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SPECIAL TOPIC — Phononics and phonon engineering

# Nonequilibrium reservoir engineering of a biased coherent conductor for hybrid energy transport in nanojunctions*

Bing-Zhong Hu(胡柄中), Lei-Lei Nian(年磊磊),† and Jing-Tao Lü(吕京涛)‡

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We show that a current-carrying coherent electron conductor can be treated as an effective bosonic energy reservoir involving different types of electron–hole pair excitations. For weak electron–boson coupling, hybrid energy transport between nonequilibrium electrons and bosons can be described by a Landauer-like formula. This allows for unified account of a variety of heat transport problems in hybrid electron–boson systems. As applications, we study the non-reciprocal heat transport between electrons and bosons, thermoelectric current from a cold-spot, and electronic cooling of the bosons. Our unified framework provides an intuitive way of understanding hybrid energy transport between electrons and bosons in their weak coupling limit. It opens the way of nonequilibrium reservoir engineering for efficient energy control between different quasi-particles at the nanoscale.

**Keywords:** electron–hole pair, hybrid energy transport, nanojunction, electron–phonon interaction

**PACS:** 05.70.Ln, 63.20.kd, 63.22.–m, 85.65.+h **DOI:** 10.1088/1674-1056/abb3ee

## 1. Introduction

Understanding nonequilibrium energy transport at the nanoscale is of crucial importance both for the fundamental development of quantum thermodynamics and for the practical application of nanoscale thermal, thermoelectric, and optoelectronic devices. For phase coherent transport, the celebrated Landauer–Büttiker formalism has been successfully applied to study quasi-particle energy transport following different statistics, including electrons,[1] photons,[2–5] phonons,[6–15] and magnons.[16] Wherein, the baths connecting to the system are assumed to be in thermal equilibrium with given temperature and/or chemical potential, where the quasi-particle distribution function is determined by its statistics, i.e., the Fermi–Dirac distribution for fermions, and the Bose–Einstein distribution for bosons. A difference in the distribution drives an energy current flow between the two thermal baths.

However, the same approach is difficult to describe energy transport between quasi-particles following different statistics, which is ubiquitous in thermoelectric and optoelectronic processes of nano-junctions. Examples of such processes include electroluminescence,[17–20] Joule heating,[21–26] current-induced[24,27–29] or radiative cooling.[30] Another difficulty arising in these processes is that the quasi-particles may be in nonequilibrium state due to driving from external bias.

In this work, we show that these processes can be conveniently analyzed by treating a voltage-biased coherent electron conductor as effective bosonic reservoirs with non-zero chemical potentials. In the limit of weak electron–boson coupling, to the second order in their coupling, we obtain a Landauer formula to describe energy transport between electrons and bosons. This is possible since energy transport between electrons and bosons is always accompanied by the generation or annhilation of different kinds of electron–hole pairs (EHPs).[31,32] This provides a simple physical picture to understand qualitatively energy transport in different thermal, thermoelectric, and optoelectronic processes.

## 2. Theory

### 2.1. System setup

We consider a model system schematically shown in Fig. 1(a). The system is composed of an independent set of bosonic degrees of freedom (DOF) taken as a set of harmonic oscillators. It couples to two kinds of baths. One is an equilibrium boson bath (ph-bath), modeled by an infinite number of harmonic oscillators (bosonic modes). The other is an electron bath (e-bath), which itself includes a central part ($C$) and two electrodes ($L$ and $R$). The e-bath may be driven into a nonequilibrium steady state by a voltage bias applied between the two electrodes. We consider the situation where transport in the e-bath can be treated using the coherent

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Landauer–Büttiker picture. Without loss of generality, we assume that the system bosons couple only to the central region of the e-bath. Energy transport between the e-bath and ph-bath takes place through their simultaneous coupling to the system bosons. We limit ourselves to non-interacting electrons and weak electron–boson interaction such that the lowest order expansion is valid.[33] The electrons couple to the 'displacement' of the system harmonic oscillators

$$H_{\rm es} = \sum_{i,j,k} M_{ij}^{k} c_i^{\dagger} c_j u_k. \tag{1}$$

Here, $M_{ij}^{k}$ describes the coupling of the system mode $k$ to the electronic transition between states $i$ and $j$, and $u_k$ is the 'displacement' operator of the system mode $k$. For phonons, it is the displacement, while for photons it is the vector potential. The system–ph-bath coupling is linear between harmonic oscillators and can be treated exactly.

### 2.2. Electron–hole pair excitation

Our key observation is that the energy transport between the system and the e-bath can be modeled by different kinds of reactions between EHPs in the e-bath and the bosonic modes in the system. The creation (annihilation) of the bosonic mode is always accompanied by the inelastic electronic transitions from a filled high (low) energy state to an empty low (high) energy state. These processes can be considered as recombination and creation of EHPs that have the same energy as the bosonic mode, which we take as positive. They can be expressed in the form of reactions

$$e_\alpha + h_\beta \rightleftharpoons b_n, \tag{2}$$

where $e_\alpha$, $h_\beta$, and $b_n$ represent electron in electrode $\alpha$, hole in electrode $\beta$, and bosonic mode $b_n$ of the system, respectively. Equivalently, we can write

$$e_\alpha \rightleftharpoons e_\beta + b_n, \tag{3}$$

representing an inelastic electronic transition from electrode $\alpha$ to $\beta$, accompanied by emission of bosonic mode $n$ (forward process). The backward direction corresponds to an absorption process.

There are four types of EHPs which we label by the spatial location of the electron ($\alpha$) and hole ($\beta$) states. They are schematically shown in Figs. 1(c) and 1(d) for the recombination and creation processes, respectively. They are denoted by EHP-$i$ ($i = 1,2,3,4$) and are further divided into two groups. The intra-electrode type includes $1/LL$ and $2/RR$, and the inter-electrode type includes $3/RL$, $4/LR$. Additional to energy transfer between e-bath and system, the generation and recombination of inter-electrode EHPs also involve charge transport across the system. As mentioned, we take the energy of the mode and the EHPs to be positive.

A generalized detailed balance relation applies to each of the reactions

$$\frac{\tau_{\alpha\rightharpoonup\beta}}{\tau_{\alpha\rightharpoondown\beta}} = \exp\left[-\beta_{\rm B}(\hbar\Omega - \mu_{\alpha\beta})\right]. \tag{4}$$

Here, $\tau_{\alpha\rightharpoonup\beta}$ and $\tau_{\alpha\rightharpoondown\beta}$ are the reaction rates for the forward (boson emission) and backward (boson absorption) processes in Eq. (2), respectively. They are obtained from the Fermi golden rule

$$\begin{aligned}\tau_{\alpha\rightharpoonup\beta} = {} & \frac{2\pi}{\hbar}\sum_{i\in\alpha, f\in\beta} |M_{ij}^{m}|^2 \delta(\varepsilon_i - \varepsilon_f - \hbar\Omega) \\ & \times n_{\rm F}(\varepsilon_i - \mu_\alpha)(1 - n_{\rm F}(\varepsilon_f - \mu_\beta)).\end{aligned} \tag{5}$$

Here, $n_{\rm F/B}(\varepsilon, T) = [\exp(\beta_{\rm B}\varepsilon) \pm 1]^{-1}$ is the Fermi–Dirac/Bose–Einstein distribution, with $\beta_{\rm B} = (k_{\rm B}T)^{-1}$, $\mu_{\alpha\beta} = \mu_\alpha - \mu_\beta$, and $M_{ij}^{m} = \langle\psi_i(\varepsilon_i)|M|\psi_f(\varepsilon_f)\rangle$ is the transition matrix element from initial state $i$ in electrode $\alpha$ to final state $f$ in electrode $\beta$. The reverse rate $\tau_{\alpha\rightharpoondown\beta}$ can be written similarly. Thus, when reaching equilibrium with the EHP bath $\alpha\beta$, the bosonic mode follows a Bose–Einstein distribution at temperature $T_{\rm e}$ and chemical potential $\mu_{\alpha\beta}$. For intra-electrode processes, $\mu_{\alpha\beta} = 0$, we have the normal detailed balance relation, while for inter-electrode processes $\mu_{\alpha\beta}$ is determined by the applied voltage bias. Thus, the bosonic mode may acquire a non-zero chemical potential in nonequilibrium. This is consistent with the equilibrium condition for reaction (2).

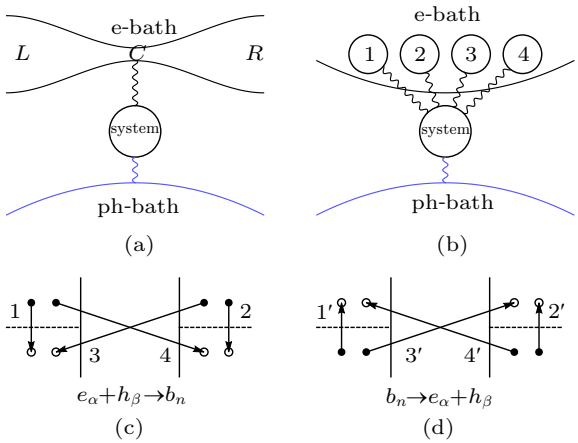



**Fig. 1.** (a) Schematics of the model we consider. The system consists a set of independent bosonic modes. It couples to an electron bath (e-bath), which is modeled as a conductor including a left ($L$) and a right ($R$) electrode, with temperature $T_{\rm e}$ and chemical potential $\mu_L$ and $\mu_R$, respectively. The system further couples to an external thermal bath (ph-bath) at temperature $T_{\rm ph}$. (b) The electron bath can be treated as four different kinds of electron–hole pair (EHP) baths (1–4), shown in (c). (c)–(d) Four kinds of EHP recombination (c) and excitation (d) processes. The EHPs are classified according to the spatial location of the electron ($e_\alpha$) and the hole ($h_\beta$).

The key quantity to describe the EHP baths is the coupling-weighted power spectrum. It can be written as

$$\tilde{\Pi}_{mn}^{\alpha\beta}(\omega) = \left[n_{\rm B}(\hbar\omega - \mu_{\alpha\beta}, T_{\rm e}) + \frac{1}{2}\right]\Lambda_{mn}^{\alpha\beta}(\omega). \tag{6}$$

We have introduced the coupling-weighted EHP density of states (DOS)[34,35]

$$\begin{aligned}\Lambda_{mn}^{\alpha\beta}(\omega) &= -\sum_{i\in\alpha, f\in\beta} M_{fi}^{m} M_{if}^{n} \delta(\varepsilon_i - \varepsilon_f - \hbar\omega) \\ &\quad \times (n_{\mathrm{F}}(\varepsilon_\alpha - \mu_\alpha, T_\alpha) - n_{\mathrm{F}}(\varepsilon_\beta - \mu_\beta, T_\beta)) \\ &= -\int \frac{\mathrm{d}\varepsilon}{2\pi} \mathrm{tr}[M^m A_\alpha(\varepsilon) M^n A_\beta(\varepsilon - \hbar\omega)] \\ &\quad \times (n_{\mathrm{F}}(\varepsilon - \mu_\alpha, T_\alpha) - n_{\mathrm{F}}(\varepsilon - \hbar\omega - \mu_\beta, T_\beta)), \quad (7)\end{aligned}$$

which also characterizes the system dissipation due to coupling to the e-bath.[34] Here, $A_\alpha$ is the electrode spectrum functional. Equation (6) follows a form of the fluctuation–dissipation relation for an equilibrium ph-bath, albeit with a possibly non-zero chemical potential $\mu_{\alpha\beta}$. The intra-electrode EHPs ($i = 1,2$) are always in equilibrium with $\mu_{\alpha\alpha} = 0$ and temperature $T_{\mathrm{e}}$. But the two inter-electrode EHPs ($i = 3,4$) have opposite chemical potentials $\mu_{RL} = -\mu_{LR}$. They are non-zero when there is a voltage bias applied. To this end, we have shown that the nonequilibrium e-bath can be divided into four EHP baths with different chemical potentials. This effective model is shown in Fig. 1(b).

### 2.3. Energy transport

Within this effective EHP model, hybrid energy transport between electrons and system bosons can be treated as bosonic transport. To the lowest order approximation, we arrive at a Landauer-like formula for the energy and particle transport from e-bath to the system as a summation of contributions from all the EHP baths

$$\begin{aligned}J &= \sum_{\alpha,\beta} \int_0^{+\infty} \frac{\mathrm{d}\omega}{2\pi} \hbar\omega \, \mathrm{Tr}[\Lambda^{\alpha\beta}(\omega) \mathcal{A}_{\mathrm{ph}}(\omega)] \\ &\quad \times [n_{\mathrm{B}}(\omega - \mu_{\alpha\beta}, T_{\mathrm{e}}) - n_{\mathrm{B}}(\omega, T_{\mathrm{ph}})]. \quad (8)\end{aligned}$$

Here, $J$ is the energy flux from e-bath to ph-bath. We consider only one bosonic mode, such that mode $k = 1$ and the transition matrix can be expressed as

$$M = \begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix}.$$

The $T_{\mathrm{e}}$ and $T_{\mathrm{ph}}$ are the temperatures of the e-bath and ph-bath, respectively. The trace Tr is over system DOF, with $\mathcal{A}_{\mathrm{ph}}$ the spectral function of the system due to coupling to the ph-bath. We can write it in terms of the non-interacting boson Green's function $D^{\mathrm{r/a}}$ and self-energy $\Pi_{\mathrm{ph}}^{\mathrm{r/a}}$ as $\mathcal{A}_{\mathrm{ph}} = \mathrm{i}D^{\mathrm{r}}(\Pi_{\mathrm{ph}}^{\mathrm{r}} - \Pi_{\mathrm{ph}}^{\mathrm{a}})D^{\mathrm{a}}$.[35] Here, $\Pi_{\mathrm{ph}}^{\mathrm{r,a}} = \mp 2\mathrm{i}\gamma_{\mathrm{ph}}\omega$, with $\gamma_{\mathrm{ph}}$ being the coupling strength. The summation over $\alpha\beta$ includes contributions from all the four types of EHPs. Each of them contributes to one transport channel.

In the following we show several applications of this central result. To be more specific, we consider a minimum model of the e-bath shown in Fig. 2. We have two electronic states 1 and 2 (on-site energies $\varepsilon_1$ and $\varepsilon_2$) couple to the electrodes $L$ and $R$ with coupling parameters $\gamma_1$ and $\gamma_2$, respectively. Electron hopping between the two states is assisted by one bosonic mode, which at the same time couples to a ph-bath with coupling constant $\gamma_{\mathrm{ph}}$.

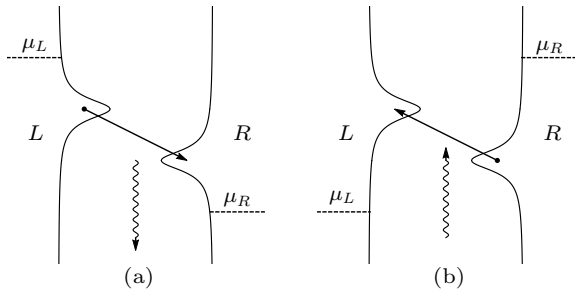



**Fig. 2.** Two limiting cases of nonequilibrium reservoir engineering. In panel (a), we have a filled electronic level $\varepsilon_L$ that couples to the left electrode with chemical potential $\mu_L$, and an empty level $\varepsilon_R$ that couples to the right electrode with chemical potential $\mu_R$. We have $\mu_L > \mu_R$. Heating of the bosonic mode is due to resonant recombination of inter-electrode EHPs (process 4 in Fig. 1). In panel (b), the situation is reversed. The left state $\varepsilon_L$ is empty, while the right state $\varepsilon_R$ is filled. When $\mu_R > \mu_L$, the e-bath can be used to cool the bosonic mode through creation of inter-electrode EHPs (process 4′ in Fig. 1).

Several comments are noteworthy before presenting the numerical results.

**Remark 1** We limit ourselves to the case of weak electron–boson coupling such that the lowest order result (Eq. (8)) is valid. There are cases where the lowest order result fails qualitatively, see for example Ref. [36]. In that case, the lowest order result shows some instability, indicating its failure. Thus, we have checked our results against more accurate self-consistent Born approximation (SCBA) (dots in Figs. 3–7) to ensure that we are always in the weak coupling limit.

**Remark 2** One related issue is the energy conservation. Equation (8) can also be obtained from the Born approximation in the nonequilibrium Green's function method, if one considers energy transfer from the system to the ph-bath and includes the electron 'bubble' self-energy to the phonon Green's function.[37] However, if one considers the energy current from e-bath to the system using Born approximation, the result will be different from that of Eq. (8). Thus, we conclude that the Born approximation does not conserve energy. However, if one always keeps only the terms to the lowest order in the electron–boson coupling (the second order),[33,35] the energy current from the above two points of view is still the same and given by Eq. (8). In conclusion, the lowest order approximation is energy conserving, although it only works for weak electron–boson coupling.

**Remark 3** Equation (8) has been derived before.[35] The focus of this work is to provide a physical transparent picture to understand this result, such that the numerical results obtained from it can be understood from the coherent bosonic energy transport point of view. In doing so, it can be applied to understand more cases including optoelectronic processes,

heat rectification, and others, as we demonstrate in the next section.

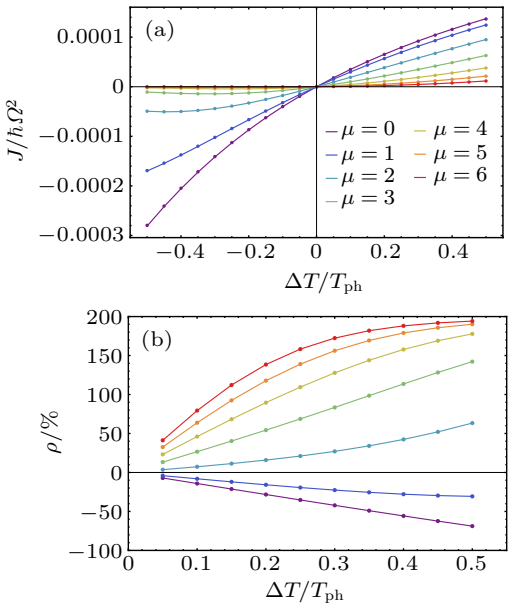



**Fig. 3.** Non-reciprocal heat transport in a double-dot junction shown in Fig. 2(a). (a) Heat current as a function of temperature difference $\Delta T/T_{\rm ph}$ for different chemical potentials. The lines and dots correspond to results from Eq. (8) and SCBA, respectively. The same applies to Figs. 6 and 7. (b) Rectification ratio $\rho = (|J(|\Delta T|)| - |J(-|\Delta T|)|)/(|J(|\Delta T|)| + |J(-|\Delta T|)|)/2$ as a function of $\Delta T/T_{\rm ph}$ for different chemical potentials. We consider only one bosonic mode, whose energy is taken as unit energy. The following parameters are used in the calculation: $\varepsilon_L = 0.5$, $\varepsilon_R = -0.5$, $\gamma_L = \gamma_R = 0.5$, $m = 0.05$, $\hbar\Omega = 1$, $k_{\rm B} = 1$.

## 3. Applications

### 3.1. Non-reciprocal heat transport

Firstly, we consider the situation where the e-bath and ph-bath are in their own thermal equilibrium at two different temperatures $T_{\rm e}$ and $T_{\rm ph}$. This indicates that $\mu_L = \mu_R$ and $T_L = T_R = T_{\rm e}$. If we ignore the energy dependence of $A$ in Eq. (7), $\Lambda_{mn}(\omega) = \hbar\omega{\rm tr}[M^m A M^n A]$ with $A = A_L + A_R$. Consequently, the transmission $\mathcal{T} = {\rm Tr}[\Lambda\mathcal{A}_{\rm ph}]$ does not depend on $T_{\rm e}$. Equation (8) reduces to the Landauer formula for heat transport between two harmonic thermal baths. Thus, the EHPs behave as linear harmonic oscillator thermal baths with a common temperature $T_{\rm e}$.

On the other hand, if we consider the energy dependence of $A(\varepsilon)$ and $\Gamma(\omega)$, $\mathcal{T}$ will depend on $T_{\rm e}$. Energy transport becomes anharmonic. In this case, non-reciprocal energy flow is possible, i.e., $J(\Delta T) \neq J(-\Delta T)$, with $\Delta T = T_{\rm e} - T_{\rm ph}$. We thus find a necessary condition for non-reciprocal energy transport in a hybrid electron–boson system: the electron DOS in the thermal window near the chemical potential has to be energy dependent.[38,39] For a normal metal electrode, the energy scale of electrons is much larger than the thermal energy, leading to a flat DOS. The energy dependence of $A(\varepsilon)$ can be engineered by changing the electronic states of the central part. For example, discrete energy levels of a molecular junction or quantum dot can be used.

In Fig. 3, we have considered a two-dot junction shown in Fig. 2(a). We set $\mu_L = \mu_R = \mu$ and $T_{\rm e} \neq T_{\rm ph}$ to consider heat transport. The energy dependent EHP DOS gives rise to temperature dependent transmission function $\mathcal{T}$ and consequently heat rectification. Thus, we can manipulate energy current via engineering the e-bath parameters. This shows the advantage of hybrid energy transport compared to the pure bosonic systems.

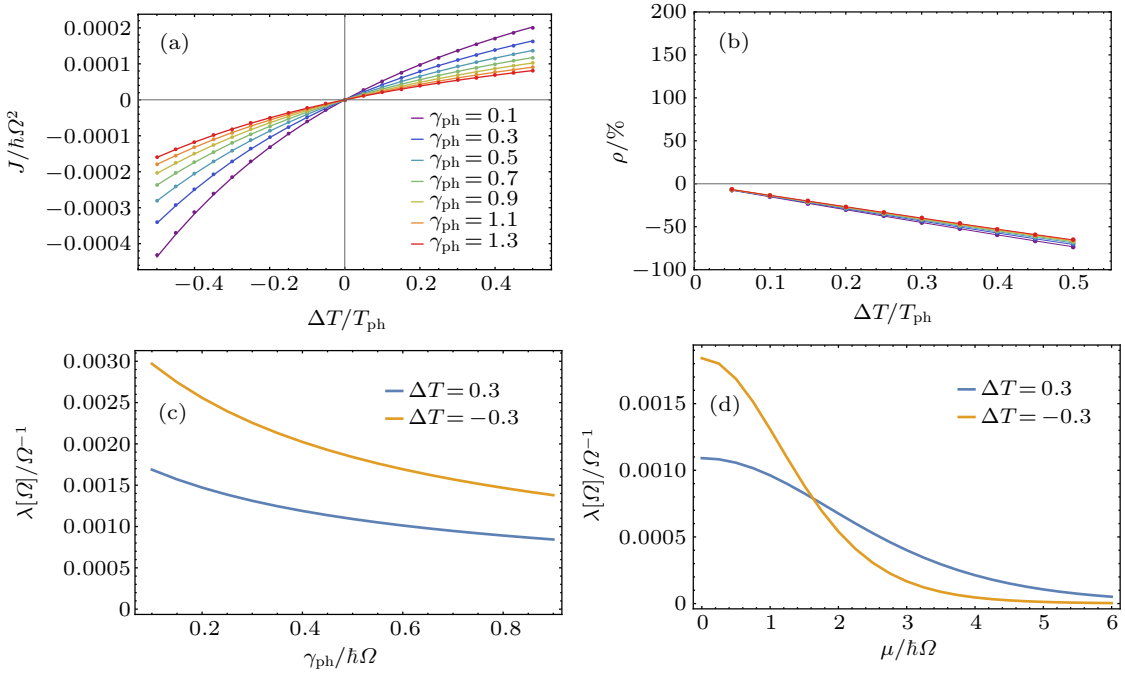



**Fig. 4.** (a) Heat current as a function of temperature difference $\Delta T/T_{\rm ph}$ for different coupling to phonon bath $\gamma_{\rm ph}$. (b) Rectification ratio $\rho$ as a function of $\Delta T/T_{\rm ph}$ for different $\gamma_{\rm ph}$. (c) EHP DOS $\Lambda(\Omega)$ as a function of $\gamma_{\rm ph}$ for $\Delta T = 0.3$ and $\Delta T = -0.3$. (d) Similar to (c), but for $\Lambda(\Omega)$ vs. $\mu$. The other parameters are the same as those in Fig. 3.

In Fig. 4(a), we show the heat current as a function of temperature difference, but rectification ratio $\rho$ is only weakly dependent on the phonon–bath coupling strength $\gamma_{\rm ph}$ in Fig. 4(b). This is clearly seen in Fig. 4(c), where EHP DOS ratios (the ratio of $\Lambda(\Omega)$ for $\Delta T = 0.3$ and $\Delta T = -0.3$) share a similar trend. In Fig. 4(d), one can find that EHP DOS ratio vs. $\mu$ is remarkable different. This is consistent with Fig. 3.

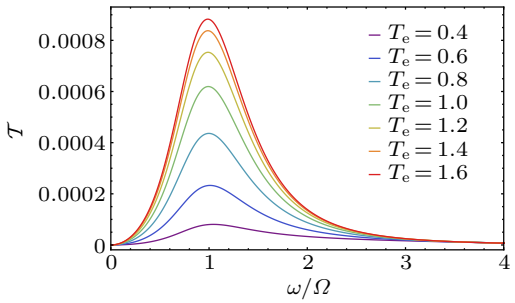



**Fig. 5.** Transmission $\mathcal{T}$ as a function of phonon frequency $\omega$ at different e-bath temperature $T_e$, $\Delta T/T_{\rm ph}$ for $\mu = 3$. The other parameters are the same as those in Fig. 3.

### 3.2. Hybrid thermoelectric transport

We can also study the thermoelectric transport of the temperature-biased electron–boson junction. When $T_{\rm ph} \neq T_e$, in addition to the heat transport between system and e-bath, an electrical current may also be induced between the two electrodes.[40,41] In our EHP picture, this is realized through coupling of the bosonic mode with two inter-electrode EHPs. Since they contribute to the electrical currents of opposite directions, in order to get a non-zero electrical current, these two channels should not get canceled. We can write the electron particle flux as

$$I^{\rm e} = \sum_{\alpha,\beta}(\delta_{\alpha L}\delta_{\beta R} - \delta_{\alpha R}\delta_{\beta L}) \int_0^{+\infty} \frac{{\rm d}\omega}{2\pi} {\rm Tr}[\Lambda^{\alpha\beta}(\omega)\mathcal{A}_{\rm ph}(\omega)] \times [n_{\rm B}(\omega - \mu_{\alpha\beta}, T_e) - n_{\rm B}(\omega, T_{\rm ph})]. \quad (9)$$

Here, $\delta_{\alpha/\beta,L/R}$ are the Kronecker delta functions. For simplicity, we introduce thermoelectric efficiency $\eta$ as the ratio between electron particle flux and phonon particle flux, $\eta = I^{\rm e}/(J^{\rm ph}/\hbar\Omega)$.

The resonant situation in Fig. 2 can be used to enhance one of the two channels. In Fig. 6(b), we show the thermoelectric current induced by the temperature different $\Delta T$ for different chemical potentials $\mu_L = \mu_R$ in the case of Fig. 2(a). The efficiency $\eta$ in Fig. 6(a) is the largest when the chemical potential is in between $\varepsilon_L$ and $\varepsilon_R$, where the resonant enhancement is the most prominent. Previously, the electrical current generated from a phonon hot-spot ($T_{\rm ph} > T_e$) has been considered.[40] Our results show that the opposite ($T_{\rm ph} < T_e$) is also possible, where electricity is generated by cooling the ph-bath.

Although it sounds a bit counter intuitive, we note that generating electricity from a cold bosonic bath does not violate the second law of thermodynamics. The energy flows from the high temperature e-bath to the low temperature ph-bath in accordance with the second law. The energy source comes from the recombination of the inter-electrode EHPs. During this process, electronic charge is transferred from one electrode to the other. Indeed, this kind of setup demonstrates the decoupling of heat and charge transport directions as an advantage of thermoelectricity in hybrid nano-junctions.

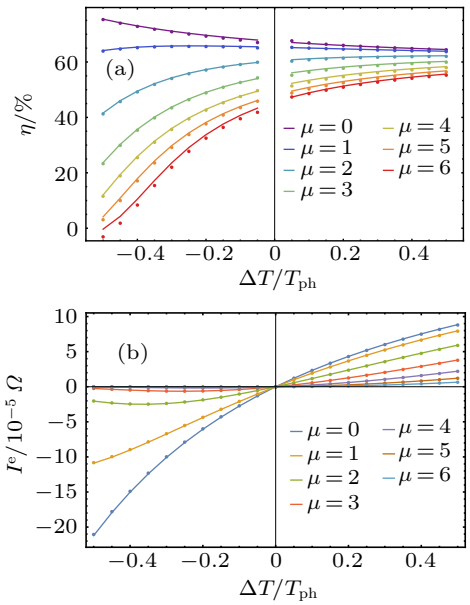



**Fig. 6.** (a) Thermoelectric efficiency $\eta$ and (b) electron particle flux $I^{\rm e}$ as a function of temperature difference between e-bath and ph-bath $\Delta T/T_{\rm ph}$. The parameters are the same as those in Fig. 3.

### 3.3. Electronic cooling of bosonic mode

We now turn on the voltage bias in the e-bath. The applied voltage changes the initial and final electron states of the EHP excitation. Thus, the EHP DOS can be modified by voltage. More importantly, the inter-electrode EHPs acquire a non-zero chemical potential, EHP-4 has a chemical potential of $\mu_{LR}$, while EHP-3 gets a chemical potential with opposite value $\mu_{RL}$. The change of the chemical potential breaks the equilibrium in the reaction, and drives the energy transport between e-bath and the system. Direction of energy flow depends on the relative magnitude of two fluxes. It can be engineered by tuning the electronic band structure, or more specifically, the transition probability of the two types of EHP excitations.

In the case shown in Fig. 2(b), process $4'$ (accompanied with phonon annihilation) is resonantly enhanced. The EHP corresponding to this dominant process has a negative effective chemical potential $\mu_{RL} < 0$ (gray shaded area in Fig. 7), lower than that of the ph-bath, which is always zero. This unbalance in the chemical potential drives an energy current from the ph-bath to the e-bath. Electronic cooling becomes possible using this resonant enhancement. This is demonstrated in Fig. 7, where the heat current from the e-bath to the system $J$ is plotted as a function of voltage bias $\mu_{LR}$ while keeping the

temperature fixed $T_e = T_{ph} = T$. For the negative bias, we observe a negative $J$ regime. The range of this regime gets larger for higher temperature $T$. This is the electronic cooling of the bosonic mode. Very recently, experimental demonstration of near field radiative cooling using a reversely biased p–n junction has been demonstrated.[30] The experimental results can be understood using this simple model.

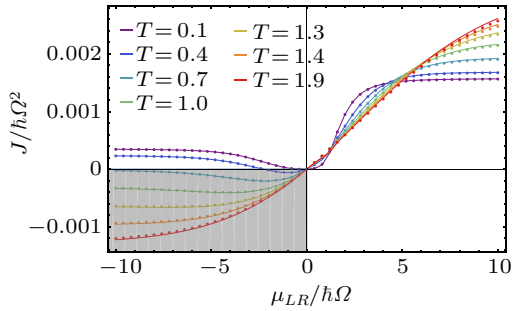



**Fig. 7.** Energy current $J$ from the e-bath to the bosonic mode as a function of chemical potential $\mu_{LR}$, corresponding to the situation in Fig. 2(b). Negative $J$ (gray shaded area) means cooling of the bosonic mode.

## 4. Conclusions

In summary, we have shown that a normal two-probe coherent electron conductor can be effectively viewed as EHP baths with chemical potential determined by the applied voltage bias. This is made possible by introducing the inter-electrode charge transfer EHPs. Properties of the EHP baths can be engineered through tuning the parameters of the conductor and the external voltage bias. This bath engineering provides an efficient way of controlling hybrid energy and thermoelectric transport in electron–boson junctions. Moreover, this point of view can help one to understand qualitatively numerical results from more accurate calculations, i.e., SCBA, although it is quantitatively only applicable in the weak electron–boson interaction regime.

## Acknowledgement

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