

Entropy squeezing for a V-type three-level atom interacting with a single-mode field and passing through the amplitude damping channel with weak measurement

Cui-Yu Zhang(张翠玉) and Mao-Fa Fang(方卯发)

Citation: Chin. Phys. B, 2021, 30 (**1**): 010303. DOI: 10.1088/1674-1056/abb304
Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Steady and optimal entropy squeezing for three types of moving three-level atoms coupled with a single-mode coherent field

Wen-Jin Huang(黄文进) and Mao-Fa Fang(方卯发)
Chin. Phys. B, 2021, 30 (**1**): 010304. DOI: 10.1088/1674-1056/abc542

Optimal parameter estimation of open quantum systems

Yinghua Ji(嵇英华), Qiang Ke(柯强), and Juju Hu(胡菊菊)
Chin. Phys. B, 2020, 29 (**12**): 120303. DOI: 10.1088/1674-1056/abc0d1

Protecting the entanglement of two-qubit over quantum channels with memory via weak measurement and quantum measurement reversal

Mei-Jiao Wang(王美姣), Yun- Jie Xia(夏云杰), Yang Yang(杨阳), Liao-Zhen Cao(曹连振), Qin-Wei Zhang(张钦伟), Ying-De Li(李英德), Jia-Qiang Zhao(赵加强)
Chin. Phys. B, 2020, 29 (**11**): 110307. DOI: 10.1088/1674-1056/aba614

Entropy squeezing for three-level atom interacting with a single-mode field

Fei-Fan Liu(刘非凡), Mao-Fa Fang(方卯发), Xiong Xu(许雄)
Chin. Phys. B, 2019, 28 (**6**): 060304. DOI: 10.1088/1674-1056/28/6/060304

Preparation of optimal entropy squeezing state of atomic qubit inside the cavity via two-photon process and manipulation of atomic qubit outside the cavity

Zhou Bing-Ju, Peng Zhao-Hui, Jia Chun-Xia, Jiang Chun-Lei, Liu Xiao-Juan
Chin. Phys. B, 2014, 23 (**12**): 120305. DOI: 10.1088/1674-1056/23/12/120305

Entropy squeezing for a V-type three-level atom interacting with a single-mode field and passing through the amplitude damping channel with weak measurement*

Cui-Yu Zhang(张翠玉) and Mao-Fa Fang(方卯发)†

Synergetic Innovation Center for Quantum Effects and Applications, Key Laboratory of Low-dimensional Quantum Structures and Quantum Control of Ministry of Education, School of Physics and Electronics, Hunan Normal University, Changsha 410081, *China*

(Received 24 June 2020; revised manuscript received 13 August 2020; accepted manuscript online 27 August 2020)

The entropy squeezing of a V-type three-level atom interacting with a single-mode field and passing through the amplitude damping channel is investigated in detail. Our results show that when coupled to the single-mode field, the atom in appropriate initial states can not only generate obvious entropy squeezing but also keep in the optimal squeezing state, while passing through the amplitude damping channel, the atom can generate entropy squeezing under the control of the weak measurement. Besides, it is proved again that as a measurement method for atomic squeezing, the entropy squeezing is precise and effective. Therefore our work is instructive for experiments in preparing three-level system information resource with ultra-low quantum noise.

Keywords: entropy squeezing, V-type three-level atom, single-mode field, weak measurement

PACS: 03.67.–a, 03.65.Yz, 42.50.–p **DOI:** 10.1088/1674-1056/abb304

1. Introduction

Squeezing effect, as an effective method to suppress quantum noise in quantum systems, has attracted great attention during the last decades due to its potential application in optical communication,[1] detections of weak signal like a gravitational wave,[2] high-resolution spectroscopy,[3] the high-precision atomic fountain clock,[4,5] high-precision spin polarization measurements,[6] and the quantum information processing. It is well known that the research on squeezing effects mainly includes the generation of the squeezing and the measurement of the squeezing. As for the measure of squeezing, many studies[7–9] are based on the Heisenberg uncertainty relation, which is regarded as the standard limitation on measurements of quantum fluctuations. However, the variance squeezing (VS) based on the Heisenberg uncertainty has its own limitations in the following aspects. On the one hand, standard deviation only contains the second-order statistical moment for the state density matrix and the higher-order statistical moments are neglected. On the other hand, for some particular cases, the Heisenberg uncertainty relation is trivial and fail to provide any useful information. Fortunately, one of our authors[10] defines the entropy squeezing (ES) firstly for a two-level atom via quantum information entropy, which not only captures all information about the squeezed states and provides a general measure of the quantum uncertainty but also overcomes the limitations of the definition of the variance squeezing. Since the entropy squeezing was presented, its properties and schemes for generating entropy squeezing have been widely investigated. Xiao *et al.*[11] have studied the entropy squeezing properties of a two-level atom off-resonantly coupled to a non-Markovian reservoir. Yu *et al.*[12] have obtained the optimal and steady entropy squeezing of a two-level atom in a dissipative cavity under quantum-jump-based feedback and classical field. Via the non-Hermitian operation, Wang *et al.*[13] have provided a scheme for generating sustained optimal entropy squeezing of a two-level atom. However, all of the researches mentioned above are aimed at two-level atom while there is little attention devoted to the entropy squeezing of three-level atom until Liu *et al.*[14] made a general definition of entropy squeezing for a Ξ-type three-level atom recently.

Nowadays, it is well known that quantum systems with higher dimensions may have lots of obvious advantages than ones with lower dimensions in quantum information processing,[15,16] since they provide higher channel capacities, more secure cryptography as well as superior quantum gates.[17–19] More importantly, compared with qubits, maximally high-dimensional entanglement states are less influenced by noise effects.[20] Motivated by the above-mentioned advantages and considering one of the author in Refs. [14] has present the entropy squeezing of the Ξ-type atom only, we focus our attention on the squeezing properties of a V-type three-level atom in this paper. Firstly, we conduct a review of the definition of the entropy squeezing of three-level atom. Then we investigate the entropy squeezing of the V-type three-level atom in different conditions: one is interacting with the single-

*Project supported by the National Natural Science Foundation of China (Grant Nos. 12064012 and 11374096).

†Corresponding author. E-mail: mffang@hunnu.edu.cn

© 2021 Chinese Physical Society and IOP Publishing Ltd

mode field while the other is in the noisy environment and under the control of the weak measurement technology. The numerical results show that when coupled to the single-mode field, the V-type atom can not only generate obvious squeezing entropy but also keep in the optimal entropy squeezing state if only the system is prepared in the appropriate initial state. While passing through the amplitude damping channel, the atom can also generate entropy squeezing via weak measurement with a certain strength.

The paper is organized as follows. In Section 2, we review the general definition of the entropy squeezing for a three-level atom. In Section 3, we firstly investigate the entropy squeezing properties of a V-type three-level atom coupled to a single-mode field and compare it to variance squeezing. Then we analyse the effects of the weak measurement as well as measurement reversal on the atomic entropy squeezing when the V-type three-level atom is in noisy environment simulated by amplitude damping channels in Section 4. Finally, a brief summary is presented in Section 5.

2. Entropy squeezing for a three-level atom

Before analyzing the entropy squeezing properties of the atom in our systems, we firstly review the general definition of the entropy squeezing of a three-level atom given by Ref. [14]. As is well known, a three-level atom can be described by dipole operators S_α $(\alpha = x,y,z)$, which satisfy the commutation relationship $[S_x, S_y] = \mathrm{i}S_z$. Besides, since the definition of the squeezing depends on them, we choose a universally adapted spin operators in this paper, which are shown as

$$S_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$
$$S_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -\mathrm{i} & 0 \\ \mathrm{i} & 0 & -\mathrm{i} \\ 0 & \mathrm{i} & 0 \end{pmatrix},$$
$$S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{1}$$

Then the Heisenberg uncertainty relation is shown as

$$\Delta S_x \Delta S_y \geq \frac{1}{2}|\langle S_z\rangle|, \tag{2}$$

where $\Delta S_\alpha = [\langle S_\alpha^2\rangle - \langle S_\alpha\rangle^2]^{1/2}$ and $\langle S_\alpha\rangle$ is the expectation of the observable S_α. For the convenience of the following discussion, the definition of the variance squeezing based on the Heisenberg uncertainty relation is given firstly. If the variance squeezing factor in component S_α satisfies the condition

$$V(S_\alpha) = \Delta S_\alpha - \left(\frac{|\langle S_z\rangle|}{2}\right)^{1/2} < 0, \quad (\alpha = x,y), \tag{3}$$

the variance squeezing in component S_α occurs.

Similarly with the definition of the variance squeezing, we start with the entropy uncertainty relation. Accordingly, the information entropy of the operators S_α $(\alpha = x,y,z)$ for a three-level atom is

$$H(S_\alpha) = -\sum_{i=1}^{3} P_i(S_\alpha)\log_2 P_i(S_\alpha), \tag{4}$$

where $P_i(S_\alpha) = \langle\psi_{\alpha i}|\rho|\psi_{\alpha i}\rangle$ $(i = 1,2,3)$ is the probability distribution for each possible outcome of measurement on the operator S_α, while $\psi_{\alpha i}$ represents the eigenvector of the operator S_α and ρ is the density matrix for an arbitrary quantum state. The information entropies satisfy the entropy uncertainty relation for three observables of the system with dimension $d = 3$ derived by Alberto *et al*. in Ref. [21]

$$H(S_x) + H(S_y) + H(S_z) \geq 2. \tag{5}$$

This inequality can also be written as

$$\delta H(S_x)\delta H(S_y) \geq \frac{\mathrm{e}^2}{\delta H(S_z)}, \tag{6}$$

where $\delta H(S_\alpha) = \exp[H(S_\alpha)]$. When the entropy squeezing factor in component S_α satisfies the condition

$$E(S_\alpha) = \delta H(S_\alpha) - \frac{\mathrm{e}}{\sqrt{\delta H(S_z)}} < 0, \quad (\alpha \equiv x \text{ or } y), \tag{7}$$

the fluctuation in component S_α is said to be squeezed in entropy. Using the atomic density operator $\rho(t)$, we can obtain the information entropies of the V-type three-level atomic operators $S_\alpha(\alpha = x,y,z)$ directly by Eq. (4) as

$$\begin{aligned} H(S_x) &= H(S_{x1}) + H(S_{x2}) + H(S_{x3}), \\ H(S_y) &= H(S_{y1}) + H(S_{y2}) + H(S_{y3}), \\ H(S_z) &= H(S_{z1}) + H(S_{z2}) + H(S_{z3}), \end{aligned} \tag{8}$$

where $H(S_{\alpha i}) = -P_i(S_\alpha)\log_2 P_i(S_\alpha)$, $(\alpha = x,\ y,\ z,\ i = 1,2,3)$,

$$\begin{aligned} P_1(S_x) &= \frac{1}{4}(\rho_{11} + 2\rho_{22} + \rho_{33} - 2\sqrt{2}\,\mathrm{Re}\,(\rho_{12}) \\ &\quad - 2\sqrt{2}\,\mathrm{Re}\,(\rho_{23}) + 2\,\mathrm{Re}\,(\rho_{13})), \\ P_2(S_x) &= \frac{1}{4}(\rho_{11} + 2\rho_{22} + \rho_{33} + 2\sqrt{2}\,\mathrm{Re}\,(\rho_{12}) \\ &\quad + 2\sqrt{2}\,\mathrm{Re}\,(\rho_{23}) + 2\,\mathrm{Re}\,(\rho_{13})), \\ P_3(S_x) &= \frac{1}{2}(\rho_{11} + \rho_{33} - 2\,\mathrm{Re}(\rho_{13})), \\ P_1(S_y) &= \frac{1}{4}(\rho_{11} + 2\rho_{22} + \rho_{33} + 2\sqrt{2}\,\mathrm{Im}\,(\rho_{12}) \\ &\quad + 2\sqrt{2}\,\mathrm{Im}\,(\rho_{23}) - 2\,\mathrm{Re}\,(\rho_{13})), \\ P_2(S_y) &= \frac{1}{4}(\rho_{11} + 2\rho_{22} + \rho_{33} - 2\sqrt{2}\,\mathrm{Im}\,(\rho_{12}) \\ &\quad - 2\sqrt{2}\,\mathrm{Im}\,(\rho_{23}) - 2\,\mathrm{Re}\,(\rho_{13})), \\ P_3(S_y) &= \frac{1}{2}(\rho_{11} + \rho_{33} + 2\,\mathrm{Re}\,(\rho_{13})), \\ P_1(S_z) &= \rho_{11},\ P_2(S_z) = \rho_{22},\ P_3(S_z) = \rho_{33}. \end{aligned}$$

Next, we will investigate the properties of the entropy squeezing for a V-type three-level atom.

3. Entropy squeezing for a V-type three-level atom

3.1. Entropy squeezing for a V-type three-level atom interacting with a single-mode field

In this section, the system we considered is a V-type three-level atom (here $|0\rangle$ is ground state while $|1\rangle$ and $|2\rangle$ are two excited states) interacting with a single-mode field. The transitions between $|0\rangle$ and $|1\rangle$, $|0\rangle$ and $|2\rangle$ are interacting with a single-mode field resonantly with interaction strength g_j $(j=1,2)$. In the interaction picture, the system Hamiltonian can be expressed as ($\hbar=1$)

$$H_{\rm I}=g_1a|1\rangle\langle 0|+g_2a|2\rangle\langle 0|+{\rm H.c}, \tag{9}$$

where a is the annihilation operator of the field. For simplicity, we consider the resonant case between the atom and the field, for the atom initially in the superposition state

$$|\Psi_A(0)\rangle=\cos\theta|2\rangle+{\rm e}^{-{\rm i}\phi}\sin\theta\cos\varphi|1\rangle + {\rm e}^{-{\rm i}\xi}\sin\theta\sin\varphi|0\rangle, \tag{10}$$

and the initial field in a coherent state

$$|\alpha\rangle=\sum_{n=0}^{\infty}F_n|n\rangle=\exp\left(-\frac{1}{2}|\alpha^2|\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{(n!)^{1/2}}|n\rangle, \tag{11}$$

where $\alpha=\bar{n}^{1/2}{\rm e}^{{\rm i}\beta}$. Then if the atom and the field are decoupled initially, the initial state vector can be expressed as $|\Psi_{AF}(0)\rangle=|\Psi_A(0)\rangle\otimes|\alpha\rangle$ accordingly. By solving the Schrödinger equation of the entire system in the interaction picture, the state vector $|\Psi_{\rm AF}^{\rm I}(t)\rangle$ at time t can be obtained as

$$|\Psi_{\rm AF}^{\rm I}(t)\rangle=\sum_{n=0}^{\infty}(a_n(t)|2,n\rangle+b_n(t)|1,n\rangle+c_n(t)|0,n\rangle), \tag{12}$$

where

$$\begin{aligned}
a_n(t)&=-{\rm i}g_2\sqrt{n+1}\left[\frac{Y_{n+1}}{Z_{n+1}}\sin(Z_{n+1}t)-\frac{X_{n+1}}{Z_{n+1}^2}\cos(Z_{n+1}t)+\frac{X_{n+1}}{Z_{n+1}^2}\right]+F_n\cos\theta,\\
b_n(t)&=-{\rm i}g_1\sqrt{n+1}\left[\frac{Y_{n+1}}{Z_{n+1}}\sin(Z_{n+1}t)-\frac{X_{n+1}}{Z_{n+1}^2}\cos(Z_{n+1}t)+\frac{X_{n+1}}{Z_{n+1}^2}\right]+F_n{\rm e}^{-{\rm i}\phi}\sin\theta\cos\varphi,\\
c_n(t)&=Y_n\cos(Z_nt)+\frac{X_n}{Z_n}\sin(Z_nt)
\end{aligned}$$

with

$$\begin{aligned}
X_n&=-{\rm i}\sqrt{n}(g_1F_n{\rm e}^{-{\rm i}\phi}\sin\theta\cos\varphi+g_2F_{n-1}\cos\theta),\\
Y_n&=F_{n-1}{\rm e}^{-{\rm i}\xi}\sin\theta\sin\varphi,\\
Z_n&=\sqrt{n(g_1^2+g_2^2)}.
\end{aligned}$$

Accordingly, the atomic reduced density matrix can be given by

$$\rho_A(t)={\rm Tr}_F[|\Psi_{\rm AF}^{\rm I}(t)\rangle\langle\Psi_{\rm AF}^{\rm I}(t)|]. \tag{13}$$

The elements of $\rho_A(t)$ can be written as

$$\begin{aligned}
&\rho_{11}=\sum_{n=0}^{\infty}a_na_n^*,\ \ \rho_{12}=\sum_{n=0}^{\infty}a_nb_n^*,\ \ \rho_{13}=\sum_{n=0}^{\infty}a_nc_n^*,\\
&\rho_{22}=\sum_{n=0}^{\infty}b_nb_n^*,\ \ \rho_{23}=\sum_{n=0}^{\infty}b_nc_n^*,\ \ \rho_{33}=\sum_{n=0}^{\infty}c_nc_n^*,\\
&\rho_{21}=\rho_{12}^*,\ \ \rho_{31}=\rho_{13}^*,\ \ \rho_{32}=\rho_{23}^*.
\end{aligned} \tag{14}$$

According to the Eqs. (3), (7), (8), (12), and (14), $V(S_\alpha)$ and $E(S_\alpha)$ $(\alpha=x,\ y)$ can be derived respectively. Next, we will investigate the properties of entropy squeezing effects in detail with the atom in different initial states.

Firstly, we explore the case that the atom is initially in the superposition state of the two exited states $|1\rangle$ and $|2\rangle$ $(\theta=\pi/4,\varphi=0)$ with the same phase $(\phi=0,\xi=0)$ while the field is in a coherent state with the average photon number $\bar{n}=225$ and the phase $\beta=0$. Figure 1 shows the entropy squeezing and variance squeezing as well as atomic inversion $\langle S_z\rangle$ as a function of the scaled time gt ($g_1=g_2=g$). From Fig. 1 we can observe that when the atom is in the superposition of the two exited states with equal phase initially, variance squeezing exhibits no squeezing in both variable S_x and S_y as shown in Figs. 1(c) and 1(d), while entropy squeezing occurs only in the initial time in the variable S_y as shown in Figs. 1(a) and 1(b), which indicates that the atom in this initial state can not generate entropy squeezing expect for initial time. In addition, we also can see that when atomic inversion $\langle S_z\rangle$ evolves to zero, the variance squeezing cannot reflect accurately the quantum fluctuations of the observable variables.

Secondly, a superposition state of $|0\rangle$, $|1\rangle$, and $|2\rangle$ is prepared by setting $\theta=\pi/3$, $\varphi=\arccos(\sqrt{6}/3)$ and the phase $\phi=0$, $\xi=0$ initially and other parameters are the same as those in Fig. 1. Similarly, the entropy squeezing and variance squeezing as well as atomic inversion $\langle S_z\rangle$ as a function of the scaled time gt are shown in Fig. 2.

It is noted from Figs. 2(c) and 2(d) that variance squeezing cannot predict squeezing information in neither S_x nor S_y components while the entropy squeezing can capture the squeezing information only in S_x component in certain time ranges as shown in Figs. 2(a) and 2(b). The reason is that the

atomic inversion $\langle S_z\rangle$ keeps zero all the time so the Heisenberg uncertainty relation is trivial.

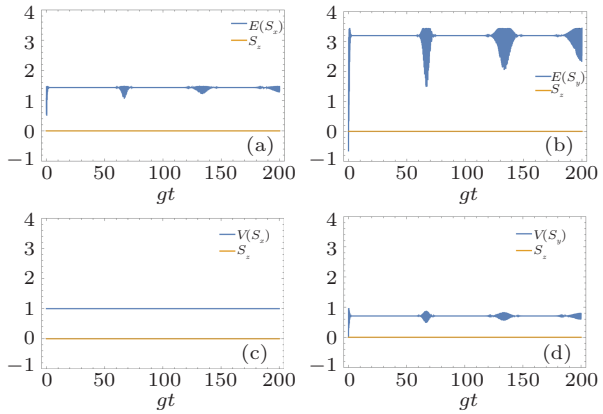


Fig. 1. Squeezing factors and the atomic inversion of a V-type atom interacting with single-mode field as a function of scaled time gt when the atom is initially in the exited state with $\theta=\pi/4$, $\varphi=0$, and $\phi=0$, $\xi=0$ while the field is initially in the coherent state with the average photon number $\bar{n}=225$. (a) $E(S_x)$ and $\langle S_z\rangle$; (b) $E(S_y)$ and $\langle S_z\rangle$; (c) $V(S_x)$ and $\langle S_z\rangle$; (d) $V(S_y)$ and $\langle S_z\rangle$.

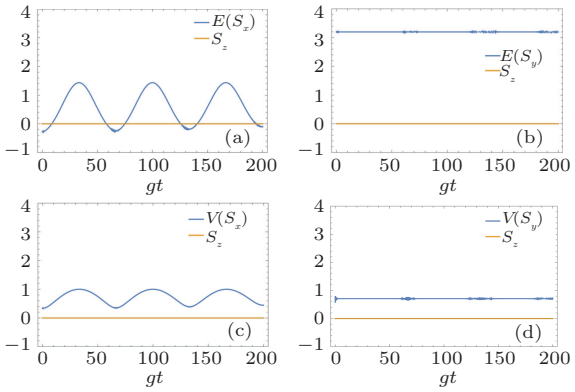


Fig. 2. Entropy squeezing and variance squeezing of the atomic polarization components S_x and S_y as a function of scaled time gt when the atom is initially in a superposition state with $\theta=\pi/3$, $\varphi=\arccos(\sqrt{6}/3)$ and the phase $\phi=0$, $\xi=0$ while the field is initially in the coherent state with the average photon number $\bar{n}=225$. (a) $E(S_x)$ and $\langle S_z\rangle$; (b) $E(S_y)$ and $\langle S_z\rangle$; (c) $V(S_x)$ and $\langle S_z\rangle$; (d) $V(S_y)$ and $\langle S_z\rangle$.

Finally, we prepare a special state initially, which is the superposition state of excited states $|1\rangle$ and $|2\rangle$ with different phase. By setting the parameters of the initial atom state: $\theta=\pi/4$, $\varphi=0$, and the phase $\phi=\pi$, $\xi=0$ while the other parameters are the same as those given before, we obtain the density matrix elements of the atom respectively

$$\begin{aligned}&\rho_{11}=\rho_{33}=-\rho_{13}=\sum_{n=0}^{\infty}\frac{\bar{n}^n}{2\mathrm{e}^{\bar{n}}n!},\\&\rho_{12}=\rho_{22}=\rho_{23}=0,\\&\rho_{21}=\rho_{12}^*,\ \ \rho_{31}=\rho_{13}^*,\ \ \rho_{32}=\rho_{23}^*.\end{aligned}\tag{15}$$

Substituting Eq. (15) into Eqs. (7), (8), and (3), the entropy squeezing factor and variance squeezing factor of the atom can be obtained. The results are shown in Fig. 3, from which we can see that although $E(S_x)$, $E(S_y)$, $V(S_x)$, $V(S_y)$, and $\langle S_z\rangle$ all remain unchanged, there is an obvious difference between $E(S_x)$ and $V(S_x)$: the former stabilizes in the optimal squeezing while the latter predicts no squeezing at all for $\langle S_z\rangle$ keeps zero. In other words, when the atom is prepared in this optimal entropy squeezing state initially, the field can keep the atom in this optimal entropy squeezing state, which is a great advantage over the Ξ-type three-level atom in Ref. [14].

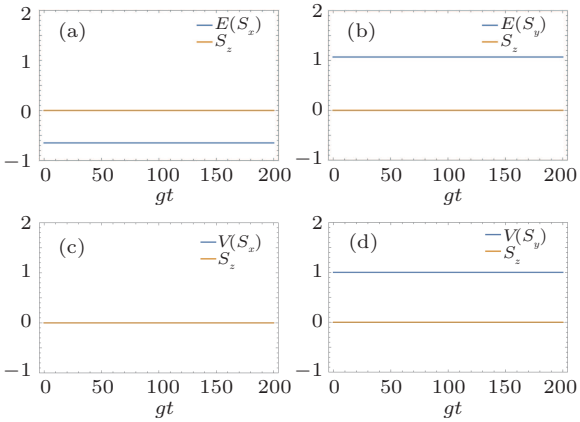


Fig. 3. Entropy squeezing and variance squeezing of the atomic polarization components S_x and S_y as a function of scaled time gt when the atom is initially in a superposition state with $\theta=\pi/4$, $\varphi=0$ and $\phi=\pi$, $\xi=0$ while the field is initially in the coherent state with the average photon number $\bar{n}=225$. (a) $E(S_x)$ and $\langle S_z\rangle$; (b) $E(S_y)$ and $\langle S_z\rangle$; (c) $V(S_x)$ and $\langle S_z\rangle$; (d) $V(S_y)$ and $\langle S_z\rangle$.

3.2. Entropy squeezing for a V-type three-level atom under the amplitude damping channel via weak measurement and measurement reversal

In this section, we investigate the entropy squeezing of a V-type three-level atom under decoherence simulated by amplitude damping channels via the control of the weak measurement and the measurement reversal at finite temperatures, before proceeding, we first review the knowledge of amplitude damping channels, the weak measurement, and the measurement reversal.

(i) **Amplitude damping channel**

As we know, the environment noisy can be simulated well by noisy quantum channels, under which various quantum effects have always been investigated widely.[22–24] The amplitude damping (AD) channel we considered is a dissipative channel with the probability r of losing the excitation, and the probability $1-r$ of absorbing the excitation, where $0\le r\le 1$ is usually a function of temperature.[25] This process can be characterized by a quantum operation

$$\varepsilon(\rho_A(0))=\sum_{i=1}^{6}E_i\rho_A(0)E_i^{\dagger},\tag{16}$$

where $\rho_A(0)=|\psi_A(0)\rangle\langle\psi_A(0)|$ is the atomic initial density matrix and E_i represents the single-qutrit Kraus operator[26] for the AD noise which are given as

$$E_1=\sqrt{r}\begin{pmatrix}1&0&0\\0&\sqrt{1-d_1}&0\\1&0&\sqrt{1-d_2}\end{pmatrix},$$

$$E_2=\sqrt{r}\begin{pmatrix}0&\sqrt{d_1}&0\\0&0&0\\0&0&0\end{pmatrix},$$

$$E_3=\sqrt{r}\begin{pmatrix}0&0&\sqrt{d_2}\\0&0&0\\0&0&0\end{pmatrix},$$

$$E_4=\sqrt{1-r}\begin{pmatrix}\sqrt{1-d_1-d_2}&0&0\\0&1&0\\0&0&1\end{pmatrix},$$

$$E_5=\sqrt{1-r}\begin{pmatrix}0&0&0\\\sqrt{d_1}&0&0\\0&0&0\end{pmatrix},$$

$$E_6=\sqrt{1-r}\begin{pmatrix}0&0&0\\0&0&0\\\sqrt{d_2}&0&0\end{pmatrix},$$

such that d_i $(i=1,2)$ represents the decoherence constant induced by the spontaneous emission of the exited state $|i\rangle$ $(i=1,2)$ to the ground state $|0\rangle$, respectively.

(ii) **Weak measurement and measurement reversal protocol**

The weak measurement we considered is a positive-operator-valued measurement (POVM) originally discussed in Refs. [27,28]. Sun *et al.*[29] and Kim *et al.*[30] have pointed out that weak measurement and weak measurement reversal can actively combat decoherence. Xiao *et al.* generalized their studies from qubits to qutrits to protect entanglement under the amplitude damping decoherence.[31] We will perform prior weak measurements and measurement reversal on the atom before and after sending it to the noisy channels to generate the entropy squeezing. Usually the expression for the weak measurement and quantum measurement reversal can be written as $M_{\rm w}=\mathrm{diag}\{1,\sqrt{1-p},\sqrt{1-p}\}$, $M_{\rm r}=\mathrm{diag}\{1-q,\sqrt{1-q},\sqrt{1-q}\}$, where p and q represent the weak measurement strength and measurement reversal strength, respectively.

Then we will discuss the generation of the entropy squeezing of the V-type three-level atom exposed to amplitude damping channel. Under the weak measurement and measurement reversal, the final state subjected to the AD channel can be expressed as

$$\rho=\left(M_r\sum_{i=1}^{6}(E_iM_{\rm w}\rho_A(0)M_{\rm w}^{\dagger}E_i^{\dagger})M_r^{\dagger}\right)\Big/T, \tag{17}$$

where $T=\mathrm{Tr}[M_r\sum_{i=1}^{6}(E_iM_{\rm w}\rho_A(0)M_{\rm w}^{\dagger}E_i^{\dagger})M_r^{\dagger}]$. According to the Eqs. (3), (6), (7), and (17), $E(S_x)$ and $E(S_y)$ can be derived, but their general analytic expressions are too complicated to present. So we will provide the numerical results in the discussion below.

In Fig. 4, we show entropy squeezing for a V-type three-level atom under decoherence at finite temperature by weak measurement technology. One can observe that when there is no weak measurement reversal $(q=0)$, entropy squeezing can occur under control of weak measurement with certain strength. However, when there is only weak measurement reversal $(p=0)$, there is no entropy squeezing at all, and it does not work even if both of the weak measurement and the measurement reversal $(p=q)$ are used on the atom at the same time. This indicates that the weak measurement technology is more helpful than measurement reversal in generating the entropy squeezing.

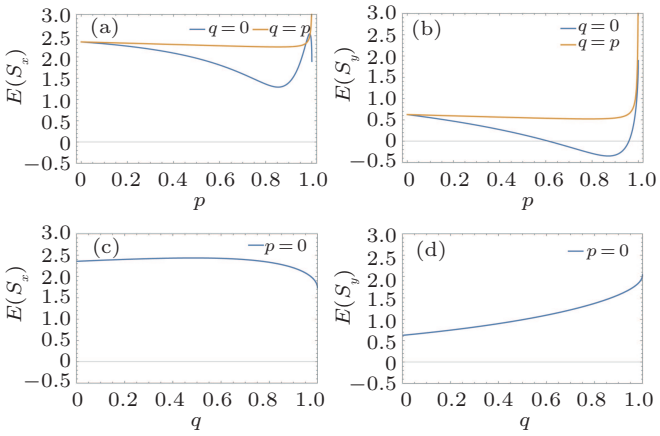


Fig. 4. Entropy squeezing of the V-type three-level atom under decoherence via weak measurement and measurement reversal when the atom is initially in a superposition state with $\theta=\pi/8$, $\varphi=\pi/8$, and $\phi=0$, $\xi=0$. Other parameters are chosen to be $r=0.9$, and $d_1=d_2=0.05$. (a) $E(S_x)$ *versus* the weak measurement strength p; (b) $E(S_y)$ *versus* the weak measurement strength p; (c) $E(S_x)$ *versus* measurement reversal strength q; (d) $E(S_y)$ *versus* measurement reversal strength q.

4. Conclusion

Suppressing the quantum noise (quantum uncertainty) via squeezing effects is crucial for the development of the quantum information technology. To make contributions to prepare three-level system information resources with ultra-low quantum noise, we have discussed the squeezing properties of a V-type three-level atom interacting with the single-mode field or passing through the amplitude damping channels under the control of the weak measurement. First of all, by calculating the entropy squeezing of a V-type three-level atom interacting with a single-mode field, we found that the single-mode field in coherent state can keep the atom in the optimal entropy squeezing state when the atom is initially in the eigenstate of S_x, which can be interpreted by the steady-state solution shown in Eq. (15); And we also studied the variance squeezing for comparisons, which demonstrates the general validity of the entropy squeezing as a measure of quantum uncertainty. Then under the help of the weak measurement, the V-type three-level atom exposed to amplitude damping channel can generate obvious entropy squeezing in appropriate initial atomic state. That is to say, the weak measurement technology is effective in the generation of squeezing effect via suppressing quantum noise, which is meaningful for the suppression of quantum noise with current experimental capabilities.

References

[1] Furusawa A, Sorensen J L, Braunstein S L, Fuchs C A, Kimble H J and Polzik E S 1998 *Science* **282** 706
[2] Caves C M 1980 *Phys. Rev. D* **23** 1693
[3] Kitagawa M and Ueda M 1993 *Phys. Rev. A* **47** 5138
[4] Wineland D J, Bollinger J J, Itano W M and Heinzen D J 1994 *Phys. Rev. A* **50** 67
[5] Sorensen A and Molmer K 1999 *Phys. Rev. Lett.* **83** 2274
[6] Sorensen J L, Hald J and Polzik E S 1998 *Phys. Rev. Lett.* **80** 3487
[7] Li X S, Lin D L and George T F 1989 *Phys. Rev. A* **40** 2504
[8] Ashraf M M and Razmi M S K 1992 *Phys. Rev. A* **45** 8121
[9] Zhou P and Peng J S 1991 *Phys. Rev. A* **44** 3331
[10] Fang M F, Zhou P and Swain S 2000 *J. Mod. Opt.* **47** 1043
[11] Xiao X, Fang M F and Hu Y M 2011 *Phys. Scr.* **84** 045011
[12] Yu M and Fang M F 2016 *Quantum Inf. Process.* **15** 4175
[13] Wang Y Y and Fang M F 2018 *Chin. Phys. B* **11** 114207
[14] Liu F F, Fang M F and Xu X 2019 *Chin. Phys. B* **6** 060304
[15] Walborn S P, Lemelle D S, Almeida M P and Ribeiro P H S 2006 *Phys. Rev. Lett.* **96** 090501
[16] Bourennane M, Karlsson A and Bjork G 2001 *Phys. Rev. A* **64** 012306
[17] Cerf N J, Bourennane M, Karlsson A and Gisin N 2002 *Phys. Rev. Lett.* **88** 127902
[18] Durt T, Cerf N J, Gisin N and Zukowski M 2003 *Phys. Rev. A* **67** 012311
[19] Jafarpour M and Ashrafpour M 2013 *Quantum Inf. Process.* **12** 761
[20] Kaszlikowski D, Gnaciski P, Zukowski M, Miklaszewski W and Zeilinger A 2000 *Phys. Rev. Lett.* **85** 4418
[21] Riccardi A, Macchiavello C and Maccone L 2017 *Phys. Rev. A* **95** 032109
[22] Hu M L, Hu X Y, Wang J C, Peng Y, Zhang Y R and Fan H 2018 *Phys. Rep.* **762** 1
[23] Basit A, Badshah F, Ali H and Ge G Q 2017 *Europhys. Lett.* **118** 30002
[24] Guo Y N, Tian Q L, Zeng K and Chen P X 2020 *Quantum Inf. Process.* **19** 182
[25] Wang S C, Yu Z W, Zou W J and Wang X B 2014 *Phys. Rev. A* **89** 022318
[26] Guo Y N, Fang M F, Wang G Y and Zeng K 2016 *Quantum Inf. Process.* **15** 2851
[27] Korotkov A N 1999 *Phys. Rev. B* **60** 5737
[28] Korotkov A N and Jordan A N 2006 *Phys. Rev. Lett.* **97** 166805
[29] Sun Q Q, Amri M A, Davidovich L and Zubairy M S 2010 *Phys. Rev. A* **82** 052323
[30] Kim Y S, Lee J C, Kwon O and Kim Y H 2012 *Nat. Phys.* **8** 117
[31] Xiao X and Li Y L 2013 *Eur. Phys. J. D* **67** 204