\sigma}_y$ direction, $\varphi=\pi/2$, (c) driving the $\hat{\sigma}_z$–$\hat{\sigma}_x$ direction, $\varphi=0$.

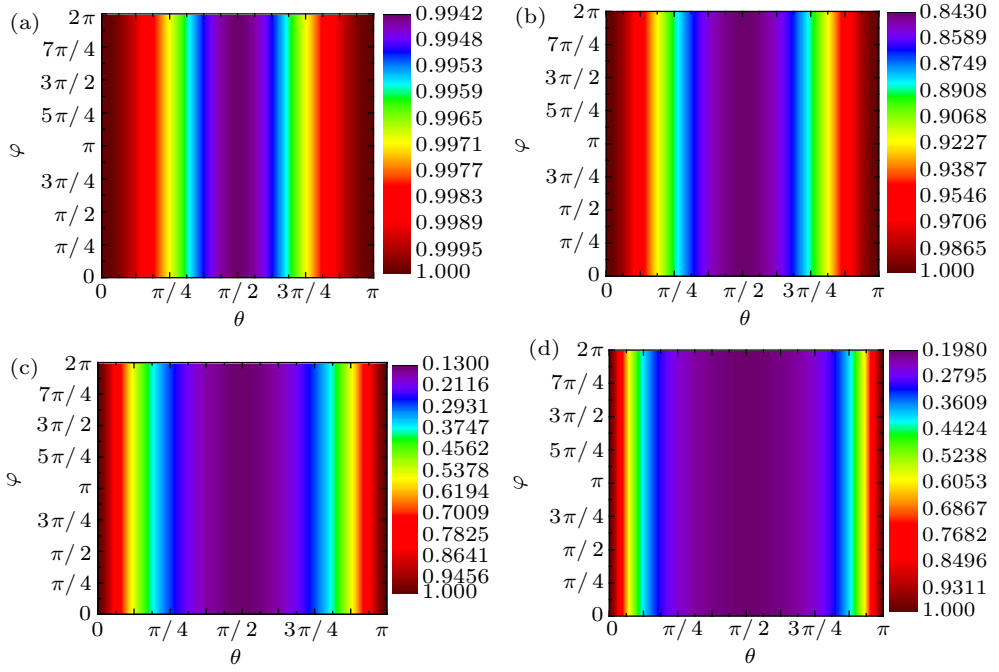


Fig. 4. The relationship between QSLT and driving angles of the coherent driving in the amplitude damping channel with $\tau=1$, $N=0$: (a) $\omega/\gamma=0.2$, (b) $\omega/\gamma=1$, (c) $\omega/\gamma=3$, (d) $\omega/\gamma=5$.

Here, by choosing different certain driving angles in the amplitude damping channel, we firstly analyze the relationship between the QSLT and the driving strength ω in Fig. 3. It can be clear to find that, in the case of no coherent driving term acting on the system ($\omega=0$), no-speedup evolution still exists when the dynamics process is considered from ρ_0 to ρ_τ

under the Markovian dynamics. Furthermore, with increasing ω, $\tau_{\rm qsl}/\tau$ has the tendency of oscillation attenuation. That is to say, the addition of coherent driving can also speedup the evolution of the system in the amplitude damping channel. Moreover, as shown by the green lines in Figs. 3(b) and 3(c), only the driving in the $\hat{\sigma}_z$ direction when $\theta = 0$, the quantum process cannot be speedup regardless of the value of the driving parameters. Especially, the QSLT is exactly the same in both the cases of $\varphi = 0$ and $\varphi = \pi/2$. This means that the driving $\hat{\sigma}_x$ or $\hat{\sigma}_y$ to the qubit alone can lead to the same effect on the quantum evolution speed of the system.

To further investigate the effects of various coherent driving parameters on the QSLT, Fig. 4 shows the ratio $\tau_{\rm qsl}/\tau$ as a function of two driving direction angles θ and φ in the amplitude damping channel. It is worth noting that the value of $\tau_{\rm qsl}/\tau$ is symmetric about $\theta = \pi/2$, as shown in Fig. 4. In the region $0 \le \theta \le \pi/2$, QSLT would decrease monotonically with increasing the driving direction angle θ. Moreover, under the different driving strength ω, QSLT is independent of the driving angle φ. That is, the value of the driving angle φ has no effect on the quantum evolution speed of the dynamics process. However, different from the phase-flip qubit channel, the minimum quantum speed limit time is always obtained when adding the driving $\hat{\sigma}_x$–$\hat{\sigma}_y$ direction to the qubit in $\theta = \pi/2$. According to the above analysis of QSLT in the amplitude damping channel, we therefore reach the interesting result that the larger quantum speedup of the system from ρ_0 to ρ_τ can be achieved by manipulating the larger ω in the coherent driving Hamiltonian $\omega\hat{n}\cdot\hat{\sigma}$ on the system with $\theta = \pi/2$.

3.3. Depolarizing channel

Depolarizing channel is a familiar quantum noisy channel, in which quantum system has a certain probability to evolve into the maximum mixed state and has a certain probability to remain unchanged. The Lindblad operators of depolarizing channel are $L_1 = \hat{\sigma}_x/2$, $L_2 = \hat{\sigma}_y/2$, $L_3 = \hat{\sigma}_z/2$. Now setting the initial state of the qubit also in the pure maximal coherent state with $r_x = 1$, $r_y = r_z = 0$, Eq. (1) would become

$$\begin{aligned}\mathscr{L}[\rho_t] = {} & \frac{\gamma}{4}(\hat{\sigma}_x\rho_t\hat{\sigma}_x + \hat{\sigma}_y\rho_t\hat{\sigma}_y + \hat{\sigma}_z\rho_t\hat{\sigma}_z - 3\rho_t) \\ & - \mathrm{i}\omega[\hat{n}\cdot\hat{\sigma}, \rho_t]_-. \end{aligned} \tag{16}$$

The Lindblad superoperator[49] in the depolarizing channel is as follows:

$$\mathscr{L} = \begin{pmatrix} -\frac{\gamma}{2} & -\mathrm{i}\omega(\alpha+\mathrm{i}\beta) & -\mathrm{i}\omega(-\alpha+\mathrm{i}\beta) & \frac{\gamma}{2} \\ -\mathrm{i}\omega(\alpha-\mathrm{i}\beta) & -\gamma-2\mathrm{i}\omega\cos\varphi & 0 & -\mathrm{i}\omega(-\alpha+\mathrm{i}\beta) \\ -\mathrm{i}\omega(-\alpha-\mathrm{i}\beta) & 0 & -\gamma+2\mathrm{i}\omega\cos\varphi & -\mathrm{i}\omega(\alpha+\mathrm{i}\beta) \\ \frac{\gamma}{2} & -\mathrm{i}\omega(-\alpha-\mathrm{i}\beta) & -\mathrm{i}\omega(\alpha-\mathrm{i}\beta) & -\frac{\gamma}{2} \end{pmatrix}. \tag{17}$$

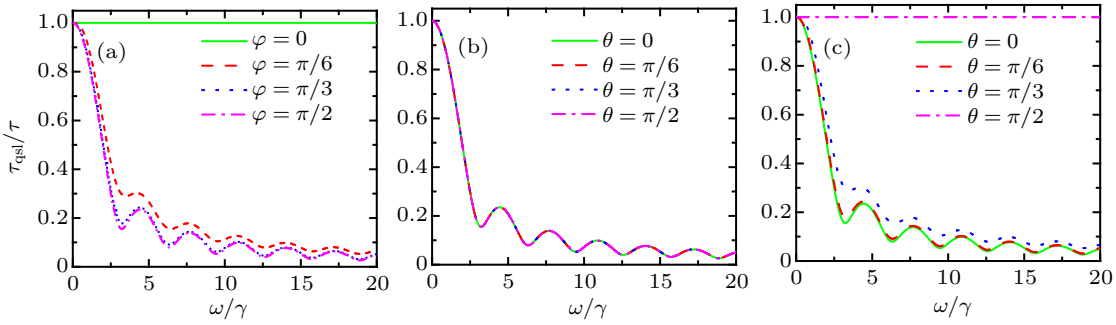


Fig. 5. The relationship between QSLT and the driving strength ω for the qubit in the depolarizing channel with $\gamma = 1$: (a) driving the $\hat{\sigma}_x$–$\hat{\sigma}_y$ direction, $\theta = \pi/2$, (b) driving the $\hat{\sigma}_z$–$\hat{\sigma}_y$ direction, $\varphi = \pi/2$, (c) driving the $\hat{\sigma}_z$–$\hat{\sigma}_x$ direction, $\varphi = 0$.

In the depolarizing channel, from Fig. 5, it is noted that, except at certain angles ($\theta = \pi/2$, $\varphi = 0$), the QSLT oscillates down as the driving strength increases. This means that only inducing the driving direction $\hat{\sigma}_x$ in the depolarizing channel can not lead to quantum speedup of the dynamics process of the qubit. In addition, with increasing the proportion of the driving $\hat{\sigma}_y$ or $\hat{\sigma}_z$ direction to the qubit, the greater capacity for potential speedup would be achieved. For a given driving strength ω, the value of $\tau_{\rm qsl}/\tau$ would decrease with the proportion $|\sin\varphi|^2$ of the driving $\hat{\sigma}_y$ increasing, as shown in Fig. 5(a). While in the $\hat{\sigma}_z$–$\hat{\sigma}_x$ driving plane in Fig. 5(c), the smaller $|\cos\theta|^2$ is corresponding to the smaller $\tau_{\rm qsl}/\tau$. Therefore, as long as the qubit is driven on the $\hat{\sigma}_z$–$\hat{\sigma}_y$ plane, the evolution of the system must be accelerated ($\tau_{\rm qsl}/\tau < 1$ is always satisfied, no matter how θ is adjusted, as shown in Fig. 5(b)).

By considering the different values of the driving strength ω, in order to clearly show how to coherently drive the system to the maximum capacity for quantum speedup, Fig. 6 describes $\tau_{\rm qsl}/\tau$ as a function of two driving direction angles θ and φ for a fixed target state ρ_τ at $\tau = 1$. Under the Markovian

dynamics process in the depolarizing channel, the evolution speed of the system cannot be accelerated in the case of $\omega = 0$. From Fig. 6, it can be concluded that QSLT is axisymmetric with the driving angle $\theta = \pi/2$ and $\varphi = \pi$. Moreover, we further confirm that driving $\hat{\sigma}_z$ direction has exactly the same effect on the quantum evolution speed of the qubit as driving $\hat{\sigma}_y$ direction. Hence, for the driving direction in the $\hat{\sigma}_z$–$\hat{\sigma}_y$ plane, the effect is always the same. Then, for the distribution of the maximum and minimum points of the QSLT, only the driving $\hat{\sigma}_x$ direction does not speedup the quantum process, i.e., $\tau_{\rm qsl}/\tau = 1$. The minimum value of QSLT is obtained when the $\hat{\sigma}_z$–$\hat{\sigma}_y$ direction is driven with the larger driving strength ω.

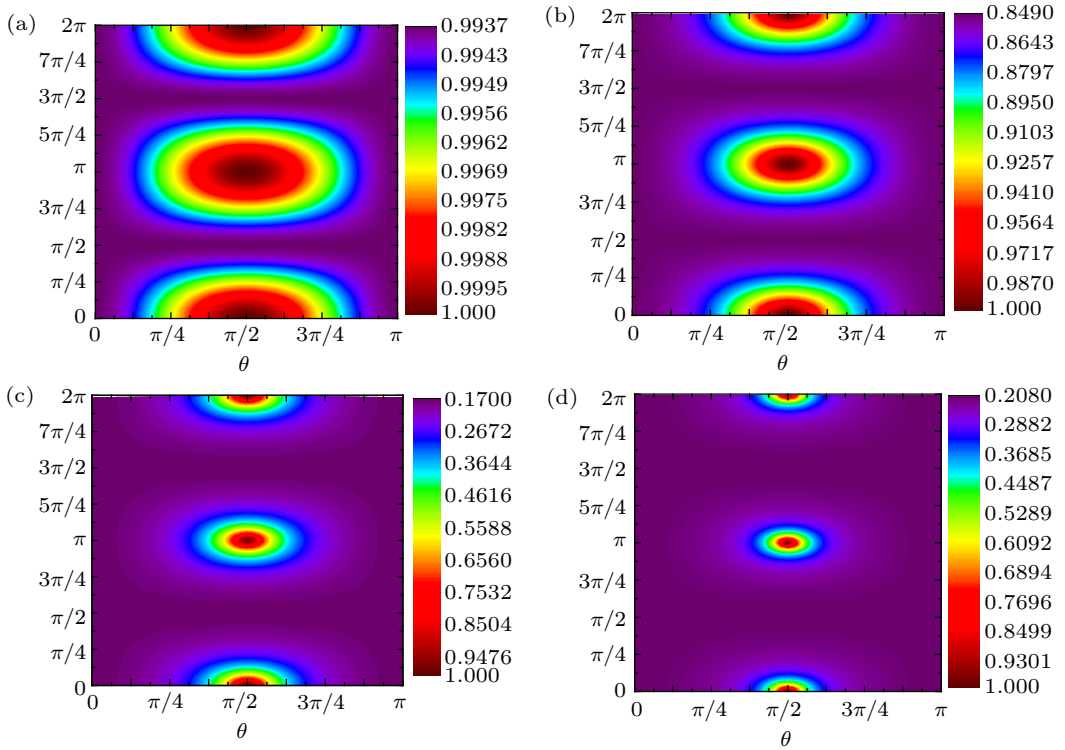


Fig. 6. The relationship between QSLT and driving angles of the coherent driving in the depolarizing channel with $\tau = 1$: (a) $\omega/\gamma = 0.2$, (b) $\omega/\gamma = 1$, (c) $\omega/\gamma = 3$, (d) $\omega/\gamma = 5$.

3.4. Physical reason for speedup

In the above subsections, we mainly discuss the quantum speedup under the coherent Hamiltonian driving in three different Markovian channels. As we all know, quantum speedup evolution of an open system is usually induced by the non-Markovianity[55–57] of the environment. However, for the evolution of this model under the Lindblad main equation described by Eq. (1), no matter how the driving term is adjusted, the dynamics of the system cannot exhibit the transition from Markovian behavior to non-Markovian behavior. To prove it, we have used the Breuer–Laine–Piilo method[57] to calculate the non-Markovianity of the above three channels. We confirm that the dynamics of the system still maintains the Markovian property under the coherent driving. Interestingly, unlike other speedup models mentioned previously,[14,18,39,45,46] this speedup scheme in our work can be achieved in the Markovian regime, and does not have to be generated the non-Markovian behavior of the dynamcis.

In Ref. [29], the authors described the geometric distance as the numerator of state space by generalized Bloch vectors. This geometric distance $D(\rho,\sigma) = \|\boldsymbol{r} - \boldsymbol{s}\|_{\rm hs}$ is also defined as the numerator of $\tau_{\rm qsl}$. That connects $\boldsymbol{r} = \rho_0$, $\boldsymbol{s} = \rho_\tau$. To further make the analysis of the intrinsic physical reason for quantum speedup induced by the coherent driving operation on the system, we concern the geometric distance $D(\rho_0,\rho_\tau)$, which can be written as $D(\rho_0,\rho_\tau) = \|\rho_0 - \rho_\tau\|_{\rm hs}$. We compare $\tau_{\rm qsl}/\tau$ and $D(\rho_0,\rho_\tau)$ in the above three different channels. In the Fig. 7, we take three different sets of driving direction angles for the above three channels. For the green solid line in Fig. 7, we choose the driving direction angles that cannot lead to speedup in each channel, i.e., $\tau_{\rm qsl}/\tau = 1$. The geometric distance $D(\rho_0,\rho_\tau)$ is going to increase smoothly and monotonically. While for the red dashed line in Fig. 7, we choose the driving angles that can acquire the maximum accelerated evolution of the dynamics process, i.e., $\tau_{\rm qsl}/\tau$ is the minimum. The geometric distance $D(\rho_0,\rho_\tau)$ increases in oscillating with the actual evolution time τ. For the blue dash-dotted line, we take an arbitrary set of angles for comparison. We can find that the geometric distance $D(\rho_0,\rho_\tau)$ is also oscillating up.

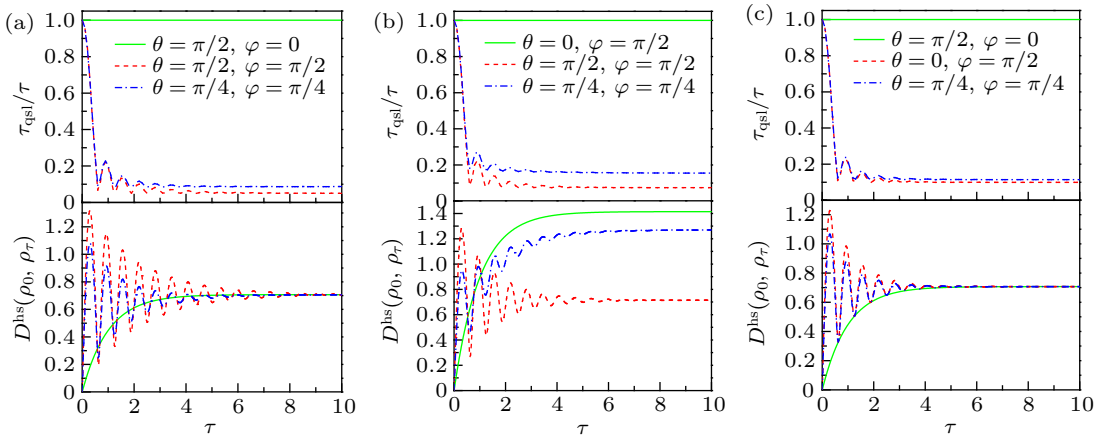


Fig. 7. The QSLT $\tau_{\rm qsl}/\tau$ and the geometric distance $D(\rho_0,\rho_\tau)$ of the dynamics evolution from the initial state to the final state as a function of the actual evolution time τ, with $\omega/\gamma = 5$: (a) the phase-flip channel, (b) the amplitude damping channel, (c) the depolarizing channel.

This phenomenon shows that for a speedup quantum process, the geometric distance between the initial state and the target final state would oscillate with the actual evolution time. Especially, the larger the oscillation, the smaller the QSLT of the quantum process, indicating that the system can be speedup more. Similarly, we can find that with the increase of the actual evolution time τ, both $\tau_{\rm qsl}/\tau$ and the geometric distance $D(\rho_0,\rho_\tau)$ tend to the stable value. The reason is that, when the actual evolution time increases to a certain extent, the density matrix of the qubit hardly changes, and the decoherence process has been completed. Consequently, it is worth noting that the coherent driving term added to the open system does not lead to the non-Markovian behavior. However, the coherent driving parameters could cause the oscillation changes of the geometric distance $D(\rho_0,\rho_\tau)$ between the initial state ρ_0 and the target final state ρ_τ. Somewhat contrary to the previous speedup scenarios by the controllable non-Markovianity, our results clearly show that the oscillation changes of the geometric distance $D(\rho_0,\rho_\tau)$ caused by the coherent driving are the intrinsic physical reason for quantum speedup in our scheme.

4. Conclusion

As we all know, the quantum evolution speed of an open system can never be accelerated in the Markovian regime without any operators on the system. One unitary coherent driving Hamiltonian corresponding to a simple quantum rotation operation on the qubit can be easily accomplished in a variety of actual physical systems in experiments, so in this work we mainly demonstrate that a speedup evolution of the system could be implemented by the action of the coherent driving on the system in the phase-flip channel, the amplitude damping channel and the depolarizing channel.

It is interesting to show that, in the presence of the coherent driving, no-speedup evolution in the Markovian dynamics process can be transformed into quantum speedup evolution. We also analyze the best driving way to achieve the most degree of quantum speedup in the considered three noisy channels, respectively. For instance, in the phase-flip channel, we therefore reach that, for the smaller driving strength ω, the best driving way to achieve the most degree of quantum speedup can be reported by manipulating the optimal driving $\hat{\sigma}_z$ direction. For larger driving strength ω, the optimal driving direction is obtained near the $\hat{\sigma}_y$ direction. In the the amplitude damping channel, the best way corresponds to the coherent driving in the $\hat{\sigma}_x$–$\hat{\sigma}_y$ direction plane on the system with the larger ω. While in the depolarizing channel, the coherent driving in the $\hat{\sigma}_z$–$\hat{\sigma}_y$ direction plane with the larger driving strength ω could lead to the best way to achieve the most degree of quantum speedup. Furthermore, under the condition of the Markovian regime, it should be pointed out that the coherent driving term added to the open system does not lead to the non-Markovian behavior, and the quantun speedup dynamics of the system still shows the Markovian behavior. Unlike most of the literature, the acceleration under our scheme does not comes from the non-Markovian dynamical behavior but from the geometric distance oscillation.

In addition, in order to better illustrate our manipulation of system dynamics via the external unitary coherent driving, firstly we need to compare this work with previous related research, e.g., Hegerfeldt discussed a time-dependent driving Hamiltonian[25,26] and pointed out that the shortest evolution time of this quantum process depends on the maximum driving strength that can be achieved in experiment. Their main results are much similar to the analyses illustrated in Figs. 1, 3 and 5 in our paper. In Ref. [27] the authors studied theoretically and experimentally the time-optimal construction of arbitrary single-qubit rotations under a single strong driving field of finite amplitude. In these previous studies, the driving Hamiltonian in different directions ($\hat{\sigma}_x$, $\hat{\sigma}_y$, $\hat{\sigma}_z$) acting on the system are all time-dependent, and have different driving strengths. While for our driving model, as shown

in Eq. (4), we mainly have considered the coherent driving Hamiltonian with uniform driving strength in different driving directions on the quantum system. We use two angles to characterize the different weights in different driving directions. Secondly, at present, the coherent driving Hamiltonian can be constructed experimentally in actual physical systems, such as cavity QED,[60] trapped ions,[61] superconducting qubits[62] and the nitrogen-vacancy center of diamond.[63] A strongly driven two-level system with driving frequencies four times larger than its precession frequency is realized using radiation-dressed states of nitrogen-vacancy centers in diamond.[27] Dai *et al.* experimentally realized the driving Hamiltonian in a superconducting transmon system.[64] In 2018, Magazzu *et al.* conducted experimental research on a strongly driven spin-boson model of a superconducting quantum circuit.[65] They pointed out that the driving reinforces environmental suppression of quantum coherence and a coherent-to incoherent transition could be achieved by tuning the driving strength. Therefore, the theoretical construction of coherent driving Hamiltonian in our work is experimentally feasible.

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