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# Jeans gravitational instability with $\kappa$-deformed Kaniadakis distribution in Eddington-inspired Born–Infield gravity*

Wei-Heng Yang(杨伟恒)[1,2], Yu-Zhen Xiong(熊玉珍)[1,2], Hui Chen(陈辉)[1,2,†], and San-Qiu Liu(刘三秋)[1,2]

[1] *Department of Physics, Nanchang University, Nanchang* 330031, *China*
[2] *Jiangxi Provincial Key Laboratory of Fusion and Information Control, Nanchang* 330031, *China*

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Based on the framework of Kaniadakis' statistics and its related kinetic theory, the Jeans instability for self-gravitational systems in the background of Eddington-inspired Born–Infield (EiBI) gravity is revisited. A dispersion relation generalizing the Jeans modes is derived by modifying the Maxwellian distribution to a family of power law distributions parameterized by the $\kappa$ parameter. It is established that the $\kappa$-deformed Kaniadakis distribution has significant effects on the Jeans modes of the collisionless EiBI-gravitational systems. And as expected, in the limitation $\kappa \to 0$, the corresponding results for Maxwellian case are recovered. The related result in the present work is valuable for the investigations involving the fields of astrophysics such as neutron stars, accretion disks, and relevant plasma physics, *etc*.

**Keywords:** Jeans instability, EiBI gravity, $\kappa$-deformed Kaniadakis distribution

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Jeans instability, proposed by Jeans[1] in 1929, is one of the key mechanisms to explain the gravitational formation of structures and their evolution in linear regime, which is ubiquitous in the fields of astrophysics,[2–6] plasma,[7–16] and complex fluid community.[17,18] A significant conclusion of Jeans' work is that perturbations which have wave-length $\lambda$ greater than a critical wave-length $\lambda_{\rm J}$ (Jeans wave-length) may grow exponentially thereby producing gravitating instability, whereas perturbations with wave-length smaller than $\lambda_{\rm J}$ will be suppressed and its physical behaviors like acoustic waves. Such Jeans' criterion is a fundamental theory and remains basically valid even in the expanding Universe, which follows from a kinetic theoretical approach involving the Vlasov equation.[1,2] However, more and more challenges to traditional Jeans' theory have emerged as the door to alternatives theories of gravity has been opened and the observation data involving the statistical description become increasingly more precise.[19–21] From the perspective of research methods, Jeans instabilities in a self-gravitational granular gas based on the hydrodynamic equations which is different from Boltzmann equation were also studied by Kremer.[22] It is, therefore, necessary to make some amendments for the theory of Jeans instability, especially on some astronomical systems where traditional theories cannot be explained. Recently, there has been a growing interest[23,24] in investigations of the Jeans instability in a variety of alternatives theories of gravity. For example, inspired by some studies involving the alternatives theories of gravity,[21,25] Martino *et al*. have researched the effect of EiBI gravity on the Jeans instability and found that the EiBI gravity influences the Jeans instability only in high densities environments like the hyper massive neutron star,[26] where the particle number density is about $10^{39}$ cm$^{-3}$ and high temperature $5\times10^{10}$ K–$5\times10^{11}$ K. It is one of the high density systems known, but, more importantly, it is considered as one of the most promising laboratories to probe EiBI gravity. However, the effect of EiBI gravity can be neglected for the low densities environments because of the condition $\lambda_{\rm EiBI} \ll \lambda_{\rm J}$. In order to get a deeply insight of this key ingredient of the present paper, EiBI gravity, it is necessary to introduce its detailed information. In fact, EiBI gravity, inspired by the Born–Infeld action for non-linear electrodynamics and developed based on the Eddington's theory of gravity,[19] is considered as a modern formulation of Eddington's theory, which is proposed to solve some hot problems, such as, the dynamics of self-gravitational systems, the accelerated expansion of the Universe, and a non-singular description of the Universe, *etc*.

On the other hand, a noteworthy phenomenon is that several non-Maxwellian distribution functions related to different statistics, such as $q$-nonextensive statistics[27,28] and Kaniadakis' statistics,[29] have been proposed to fit observed particles' distribution in space environment during the last decades. Among them, the $\kappa$-deformed Kaniadakis distribution, one of the more general distribution functions,[30] has been established by Kaniadakis[29] in 2001 and developed over the

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†Corresponding author. E-mail: hchen61@ncu.edu.cn

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past decades, which is now widely applied to the field of black-body radiation,[31] quantum entanglement,[32] cosmic rays,[33] plasma physics,[34,35] kinetics of interaction atoms and photons,[36] nonlinear kinetics,[37,38] and even the financial systems,[39] *etc*. It should be point out that some studies recently have used the $\kappa$-deformed distribution to examine the Jeans instability of self-gravitational systems and showed that it plays a significant role in the process of gravitating collapse of matter systems.[5,40]

Therefore, with the above-mentioned contents, one can conclude that the study with regard to the effect of $\kappa$-deformed Kaniadakis distribution on the Jeans instability in the EiBI-gravitational systems is theoretically feasible. Of particular interest is its theoretical value for further studies in the fields of astrophysics such as neutron stars, accretion disks, and relevant plasma physics, *etc*. The consideration of the Kaniadakis' statistics allows us to use a family of power law distributions parameterized by the $\kappa$ parameter for inferring the dispersion relation of Jeans instability in high-density, high-temperature, collisionless EiBI-gravitational systems.

In the present work, a collisionless, high-density, high-temperature, isotropic EiBI-gravitational matter system is taken into consideration. Along the standard lines, the collisionless Botlzmann equation for EiBI-gravitational systems is written as

$$\frac{\partial f(\boldsymbol{v},\boldsymbol{r},t)}{\partial t}+\boldsymbol{v}\cdot\frac{\partial f(\boldsymbol{v},\boldsymbol{r},t)}{\partial \boldsymbol{r}}=\nabla\phi\cdot\frac{\partial f(\boldsymbol{v},\boldsymbol{r},t)}{\partial \boldsymbol{v}}, \tag{1}$$

where $f(\boldsymbol{v},\boldsymbol{r},t)$ and $\phi$ are the matter–particle distribution function and gravitational potential, respectively. The consideration of EiBI-gravitational model allows one to modify gravitational Poisson equation as[23]

$$\begin{aligned}\nabla^2\phi &= \left(4\pi G+\frac{\chi}{4}\nabla^2\right)\int f(\boldsymbol{v},\boldsymbol{r},t)\,\mathrm{d}\boldsymbol{v}\\ &= \left(4\pi G+\frac{\chi}{4}\nabla^2\right)\rho(\boldsymbol{r},t),\end{aligned} \tag{2}$$

where $\rho(\boldsymbol{r},t)$, $\boldsymbol{v}$, and $\chi$ denote the mass density, velocity of particles, and the parameter of EiBI gravity, respectively. When the effect of EiBI gravity is neglected, *i.e*., $\chi=0$, then the standard Poisson equation $\nabla^2\phi=4\pi G\rho(\boldsymbol{r},t)$ is recovered. Along the standard procedure, the distribution function and gravitational potential can be divided into equilibrium and perturbation, *i.e*.,

$$f(\boldsymbol{v},\boldsymbol{r},t)=f_0(\boldsymbol{v})+f_1(\boldsymbol{v},\boldsymbol{r},t),\quad \phi(\boldsymbol{r})=\phi_0(\boldsymbol{r})+\phi_1(\boldsymbol{r}).$$

And involving the Jeans swindle,[41] one can assume the unperturbed potential $\phi_0(\boldsymbol{r})=0$. Then combining Eqs. (1) and (2), as well as using the the Fourier transformation of the perturbed quantities, *i.e*.,

$$X_1\propto\exp[-\mathrm{i}(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})](X_1=f_1,\phi_1),$$

one can get the dispersion equation for the perturbation mode with its frequency $\omega$ and wavenumber $\boldsymbol{k}$ as

$$1-\left(\frac{4\pi G}{k^2}-\frac{\chi}{4}\right)\int\frac{\boldsymbol{k}\cdot\dfrac{\partial f_0(\boldsymbol{v})}{\partial \boldsymbol{v}}}{\omega-\boldsymbol{k}\cdot\boldsymbol{v}+\mathrm{i}\varepsilon}\mathrm{d}\boldsymbol{v}=0. \tag{3}$$

Without loss of generality, one can choose $\boldsymbol{v}$ in the direction of $x$ and setting $\boldsymbol{k}=(k,0,0)$, then equation (3) is rewritten as

$$1-\left(\frac{4\pi G}{k^2}-\frac{\chi}{4}\right)\int\frac{k\dfrac{\partial f_0(v_x)}{\partial v_x}}{\omega-kv_x+\mathrm{i}\varepsilon}\mathrm{d}v_x=0. \tag{4}$$

In the framework of Kaniadakis' statistics, the background distribution function of the mass-particles can be modeled by the $\kappa$-deformed distribution[29,33,35]

$$f_0(\boldsymbol{v})=A_\kappa\left(\sqrt{1+\kappa^2\frac{v^4}{4\sigma^4}}-\kappa\frac{v^2}{2\sigma^2}\right)^{1/\kappa}, \tag{5}$$

where $\sigma$ is the velocity dispersion, which is viewed as the thermal velocity of the matter systems. Through the normalized condition

$$\int_{-\infty}^{\infty}f_0(\boldsymbol{v})\,\mathrm{d}\boldsymbol{v}=\rho_0,$$

the normalized coefficient $A_\kappa$ is presented as

$$A_\kappa=\rho_0\frac{\left|\dfrac{\kappa}{\pi}\right|^{3/2}}{\sigma^3}\left(1+\frac{3}{2}|\kappa|\right)\frac{\Gamma\left(\dfrac{1}{|2\kappa|}+\dfrac{3}{4}\right)}{\Gamma\left(\dfrac{1}{|2\kappa|}-\dfrac{3}{4}\right)}, \tag{6}$$

where $\rho_0$ represents the equilibrium matter density. Putting Eq. (5) into the definition of one-dimensional distribution function, *i.e*.,

$$f_0(v_x)=\int f_0(\boldsymbol{v})\,\mathrm{d}v_y\,\mathrm{d}v_z,$$

one can obtain the one-dimensional $\kappa$-deformed Kaniadakis distribution function[5,35]

$$\begin{aligned}f_0(v_x)=-\frac{2\sigma^2A_\kappa\pi}{\kappa^2-1}&\left[-\kappa\frac{v_x^2}{2\sigma^2}+\sqrt{1+\kappa^2\left(\frac{v_x^2}{2\sigma^2}\right)^2}\right]^{1/\kappa}\\ &\times\left[\kappa^2\frac{v_x^2}{2\sigma^2}+\sqrt{1+\kappa^2\left(\frac{v_x^2}{2\sigma^2}\right)^2}\right].\end{aligned} \tag{7}$$

Substituting $\partial f_0(v_x)/\partial v_x$ into Eq. (4) and with the aid of the following integral relation[33,40,42]

$$\int_0^\infty x^{r-1}(-\kappa x+\sqrt{1+\kappa^2x^2})^{1/\kappa}\mathrm{d}x=\frac{|2\kappa|^{-r}}{1+r|\kappa|}\frac{\Gamma\left(\frac{1}{|2\kappa|}-\frac{r}{2}\right)}{\Gamma\left(\frac{1}{|2\kappa|}+\frac{r}{2}\right)}\Gamma(r), \tag{8}$$

then the modified dispersion equation is obtained as

$$1-\frac{1}{C_\kappa}\left(\frac{k_\mathrm{J}^2}{k^2}-\frac{k_\mathrm{J}^2}{k_\mathrm{EiBI}^2}\right)[B_\kappa-Z(\xi)]=0, \tag{9}$$

where $k_\mathrm{J}(\equiv\sqrt{4\pi G\rho_0/\sigma^2})$ and $k_\mathrm{EiBI}(\equiv\sqrt{16\pi G/\chi})$ denote the classical Jeans wave-number[41] and EiBI Jeans wave-number,[23] respectively; the modified dispersion function $Z(\xi)$ is written as

$$Z(\xi)=\int_{-\infty}^{\infty}\frac{\xi}{\xi-X+\mathrm{i}\varepsilon}\left(-\kappa X^2+\sqrt{1+\kappa^2X^4}\right)^{1/\kappa}\mathrm{d}X, \tag{10}$$

where $\xi=\omega/\sqrt{2}k\sigma$. Using Plemelj relation,[43] one has

$$Z(\xi)=\wp\int_{-\infty}^{\infty}\frac{\xi}{\xi-X}\left(-\kappa X^2+\sqrt{1+\kappa^2X^4}\right)^{1/\kappa}\mathrm{d}X-\mathrm{i}\pi\xi\left(-\kappa\xi^2+\sqrt{1+\kappa^2\xi^4}\right)^{1/\kappa}. \tag{11}$$

In addition, the coefficient $B_\kappa$ and $C_\kappa$ are given by[5,35]

$$B_\kappa=\frac{|2\kappa|^{-1/2}}{1+\frac{1}{2}|\kappa|}\frac{\Gamma\left(\frac{1}{|2\kappa|}-\frac{1}{4}\right)}{\Gamma\left(\frac{1}{|2\kappa|}+\frac{1}{4}\right)}\sqrt{\pi}, \tag{12}$$

$$C_\kappa=\frac{|\kappa|^{-3/2}}{1+\frac{3}{2}|\kappa|}\frac{\Gamma\left(\frac{1}{|2\kappa|}-\frac{3}{4}\right)}{\Gamma\left(\frac{1}{|2\kappa|}+\frac{3}{4}\right)}\frac{\sqrt{\pi}}{2\sqrt{2}}. \tag{13}$$

According to physical content of Jeans instability, the boundary between stable and unstable modes is achieved by setting $\omega=0$ (*i.e.*, $\xi=0$) in Eq. (9), and thus the critical Jeans wave-number is written by

$$k_{\mathrm{J}\kappa}^2=k_\mathrm{J}^2\left(\frac{C_\kappa}{B_\kappa}+\frac{k_\mathrm{J}^2}{k_\mathrm{EiBI}^2}\right)^{-1}, \tag{14}$$

*i.e.*, the unstable mode of the systems will occur when $k<k_{\mathrm{J}\kappa}$, whereas perturbations with $k>k_{\mathrm{J}\kappa}$ are quickly damped. Equation (14) shows that the Jeans threshold of collisionless EiBI-gravitational systems in the context of Kaniadakis distribution is modified by distribution index $\kappa$ and EiBI-gravity parameter $\chi$.

In what follows, we focus our attention on the analysis of dispersion equation within the high and low frequency perturbations, respectively. Firstly, in the limitation of high frequency perturbations ($\xi\gg1$), the modified dispersion function $Z(\xi)$ can be written by

$$Z(\xi)=B_\kappa+\frac{1}{2\xi^2}C_\kappa+\frac{3}{4\xi^4}D_\kappa+\cdots. \tag{15}$$

Then from Eq. (9), one can obtain the dispersion relation for Jeans modes as

$$\omega_*^2=\left[k_*^2\left(\frac{k_\mathrm{J}^2}{k_\mathrm{EiBI}^2}+3\frac{D_k}{C_k}\right)-1\right], \tag{16}$$

where the coefficient $D_\kappa$ is expressed as

$$D_\kappa=\frac{|2\kappa|^{-5/2}}{1+\frac{5}{2}|\kappa|}\frac{\Gamma\left(\frac{1}{|2\kappa|}-\frac{5}{4}\right)}{\Gamma\left(\frac{1}{|2\kappa|}+\frac{5}{4}\right)}\sqrt{\pi}, \tag{17}$$

and $k_*\ (=\sigma k/\sqrt{4\pi G\rho_0})$, $\omega_*\ (=\omega_\mathrm{r}/\sqrt{4\pi G\rho_0})$ correspond to the normalized wave-number and frequency of real part, respectively. Equation (16) reduces to the result of Martino *et al.*[23] in the limitation $\kappa=0$, and it also recovers to the classical Newtonian case when $\chi=0$ and $\kappa=0$, that is, $\omega^2=c_\mathrm{s}^2k^2-4\pi G\rho_0$. The comparison between the electromagnetic and gravitational interactions inside atomic nuclei allows us to limit the EiBI parameter in the range of $|\chi|<10^{-3}\ \mathrm{kg}^{-1}\cdot\mathrm{m}^5\cdot\mathrm{s}^{-2}$,[44] while we will only adopt the classical value $|\chi|=5\times10^{-5}\ \mathrm{kg}^{-1}\cdot\mathrm{m}^5\cdot\mathrm{s}^{-2}$ to simplify our result. By substituting $\rho_0=10^{18}\ \mathrm{kg}\cdot\mathrm{m}^{-3}$ and $|\chi|=5\times10^{-5}\ \mathrm{kg}^{-1}\cdot\mathrm{m}^5\cdot\mathrm{s}^{-2}$ into expression (16), one can easily find that the value of the Jeans wave-number $k_\mathrm{J}^2=8.37\times10^{-6}\ \mathrm{m}^{-1}$ and the coefficient $(k_\mathrm{J}^2/k_\mathrm{EiBI}^2+3D_k/C_k)$ is always greater than zero in the present work because the value of $k_\mathrm{J}^2/k_\mathrm{EiBI}^2\ (=\rho_0\chi/4\sigma^2)$ is small than the value of $3D_k/C_k$ even in the case of negative EiBI parameter, which can be shown in Fig. 1.

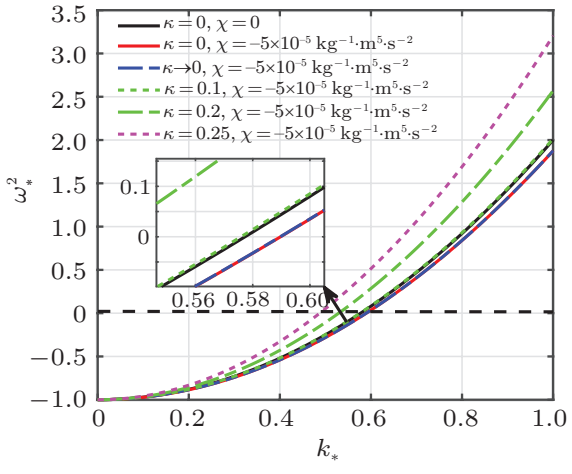



**Fig. 1.** The square of normalized growth rate ($\omega_*^2=\omega_\mathrm{r}^2/4\pi G\rho_0$) *versus* the normalized wave-number ($k_*=k/k_\mathrm{J}$) in high frequency regime with different $\kappa$ and $\chi<0$.

Figure 1 shows the effect of the distribution index $\kappa$ and negative EiBI gravity parameter $\chi$ on the Jeans instabilities of high-density, high-temperature, collisionless self-gravitational systems, *i.e.*, the numerical results of Eq. (16) in the case of negative EiBI parameter $\chi$. The black line ($\kappa = 0$, $\chi = 0$) represents the conventional Jeans instability for comparisons. It can be seen that the red line ($\kappa = 0$, $\chi = -5\times10^{-5}\ \mathrm{kg^{-1}\cdot m^5\cdot s^{-2}}$) which stands for the Maxwellian case is highly overlapped by the blue one ($\kappa \to 0$, $\chi = -5\times10^{-5}\ \mathrm{kg^{-1}\cdot m^5\cdot s^{-2}}$), which means the growth rate derived by Kaniadakis distribution can degenerate to the Maxwellian case in the limitation $\kappa \to 0$, as expected. Actually, it can be easily understood from Eq. (16) that the value of $C_\kappa/D_\kappa = 1$ when $\kappa \to 0$, so the Maxwellian case will be restored in the limitation of $\kappa \to 0$. Furthermore, from the comparisons of the black and red lines, one can find that the growth rate of the Jeans instability in negative EiBI gravity is higher than that of the Newtonian case. It is also noticeable from Fig. 1 that the range of the unstable mode and the growth rate decrease with the increased $\kappa$ for the same value of $\chi$.

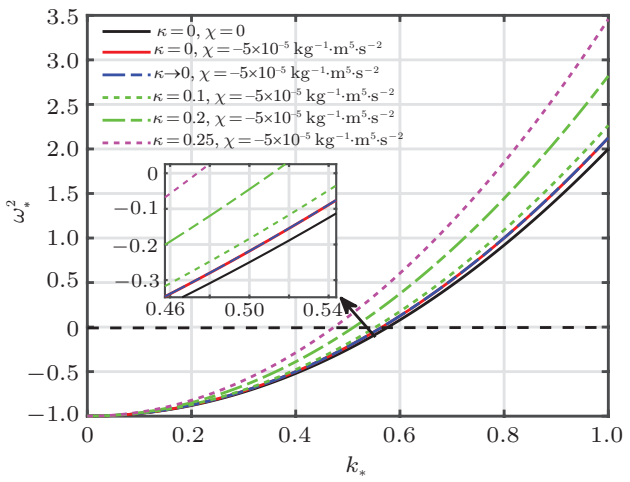



**Fig. 2.** The square of normalized growth rate ($\omega_*^2 = \omega_r^2/4\pi G\rho_0$) *versus* the normalized wave-number ($k_* = k/k_\mathrm{J}$) in high frequency regime with different $\kappa$ and $\chi > 0$.

Figure 2 shows the behavior of the growth rate of Jeans instability vs the normalized wave-number $k_*$ for different $\kappa$ and positive EiBI gravity parameter $\chi = 5\times10^{-5}\ \mathrm{kg^{-1}\cdot m^5\cdot s^{-2}}$. Comparing to Fig. 1, one can find from Fig. 2 that the presence of positive EiBI gravity parameter decreases the growth rate of Jeans instability, inversely. However, the growth rate also decrease with increased $\kappa$. Thus, one can concluded that in the background of EiBI gravity, the $\kappa$-deformed Kaniadakis distribution may suppress the Jeans instability of self-gravitational system, no matter the EiBI parameter is negative or positive. In other words, this kind of $\kappa$-deformed Kaniadakis distributed EiBI-gravitational system is more stable than the one described by Maxwellian distribution. Physically, Jeans instability can be viewed as a result of competition between thermal pressure and self-gravity, that is, $\omega^2 = c_\mathrm{s}^2k^2 - 4\pi G\rho_0$. A larger $\kappa$ parameter means a larger thermal pressure, so the growth rate of Jeans instability decreases.

On the other hand, it is also interesting to study the limit case of low frequency ($\xi \ll 1$) perturbations. Then the modified dispersion function $Z(\xi)$ is expressed by[5,35]

$$Z(\xi) = 2\xi^2\wp\int_{-\infty}^{\infty}\frac{\left(\sqrt{1+\kappa^2x^4}-\kappa x^2\right)^{1/\kappa}}{\sqrt{1+\kappa^2x^4}}\mathrm{d}x - \mathrm{i}\sqrt{\pi}\xi \simeq -\mathrm{i}\sqrt{\pi}\xi, \tag{18}$$

then dividing $\omega$ into its real and imaginary parts, *i.e.*, $\omega = \omega_r + \mathrm{i}\omega_i$, and setting $\omega_r = 0$, we have

$$\Omega_* = \frac{\sqrt{2}}{\sqrt{\pi}}k_*\left[B_\kappa - C_\kappa\left(\frac{1}{k_*^2} - \frac{k_\mathrm{J}^2}{k_\mathrm{EiBI}^2}\right)^{-1}\right], \tag{19}$$

where $k_* = \sigma k/\sqrt{4\pi G\rho_0}$ and $\Omega_* = \omega_\mathrm{i}/\sqrt{4\pi G\rho_0}$ are the normalized wave-number and frequency of image part, respectively. It can be found that when $\kappa = 0$, then $B_\kappa = \sqrt{\pi}$ and $C_\kappa = \sqrt{\pi}$,[35] equation (19) degenerates to the results of Martino *et al.*[23]

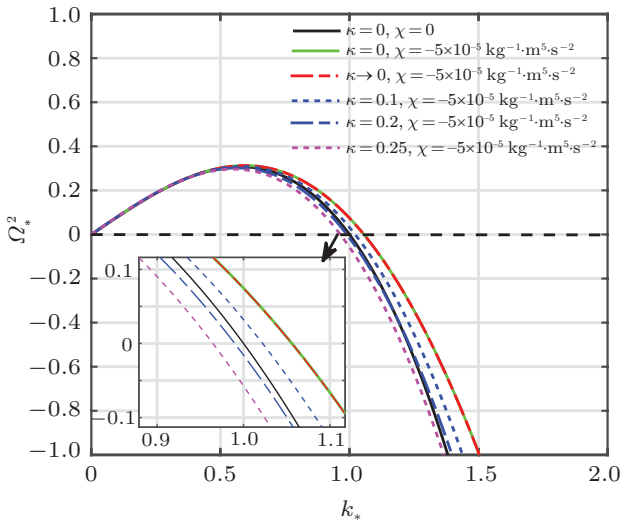



**Fig. 3.** The normalized growth rate ($\Omega_* = \omega_\mathrm{i}/4\pi G\rho_0$) *versus* the normalized wave-number ($k_* = k/k_\mathrm{J}$) in low frequency regime with different $\kappa$ and $\chi < 0$.

Figures 3 and 4 show the normalized growth rate ($\Omega_*$) *versus* the normalized wave-number ($k_*$) with different $\kappa$ and $\chi$ within the low frequency regime. In particulary, figure 3 illustrates the growth rate of Jeans mode with negative values of $\chi$, while figure 4 is devoted to positive values of $\chi$. It should be point out that the black and green lines in Fig. 3 are identical to the black and blue lines in Fig. 2 of Martino *et al.*, respectively.[23] Furthermore, the red line ($\kappa \to 0$) also highly overlaps with the green line (the Maxwellian case).

One can also find from Fig. 3 that for the fixed value of $\chi = -5 \times 10^{-5}\ \mathrm{kg}^{-1} \cdot \mathrm{m}^5 \cdot \mathrm{s}^{-2}$, an increasement in the $\kappa$ parameter tends to a decrease of the growth rate and critical wave-number, which can be easily comprehended looking at Eq. (14). Therefore, one can concluded that the collapse of EiBI-gravitational systems in high density environments is disfavored with respect to the Kaniadakis' distribution case, which may be due to an increased thermal pressure to prevent gravitational collapse of the matter system.

The influence of the distribution index $\kappa$ on Jeans instability in the presence of positive EiBI parameter $\chi$ is shown in Fig. 4. It can be found that there is a singularity at $k = k_{\mathrm{EiBI}}$, which have been explain by Martino and Capolupo for the Maxwellian case.[23] However, the presences of both the black solid and dashed lines illustrates that this singularity will disappear if $\chi = 0$. Also, it is clear from Fig. 4 that the growth rate of Jeans instability are enhanced by the increasing value of parameter $\kappa$. That is, comparing to the Maxwellian case, the system with $\kappa$-deformed Kaniadakis distribution is more conducive to gravitational collapse. Then from Figs. 3 and 4, one can find that the effect of the $\kappa$-deformed Kaniadakis distribution on the collisionless, high-density, high-temperature, isotropic EiBI-gravitational systems displays double-edged, enhancing the growth rate when $\chi$ is positive but damping the modes in the case of negative value.

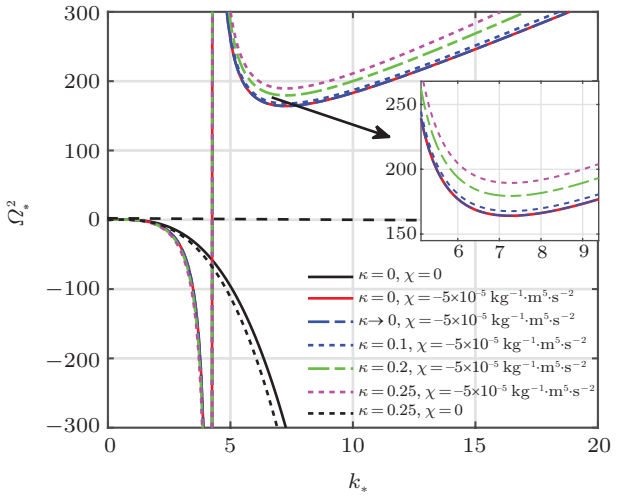



**Fig. 4.** The normalized growth rate ($\Omega_* = \omega_i/4\pi G\rho_0$) *versus* the normalized wave number ($k_* = k/k_{\mathrm{J}}$) in low frequency regime with different $\kappa$ and $\chi > 0$.

In conclusion, based on the kinetic theory, we have studied the influence of the $\kappa$-deformed Kaniadakis distribution on Jeans instability for the collisionless, high-density, high-temperature, isotropic self-gravitational systems in the background of EiBI gravity. In this framework, we have made a step forward by changing the Maxwellian distributions to a family of power law distributions parameterized by the $\kappa$ parameter to study what extent the Kaniadakis distribution modifies the threshold and growth rate of the Jeans instability in EiBI-gravitational systems. For the limit of high frequency perturbations, the results of this work show that $\kappa$-deformed Kaniadakis velocity distribution tends to decrease the range of unstable modes and growth rate of Jeans instability in EiBI-gravitational systems and, therefore, opposes the gravitational collapse of EiBI-gravitational systems. Further more, under the limitation of low frequency perturbations, the role of the $\kappa$-deformed Kaniadakis velocity distribution on the EiBI-gravitational systems depends on the value of EiBI parameter. It shows that Kaniadakis distribution behaves as a stimulative factor in the presence of positive EiBI parameter, on the other hand, a comparatively higher value of $\kappa$ damps the range of unstable modes and growth rate for Jeans instability and tries to depress the gravitating collapse of matter systems in the case of negative EiBI parameter. The present work maybe useful to examine the instabilities presented in high density and temperature star systems, such as a hyper massive neutron star.[26] The significant background of the Kaniadakis' statistics is also meaningful for celestial system. However, the present work can be possibly extended with the consideration of the intrinsic mechanism of the singularity. We plan to research these structures in the forthcoming paper.

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