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Generation of atomic spin squeezing via quantum coherence: Heisenberg–Langevin approach

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Taking into account the dephasing process in the realistic atomic ensemble, we theoretically study the generation of atomic spin squeezing via atomic coherence induced by the coupling and probe fields. Using the Heisenberg–Langevin approach, we find that the perfect spin squeezing in the X component can be obtained while the coupling and probe fields produce the maximum coherence between the ground state sublevels 1 and 2. Moreover, the degree of atomic spin squeezing in the X component can be strengthened with the increasing atomic density and/or Rabi frequency of the mixing field. The theoretical results provide a proof-of-principle demonstration of generating the atomic spin squeezing via quantum coherence in the realistic atomic ensemble which may find potential applications in quantum information processing and quantum networks.

Keywords: atomic spin squeezing, quantum coherence

PACS: 42.50.Lc, 42.50.Gy, 42.50.Dv **DOI:** 10.1088/1674-1056/abab7c

1. Introduction

Squeezed spin states[1–3] are a very fundamental physical resource in quantum information processing. They can be used not only in detection of quantum entanglement,[4] which plays a key role in the foundations of quantum physics, but also in the high-precision measurement.[5,6] Squeezed spin states have attracted extensive attention in recent decades. Various schemes to generate squeezed spin states in atomic systems have been proposed.[7–18] Generally, performing a quantum non-demolition (QND) measurement,[7,8] transferring squeezing from light to atoms[9–14] and nonlinear interactions between atoms and fields in optical cavities or in free space[15–18] have been extensively verified for generation of atomic spin squeezing. In the third category, the four-wave mixing (FWM) process based on electromagnetically induced transparency (EIT) is a convenient way for creating squeezed or entangled states.[19–28] Recently, we studied the generation and control of atomic spin squeezing via atomic coherence in the Λ-type EIT configuration by solving the Heisenberg equations of motion.[27] However, in the previous treatment of the interaction between the laser fields and the atoms, only the mean response of the Stokes field far less than the mixing field was considered, whereas the quantum fluctuation of the field and the Langevin noise of the atoms were ignored. In this paper, we take into account the more realistic experiment condition in two-photon Raman non-resonance configuration and theoretically study the generation of the atomic spin squeezing using the Heisenberg–Langevin approach. It is found that the perfect spin squeezing in the X component can be obtained while the coupling and probe fields produce the maximum coherence. Moreover, the degree of atomic spin squeezing in the X component will strengthen with the increasing atomic density and/or Rabi frequency of the mixing field in such a configuration. Compared to the other methods for creating spin squeezing, the present proposal doe not require the projection by the measurement and the squeezed field, which is much easier to implement in experiments. The atomic spin squeezing is realized only by a single pass interaction between the atoms and the fields, and the nearly perfect atomic spin squeezing in the X component can be achieved conveniently.

2. Heisenberg–Langevin equations

We consider an ensemble of N three-level atoms in a Λ-type configuration. As shown in Fig. 1(a), each atom has two ground states $|1\rangle$ and $|2\rangle$, and one excited state $|3\rangle$, corresponding to the hyperfine levels $5S_{1/2}$ ($F=2$), $5S_{1/2}$ ($F=3$) and fine structure energy level $5P_{1/2}$ in D_1 line of the ^{85}Rb atom, respectively. The atoms interact with two fields firstly, a probe field E_p (with frequency ω_p and Rabi frequency Ω_p) is tuned to resonance with the transition $|1\rangle-|3\rangle$, and a coupling field E_c (with frequency ω_c and Rabi frequency Ω_c) is tuned close to resonant transition $|2\rangle-|3\rangle$ with a finite detuning $\delta=\omega_c-\omega_{32}$ with ω_{ij} $(i\neq j)$ being the atomic resonant frequency between levels i and j. A third mixing field E_m (with frequency ω_m and Rabi frequency Ω_m) off-resonantly couples levels $|1\rangle$ and $|3\rangle$ with the detuning Δ_1. Then a Stokes field E_1 (with frequency ω_1) can be generated through the non-degenerate FWM process. Actually, as presented in Ref. [24], the Stokes field can be equivalently viewed as scattering the mixing field off the atomic spin wave S, where the atomic spin wave S acts as a frequency converter with the frequency equal

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to the separation between the lower doublet. The equivalent configuration is shown in Fig. 1(b). On the other hand, the four fields drive a double-Λ scheme in the atoms. One of the Λ schemes combined by coupling field and probe field is designed to produce the initial population and the atomic spin wave S between the ground state levels $|1\rangle$ and $|2\rangle$. However, generally speaking, the EIT alone does not produce atomic spin squeezing. Consequently, in order to squeeze the ground state atomic spin, we introduce an additional pair of fields, mixing field and Stokes field, interacting with the second Λ system.

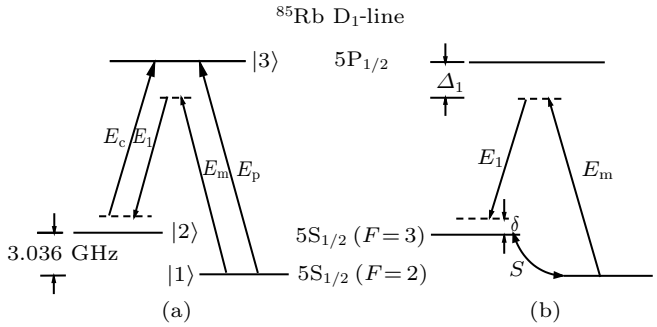


Fig. 1. (a) The double-Λ scheme of the D_1 line in the ^{85}Rb atom interacting with four fields. The coupling field E_c is tuned close to resonant transition $|2\rangle - |3\rangle$ with a finite detuning, the probe field E_p is tuned to resonance with the transition $|1\rangle - |3\rangle$. The mixing field E_m off-resonantly couples levels $|1\rangle$ and $|3\rangle$, and the corresponding Stokes field E_1 is generated through nonlinear FWM process. (b) The equivalent configuration of (a) with the two ground states driven by the atomic spin wave S pre-established by E_c and E_p fields in the Λ-type EIT configuration.

In the present proposal, we assume that the mixing field is much stronger than the generated Stokes field, thus the mixing field can be treated classically, whereas the Stokes field E_1 can be treated quantum mechanically. Meanwhile, in order to ensure that the mixing field has little influence on the atomic coherence, here we assume that the Rabi frequency of the mixing field Ω_m is far smaller than its frequency detuning Δ_1 and the coupling and probe fields. Under this condition, the evolution of the collective atomic operators and the Stokes field E_1 can be written as[13,29]

$$\frac{\partial}{\partial t}\sigma_{23} = -[(\gamma_1+\gamma_2)/2+\mathrm{i}\Delta_2]\sigma_{23}+\mathrm{i}ga_1(\sigma_{22}-\sigma_{33}) + \mathrm{i}\Omega_m\sigma_{21}+f_{23}, \quad (1)$$

$$\frac{\partial}{\partial t}\sigma_{13} = -[(\gamma_1+\gamma_2)/2+\mathrm{i}\Delta_1]\sigma_{13}+\mathrm{i}\Omega_m(\sigma_{11}-\sigma_{33}) + \mathrm{i}ga_1\sigma_{12}+f_{13}, \quad (2)$$

$$\frac{\partial}{\partial t}\sigma_{21} = -(\gamma_0+\mathrm{i}\delta)\sigma_{21}+\mathrm{i}\Omega_m\sigma_{23}-\mathrm{i}ga_1\sigma_{31}+f_{21}, \quad (3)$$

$$\left(\frac{\partial}{\partial t}+c\frac{\partial}{\partial z}\right)a_1 = \mathrm{i}gN\sigma_{23}, \quad (4)$$

where γ_1 and γ_2 are the dipole decay rates from level $|3\rangle$ to levels $|1\rangle$ and $|2\rangle$ (here make them equal for simplicity), γ_0 the decay of the ground-state coherence due to collisions between atoms or finite interaction time between atoms and fields, $\Delta_{1(2)} = \omega_{m(1)} - \omega_{13(23)}$ one-photon detuning between mixing (Stokes) field and energy level $|1\rangle - |3\rangle$ ($|2\rangle - |3\rangle$), $\delta = \Delta_2 - \Delta_1$ the two-photon detuning of the Stokes field and mixing field. The annihilation operator of the Stokes field E_1 is represented by a_1. The atom-field coupling constant is $g = \mu_{23}\varepsilon_1/\hbar$, where μ_{23} is the dipole moment for the $|2\rangle - |3\rangle$ transition, and $\varepsilon_1 = \sqrt{\hbar\omega_1/2\varepsilon_0 V}$ corresponds to the electric field of a single Stokes photon with V as the interaction volume with length L and beam radius r. Here f_{ij} are the collective atomic δ-correlated Langevin noise operators, the correlation functions are of the form $\langle f_{ij}(z,t)f_{\alpha\beta}(z',t')\rangle = \frac{L}{N}\langle D(\sigma_{ij}\sigma_{\alpha\beta}) - D(\sigma_{ij})\sigma_{\alpha\beta} - \sigma_{ij}D(\sigma_{\alpha\beta})\rangle\,\delta(t-t')\delta(z-z')$, here $D(\sigma_{ij})$ is the expression obtained from the Heisenberg–Langevin equation for σ_{ij} omitted the Langevin force term.

3. Results and discussion

The definition of spin squeezing is not unique, the most popular definition were proposed by Kitagawa and Ueda.[1] The atoms are said to be spin squeezed when the variance of one spin component in the plane orthogonal to the mean spin direction (MSD) is less than its coherent state value. That means that the variance of the atomic spin component is less than the absolute value of half the spin mean value. Consequently, the parameter $\xi_S^2 = \frac{\min(\Delta S_{\boldsymbol{n}_\perp}^2)}{\langle S_Z\rangle/2}$ is used to determine the existence of atomic spin squeezing,[3] where $\boldsymbol{n}_\perp$ refers to an axis perpendicular to the MSD. If $\xi_S^2 < 1$, the atomic spin state will be squeezed. As performed in Ref. [26], we treat the interaction of the atoms with the fields using the perturbation analysis, and obtain the steady-state mean values of the atomic operators by semi-classically treating the interaction between atoms and fields in the first Λ-type EIT configuration. By Fourier transforming the Heisenberg–Langevin equations (1)–(4) for the atomic operators and evolution equation of quantum field a_1, then adiabatic eliminating the excited state under the large detuning of Δ_1 and solving the simplified equations, we obtain the analytical expression of atomic ground state coherence for the operator $\sigma_{21}(z,\omega)$. Actually, by adiabatic eliminating the excited state, the three-level atom can be regarded as a quasi-two-level atom, and then we discuss the squeezing of the ground state atomic spin. After integrating the continuous operators over the atomic length we obtain the collective atomic observable $\sigma_{21}(\omega) = N\int_0^L \frac{\mathrm{d}z}{L}\sigma_{21}(z,\omega)$. Then the atomic collective spin components $S_x = (\sigma_{21}(\omega)+\sigma_{12}(-\omega))/2$, $S_y = (\sigma_{21}(\omega)-\sigma_{12}(-\omega))/2\mathrm{i}$, and $S_z = [\sigma_{21}(\omega),\sigma_{12}(-\omega)]/2$ are obtained.

According to the definition of spin squeezing parameter ξ_S^2, the first step is to compute the MSD determined by the expectation values $\langle S_i\rangle$, $i \in \{x,y,z\}$. The MSD can be written in spherical coordinates as $\boldsymbol{n}_0 = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$, where θ and φ are polar and azimuthal angles, respectively.

The θ and φ are given by

$$\theta = \arccos\left(\frac{\langle S_z\rangle}{|\boldsymbol{S}|}\right), \quad \varphi = 2\pi - \arccos\left(\frac{\langle S_x\rangle}{|\boldsymbol{S}|\sin\theta}\right),$$

where $|\boldsymbol{S}| = \sqrt{\langle S_x\rangle^2 + \langle S_y\rangle^2 + \langle S_z\rangle^2}$ is the magnitude of the mean spin. As mentioned above, the atomic ground state coherence (atomic spin wave S) is pre-established by the coupling and probe fields in the first Λ-type EIT configuration. This means that the collective atomic state is in a coherent superposition state $|\varphi_a\rangle = (1/\sqrt{N_a})\sum_i(\cos\theta'|2\rangle_i - \sin\theta'|1\rangle_i)$ with $\mathrm{tg}\,\theta' = \Omega_\mathrm{p}/\Omega_\mathrm{c}$, where θ' represents the relative intensity of the coupling and probe fields. After calculating the expectation values, we find that $\langle S_x\rangle = -\frac{1}{2}\sin 2\theta'$, $\langle S_y\rangle = 0$, $\langle S_z\rangle = \frac{1}{2}\sin 2\theta'$, which means that the collective MSD is not polarized along the z-axis. Thus we define a new mean spin direction, denoted by S_Z, which is now aligned along the (θ,φ) direction with $\theta = 2\theta'$ and $\varphi = \pi$. In the new coordinate frame, the mean value of atomic spin is defined as $S_Z = -S_x\sin\theta + S_z\cos\theta$, and the other two atomic spin components in the plane orthogonal to the MSD are $S_X = S_x\cos\theta + S_z\sin\theta$ and $S_Y = S_y$. Then it yields the variance of the atomic spin component in the frequency-domain $\langle\Delta S_i(\omega)\Delta S_i(\omega')\rangle = 2\pi\delta(\omega+\omega')S_{S_i}(\omega)$ $(i = X,Y)$ with $\Delta S_i(\omega) = S_i(\omega) - \langle S_i(\omega)\rangle$. The exact expression is not given here since it is complicated. In what follows, we discuss the evolution of $\langle\Delta S_X(\omega)\Delta S_X(\omega')\rangle/|\langle S_Z\rangle/2|$ in the new coordinate frame with the variation of parameters and find the appropriate parameters to realize the ideal atomic spin squeezing via quantum coherence. The relevant parameters are scaled with m and MHz, or m^{-1} and MHz^{-1}. According to the experimental conditions,[24,30,31] we set the parameters as follows: $n_0 = 10^{16}$, $L = 0.06$, $r = 1\times10^{-4}$, $\gamma_1 = \gamma_2 = 3$, $\gamma_0 = 0.001$, $\Omega_\mathrm{c} = \Omega_\mathrm{p} = 700$, $\Omega_\mathrm{m} = 15$.

Figure 2 depicts the 3D plot of the evolution of $\langle\Delta S_X(\omega)\Delta S_X(\omega')\rangle/|\langle S_Z\rangle/2|$ as functions of θ and two-photon detuning between the Stokes field and mixing field. The evolution of the ratio with θ is similar to the rule discussed in Ref. [32], it is robust to θ (exactly the initial atomic coherence) in the vicinity of $\pi/2$. Compared to the results in Ref. [27], the spin squeezing in the X component with θ shows no periodicity, which is caused by consideration of the noise and dissipation of the atoms. It can be seen that, when θ closes to 0 or π, which means that the Rabi frequency of the probe field or the coupling field approaches to zero, no atomic coherence and no atomic spin squeezing can be obtained in the X component. However, when θ closes to $\pi/2$, there is the maximum coherence between the ground states and the perfect spin squeezing in the X component can be obtained. On the other hand, the two-photon detuning also influences the degree of squeezing. According to the criterion of the squeezing parameter $\xi^2 = \langle\Delta S_X(\omega)\Delta S_X(\omega')\rangle/|\langle S_Z\rangle/2|$, the smaller ratio means the higher squeezing. We find that the higher squeezing in the X component can be obtained when the δ is not equal to 0 under the same condition of θ value. For example, under the same condition of θ ($\theta = 1.005$), when δ is equal to 0 and -100, the ratio is equal to 0.8588 and 0.6994, respectively. The similar situation occurred in the discussion of entanglement in Ref. [33]. Thus, in the following discussion we set the parameters $\theta = 0.49\pi$ and $\delta = -100$, and investigate the effects of the atomic density, the Rabi frequency of the mixing field on the collective atomic spin squeezing.

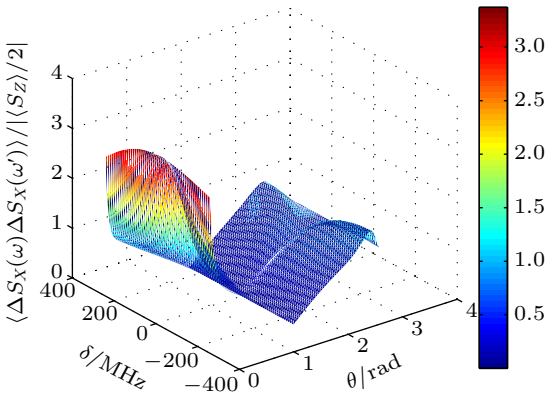


Fig. 2. The 3D plot of evolution of $\langle\Delta S_X(\omega)\Delta S_X(\omega')\rangle/|\langle S_Z\rangle/2|$ as functions of θ and the two-photon detuning between the Stokes field and mixing field. The Fourier frequency ω is equal to the two-photon detuning.

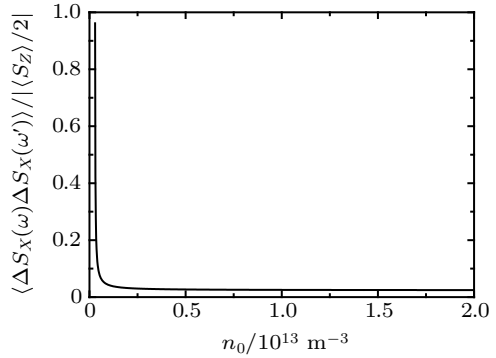


Fig. 3. The evolution of $\langle\Delta S_X(\omega)\Delta S_X(\omega')\rangle/|\langle S_Z\rangle/2|$ as a function of the atomic density with $\theta = 0.49\pi$, $\omega = \delta = -100$. The other parameters are the same as those in Fig. 2.

It is known that the optical depth of the atomic ensemble is a critical parameter in the discussion of squeezed or entangled state.[19–27] In the present case, the dependence of the ratio $\langle\Delta S_X(\omega)\Delta S_X(\omega')\rangle/|\langle S_Z\rangle/2|$ on the atomic density n_0 is depicted in Fig. 3 with Fourier frequency $\omega = -100$. It is clear that the ratio decreases rapidly with the increasing atomic density, which indicates that the degree of the atomic spin squeezing of the X component is increased. When the atomic density is about larger than $1\times10^{13}/\mathrm{m}^3$, the ratio nearly closes to zero. This means that the perfect squeezing can be obtained. Moreover, the similar evolution of the ratio with the Rabi frequency of the mixing field is observed, as shown in Fig. 4. The degree of the atomic spin squeezing is strengthened with the

increasing Rabi frequency of the mixing field. However, the Rabi frequency of the mixing field should be far less than the coupling and probe fields in order to satisfy the above hypothesis. Therefore, the $\Omega_{\rm m} = 15$ is still used in this study. In addition, the spin squeezing in one component is at the expense of excess noise in the perpendicular direction; that is, the Heisenberg uncertainty relationship must be satisfied. In all the cases discussed above, the variances in the X and Y components satisfy the Heisenberg uncertainty relationship. Here we do not depict them separately.

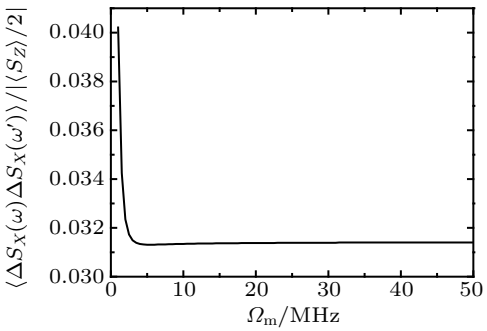


Fig. 4. The evolution of the ratio as a function of the Rabi frequency of the mixing field with $\theta = 0.49\pi$, $\omega = \delta = -100$. The other parameters are the same as those in Fig. 2.

As is well known, the essence of atomic spin squeezing is the establishment of quantum correlations among the elementary spins in the atomic ensemble. In the present scheme, the generation of quantum correlations can be interpreted as the nonlinear interaction between the laser fields and atoms. As seen in Fig. 1(a), the Stokes field is produced through FWM process, which has proven to be an efficient way to generate the squeezed and entangled state for both atoms and fields. The generation of every Stokes photon accompanied by absorbing one mixing photon and one coupling photon and emitting one probe photon, and creating an atomic spin wave simultaneously. Making use of the equivalent Fig. 1(b), we can regard the generated Stokes field as the product of frequency down-conversion process by scattering the mixing field off the atomic spin wave, which is similar to the parametric down-conversion process in a nonlinear optical crystal. The Stokes photon and the atomic spin wave always generate simultaneously, so they are strongly quantum correlated. In other words, the generation of atomic spin squeezing is realized through the EIT-based FWM process.

4. Conclusion

In summary, we have theoretically derived the atomic spin fluctuation spectra and shown the generation of atomic spin squeezing in the realistic Λ-type three-level atomic ensemble using the Heisenberg–Langevin approach. The perfect spin squeezing in the X component could be obtained when the coupling and probe fields produce the maximum coherence between the ground states. Moreover, we depict the effects of the atomic density and Rabi frequency of the mixing field on the degree of spin squeezing, and find the optimum condition of the atomic spin squeezing which has potential applications in quantum information processing and quantum networks.

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