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Linear and nonlinear propagation characteristics of multi-Gaussian laser beams

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Theoretical investigation on the propagation characteristics of a new class of laser beams known as multi Gaussian (M.G) laser beams has been presented. To investigate the linear characteristics, propagation of the laser beam in vacuum has been considered. Whereas, the nonlinear characteristics have been investigated in plasmas. Optical nonlinearity of the plasma has been modeled by relativistic mass nonlinearity of the plasma electrons in the field of laser beam. Formulation is based on finding the semi analytical solution of the wave equation for the slowly varying envelope of the laser beam. Particularly, the dynamical evolutions of the beam width and longitudinal phase of the laser beam have been investigated in detail.

Keywords: self-focusing, multi Gaussian, self phase modulation, self-channeling

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1. Introduction

The expansion of the transverse dimensions of a propagating beam is ubiquitous for all kinds of waves including electro-magnetic waves and even matter waves. It would seem that this kind of spreading is inevitable and therefore irreducible, since it originates at a fundamental level from light's natural wave property of diffraction. However, in 1964 Chio *et al*.[1] showed that in media whose index of refraction depends on the intensity of light, and the spreading of an optical beam in principle can be obviated. Hence, the expansion of optical beam due to diffraction is neither inevitable nor irreducible. In nonlinear media, the presence of an optical beam modifies their optical properties like index of refraction, absorption, or conversion to higher frequencies. The resulting change in index of refraction resembles to that of a graded index fiber and thus the beam automatically becomes accumulated towards its axis. This phenomena is known as self-focusing.[2] When self focusing exactly balances the diffraction of the laser beam, the beam propagates in a stable self trapped mode.

Since its discovery by Askaryan,[2] the phenomenon of self-focusing is at the vanguard of experimental as well as theoretical investigations due to its relevance in number of applications. In the design of ultra intense laser systems such as those being used in laser driven nuclear fusion, the phenomenon of self focusing plays an important role. By producing intra cavity losses, it can limit the cavity intensity or can slow down the release of optical energy. It can also alter the transverse profile of the laser beam by producing wavefront distortions.

It is well known fact that with respect to a reference on axis plane wave, a converging/diverging electromagnetic beam undergoes modulation of its longitudinal phase.[3] This self-phase modulation of the longitudinal phase of a laser beam is also known as Gouy phase shift, which is a matter of debate over past few years. Since this discovery,[3] various theories (ranging from classical[4] to quantum[5,6]) have been used to explain its origin. Classically, self phase modulation of the laser beam arises due to the contribution of an additional phase per unit length in the neighbourhood of beam focal spot arising from the second order derivative of field amplitude with respect to transverse coordinates. However, in quantum mechanical terms, the longitudinal phase shift is considered as a purely geometrical effect, resulting as a consequence of modification of volume of space available for the propagation of the beam. The consequent change in the transverse momentum of the photon changes the longitudinal momentum as well that in turn modifies the longitudinal phase of the laser beam. Precise knowledge of the behaviour of the overall phase of the laser beam is important to control and understand laser driven processes. In laser cavities, it plays a significant role in determining the resonant frequencies of the transverse modes.[7] By increasing the frequency degeneracy of the resonant modes of the cavity, longitudinal phase shift can influence the beam quality achieved in the laser resonator.

Plasmas can respond nonlinearly to the intense laser beams mainly through three mechanisms. These mechanisms are: 1) nonlinearity due to Ponderomotive force,[8,9] 2) nonlinear Ohmic heating,[10,11] 3) relativistic mass nonlinearity.[12,13] First two mechanisms give nonlinear response of the plasma via modification of electron density due to ponderomotive force and Ohmic heating of plasma electrons, respectively. In relativistic mechanism, modification in

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optical properties of plasma occurs without any modification in electron density. This nonlinearity immediately comes into picture when the power of incident laser is greater than the threshold power required for self-focusing and thus it does not show any transient behaviour. In this mechanism optical properties of plasma become a function of intensity of the electromagnetic beam due to change in electron mass when it starts oscillating at a velocity comparable to that of light.

Since the seminal works of Chiao *et al.*,[1] Kelley,[14] Alfano and Shapiro[15] and Gustafson *et al.*,[16] extensive literature has been reported by several researchers highlighting various aspects of propagation characteristics of laser beams in various nonlinear media. Karlsson[17] investigated the phenomena of self-focusing and self-phase modulation of laser beams in media possessing saturable nonlinearity. Manassah *et al.*[18] investigated the same phenomena for Gaussian laser beams propagating through graded index optical fibers. Karlsson *et al.*[19] reported an investigation on self action effects in parabolic index optical fibers. Habibi and Ghamari[20,21] investigated nonlinear propagation characteristics of Gaussian laser beams in cold quantum plasmas with density ramp. The effect of external magnetic field on self-focusing of laser beams under dense plasmas was investigated by Gupta and Hur.[22]

Laser beams differing in intensity profile behave differently in nonlinear media. The reported review reveals the fact that most of the earlier theoretical investigations on nonlinear optical phenomena of laser beams have been directed towards revealing the propagation characteristics of laser beams with ideal Gaussian profile. However, in context of the ICF, there is growing interest in class of laser beams those are having uniform irradiance over wider area of their cross sections. In comparison to Gaussian beams, laser beams with uniform irradiance possess low divergence and thus can improve the efficiency of laser plasma coupling during the ICF. Also, these beams can deliver high power densities to the target while keeping power density below the threshold for other parametric instabilities. Mathematically such beam profiles are modeled by super Gaussian or the M.G[23,24] beam profiles. Most of the earlier investigations on propagation characteristics of the M.G laser beams in plasmas have been carried out in the framework of paraxial theory. Paraxial theory oversimplifies the analysis by taking into consideration the effect of only the paraxial region of the cross section of the laser beam towards the optical nonlinearity of the medium. This approximation is valid up to some extent for Gaussian laser beams. But for the M.G laser beams those possess uniform irradiance over their cross section, this approximation is not valid. The aim of this article is give more detailed analysis of the propagation characteristics of the M.G laser beams with the help of moment theory approach which is free form the limitations of paraxial theory.

2. Intensity profile of the M.G laser beam

The M.G laser beams are the laser beams whose intensity profile, for a suitable choice of parameters, resembling to that with fundamental TEM_{00} Gaussian beams, super Gaussian beam and four petal beams. The intensity profile of such beams is modeled by the function[23]

$$AA^{\star}|_{z=0} = E_{00}^2\left[\mathrm{e}^{-\frac{(x-x_0)^2+y^2}{2r_0^2}} + \mathrm{e}^{-\frac{(x+x_0)^2+y^2}{2r_0^2}} + \mathrm{e}^{-\frac{x^2+(y-x_0)^2}{2r_0^2}} + \mathrm{e}^{-\frac{x^2+(y+x_0)^2}{2r_0^2}}\right]^2, \tag{1}$$

where $A(x,y,z)$ is the slowly varying beam envelope, r_0 is the beam width at focal plane, i.e. at $z=0$, and E_{00} is the axial amplitude of the electric field, i.e. amplitude at the center ($x=0$, $y=0$) of the focal plane. Such laser beams can be obtained by in phase superposition of four Gaussian laser beams that are having same spot size but the coordinates of whose intensity maxima lie at $(-x_0,0)$, $(x_0,0)$, $(0,x_0)$, and $(0,-x_0)$.

Figure 1 displays the initial beam profile for different values of x_0/r_0. The plots in Fig. 1 illustrate that the laser beams for which the value of x_0/r_0 lies in the range $0 \leqslant x_0/r_0 \leqslant 1.5$ possess uniform irradiance over a wider area of their cross section and the area of this uniform illumination region increases with an increase in the value of x_0/r_0. For $x_0/r_0 > 1.5$, the peaks of intensity profile appear in off-axial parts of the cross section of the laser beam. Surrounded by four off axial peaks a valley of intensity appear in the axial region of the laser beam.

For $z>0$, the energy conserving beam profile can be written as

$$AA^{\star} = \frac{E_{00}^2}{f^2}\left[\mathrm{e}^{-\frac{(x-x_0f)^2+y^2}{2r_0^2f^2}} + \mathrm{e}^{-\frac{(x+x_0f)^2+y^2}{2r_0^2f^2}} + \mathrm{e}^{-\frac{x^2+(y-x_0f)^2}{2r_0^2f^2}} + \mathrm{e}^{-\frac{x^2+(y+x_0f)^2}{2r_0^2f^2}}\right]^2, \tag{2}$$

where the parameter $f(z)$ is currently undetermined and upon multiplication with initial beam width r_0 and gives the instantaneous beam width of the laser beam. Hence, $f(z)$ is termed as dimensionless beam width parameter.

3. Relativistic nonlinearity of plasma

The dielectric function of plasma can be written as[25]

$$\varepsilon = 1 - \omega_{\mathrm{p}}^2/\omega_0^2, \tag{3}$$

where

$$\omega_{\mathrm{p}}^2 = \frac{4\pi e^2}{m_{\mathrm{e}}}n, \tag{4}$$

e, m_{e}, and n are electronic charge, mass and density, respectively. The electric field vector

$$\boldsymbol{E} = A(r,z)\,\mathrm{e}^{\iota(\omega_0 t - k_0 z)}\boldsymbol{e}_x + \iota\boldsymbol{e}_y, \tag{5}$$

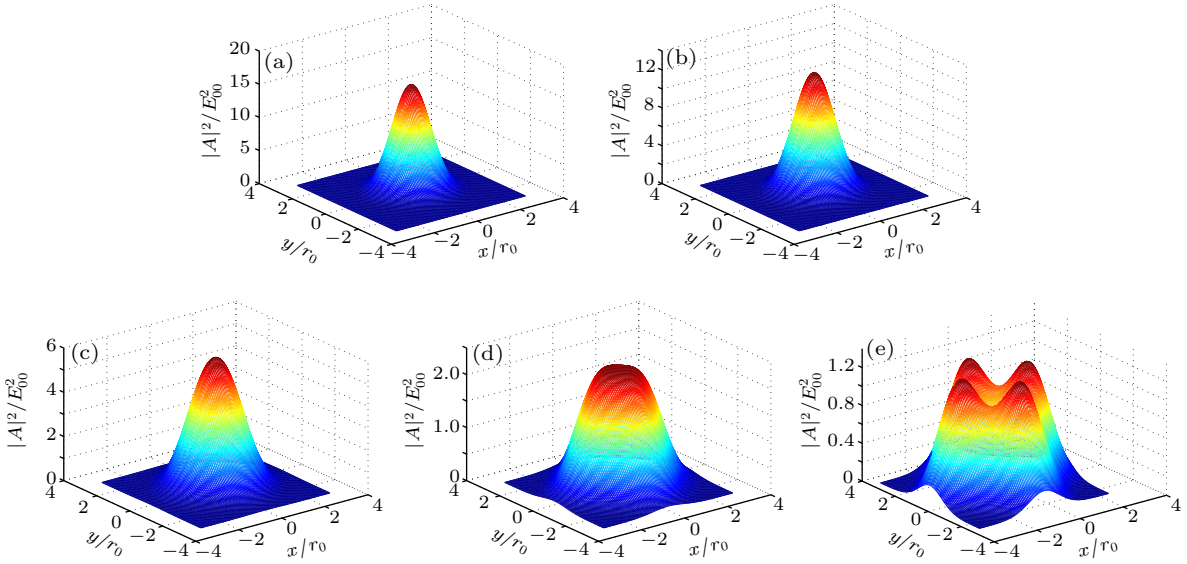


Fig. 1. 3D intensity profile of the M.G laser beam for (a) $x_0/r_0 = 0$, (b) $x_0/r_0 = 0.50$ (c) x_0/r_0=1, (d) $x_0/r_0 = 1.50$, and (e) $x_0/r_0 = 1.75$.

imparts oscillatory velocity

$$\boldsymbol{v} = \frac{e\boldsymbol{E}}{\imath m\omega_0^2} \tag{6}$$

to the plasma electrons. When this quiver velocity of the carriers approaches to that of light in vacuum under the effect of ultra intense laser beam, the effective mass m_e of the electrons in Eq. (4) needs to be replaced by $m_0\gamma$, where m_0 is the rest mass of electron and γ is the relativistic Lorentz factor. Following Akhiezer and Polovin,[12] the relativistic Lorentz factor γ is related to laser field amplitude as

$$\gamma = (1+\beta AA^{\star})^{\frac{1}{2}}, \tag{7}$$

where the coefficient $\beta = e^2/(m_0^2c^2\omega_0^2)$ gives the strength of relativistic nonlinearity. Using Eqs. (2), (4) and (7) in Eq. (3), the effective dielectric function of plasma for the M.G laser beams can ne written as

$$\varepsilon = 1 - \frac{\omega_{\mathrm{p0}}^2}{\omega_0^2}\left\{1+\frac{\beta E_{00}^2}{f^2}\left(\mathrm{e}^{-\frac{(x-x_0f)^2+y^2}{2r_0^2f^2}} + \mathrm{e}^{-\frac{(x+x_0f)^2+y^2}{2r_0^2f^2}} + \mathrm{e}^{-\frac{x^2+(y-x_0f)^2}{2r_0^2f^2}} + \mathrm{e}^{-\frac{x^2+(y+x_0f)^2}{2r_0^2f^2}}\right)^2\right\}^{-\frac{1}{2}}, \tag{8}$$

where $\omega_{\mathrm{p0}}^2 = \frac{4\pi e^2}{m_0}n$ is the equilibrium plasma frequency, i.e. the plasma frequency in the absence of laser beam. For linearly polarized laser beam the relativistic Lorentz factor is given by

$$\gamma = \left(1+\frac{1}{2}\beta AA^{\star}\right)^{\frac{1}{2}}.$$

Thus it can be predicted that for a given intensity circularly polarized laser beam produces more nonlinearity compared to that by linearly polarized laser beam. This is due to the fact that in case of circularly polarized laser beam propagating through relativistic plasma, a quasi stationary magnetic field is produced. The pinching effect of this produced magnetic field gets added to plasma nonlinearity. Thus, due to this additional contribution of the pinching effect of the magnetic field, circularly polarized laser beam produces larger nonlinearity in the index of refraction of plasma compared to that by linearly polarized beam.

Separating effective dielectric function of plasma into linear ε_0 and nonlinear $\phi(AA^{\star})$ parts as

$$\varepsilon = \varepsilon_0 + \phi(AA^{\star}), \tag{9}$$

we obtain

$$\varepsilon_0 = 1 - \omega_{\mathrm{p0}}^2/\omega_0^2, \tag{10}$$

$$\phi(AA^{\star}) = \frac{\omega_{\mathrm{p0}}^2}{\omega_0^2}\left[1-\left\{1+\frac{\beta E_{00}^2}{f^2}\left(\mathrm{e}^{-\frac{(x-x_0f)^2+y^2}{2r_0^2f^2}} + \mathrm{e}^{-\frac{(x+x_0f)^2+y^2}{2r_0^2f^2}} + \mathrm{e}^{-\frac{x^2+(y-x_0f)^2}{2r_0^2f^2}} + \mathrm{e}^{-\frac{x^2+(y+x_0f)^2}{2r_0^2f^2}}\right)^2\right\}^{-\frac{1}{2}}\right]. \tag{11}$$

4. Evolution of beam width

The propagation of the laser beams in nonlinear media is governed by nonlinear Schrodinger wave equation (NLSE)[26,27]

$$\imath\frac{\mathrm{d}A}{\mathrm{d}z} = \frac{1}{2k_0}\nabla_{\perp}^2A + \frac{k_0}{2\varepsilon_0}\phi(AA^{\star})A, \tag{12}$$

where $\nabla_{\perp}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplacian along the transverse directions.

Equation (12) is termed as Schrodinger equation because by replacing time coordinate t replaced by spatial coordinate z it can be exactly mapped with Schrodinger wave equation of quantum mechanics. It is nonlinear because here potential ϕ is itself a function of field amplitude A. Like Schrodinger equation is statement of conservation of energy, equation (12) is statement of interplay between diffraction and nonlinear refraction.

Being nonlinear in nature, superposition principle does not apply to Eq. (12), i.e., sum of two solutions is not a solution itself. From mathematical point of view, this implies that the traditional means of solving partial differential equations, i.e. expansion in power series are not applicable to the NLSE. In fact for potential function given by Eq. (11) no exact solution exists for Eq. (12). In order to obtain physical insight into the propagation dynamics of the laser beam, we use a semi analytical approach known as method of moments.[26–28] This method replaces a partial differential equation with a set of coupled differential equations governing the dynamics of the coefficients of an ansatz describing the full solution of wave Eq. (12).

Moment theory defines the effective beam width of an arbitrary laser beam in (r.m.s) sense as

$$\sigma^2 = \langle x^2 \rangle + \langle y^2 \rangle, \tag{13}$$

where

$$\langle x^2 \rangle = \frac{1}{I_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^2 A A^{\star} \mathrm{d}x \mathrm{d}y, \tag{14}$$

$$\langle y^2 \rangle = \frac{1}{I_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y^2 A A^{\star} \mathrm{d}x \mathrm{d}y, \tag{15}$$

with

$$I_0 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A A^{\star} \mathrm{d}x \mathrm{d}y. \tag{16}$$

Using Eq. (2) in Eq. (13), the r.m.s beam width of M.G beam is obtained as

$$\sigma_{\mathrm{M.G}} = \frac{\sqrt{2} r_0 f}{\left(1+3\mathrm{e}^{-\frac{x_0^2}{2r_0^2}}\right)^{\frac{1}{2}}} \left\{ \left(2+2\frac{x_0^2}{r_0^2}\right) + \left(2+\frac{x_0^2}{r_0^2}\right) \mathrm{e}^{-\frac{x_0^2}{2r_0^2}} + 2\mathrm{e}^{-\frac{x_0^2}{2r_0^2}} \right\}^{\frac{1}{2}}. \tag{17}$$

The r.m.s beam width of corresponding Gaussian beam is $\sigma_{\mathrm{G}} = r_0 f$. The ratio Σ of the r.m.s beam widths of the M.G beam $\sigma_{M.G}$ to that of Gaussian beam σ_{G} can be written as

$$\Sigma = \frac{\sigma_{\mathrm{M.G}}}{\sigma_{\mathrm{G}}} = \frac{\sqrt{2}}{\left(1+3\mathrm{e}^{-\frac{x_0^2}{2r_0^2}}\right)^{\frac{1}{2}}} \times \left\{ \left(2+2\frac{x_0^2}{r_0^2}\right) + \left(2+\frac{x_0^2}{r_0^2}\right) \mathrm{e}^{-\frac{x_0^2}{2r_0^2}} + 2\mathrm{e}^{-\frac{x_0^2}{2r_0^2}} \right\}^{\frac{1}{2}}. \tag{18}$$

In order to see the effect of parameter x_0/r_0 on the beam width of the M.G beam, the variation of Σ has been displayed in Fig. 2. It is evident that the effective beam width of the M.G laser beam is an monotonically increasing function of x_0/r_0.

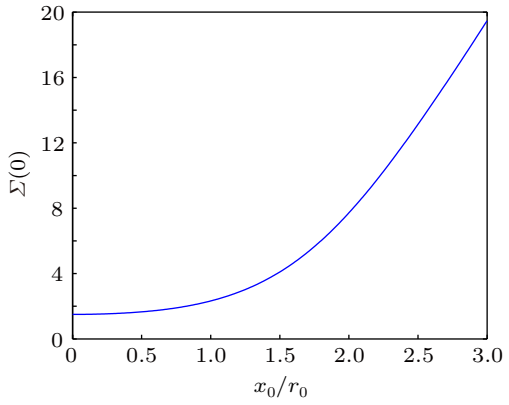


Fig. 2. Variation of the ration $\Sigma(0)$ of the r.m.s beam widths of an input M.G beam to that of a Gaussian beam with x_0/r_0.

Differentiating Eq. (13) twice with respect to z and using the values of $\frac{\partial A}{\partial z}$ and $\frac{\partial A^{\star}}{\partial z}$ from Eq. (12), we obtain

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}\sigma_{\mathrm{M.G}}^2 = \frac{1}{I_0 k_0^2} \iint \left(\left|\frac{\partial A}{\partial x}\right|^2 + \left|\frac{\partial A}{\partial y}\right|^2 \right) \mathrm{d}x\mathrm{d}y + \frac{1}{2}\frac{1}{\varepsilon_0 I_0} \iint \left(x\frac{\partial \phi}{\partial x} + y\frac{\partial \phi}{\partial y} \right) A A^{\star} \mathrm{d}x\mathrm{d}y. \tag{19}$$

Using Eqs. (2), (11) and (17) in Eq. (19) one can obtain following differential equation for the beam width of laser beam

$$\begin{aligned} &\frac{\mathrm{d}^2 f}{\mathrm{d}\xi^2} + \frac{1}{f}\left(\frac{\mathrm{d}f}{\mathrm{d}\xi}\right)^2 \\ &= \frac{1}{4f^3} \left[\frac{1+\mathrm{e}^{-\frac{x_0^2}{r_0^2}}\left(1-\frac{x_0^2}{r_0^2}\right) + \mathrm{e}^{-\frac{x_0^2}{2r_0^2}}\left(2-\frac{x_0^2}{r_0^2}\right)}{\left(2+2\frac{x_0^2}{r_0^2}\right) + \left(2+2\frac{x_0^2}{r_0^2}\right)\mathrm{e}^{-\frac{x_0^2}{2r_0^2}} + 2\mathrm{e}^{-\frac{x_0^2}{r_0^2}}} \right] \\ &- \frac{3}{4\pi}\left(\frac{\omega_{\mathrm{p0}}^2 r_0^2}{c^2}\right)\frac{\beta E_{00}^2}{f^3} \\ &\times \frac{1}{\left(2+2\frac{x_0^2}{r_0^2}\right) + \left(2+2\frac{x_0^2}{r_0^2}\right)\mathrm{e}^{-\frac{x_0^2}{2r_0^2}} + 2\mathrm{e}^{-\frac{x_0^2}{r_0^2}}} \\ &\times (K_1 + K_2 + K_3 + K_4), \end{aligned} \tag{20}$$

where

$$K_1 = \int_0^{\infty}\int_0^{\infty} t_1\left(t_1 - \frac{x_0}{r_0}\right)\mathrm{e}^{-\frac{(t_1-\frac{x_0}{r_0})^2+t_2^2}{2}} \times G_1^3(t_1,t_2)G_2(t_1,t_2)\mathrm{d}t_1\mathrm{d}t_2,$$

$$K_2 = \int_0^{\infty}\int_0^{\infty} t_1\left(t_1 + \frac{x_0}{r_0}\right)\mathrm{e}^{-\frac{(t_1+\frac{x_0}{r_0})^2+t_2^2}{2}} \times G_1^3(t_1,t_2)G_2(t_1,t_2)\mathrm{d}t_1\mathrm{d}t_2,$$

$$K_3 = \int_0^{\infty}\int_0^{\infty} t_1^2\,\mathrm{e}^{-\frac{t_1^2+(t_2-\frac{x_0}{r_0})^2}{2}} G_1^3(t_1,t_2)G_2(t_1,t_2)\mathrm{d}t_1\mathrm{d}t_2,$$

$$K_4 = \int_0^{\infty}\int_0^{\infty} t_1^2\,\mathrm{e}^{-\frac{t_1^2+(t_2+\frac{x_0}{r_0})^2}{2}} G_1^3(t_1,t_2)G_2(t_1,t_2)\mathrm{d}t_1\mathrm{d}t_2,$$

$$G_1(t_1,t_2) = \mathrm{e}^{-\frac{(t_1-\frac{x_0}{r_0})^2+t_2^2}{2}} + \mathrm{e}^{-\frac{(t_1+\frac{x_0}{r_0})^2+t_2^2}{2}}$$

$$+\mathrm{e}^{-\frac{t_1^2+(t_2-\frac{x_0}{r_0})^2}{2}}+\mathrm{e}^{-\frac{t_1^2+(t_2+\frac{x_0}{r_0})^2}{2}},$$

$$G_2(t_1,t_2)=\left\{1+\frac{\beta AA^\star}{f^2}G_1^2(t_1,t_2)\right\}^{-\frac{3}{2}},$$

$$t_1=\frac{x}{r_0 f},\quad t_2=\frac{y}{r_0 f},\quad \xi=z/(k_0 r_0^2).$$

Thus, it follows from Eq. (20) that the problem of solving a partial differential equation, i.e., the NLSE (Eq. (12)) has reduced to that of solving an ordinary differential equation. Although this reduced equation is also lacking form exact analytical solution due to its non integrability, its approximate solution can be easily obtained with the help of simple numerical techniques by assuming that initially the beam is collimated, i.e. it satisfies the initial conditions $f=1$ and $\mathrm{d}f/\mathrm{d}\xi=0$ at $\xi=0$.

5. Potential well for self-focusing

Equation (20) resembles to the equation of motion of a forced harmonic oscillator of unit mass and can be written as

$$\frac{\mathrm{d}^2 f}{\mathrm{d}\xi^2}+\frac{\partial V(f)}{\partial f}=0, \tag{21}$$

where

$$V(f)=-\int^f F(f)\mathrm{d}f, \tag{22}$$

and

$$F(f)=\frac{1}{4f^3}\left[\frac{1+\mathrm{e}^{-\frac{x_0^2}{r_0^2}}\left(1-\frac{x_0^2}{r_0^2}\right)+\mathrm{e}^{-\frac{x_0^2}{2r_0^2}}\left(2-\frac{x_0^2}{r_0^2}\right)}{\left(2+2\frac{x_0^2}{r_0^2}\right)+\left(2+2\frac{x_0^2}{r_0^2}\right)\mathrm{e}^{-\frac{x_0^2}{2r_0^2}}+2\mathrm{e}^{-\frac{x_0^2}{r_0^2}}}\right]$$
$$+\frac{1}{2\pi}\left(\frac{s}{2}-1\right)\left(\frac{\omega_{\mathrm{p}0}^2 r_0^2}{c^2}\right)\frac{\beta E_{00}^2}{f^3}$$
$$\times\frac{1}{\left(2+2\frac{x_0^2}{r_0^2}\right)+\left(2+2\frac{x_0^2}{r_0^2}\right)\mathrm{e}^{-\frac{x_0^2}{2r_0^2}}+2\mathrm{e}^{-\frac{x_0^2}{r_0^2}}}$$
$$\times(K_1+K_2+K_3+K_4)-\frac{1}{f}\left(\frac{\mathrm{d}f}{\mathrm{d}\xi}\right)^2. \tag{23}$$

From Eqs. (20)–(23), it is clear that with the help of moment theory, the problem of nonlinear wave propagation has reduced to a simple mechanical problem with potential $V(f)$, where the particle's coordinate is related to the beam width parameter f, particle velocity to the wavefront curvature $(\mathrm{d}f/\mathrm{d}\xi)$, and the time to axial distance ξ.

As $V(f)\to\infty$ for $f\to 0$ and $V(f)\to 0$ for $f\to\infty$, the potential function $V(f)$ for self-focusing resembles to that for Kepler's attractive central force problem. Hence f can be treated as radial coordinate of polar coordinate system (f,θ_f). The lagrangian L for a particle of unit mass in (f,θ_f) plane is given by

$$L=\frac{1}{2}\left[\left(\frac{\mathrm{d}f}{\mathrm{d}\xi}\right)^2+f^2\left(\frac{\mathrm{d}\theta_f}{\mathrm{d}\xi}\right)^2\right]-U(f). \tag{24}$$

Multiplying Eq. (21) by $\mathrm{d}f/\mathrm{d}\xi$ and integrating both sides we obtain

$$\frac{1}{2}\left(\frac{\mathrm{d}f}{\mathrm{d}\xi}\right)^2+V(f)=E. \tag{25}$$

where E is the constant of integration and is known as energy function.

The canonical momentum associated with polar angle θ_f is

$$p_{\theta_f}=\frac{\mathrm{d}L}{\mathrm{d}\dot{\theta}_f}=f^2\dot{\theta}_f, \tag{26}$$

where $\dot{\theta}_f=\mathrm{d}\theta_f/\mathrm{d}\xi$. The corresponding Lagrange equation

$$\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\frac{\partial L}{\partial\dot{\theta}_f}\right)-\frac{\partial L}{\partial\theta_f}=0$$

gives

$$\frac{\mathrm{d}}{\mathrm{d}\xi}\left(f^2\dot{\theta}_f\right)=0,$$

which implies that

$$f^2\dot{\theta}_f=l. \tag{27}$$

Hence, angular momentum corresponding to polar coordinate θ_f is conserved. The Lagrange equation for radial coordinate f is

$$\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\frac{\partial L}{\partial\dot{f}}\right)-\frac{\partial L}{\partial f}=0.$$

Using Eq. (24) we obtain

$$\frac{\mathrm{d}^2 f}{\mathrm{d}\xi^2}=-\frac{\mathrm{d}}{\mathrm{d}f}\left(U(f)+\frac{l^2}{2f^2}\right).$$

Integrating this equation after multiplying with $\mathrm{d}f/\mathrm{d}\xi$ we get

$$\frac{1}{2}\left(\dot{f}^2+\frac{l^2}{f^2}\right)+U(f)=E. \tag{28}$$

Comparing Eq. (25) with Eq. (28)

$$V(f)=U(f)+l^2/2f^2. \tag{29}$$

Comparing this Kepler's potential with potential function (Eq. (22)) for self-focusing we obtain

$$l=\frac{1}{4}\left[\frac{1+\mathrm{e}^{-\frac{x_0^2}{r_0^2}}\left(1-\frac{x_0^2}{r_0^2}\right)+\mathrm{e}^{-\frac{x_0^2}{2r_0^2}}\left(2-\frac{x_0^2}{r_0^2}\right)}{\left(2+2\frac{x_0^2}{r_0^2}\right)+\left(2+2\frac{x_0^2}{r_0^2}\right)\mathrm{e}^{-\frac{x_0^2}{2r_0^2}}+2\mathrm{e}^{-\frac{x_0^2}{r_0^2}}}\right].$$

This is an interesting and curious result since it explains the dependence of divergence of laser beam on x_0/r_0, as the angular momentum of the beam in (f,θ_f) plane is dependent on x_0/r_0. Since, with increase in the value of x_0/r_0, the magnitude of angular momentum l decreases (Fig. 12), hence with increase in the value of x_0/r_0, the divergence of the laser beam decreases.

6. Self-channeling of laser beam

If while entering into the medium, the beam possesses a plane wavefront, i.e. if $f=1$ and $\mathrm{d}f/\mathrm{d}\xi=0$ at $\xi=0$, then the condition $\mathrm{d}^2f/\mathrm{d}\xi^2=0$ will maintain their values throughout the journey of the beam through the medium. Such a mode of propagation, when there is no change in the beam width of the laser beam, is called self trapped mode or spatial optical soliton. Hence, for $\mathrm{d}f/\mathrm{d}\xi=\mathrm{d}^2f/\mathrm{d}\xi^2=0$, equation (20) gives the relation between dimensionless beam width $(\omega_{\mathrm{p}0}r_0/c)$ and the critical beam intensity βE_{00}^2 as

$$r_{\mathrm{e}}^2=\frac{\pi}{2\beta E_{00}^2}\left(\frac{1+\mathrm{e}^{-\frac{x_0^2}{r_0^2}}\left(1-\frac{x_0^2}{r_0^2}\right)+\mathrm{e}^{-\frac{x_0^2}{2r_0^2}}\left(2-\frac{x_0^2}{r_0^2}\right)}{K_1'+K_2'+K_3'+K_4'}\right), \tag{30}$$

where

$$K_1'=K_1|_{f=1},\quad K_2'=K_2|_{f=1},\quad K_3'=K_3|_{f=1},\quad K_4'=K_4|_{f=1},$$

and

$$r_{\mathrm{e}}=\omega_{\mathrm{p}0}r_0/c.$$

The laser beam for which the point $(\beta E_{00}^2, r_{\mathrm{e}})$ lies on the critical curve defined by Eq. (26), $\mathrm{d}^2f/\mathrm{d}\xi^2$ will vanish at $\xi=0$. This simply means that during the journey of the laser beam through the plasma, there will be no change in the curvature of the wavefront, i.e. $\mathrm{d}f/\mathrm{d}\xi$ will remain constant and value of this constant will be equal to initial value, that we have taken to be zero. Hence, $\mathrm{d}^2f/\mathrm{d}\xi^2=\mathrm{d}f/\mathrm{d}\xi=0$ at $\xi=0$ indicates that $\mathrm{d}f/\mathrm{d}\xi=0$ for $\xi>0$ also. Physically, this means that there will be no change in the spot size of the laser beam during its propagation. This mode of propagation is known as self-trapped mode. Thus, the region of space lying on the critical curve corresponds to self-channeling of the laser beam.

If the point $(\beta E_{00}^2, r_{\mathrm{e}})$ lies in the upper region of critical curve, then the initial value of $\mathrm{d}^2f/\mathrm{d}\xi^2$ will be positive and hence f will increase monotonically with distance. This mode of propagation is known as self broadening of the beam.

If the point $(\beta E_{00}^2, r_{\mathrm{e}})$ lies below the critical curve then initial value of $\mathrm{d}^2f/\mathrm{d}\xi^2$ will be negative and thus f will decrease with distance. This mode is known as self focusing of the laser beam. Thus, the region below the critical curve corresponds to self focusing.

7. Self-phase modulation of laser beam

The self-phase modulation of the laser beam occurs due to shift in its longitudinal phase occurring from transverse spatial confinement.[21] Due to its finite effective beam width, the laser beam behaves as it is passing through a narrow slit and thus through position-momentum uncertainty ($\Delta x\Delta p_x=$ constant) experiences a spread in its transverse momenta and hence a shift in expectation value of the propagation constant. The r.m.s spectral width σ_k of the laser beam is defined as

$$\sigma_{\mathrm{k,M.G}}=\sqrt{\langle k_x^2\rangle+\langle k_y^2\rangle}, \tag{31}$$

$$\langle k_x^2\rangle=\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}k_x^2S(k_x,k_y)\,\mathrm{d}k_x\,\mathrm{d}k_y}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}S(k_x,k_y)\,\mathrm{d}k_x\,\mathrm{d}k_y}, \tag{32}$$

$$\langle k_y^2\rangle=\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}k_y^2S(k_x,k_y)\,\mathrm{d}k_x\,\mathrm{d}k_y}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}S(k_x,k_y)\,\mathrm{d}k_x\,\mathrm{d}k_y}, \tag{33}$$

$$S(k_x,k_y)=\left|\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}A_0(x,y)\Big|_{z=0}\mathrm{e}^{-\imath(k_xx+k_yy)}\,\mathrm{d}x\mathrm{d}y\right|^2. \tag{34}$$

The ratio Σ_k of spectral widths of a M.G beam to that of a Gaussian beam $\sigma_{k,G}$ can be written as

$$\Sigma_k=\frac{\sigma_{k,\mathrm{M.G}}}{\sigma_{k,\mathrm{G}}}=\left\{\frac{1+\mathrm{e}^{-\frac{x_0^2}{2r_0^2}}\left(1-\frac{x_0^2}{r_0^2}\right)}{1+\mathrm{e}^{-\frac{x_0^2}{2r_0^2}}}\right\}^{\frac{1}{2}}. \tag{35}$$

The variation of Σ_k with x_0/r_0 is shown in Fig. 13. It is evident that the ratio decreases with increase in x_0/r_0 for $0\leqslant x_0/r_0\leqslant 1.5$ and the ratio starts increasing for $x_0/r_0>1.5$.

The wave number k_0 of the laser beam is related to transverse and longitudinal components through

$$k_0^2=k_x^2+k_y^2+k_z^2, \tag{36}$$

where k_z is the axial wave number, and k_x and k_y are the transverse wave numbers, respectively.

In moment theory approach the effective axial propagation constant of an optical beam is defined in r.m.s sense as

$$\bar{k_z}=\langle k_z^2\rangle/k_0=k_0-\langle k_x^2\rangle/k_0-\langle k_y^2\rangle/k_0. \tag{37}$$

The over all on axis phase $\theta(z)$ is related to effective propagation constant as[5]

$$\bar{k_z}=\partial\theta/\partial z. \tag{38}$$

The first term in Eq. (37) gives the phase k_0z of an infinite plane wave propagating along z-axis. The second term represents a phase shift of finite beam in comparison to infinite plane wave.

$$\frac{\mathrm{d}\theta_{\mathrm{p}}}{\mathrm{d}z}=-\frac{1}{k_0}(\langle k_x^2\rangle+\langle k_y^2\rangle). \tag{39}$$

Thus, it can be seen that average value of the transverse momentum of the beam is measure of shift in its longitudinal phase. Using Eqs. (32)–(34) in Eq. (39) we obtain

$$\frac{\mathrm{d}\theta_{\mathrm{p}}}{\mathrm{d}\xi}=-\left\{\frac{1+\mathrm{e}^{-\frac{x_0^2}{2r_0^2}}\left(1-\frac{x_0^2}{r_0^2}\right)}{1+\mathrm{e}^{-\frac{x_0^2}{2r_0^2}}}\right\}^{\frac{1}{2}}\frac{1}{f^2}. \tag{40}$$

8. Results and discussion

In present analysis Eqs. (20) and (40) have been solved with the help of Runge Kutta fourth order method for the following set of parameters: $\omega_0 = 1.78 \times 10^{15}$ rad/s, $\lambda = 1.06$ μm, $r_0 = 15$ μm, $(\omega_{p0} r_0/c)^2 = 12$, and $\beta E_{00}^2 = 3$.

Before actually solving the beam width equation, i.e. Eq. (20), it is important to role and origin of the various terms contained in it. The first term on the right hand side (R.H.S) that varies inversely as cube of the beam width, is the spatial dispersive term, which models the spreading of the laser beam in transverse directions occurring as a consequence of diffraction divergence. The second term which is having complex dependence on f, arises due to the relativistic mass nonlinearity of the plasma electrons and is responsible for nonlinear refraction of the laser beam. The relative magnitude of the two terms determines the behaviour of the beam width of the laser beam during its propagation.

Firstly, in order to investigate the linear characteristics of the laser beam, its propagation in vacuum has been considered. Hence, equation (20) has been solved for different values x_0/r_0 in the absence of nonlinear term. The corresponding variations of beam width with distance are plotted in Fig. 3. It can be easily seen that irrespective of the value of x_0/r_0, the beam width of the M.G laser beams diverge monotonically during its propagation in vacuum. This is due to light's natural wave property of diffraction. Due to its finite cross section, the laser beam behaves as it is passing through a slit whose width is equal to the radius of the laser beam. As an optical beam passing through a narrow slit diverge monotonically, similarly the laser beam of finite cross section undergoes monotonic increase in its radius during its propagation through a linear medium. However, the rate of divergence reduces by increasing x_0/r_0. Thus, by optimizing the value of x_0/r_0, one can control the diffraction broadening of the laser beam. From application point of view this is an important result as the M.G laser beams smaller divergence compared to that with Gaussian beams. The increase in the effective radius of the laser beam due to shifting of laser intensity to off axial regions with an increase in the value of x_0/r_0. As the phenomenon of diffraction varies inversely with its effective beam width of an optical beam, an increase in the value of x_0/r_0 leads to decrease in the divergence of the M.G laser beam due to diffraction divergence. Thus, it can be concluded that the diffraction of an optical beam depends on its effective cross section which is defined by its r.m.s beam width not by geometric radius r_0.

Figures 4 and 5 depict the effect of x_0/r_0 on the evolution of beam width of the laser beam while propagating through plasma. The plots in these figures depict that during its propagation through the plasma, the beam width of the laser beam varies harmonically. These harmonic variations of the beam width can be explained as follows.

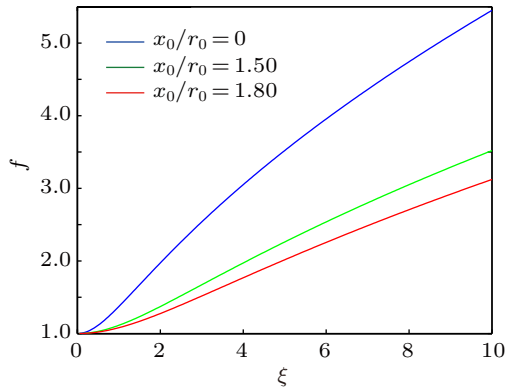


Fig. 3. Variation of beam width parameter f against the distance of propagation for different values of x_0/r_0 viz., $x_0/r_0 = 0$, 1.50, and 1.80 in the absence of nonlinear refraction.

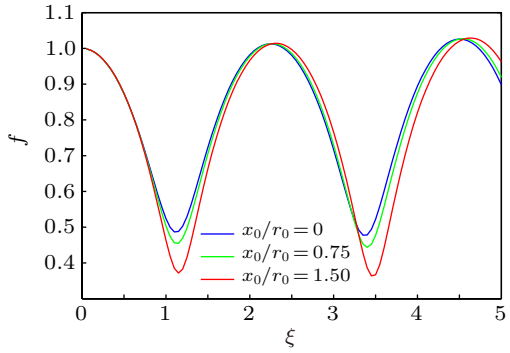


Fig. 4. Variarion of beam width parameter f with dimensionless distance of propagation ξ for different values of x_0/r_0 viz., $x_0/r_0 = 0$, 0.75, and 1.50.

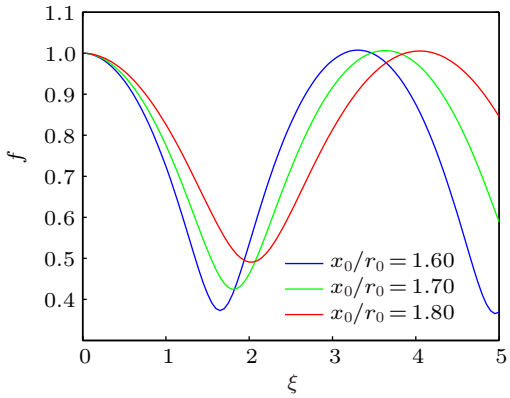


Fig. 5. Variarion of beam width parameter f with dimensionless distance of propagation ξ for different values of x_0/r_0 viz., $x_0/r_0 = 1.60$, 1.70, and 1.80.

Due to the intensity gradient along the cross section of the laser beam, there is a nonuniform mass distribution of the plasma electrons. Mass of the plasma electrons is maximum where the laser intensity is maximum and vice versa. Hence, the central part of the beam experiences larger refractive index compared to its tails. The resulting gradient in the phase velocity of the laser beam along its cross section, bends its wavefronts in such a way that the laser beam is passing through a convex lens. The nonlinear refraction of the laser beam thus tends to counter balance the effect of diffraction. Thus, during the propagation of the laser beam through the plasma, there is a competition between the two phenomena of diffraction and

nonlinear refraction. The winning phenomenon ultimately decides the behaviour of the laser beam, i.e. whether the beam will converge or diverge. Thus, there is a critical value of beam intensity (that can be obtained by equating the right hand side of Eq. (20) with zero) above which the beam will converge. In the present investigation, the initial beam intensity has been taken greater than the critical intensity, i.e. why the laser spot size of the laser beam is converging initially. As the beam width decreases, its intensity increases. When the laser intensity becomes too high, the nonlinear refractive term in Eq. (20) disappears, leaving only the diffraction effects to dominate. Hence, after focusing to minimum, the beam width bounces back to its original value. As the beam width of the laser beam starts increasing, the phenomena of diffraction divergence and nonlinear refraction again start competing each other. Now, this competition lasts till maximum value of f is obtained. These processes go on repeating themselves and thus give breather like behaviour to beam width.

Further, it is observed that an increase in the value of x_0/r_0 for $0 \leqslant x_0/r_0 < 1.5$ results in enhancement of self-focusing of the laser beam whereas beyond $x_0/r_0 = 1.5$ with an increase in the value of x_0/r_0 the extent of self-focusing of the laser beam decreases. This is because for $0 \leqslant x_0/r_0 < 1.5$ with an increase in the value of x_0/r_0 the intensity distribution over the cross section of the laser beam becomes more and more uniform. As a consequence, the laser beam gets equal contribution from the off axial parts for nonlinear refraction as provided by axial part. As the phenomenon of self-focusing is a homeostasis of nonlinear refraction, an increase in the value of x_0/r_0 in the range $0 \leqslant x_0/r_0 < 1.5$ results in the enhancement of self focusing of the laser beam.

The decrease in the extent of self-focusing of the laser beam by increasing x_0/r_0 beyond 1.5 is because for $x_0/r_0 > 1.5$ the intensity maxima of individual Gaussian laser beams constituting the M.G laser beams are so far away from each other that after superposition the intensity maxima of the resulting beam appear in the off axial parts (Fig. 1). This makes paraxial part of the laser beam weaker as compared to off axial part and hence the laser beam gets a very little contribution from the axial part for nonlinear refraction. This in turn leads to the reduced focusing of the laser beam. Thus, by optimizing the value of x_0/r_0, one can control the propagation dynamics of the laser beam in a nonlinear medium.

In order to have insight into the nature of periodicity of the oscillations of beam width of the laser beam we have plotted the phase space trajectories of the laser beam in Figs. 6 and 7. Closed phase space trajectories indicate the stable and periodic harmonic oscillations (i.e., oscillations containing a single resonant frequency) of the beam width whereas, and spiral trajectories indicate quasi periodic oscillations (i.e., oscillations containing multiple resonant frequencies). Thus, it is clear from phase space plots that the M.G laser beams possess quasi periodic oscillations in their beam width. The underlying physics behind this fact can be explained by comparing the evolution equation of beam width with equation of motion of nonlinear oscillator. In case of nonlinear oscillator, resonance does not occur at same frequencies for all amplitudes. Depending on the nature of nonlinearity, the resonance frequency is an increasing or decreasing function of amplitude. Hence, the beam width of the laser beam exhibits quasi periodic behaviour. Thus, it can be concluded that the beam width of an intense laser beam in a nonlinear medium behave like a nonlinear classical oscillator.

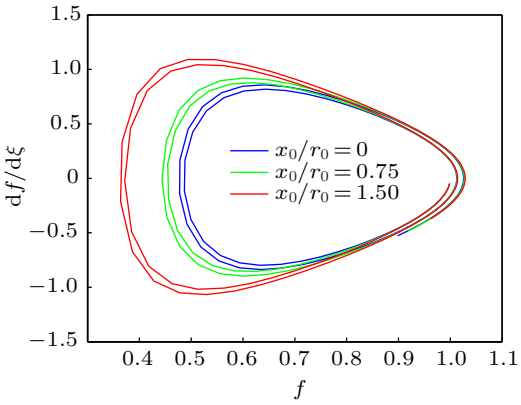


Fig. 6. Phase space plots for self-focused M.G laser beam for different values of x_0/r_0 viz., $x_0/r_0 = 0$, 0.75, and 1.50.

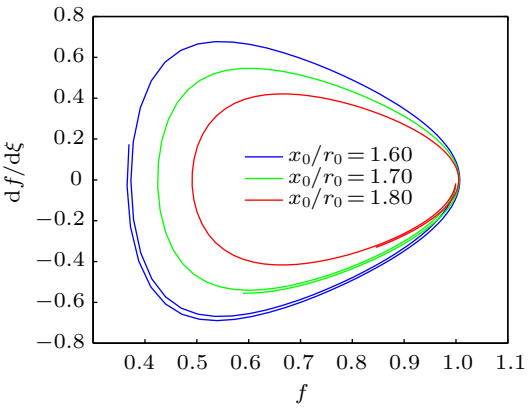


Fig. 7. Phase space plots for self-focused M.G laser beam for different values of x_0/r_0 viz., $x_0/r_0 = 1.60$, 1.70, and 1.80.

The phase space plots also depict that initially the laser beam focuses at a slower rate but as the laser beam gets more and more focused, there is an abrupt increase in the rate of focusing. This is due to the fact that with self-focusing of the laser beam, plasma becomes more and more nonlinear. Also, it is observed that the laser beams with higher value of x_0/r_0 in the range $0 \leqslant x_0/r_0 \leqslant 1.5$ span larger phase space area. Whereas, the beams with larger value of x_0/r_0 for $x_0/r_0 > 1.5$ cover lesser phase space area. This is due to the fact that area of phase space gives the measure of extent of self-focusing of the laser beam. Further it can be seen that with an increase

in x_0/r_0 for $0 \leqslant x_0/r_0 \leqslant 1.5$, the phase space trajectories become more and more spiral. This is due to an increase in self-focusing of the laser beam. An increase in self-focusing leads to enhanced amplitude of the oscillations of the beam width that makes them highly nonlinear. Similarly, with an increase in x_0/r_0 beyond 1.5, the phase space trajectories tend to become closed one. This is due to the reduction in self-focusing with an increase in x_0/r_0 beyond 1.5 that results in more and more stable oscillations of beam width.

Figures 8 and 9 illustrate the effect of x_0/r_0 on potential function $V(f)$. It is observed that $V(f) \to \infty$ for $f \to 0$. This is because of the domination of diffraction effects for $f \to 0$. For $f \to \infty$, $V(f) \to 0$ because the laser beams with very large spot size are free from diffraction effects as well as they do not produce any nonlinearity in the medium. It is also observed that with an increase in the value of x_0/r_0 for $0 \leqslant x_0/r_0 < 1.5$, the bottom of the potential well shifts upwards, and downwards with an increase in the value of x_0/r_0 for $x_0/r_0 > 1.5$. This indicates that for $0 \leqslant x_0/r_0 < 1.5$, the M.G laser beams with higher value of x_0/r_0 require lesser intensity to get self focused and for $x_0/r_0 > 1.5$, the M.G laser beams with higher value of x_0/r_0 require higher intensity to get self focused.

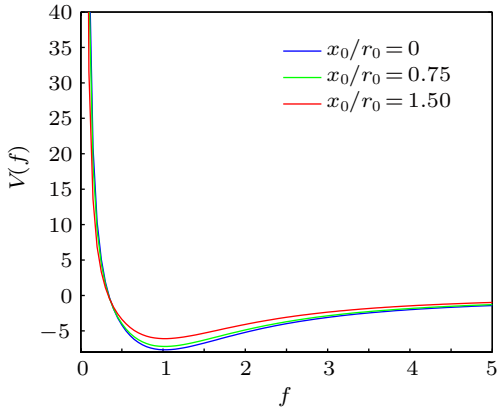


Fig. 8. Variation of potential function for self-focused M.G laser beam for different values of x_0/r_0 viz., $x_0/r_0 = 0$, 0.75, and 1.50.

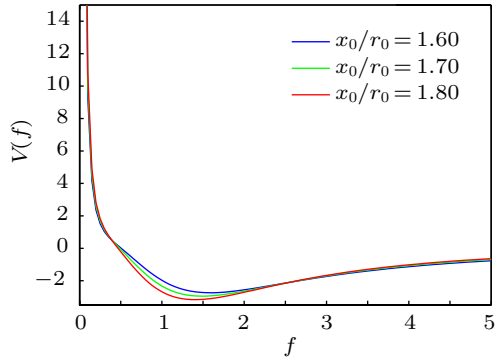


Fig. 9. Variation of potential function for self-focused M.G laser beam for different values of x_0/r_0 viz., $x_0/r_0 = 1.60$, 1.70, and 1.80.

Figures 10 and 11 illustrate the effect of x_0/r_0 on critical curves of the laser beam. It is observed that at lower laser intensity ($\beta E_{00}^2 \ll 1$), the equilibrium beam width r_e decreases very sharply and at very high laser intensity ($\beta E_{00}^2 \gg 1$), it becomes independent of laser intensity. This is because at very high intensities, in the region of plasma illuminated by the laser beam, all the electrons oscillate at same velocity and thus the relativistic nonlinearity gets saturated and hence the plasma dielectric function becomes independent of laser intensity. Hence, at high intensities, the beam width of the laser beam becomes independent of intensity. It is also observed that self-channeling can not occur for very narrow laser beams. This is due to the fact that narrow beams possess large diffraction angles. Hence, in order to get self-guided, they require larger index differences.

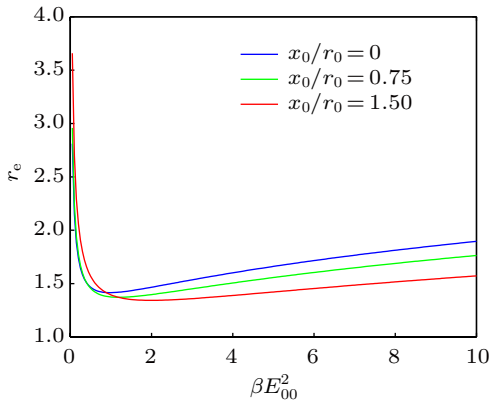


Fig. 10. Variation of equilibrium beam width r_e against the normalized intensity βE_{00}^2 for different values of x_0/r_0 viz., $x_0/r_0 = 0$, 0.75, and 1.50.

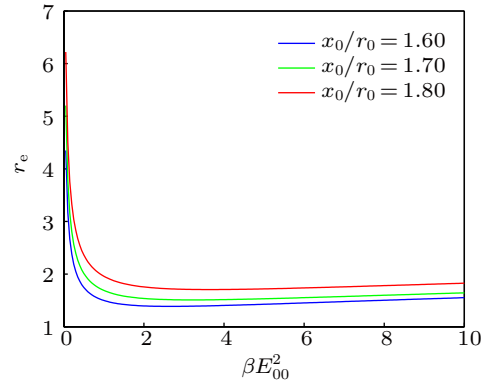


Fig. 11. Variation of equilibrium beam width r_e against the normalized intensity βE_{00}^2 for different values of x_0/r_0 viz., $x_0/r_0 = 1.60$, 1.70, and 1.80.

It is also observed that with an increase in the value of x_0/r_0 for $0 \leqslant x_0/r_0 < 1.5$, the critical curves shift downwards which means that the laser beams with higher values of x_0/r_0 in the range $0 \leqslant x_0/r_0 < 1.5$, can be self-guided at relatively lower power. This is due to the fact that these laser beams get equal contribution from off-axial rays towards nonlinear refraction.

Figures 14 and 15 illustrate that how the value of x_0/r_0 affects the evolution of longitudinal phase θ_p. It is observed that longitudinal phase θ_p decreases monotonically with distance of propagation, showing step-like behaviour. This is because self focusing of the laser beam increases its intensity and hence, the laser phase fronts start experiencing larger refractive indices. These results in decreased phase velocity of the phase fronts lead to decreased spacing between them. Hence,

the longitudinal phase of the laser beam keep on decreasing with distance.

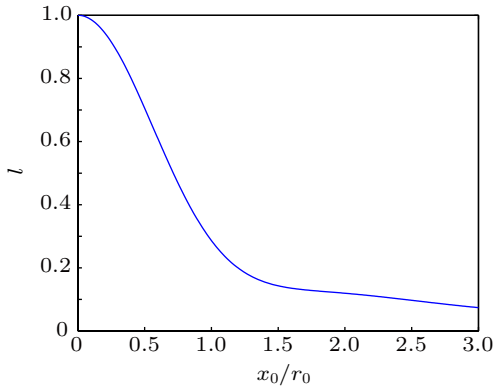


Fig. 12. Variation of angular momentum l of M.G beam in (f, θ_f) plane with x_0/r_0

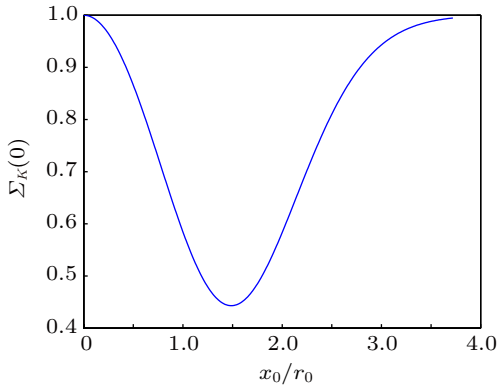


Fig. 13. Variation of the ratio $\Sigma_K(0)$ of the r.m.s beam widths of an input M.G beam to that of a Gaussian beam with x_0/r_0

Step-like behaviour of the longitudinal phase, with each step occurring at positions of minimum beam width is also observed. This indicates that there is slowest decrement in θ_p at points of minimum beam width. This is opposite to the behaviour of phase in graded index fibers, where phase decreases slowest in the positions of minimum intensity, i.e. maximum beam width. This difference in the behaviour of longitudinal phase in plasmas and that in graded index fibers is due to the fact that owing to relativistic nonlinearity, plasmas behave as oscillating linear wave guides. In linear wave guides, the growth rate of longitudinal phase is inversely proportional to the square of beam width (Eq. (40)).

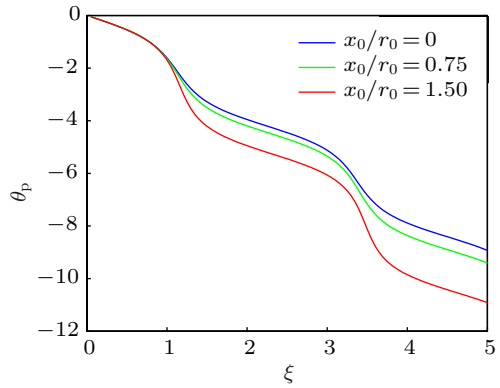


Fig. 14. Variarion of longitudinal phase θ_p with dimensionless distance of propagation ξ for different values of x_0/r_0 viz., $x_0/r_0 = 0$, 0.75, and 1.50.

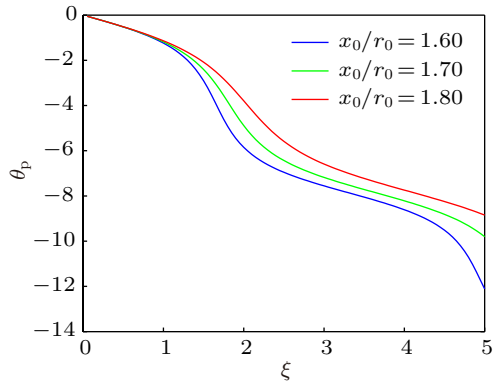


Fig. 15. Variarion of longitudinal phase θ_p with dimensionless distance of propagation ξ for different values of x_0/r_0 viz., $x_0/r_0 = 1.60$, 1.70, and 1.80.

It can also be seen that there is an increase in the rate of the decrease of longitudinal phase θ_p with an increase in the value of x_0/r_0 for $0 < x_0/r_0 \leqslant 1.5$. This is due to the dependence of longitudinal phase on beam width of the laser beam. The smaller is the spot size of the laser beam, the grater is the shit in its longitudinal phase through the position momentum uncertainty ($\Delta x \Delta k_x \geq$ constant). As, with an increase in the value of x_0/r_0 in the range $0 < x_0/r_0 \leqslant 1.5$, there is an increase in the extent of focusing of the laser beam, and there is a corresponding increase in the rate of modulation its longitudinal phase.

It is also observed that for $x_0/r_0 \geq 1.5$, an increase in the value of x_0/r_0 reduces the rate of modulation of longitudinal phase θ_p. This is due to reduced focusing of the laser beam with an increase in the vale of x_0/r_0 beyond $x_0/r_0 = 1.5$.

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