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Novel traveling wave solutions and stability analysis of perturbed Kaup–Newell Schrödinger dynamical model and its applications*

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We study the traveling wave and other solutions of the perturbed Kaup–Newell Schrödinger dynamical equation that signifies long waves parallel to the magnetic field. The wave solutions such as bright-dark (solitons), solitary waves, periodic and other wave solutions of the perturbed Kaup–Newell Schrödinger equation in mathematical physics are achieved by utilizing two mathematical techniques, namely, the extended F-expansion technique and the proposed $\exp(-\phi(\xi))$-expansion technique. This dynamical model describes propagation of pluses in optical fibers and can be observed as a special case of the generalized higher order nonlinear Schrödinger equation. In engineering and applied physics, these wave results have key applications. Graphically, the structures of some solutions are presented by giving specific values to parameters. By using modulation instability analysis, the stability of the model is tested, which shows that the model is stable and the solutions are exact. These techniques can be fruitfully employed to further sculpt models that arise in mathematical physics.

Keywords: extended F-expansion method, generalized $\exp(-\phi(\xi))$-expansion technique, perturbed Kaup–Newell Schrödinger equation, traveling wave solutions

PACS: 02.30.Jr, 02.70.Wz, 04.20.Jb, 03.67.–a **DOI:** 10.1088/1674-1056/abbbfc

1. Introduction

Analytical wave results in dissimilar kinds play an important role in realization of nonlinear physical and mathematical phenomena in the fields of hydrodynamics, plasma physics, nonlinear optics, elastic media, problems of biological, engineering and optical fibers.[1–12] The fascinating technology of sub-pico second pulses that propagate through optical fibers is modeled with the Kaup–Newell equation. Several decades ago, this famous model was first proposed by David Kaup and Alen Newell, and is one of the three forms of the derivative nonlinear Schrödinger equation (NLSE) that governs the soliton dynamics. The research of soliton solutions has been of appreciable significance for knowing nonlinear phenomena over the recent years. To know the integrability nature in ordinary and partial differential equations,[1,2] the theory of solitons has enhanced in recent decades. The derivative NLSE is

$$q_t + \mathrm{i}q_{xx} + (|q|^2 q)_x = 0. \tag{1}$$

It is also named as the Kaup–Newell (K–N) model,[3] which has several sensual applications, particularly in nonlinear optics and plasma physics. In several models, the derivative nonlinear Schrödinger equation (DNLSE) was studied to exhibit the molecule of soliton propagation throughout an optical fiber. The DNLSE has many forms which are identified, such as the Gerdjikov–Ivanov equation (GIE), the Chen–Lee–Liu equation (CLLE), and the K–N equation. Alternatively, these equations are named as DNLSE-I, DNLSE-II and DNLSE-III. Through the Gauge transformation,[9] these equations can be altered to each other. In scattering problems, the transformations cannot preserve the condition of reduction. However, the transformations cannot conserve the condition of reduction in the scattering model and extremely intricate integral is involved, and it cannot be premeditated explicitly.[10,11,13] Therefore, they merit to be investigated independently. There is an abundance of mathematical techniques that are found to tackle these assortment of nonlinear Schrödinger problems.[8,12,14–21] Recently, the perturbed Kaup–Newell (K–N) model[22,23] is introduced from the generalized Kaup–Newell spectral equation with linear perturbations as follows:

$$\begin{aligned} &\frac{\partial u}{\partial t} + \mathrm{i}\alpha_1 \frac{\partial^2 u}{\partial x^2} + \alpha_2 \frac{\partial}{\partial x}\left(|u|^2 u\right) \\ &= \beta \frac{\partial u}{\partial x} + \eta \frac{\partial}{\partial x}\left(|u|^{2m} u\right) + \delta \frac{\partial}{\partial x}\left(|u|^{2m}\right) u, \end{aligned} \tag{2}$$

where the function $u(x,t)$ indicates the wave function of complex valued. The variables x and t represent the spatial and temporal coordinates, respectively. The first term on the left-hand side of Eq. (2) indicates the temporal evaluation, and the second term indicates the group velocity dispersion having coefficient α_1. The third term together with coefficient α_2 provides nonlinear effect. On the right-hand side of Eq. (2), coefficient β indicates inter-model dispersion, η and δ indicate the self-steepening and nonlinear dispersion effects. Lastly, the parameter for full nonlinearity is m. To date, researchers[22–25]

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have been successfully shown the mathematical engineering of Eq. (2).

This article will study the exact solution of the K–N equation with some hamiltonian sort perturbation terms. F-expansion and $\exp(-\phi(\xi))$-expansion schemes executed to construct soliton solution to the current K–N equation. The achieved solutions are novel. The stability of achieved solutions are also examined via utilizing modulation instability (MI) analysis. After brief introduction, the K–N equation details are given in the rest of the paper.

2. Depiction of proposed techniques

In this section, we elucidate the algorithm of projected methods, namely, the modified F-expantion method and the generalized $\exp(-\phi(\xi))$-expantion technique for finding the exact solutions of the K–N model. A general non-linear evolution equation has the form of

$$Q(w, w_t, w_x, w_{xx}, w_{xxx}, \ldots) = 0, \tag{3}$$

where the function $w(x,t)$ is unknown, and polynomial Q has some specified functions or variables, which also contains both linear and non-linear derivative terms of the $w(x,t)$. Considering the transformation for switching independent variables into one variable yields

$$w(x,t) = \psi(\xi), \quad \xi = kx + \nu t, \tag{4}$$

where k and ν are wave length and frequency. Utilizing Eq. (4), equation (3) is diminished into the ordinary differential equation

$$P\left(\psi, \psi', \psi'', \psi''', \ldots\right) = 0, \tag{5}$$

where $\psi' = \mathrm{d}\psi/\mathrm{d}\xi$.

2.1. Modified F-expantion method

The major stages of this scheme are stated in the following.

First step Consider the solution to ODE (5) has the form of

$$\psi(\xi) = \sum_{i=-N}^{N} a_i \left(n + F(\xi)\right)^i, \tag{6}$$

where the constants a_i and n are real. $F(\xi)$ ensures the following new ansatz equation:

$$F'(\xi) = d_0 + d_1 F(\xi) + d_2 F^2(\xi) + d_3 F^3(\xi), \tag{7}$$

where d_0, d_1, d_2 and d_3 are real constants.

Second step N is a positive integer and it is generally found via utilizing homogeneous balance principle on Eq. (5), and the coefficients series a_{-N}, a_{-N+1},..., a_0, a_1,..., a_N, ν, k, n are obtained.

Third step Deputing Eqs. (6) and (7) into Eq. (5) and putting the various coefficients of $\frac{F^i(\xi)}{(n+F(\xi))^j}$ to zero yield a set of equations. The Mathematica is used to solve this set of equations and the values of constants can be attained.

Fourth step The solutions to Eq. (5) can be achieved by deputing the values achieving in the third step into Eq. (6).

2.2. Generalized $\exp(-\phi(\xi))$-expansion technique

The major stages of this scheme are given in the following.

First step Consider the solution to ODE (5) has the form of

$$\psi(\xi) = \sum_{i=0}^{N} b_i \left(\exp(-\phi(\xi))\right)^i, \tag{8}$$

where b_i $(0 \le i \le N)$ represents constants we have to calculate, such that $b_N \neq 0$ and $\phi = \phi(\xi)$ satisfies the following auxiliary ODE:

$$\phi'(\xi) = r\exp(-\phi(\xi)) + p\exp(\phi(\xi)) + q, \tag{9}$$

where r, p and q are constants.

Second step Positive integer N can be calculated by using homogeneous balance principle between nonlinear terms occurring and derivative terms of highest order in Eq. (5).

Third step Substituting Eqs. (8) and (9) into Eq. (5) yields a polynomial in $\mathrm{e}^{(-\phi(\xi))}$. Putting different powers of $(\mathrm{e}^{(-\phi(\xi))})^i$ to zero, we can reach algebraic equations. Solving this system and back substitutions, we attain a set verity of exact solutions to Eq. (3).

3. Application of proposed methods to the perturbed K–N Schrödinger equation

In this section, we construct the soliton solutions to the perturbed K–N equation by utilizing two mathematical techniques.

3.1. The modified F-expansion method

As Eq. (2) is complex, so our hypothesis of the wave solution to Eq. (2) is in the form of

$$u(x,t) = \psi(\xi)\mathrm{e}^{\mathrm{i}P}, \quad \xi = kx + \nu t, \quad P = \gamma x + \omega t + \varepsilon, \tag{10}$$

where $\psi(\xi)$ is given in Eq. (6); k, ν, γ, ω, and ε are arbitrary constants; $\psi(\xi)$ is the amplitude component of the wave profiles; P is the phase factor; γ, ω, and ε show the frequency, wave number, and phase constant, respectively. Putting Eq. (10) into Eq. (2) and making separate into parts yield

$$k\left(\eta + 2m(\delta + \eta)\right)\psi^{2m} - 3k\alpha_2\psi^2 + 2k\alpha_1\gamma + \beta k - \nu = 0, \tag{11}$$

$$k^2\alpha_1\psi'' + \gamma\alpha_2\psi^3 - \left(\beta\gamma - \omega + \gamma^2\alpha_1\right) - \gamma\eta\psi^{2m}\psi = 0. \tag{12}$$

Substituting ψ^{2m} from Eq. (11) into Eq. (12) yields

$$\alpha_1 k^3\left(\eta + 2m(\delta + \eta)\right)\psi'' + \left(\alpha_1\gamma^2 k\left(\eta - 2m(\delta + \eta)\right)\right.$$

$$
\begin{aligned}
&-\gamma\eta\nu-2\beta\gamma km(\delta+\eta)\\
&+k\omega(\eta+2m(\delta+\eta)))\psi\\
&+2\alpha_2\gamma k(m\delta+m\eta-\eta)\psi^3=0. \qquad (13)
\end{aligned}
$$

Apply balance principle of homogeneity on Eq. (13) and consider the solution to Eq. (13) to be

$$
\psi(\xi)=\frac{a_{-2}}{(n+F(\xi))^2}+\frac{a_{-1}}{n+F(\xi)}+a_0+a_1(n+F(\xi))+a_2(n+F(\xi))^2. \qquad (14)
$$

Substituting Eqs. (14) and (7) into Eq. (13) and adjusting the coefficients of $\frac{F^i(\xi)}{(n+F(\xi))^j}$ to zero, we achieve an equation set in a_{-2}, a_{-1}, a_0, a_1, a_2, d_0, d_1, d_2, d_3, k, ν, ω, η, β, and m. Mathematica was employed to solve the set of equations. We obtain the following families.

Family 1 In this family, we consider $d_0=d_3=0$,

Set 1:

$$
\begin{aligned}
&a_{-2}=a_{-1}=0,\quad a_0=\frac{k(d_1-2d_2n)\sqrt{\alpha_1(\eta+2m(\delta+\eta))}}{2\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}},\quad a_1=\frac{d_2k\sqrt{\alpha_1(\eta+2m(\delta+\eta))}}{\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}},\quad a_2=0,\\
&\nu=-\frac{k\left(\alpha_1\left(d_1^2k^2(\eta+2m(\delta+\eta))+2\gamma^2(2m(\delta+\eta)-\eta)\right)-2\eta\omega+4m(\delta+\eta)(\beta\gamma-\omega)\right)}{2\gamma\eta}. \qquad (15)
\end{aligned}
$$

Set 2

$$
\begin{aligned}
&a_{-2}=a_{-1}=a_2=0,\quad a_0=\frac{k(2d_2n-d_1)\sqrt{\alpha_1(\eta+2m(\delta+\eta))}}{2\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}},\quad a_1=-\frac{d_2k\sqrt{\alpha_1(\eta+2m(\delta+\eta))}}{\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}},\\
&\nu=-\frac{k\left(\alpha_1\left(d_1^2k^2(\eta+2m(\delta+\eta))+2\gamma^2(2m(\delta+\eta)-\eta)\right)-2\eta\omega+4m(\delta+\eta)(\beta\gamma-\omega)\right)}{2\gamma\eta}. \qquad (16)
\end{aligned}
$$

We can achieve the soliton solutions to Eq. (2) from set 1 in the form as follows:

$$
u_{11}(x,t)=\frac{d_1k\sqrt{\alpha_1(\eta+2m(\delta+\eta))}\left(1+d_2e^{d_1(\xi+\xi_0)}\right)}{2\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}\left(1-d_2e^{d_1(\xi+\xi_0)}\right)}e^{i(\gamma x+\omega t+\varepsilon)},\quad d_1>0. \qquad (17)
$$

$$
u_{12}(x,t)=\frac{d_1k\sqrt{\alpha_1(\eta+2m(\delta+\eta))}\left(1-d_2e^{d_1(\xi+\xi_0)}\right)}{2\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}\left(1+d_2e^{d_1(\xi+\xi_0)}\right)}e^{i(\gamma x+\omega t+\varepsilon)},\quad d_1<0. \qquad (18)
$$

We also achieve more soliton solutions to Eq. (2) from set 2 as

$$
u_{13}(x,t)=\frac{d_1k\sqrt{\alpha_1(\eta+2m(\delta+\eta))}\left(d_2e^{d_1(\xi+\xi_0)}+1\right)}{2\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}\left(d_2e^{d_1(\xi+\xi_0)}-1\right)}e^{i(\gamma x+\omega t+\varepsilon)},\quad d_1>0. \qquad (19)
$$

$$
u_{14}(x,t)=\frac{d_1k\sqrt{\alpha_1(\eta+2m(\delta+\eta))}\left(d_2e^{d_1(\xi+\xi_0)}-1\right)}{2\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}\left(d_2e^{d_1(\xi+\xi_0)}+1\right)}e^{i(\gamma x+\omega t+\varepsilon)},\quad d_1<0. \qquad (20)
$$

In Fig. 1, the restricted conditions for avoiding singularities are $\frac{\alpha_1(\eta+2m(\delta+\eta))}{\alpha_2\gamma(\eta-m(\delta+\eta))}>0$ and $d_2\neq\frac{1}{e^{d_1(\xi+\xi_0)}}$.

Family 2 In this family, we consider $d_1=d_3=0$,

Set 1

$$
\begin{aligned}
&a_{-2}=a_{-1}=0,\quad a_0=\frac{d_2kn\sqrt{\alpha_1(\eta+2m(\delta+\eta))}}{\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}},\quad a_1=-\frac{d_2k\sqrt{\alpha_1(\eta+2m(\delta+\eta))}}{\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}},\quad a_2=0,\\
&\omega=\frac{\gamma\eta\nu-\alpha_1k\left(2d_0d_2k^2(\eta+2m(\delta+\eta))+\gamma^2(\eta-2m(\delta+\eta))\right)+2\beta\gamma km(\delta+\eta)}{k(\eta+2m(\delta+\eta))}. \qquad (21)
\end{aligned}
$$

Set 2

$$
\begin{aligned}
&a_{-2}=a_{-1}=0,\quad a_0=-\frac{d_2kn\sqrt{\alpha_1(\eta+2m(\delta+\eta))}}{\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}},\quad a_1=\frac{d_2k\sqrt{\alpha_1(\eta+2m(\delta+\eta))}}{\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}},\quad a_2=0,\\
&\omega=\frac{\gamma\eta\nu-\alpha_1k\left(2d_0d_2k^2(\eta+2m(\delta+\eta))+\gamma^2(\eta-2m(\delta+\eta))\right)+2\beta\gamma km(\delta+\eta)}{k(\eta+2m(\delta+\eta))}. \qquad (22)
\end{aligned}
$$

The wave solutions to Eq. (2) are achieved from solution set 1 as follows:

$$u_{21}(x,t)=-\frac{k\sqrt{d_0d_2\alpha_1(\eta+2m(\delta+\eta))}\tan\left(\sqrt{d_0d_2}(\xi+\xi_0)\right)}{\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}}e^{i(\gamma x+\omega t+\varepsilon)},\quad d_0d_2>0. \tag{23}$$

$$u_{22}(x,t)=\frac{k\sqrt{-d_0d_2\alpha_1(\eta+2m(\delta+\eta))}\tanh\left(\sqrt{-d_0d_2}(\xi+\xi_0)\right)}{\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}}e^{i(\gamma x+\omega t+\varepsilon)},\quad d_0d_2<0. \tag{24}$$

We attain more exact solutions to Eq. (2) from solution set 2 as

$$u_{23}(x,t)=\frac{k\sqrt{d_0d_2\alpha_1(\eta+2m(\delta+\eta))}\tan\left(\sqrt{d_0d_2}(\xi+\xi_0)\right)}{\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}}e^{i(\gamma x+\omega t+\varepsilon)},\quad d_0d_2>0. \tag{25}$$

$$u_{24}(x,t)=-\frac{k\sqrt{-d_0d_2\alpha_1(\eta+2m(\delta+\eta))}\tanh\left(\sqrt{-d_0d_2}(\xi+\xi_0)\right)}{\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}}e^{i(\gamma x+\omega t+\varepsilon)},\quad d_0d_2<0. \tag{26}$$

In Fig. 2, the restricted condition for avoiding singularity is $\frac{\alpha_1(\eta+2m(\delta+\eta))}{\alpha_2\gamma(\eta-m(\delta+\eta))}>0$.

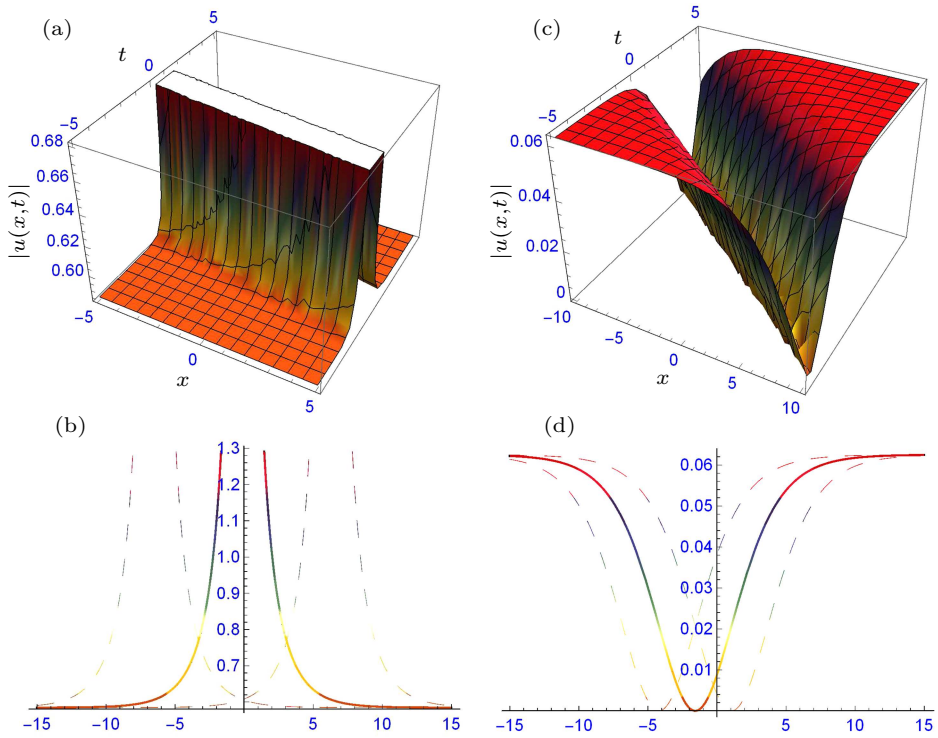


Fig. 1. The structure of solutions (17) and (18): (a) bright solitary wave and its two-dimensional profile (b), and (c) dark soliton and its two-dimensional profile (d), providing suitable values to the parameters.

Family 3 In this family, we consider $d_3=0$,

Set 1

$$a_{-2}=a_{-1}=0,\quad a_0=-\frac{k(d_1-2d_2n)\sqrt{\alpha_1(\eta+2m(\eta+\delta))}}{2\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}},\quad a_1=-\frac{d_2k\sqrt{\alpha_1(\eta+2m(\eta+\delta))}}{\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}},\quad a_2=0,$$
$$\nu=\frac{k\left(-\alpha_1\left(\left(d_1^2-4d_0d_2\right)k^2(\eta+2m(\delta+\eta))+2\gamma^2(2m(\delta+\eta)-\eta)\right)+2\eta\omega-4m(\delta+\eta)(\beta\gamma-\omega)\right)}{2\gamma\eta}. \tag{27}$$

Set 2

$$\nu=\frac{k\left(-\alpha_1\left(\left(d_1^2-4d_0d_2\right)k^2(\eta+2m(\delta+\eta))+2\gamma^2(2m(\delta+\eta)-\eta)\right)+2\eta\omega-4m(\delta+\eta)(\beta\gamma-\omega)\right)}{2\gamma\eta},$$
$$a_{-2}=a_{-1}=0,a_0=\frac{k(d_1-2d_2n)\sqrt{\alpha_1(\eta+2m(\eta+\delta))}}{2\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}},\quad a_1=\frac{d_2k\sqrt{\alpha_1(\eta+2m(\eta+\delta))}}{\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}},\quad a_2=0. \tag{28}$$

The wave solutions to Eq. (2) from sets 1 and 2 are constructed as follows:

$$u_{31}=\frac{k\sqrt{\alpha_1(\eta+2m(\eta+\delta))}\left(\sqrt{4d_0d_2-d_1^2}\tan\left(\frac{\sqrt{4d_0d_2-d_1^2}}{2}(\xi+\xi_0)\right)-2d_1\right)}{2\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}}\mathrm{e}^{\mathrm{i}P},\quad 4d_0d_2>d_1^2; \tag{29}$$

$$u_{32}=\frac{k\sqrt{\alpha_1(\eta+2m(\eta+\delta))}\left(2d_1-\sqrt{4d_0d_2-d_1^2}\tan\left(\frac{\sqrt{4d_0d_2-d_1^2}}{2}(\xi+\xi_0)\right)\right)}{2\sqrt{\alpha_2\gamma(\eta-m(\delta+\eta))}}\mathrm{e}^{\mathrm{i}P},\quad 4d_0d_2>d_1^2; \tag{30}$$

where ξ_0 is constant and $P=\gamma x+\omega t+\varepsilon$.

In Fig. 3, the restricted conditions for avoiding singularities $\frac{\alpha_1(\eta+2m(\delta+\eta))}{\alpha_2\gamma(\eta-m(\delta+\eta))}>0$ and $d_3\neq-\frac{1}{\mathrm{e}^{2d_1(\xi+\xi_0)}}$.

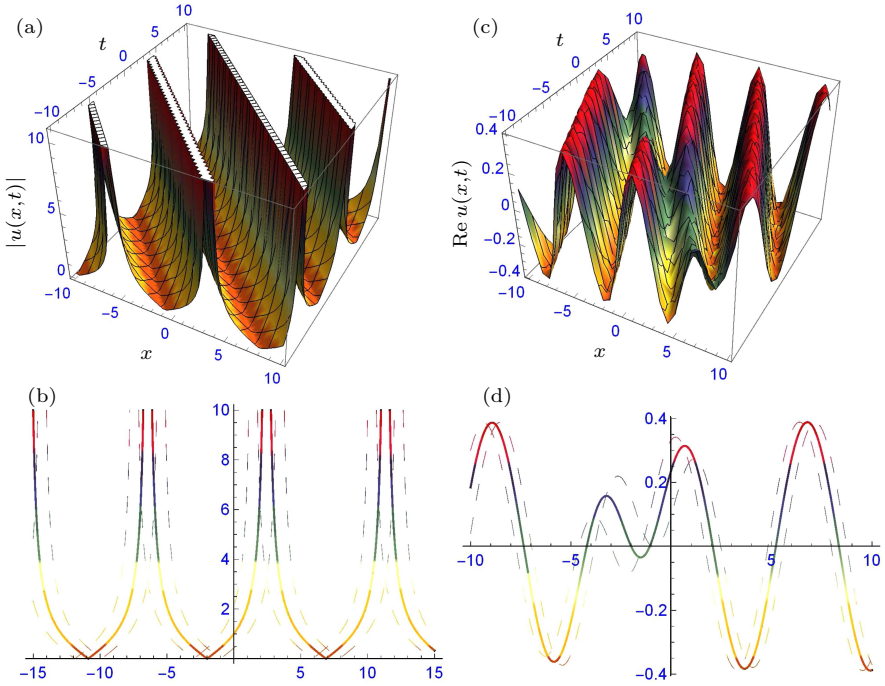


Fig. 2. The structure of solutions (26) and (27): (a) periodic solitary waves and the two-dimensional profile (b), and (c) combined bright-dark solitary wave and its two-dimensional profile (d), providing suitable values to the parameters.

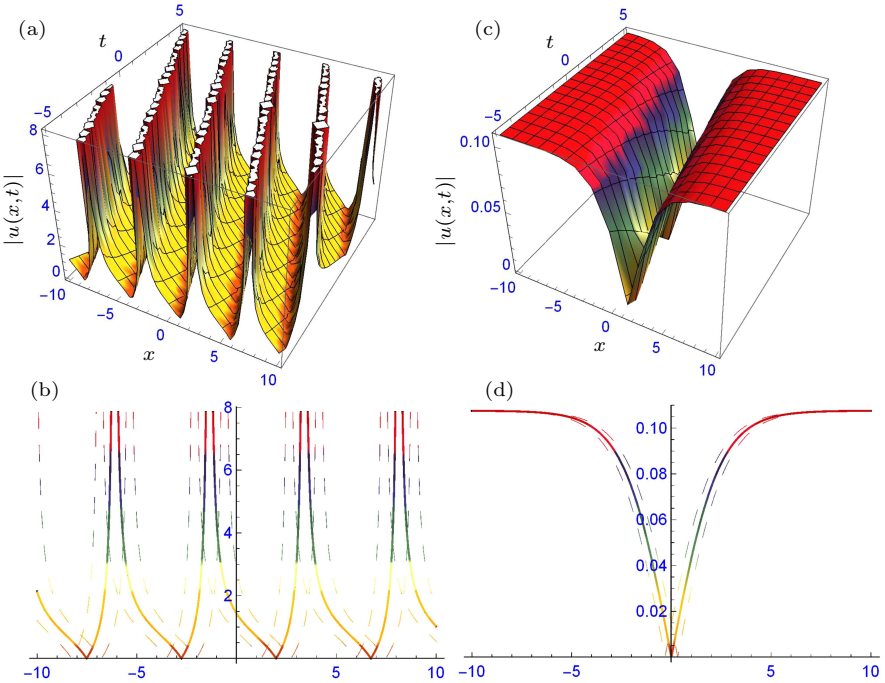


Fig. 3. The structure of solutions (29) and (34): (a) periodic waves and the two-dimensional profile (b), and (c) dark soliton and its two-dimensional profile (d), providing suitable values to the parameters.

Family 4 In this family, we consider $d_0 = d_2 = 0$,

Set 1

$$a_{-2} = a_{-1} = 0, \quad a_0 = -\frac{k\left(2d_3n^2 + d_1\right)\sqrt{\alpha_1(\eta + 2m(\eta+\delta))}}{\sqrt{\alpha_2\gamma(\eta - m(\delta+\eta))}}, \quad a_1 = \frac{4d_3kn\sqrt{\alpha_1(\eta + 2m(\eta+\delta))}}{\sqrt{\alpha_2\gamma(\eta - m(\delta+\eta))}},$$
$$\nu = \frac{k\left(-\alpha_1\left(2d_1^2k^2(\eta + 2m(\delta+\eta)) + \gamma^2(2m(\delta+\eta) - \eta)\right) + \eta\omega - 2m(\delta+\eta)(\beta\gamma - \omega)\right)}{\gamma\eta},$$
$$a_2 = -\frac{2d_3k\sqrt{\alpha_1(\eta + 2m(\eta+\delta))}}{\sqrt{\alpha_2\gamma(\eta - m(\delta+\eta))}}. \tag{31}$$

Set 2

$$a_{-2} = a_{-1} = 0, \quad a_0 = \frac{k\left(2d_3n^2 + d_1\right)\sqrt{\alpha_1(\eta + 2m(\eta+\delta))}}{\sqrt{\alpha_2\gamma(\eta - m(\delta+\eta))}}, \quad a_1 = -\frac{4d_3kn\sqrt{\alpha_1(\eta + 2m(\eta+\delta))}}{\sqrt{\alpha_2\gamma(\eta - m(\delta+\eta))}},$$
$$\nu = \frac{k\left(-\alpha_1\left(2d_1^2k^2(\eta + 2m(\delta+\eta)) + \gamma^2(2m(\delta+\eta) - \eta)\right) + \eta\omega - 2m(\delta+\eta)(\beta\gamma - \omega)\right)}{\gamma\eta},$$
$$a_2 = \frac{2d_3k\sqrt{\alpha_1(\eta + 2m(\eta+\delta))}}{\sqrt{\alpha_2\gamma(\eta - m(\delta+\eta))}}. \tag{32}$$

The following soliton solutions to Eq. (2) from set 1 are attained as follows:

$$u_{41}(x,t) = \frac{d_1k\sqrt{\alpha_1(\eta + 2m(\eta+\delta))}\left(d_3e^{2d_1(\xi+\xi_0)} + 1\right)}{\sqrt{\alpha_2\gamma(\eta - m(\delta+\eta))}\left(d_3e^{2d_1(\xi+\xi_0)} - 1\right)}e^{i(\gamma x + \omega t + \varepsilon)}, \quad d_1 > 0. \tag{33}$$

$$u_{42}(x,t) = \frac{d_1k\sqrt{\alpha_1(\eta + 2m(\eta+\delta))}\left(d_3e^{2d_1(\xi+\xi_0)} - 1\right)}{\sqrt{\alpha_2\gamma(\eta - m(\delta+\eta))}\left(d_3e^{2d_1(\xi+\xi_0)} + 1\right)}e^{i(\gamma x + \omega t + \varepsilon)}, \quad d_1 < 0. \tag{34}$$

Similarly, one can construct more soliton solutions to Eq. (2) from set 2.

3.2. Generalized exp$(-\phi(\xi))$-expansion technique

As Eq. (2) is complex, our hypothesis of the wave solution to Eq. (2) is in the form of

$$u(x,t) = \psi(\xi)e^{iP}, \quad \xi = kx + \nu t, \quad P = \gamma x + \omega t + \varepsilon, \tag{35}$$

where $\psi(\xi)$ is given in Eq. (8). Putting Eq. (35) into Eq. (2) and making separate into parts yield

$$k\left(\eta + 2m(\delta+\eta)\right)\psi^{2m} - 3k\alpha_2\psi^2 + 2k\alpha_1\gamma + \beta k - \nu = 0. \tag{36}$$

$$k^2\alpha_1\psi'' + \gamma\alpha_2\psi^3 - \left(\beta\gamma - \omega + \gamma^2\alpha_1\right) - \gamma\eta\psi^{2m}\psi = 0. \tag{37}$$

Substituting ψ^{2m} from Eq. (36) into Eq. (37) yields

$$\alpha_1k^3\left(\eta + 2m(\delta+\eta)\right)\psi'' + \left(\alpha_1\gamma^2k\left(\eta - 2m(\delta+\eta)\right)\right.$$
$$\left. - \gamma\eta\nu - 2\beta\gamma km(\delta+\eta)\right.$$
$$\left. + k\omega\left(\eta + 2m(\delta+\eta)\right)\right)\psi$$
$$+ 2\alpha_2\gamma k\left(m\delta + m\eta - \eta\right)\psi^3 = 0. \tag{38}$$

The homogeneous balance principle is applied on Eq. (38), and we consider the solution to Eq. (13) as follows:

$$\psi(\xi) = b_0 + b_1\exp(-\phi(x)). \tag{39}$$

Substituting Eqs. (39) and (9) into Eq. (38) and adjusting the coefficients of $\left(e^{(-\phi(\xi))}\right)^i$ to zero, we achieve an equation set in b_0, b_1, p, q, r, k, ν, ω, η, β, and m. Mathematica 9 was employed to solve the set of equations. We obtain the following families.

Family 1

$$\nu = \frac{k\left(\alpha_1\left(2\gamma^2(3\eta - 2\delta m + 2\eta m) - k^2(-\eta + 2\delta m - 2\eta m)\left(q^2 - 4pr\right)\right) - 2\eta\omega + 4m(\delta - \eta)(\beta\gamma + \omega)\right)}{2\gamma\eta},$$
$$b_0 = \frac{kq\sqrt{\alpha_1(\eta + 2m(\eta - \delta))}}{2\sqrt{\alpha_2\gamma(\delta m - \eta(m+2))}}, \quad b_1 = \frac{kr\sqrt{\alpha_1(\eta + 2m(\eta - \delta))}}{\sqrt{\alpha_2\gamma(\delta m - \eta(m+2))}}. \tag{40}$$

From family 1, the following kinds of soliton solutions to Eq. (2) are constructed.

Type I For $r=1$, $p\neq 0$, $q^2-4p>0$,

$$u_1(\xi)=\frac{k\sqrt{\alpha_1(\eta-2\delta m+2\eta m)}\left(q^2-4pr+q\sqrt{q^2-4p}\tanh\left(\frac{\sqrt{q^2-4p}}{2}(\xi+\xi_0)\right)\right)}{2\sqrt{\alpha_2\gamma(m(\delta-\eta)-2\eta)}\left(q+\sqrt{q^2-4p}\tanh\left(\frac{\sqrt{q^2-4p}}{2}(\xi+\xi_0)\right)\right)}\mathrm{e}^{\mathrm{i}(\gamma x+\omega t+\varepsilon)}. \tag{41}$$

Type II For $r=1$, $p\neq 0$, $q^2-4p<0$,

$$u_2(\xi)=\frac{k\sqrt{\alpha_1(\eta-2\delta m+2\eta m)}}{2\sqrt{\alpha_2\gamma(m(\delta-\eta)-2\eta)}}\left(q-\frac{4pr}{q-\sqrt{4p-q^2}\tan\left(\frac{\sqrt{4p-q^2}}{2}(\xi+\xi_0)\right)}\right)\mathrm{e}^{\mathrm{i}(\gamma x+\omega t+\varepsilon)}. \tag{42}$$

Type III For $r=1$, $p=0$, $q\neq 0$, $q^2-4p>0$,

$$u_3(\xi)=\frac{kq\sqrt{\alpha_1(\eta-2\delta m+2\eta m)}\left(\mathrm{e}^{q(\xi+\xi_0)}+2r-1\right)}{2\sqrt{\alpha_2\gamma(m(\delta-\eta)-2\eta)}\left(\mathrm{e}^{q(\xi+\xi_0)}-1\right)}\mathrm{e}^{\mathrm{i}(\gamma x+\omega t+\varepsilon)}. \tag{43}$$

Type IV For $r=1$, $p\neq 0$, $q\neq 0$, $q^2-4p=0$,

$$u_4(\xi)=-\frac{kq\sqrt{\alpha_1(-(2m(\delta-\eta)-\eta))}\,(q(r-1)(\xi+\xi_0)-1)}{2\sqrt{\alpha_2\gamma(m(\delta-\eta)-2\eta)}(q(\xi+\xi_0)+1)}\mathrm{e}^{\mathrm{i}(\gamma x+\omega t+\varepsilon)}. \tag{44}$$

Type V For $q=0$, $r>0$, $p>0$,

$$u_5(\xi)=\frac{k\sqrt{\alpha_1(\eta+2m(\eta-\delta))}}{2\sqrt{\alpha_2\gamma(\delta m-\eta(m+2))}}\left(q+2\sqrt{pr}\cot\left(\sqrt{pr}(\xi+\xi_0)\right)\right)\mathrm{e}^{\mathrm{i}(\gamma x+\omega t+\varepsilon)}. \tag{45}$$

Type VI For $q=0$, $r<0$, $p<0$,

$$u_6(\xi)=\frac{k\sqrt{\alpha_1(\eta+2m(\eta-\delta))}}{2\sqrt{\alpha_2\gamma(\delta m-\eta(m+2))}}\left(q-2\sqrt{pr}\cot\left(\sqrt{pr}(\xi-\xi_0)\right)\right)\mathrm{e}^{\mathrm{i}(\gamma x+\omega t+\varepsilon)}. \tag{46}$$

Type VII For $q=0$, $r>0$, $p<0$,

$$u_7(\xi)=\frac{k\sqrt{\alpha_1(\eta+2m(\eta-\delta))}}{2\sqrt{\alpha_2\gamma(\delta m-\eta(m+2))}}\left(q-2\sqrt{-pr}\coth\left(\sqrt{-pr}(\xi-\xi_0)\right)\right)\mathrm{e}^{\mathrm{i}(\gamma x+\omega t+\varepsilon)}. \tag{47}$$

Type VIII For $q=0,\ r<0,\ p>0$,

$$u_8(\xi)=\frac{k\sqrt{\alpha_1(\eta+2m(\eta-\delta))}}{2\sqrt{\alpha_2\gamma(\delta m-\eta(m+2))}}\left(q+2\sqrt{-pr}\coth\left(\sqrt{-pr}(\xi+\xi_0)\right)\right)\mathrm{e}^{\mathrm{i}(\gamma x+\omega t+\varepsilon)}. \tag{48}$$

Type IX For $p=0,\ q=0$,

$$u_9(\xi)=\frac{k\sqrt{\alpha_1(\eta-2\delta m+2\eta m)}(2+q(\xi+\xi_0))}{2\sqrt{\alpha_2\gamma(m(\delta-\eta)-2\eta)}(\xi+\xi_0)}\mathrm{e}^{\mathrm{i}(\gamma x+\omega t+\varepsilon)}. \tag{49}$$

Family 2

$$\begin{aligned}&\nu=\frac{k\left(\alpha_1\left(2\gamma^2(3\eta-2\delta m+2\eta m)-k^2(-\eta+2\delta m-2\eta m)\left(q^2-4pr\right)\right)-2\eta\omega+4m(\delta-\eta)(\beta\gamma+\omega)\right)}{2\gamma\eta},\\&b_0=-\frac{kq\sqrt{\alpha_1(\eta+2m(\eta-\delta))}}{2\sqrt{\alpha_2\gamma(\delta m-\eta(m+2))}},\quad b_1=-\frac{kr\sqrt{\alpha_1(\eta+2m(\eta-\delta))}}{\sqrt{\alpha_2\gamma(\delta m-\eta(m+2))}}.\end{aligned} \tag{50}$$

Similarly, one can construct more general soliton solutions to Eq. (2) from family 2.

In Fig. 4, the restricted conditions for avoiding singularities are $\frac{\alpha_1(\eta+2m(\delta+\eta))}{\alpha_2\gamma(\eta-m(\delta+\eta))}>0$, $q\neq\sqrt{4p-q^2}\tan\left(\frac{\sqrt{4p-q^2}}{2}(\xi+\xi_0)\right)$, and $q\neq-\sqrt{q^2-4p}\tanh\left(\frac{\sqrt{q^2-4p}}{2}(\xi+\xi_0)\right)$.

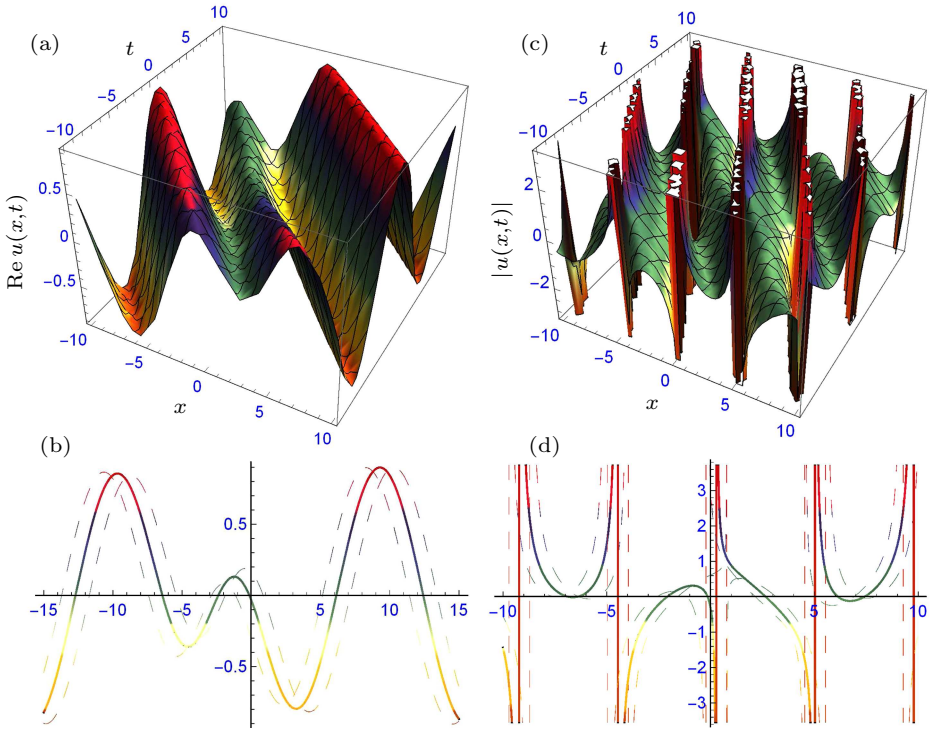


Fig. 4. The structure of solutions (41) and (42): (a combined bright-dark solitary wave and its two-dimensional profile (b), and (c) periodic traveling wave and its two-dimensional profile (d), providing suitable values to the parameters.

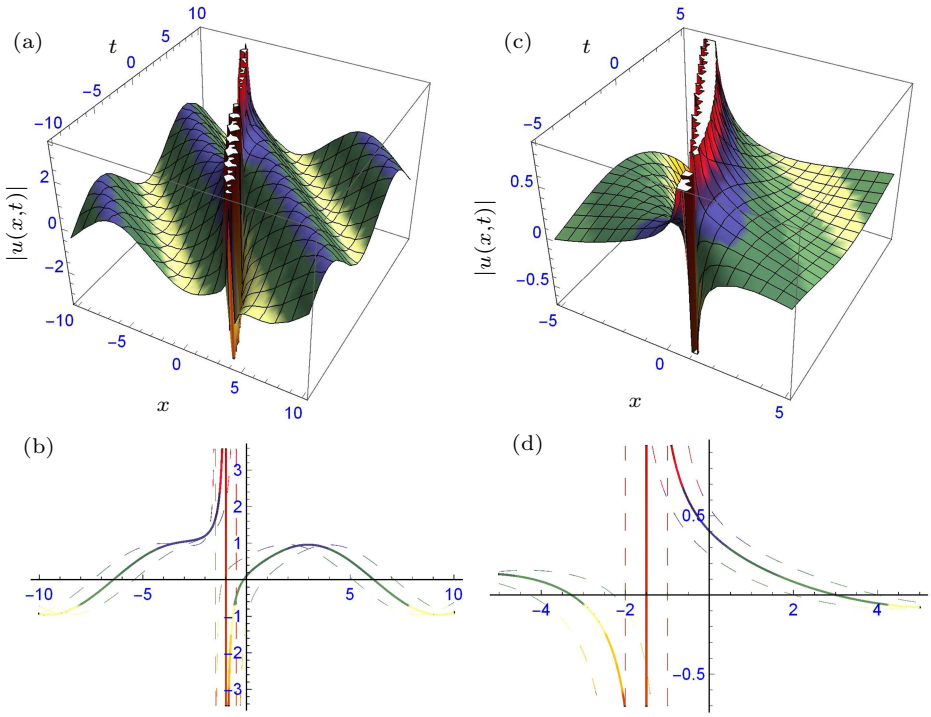


Fig. 5. The structure of solutions (43) and (44): (a) bright-dark solitary wave and its two-dimensional profile (b), and (c) traveling wave and its two-dimensional profile (d), providing suitable values to the parameters.

In Fig. 5, the restricted conditions for avoiding singularities are $\frac{\alpha_1(\eta+2m(\delta+\eta))}{\alpha_2\gamma(\eta-m(\delta+\eta))} > 0$, $e^{q(\xi+\xi_0)} \neq 1$, and $e^{q(\xi+\xi_0)} \neq -1$.

4. Modulation instability

Numerous higher order nonlinear evolution equations depicting an instability that directs to investigate the steady state modulation as an consequence of the interaction among nonlinear and dispersive effects. To examine the MI of perturbed Kaup–Newell Schröodinger Eq. (2) with using the analysis of standard linear stability[8,26–29] to scrutinize how weak and time dependent perturbations construct along the propagation distance. The steady state solution to the perturbed Kaup–Newell Schröodinger equation reads

$$u(x,t) = \left(\sqrt{P} + \phi(x,t)\right) e^{i\psi(t)}, \quad \psi(t) = P\gamma\varepsilon t, \tag{51}$$

where P is normalized optical power. The perturbation $\phi(x,t)$ is studied by employing linear stability analysis. Substituting Eq. (51) into Eq. (2) and linearizing, we obtain

$$\frac{\partial\phi}{\partial t}-(\beta+mP^m(m\delta+\eta+m\eta)-2P\alpha_2)\frac{\partial\phi}{\partial x}-(mP^m\delta+mP^m\eta-P\alpha_2)\frac{\partial\phi^*}{\partial x}+\mathrm{i}\alpha_1\frac{\partial^2\phi}{\partial x^2}+\mathrm{i}P\gamma\varepsilon(\phi+\phi^*)=0, \tag{52}$$

where $*$ denotes complex conjugate. Assume the solution to Eq. (52) as follows:

$$\phi(x,t)=\lambda_1\mathrm{e}^{\mathrm{i}(kx-\tau t)}+\lambda_2\mathrm{e}^{-i(kx-\tau t)}, \tag{53}$$

where k and τ are normalized wave number and frequency of $\phi(x,t)$. Substituting Eq. (53) into Eq. (52), we attain the dispersion relation (DR)

$$\tau=2\alpha_2kP-\beta k-kP^m(\eta+m(\delta+\eta))\pm k\sqrt{\alpha_1^2k^2+(m(\delta+\eta)P^m-\alpha_2P)^2-2\alpha_1\gamma P\varepsilon}\,. \tag{54}$$

The DR (54) signifies that stability of steady state is contingent on self-phase modulation, Raman scattering and τ. This DR is vitally dependent on the sign of the term within the square root as if $\alpha_1^2k^2+(m(\delta+\eta)P^m-\alpha_2P)^2-2\alpha_1\gamma P\varepsilon>0$. This shows the stability of the steady state against smaller perturbations. Under other conditions it becomes unstable. If $\alpha_1^2k^2+(m(\delta+\eta)P^m-\alpha_2P)^2-2\alpha_1\gamma P\varepsilon<0$, then τ will be complex and the perturbations will cultivate exponentially. One can see directly an example of MI when $\alpha_1^2k^2+(m(\delta+\eta)P^m-\alpha_2P)^2-2\alpha_1\gamma P\varepsilon<0$. Under these conditions, the rate of evolution for MI gain spectrum $H(k)$ can be expressed as

$$h(k)=2\mathrm{Im}\,g(k)=2k\sqrt{\alpha_1^2k^2+(m(\delta+\eta)P^m-\alpha_2P)^2-2\alpha_1\gamma P\varepsilon}. \tag{55}$$

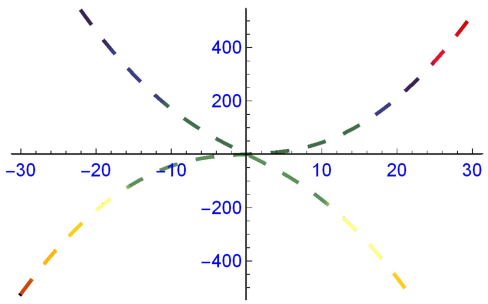


Fig. 6. The graph of dispersion relation $\tau=\tau(k)$.

5. Results and discussion

The achieved solutions are dissimilar from the attained results of numerous researchers by other prior techniques due to fact that the supposed result (14) of the proposed technique is changed from the previous technique. Equation (7) gives a few distinct type of solutions by providing dissimilar value of parameters. The authors of Ref. [23] constructed the solitons to Eq. (2) using the G'/G^2-expansion and $\exp(-\varphi(\zeta))$-expansion methods. The obtained solutions (41), (42), (44), and (35) are similar to the solutions (23), (24), (26) and (27), respectively, which were obtained by authors of Ref. [23]. The authors of Ref. [30] investigated the conservation laws of this dynamical model. Therefore, we have achieved several innovative results that have not been stated before.

Figures 1–3 signify the solitary waves in different shapes depicted. Figures 1(a)–1(d) depict the bright solitary wave and dark soliton in three and two-dimensional form of solutions (17) and (18) at $\alpha_1=0.5$, $\alpha_2=1$, $\beta=0.25$, $\gamma=0.5$, $d_1=0.5$, $d_2=0.75$, $\delta=1$, $\eta=0.5$, $k=1$, $m=1$, $n=1$, $\omega=1$, $\varepsilon=1$ and $\alpha_1=0.5$, $\alpha_2=1$, $\beta=0.25$, $\gamma=0.5$, $d_1=-0.5$, $d_2=0.75$, $\delta=1$, $\eta=0.5$, $k=1$, $m=1$, $n=1$, $\omega=1$, $\varepsilon=1$, respectively. Figures 2(a)–2(d) depict the periodic and combined bright-dark solitary waves in three- and two-dimensional forms of solutions (29) and (34) at $\alpha_1=0.5$, $\alpha_2=0.25$, $\beta=0.4$, $\gamma=0.5$, $d_0=0.5$, $d_2=1.5$, $\delta=1$, $\eta=0.25$, $k=0.5$, $\nu=0.5$, $m=1$, $n=1$, $\xi_0=1$, $\varepsilon=-1.5$ and $\alpha_1=0.5$, $\alpha_2=-1.3$, $\beta=-0.3$, $\gamma=1$, $d_0=0.5$, $d_2=-1.5$, $\delta=-1.6$, $\eta=-0.05$, $\xi_0=1$, $k=0.5$, $\nu=0.5$, $m=1$, $n=1$, $\varepsilon=-0.5$ respectively. Figures 3(a)–3(d) depict the periodic and dark solitary waves in three- and two-dimensional forms of solutions (29) and (34) at $\alpha_1=0.5$, $\alpha_2=-1.3$, $\beta=-0.3$, $\gamma=0.75$, $d_0=0.5$, $d_1=0.5$, $d_2=-1$, $\delta=1.6$, $\eta=1$, $\xi_0=-1$, $k=1$, $m=1$, $n=1$, $\omega=0.5$, $\varepsilon=-0.5$ and $\alpha_1=0.5$, $\alpha_2=-1.3$, $\beta=-0.3$, $\gamma=0.75$, $d_1=-0.5$, $d_3=1$, $\delta=-1.6$, $\eta=1$, $\xi_0=0$, $k=1$, $m=1$, $n=1$, $\omega=0.5$, $\varepsilon=-0.5$, respectively.

Figures 4 and 5 describe the soliton and solitary waves in different forms depicted. Figures 4(a)–4(d) depict the combined bright-dark and periodic traveling waves in three- and two-dimensional forms of solutions (41) and (42) at $\beta=0.5$, $p=0.5$, $q=1.5$, $r=1$, $\alpha_1=0.5$, $\alpha_2=-1.3$, $\gamma=0.5$, $\omega=0.5$, $\varepsilon=-1.5$, $k=1$, $m=1$, $\eta=-0.05$, $\xi_0=1$ and $\beta=0.5$, $p=1$, $q=1.5$, $r=1$, $\alpha_1=0.5$, $\alpha_2=-1.3$, $\gamma=0.5$, $\omega=0.5$, $\varepsilon=-1.5$, $k=1$, $m=1$, $\eta=-0.05$, $\xi_0=1$, respectively. Figures 5(a)–5(d) depict the bright-dark and traveling waves in three- and two-dimensional forms of solutions (43) and (44) at $\beta=0.5$, $p=0$, $q=1.5$, $r=1$, $\alpha_1=0.5$, $\alpha_2=-1.3$, $\gamma=0.5$, $\omega=0.5$, $\varepsilon=-1.5$, $k=1$, $m=1$, $\eta=-0.05$, $\xi_0=1$ and $\beta=0.5$, $p=1$, $q=2$, $r=1$, $\alpha_1=0.5$, $\alpha_2=-1.3$, $\gamma=0.5$, $\omega=0.5$, $\varepsilon=-1.5$, $k=1$, $m=1$, $\eta=-0.05$, $\xi_0=1$, respectively. The dispersion relation $\tau=\tau(k)$ between k and τ of perturbation is given in Fig. 6.

6. Conclusion

We have employed the extended F-expansion and generalized $\exp(-\phi(\xi))$-expansion methods on the perturbed K–N equation, where the perturbation terms appear with full nonlinearity and traveling wave solutions in dissimilar forms such as bright and dark solitons, combined dark-bright solitons, solitary waves, periodic and other wave solutions attained. These obtained results are very helpful in governing soliton dynamics. This dynamical model describes plus propagation in optical fibers and can be observed as a special case of the generalized higher order NLSE. These achieved solutions are novel and may be useful for physicians, mathematician and engineers to understand more complex physical phenomena. These achieved soliton solutions have key applications such as optical fibers and ultra short light pulses.[5,31,32] The stability of model is discussed by utilizing modulation instability analysis, which confirms that all exact solutions are stable. The moments of a few results are revealed graphically by granting suitable values to parameters. In future to extend the result, we can solve the generalized higher order NLSE in the presence of non-local perturbation terms. The computation work endorses the simplicity, effectiveness and impact of current techniques.

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