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Quantum speed limit time and entanglement in a non-Markovian evolution of spin qubits of coupled quantum dots

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Quantum speed limit time and entanglement in a system composed of coupled quantum dots are investigated. The excess electron spin in each quantum dot constitutes the physical system (qubit). Also the spin interaction is modeled through the Heisenberg model and the spins are imposed by an external magnetic field. Taking into account the spin relaxation as a non-Markovian process, the quantum speed limit and entanglement evolution are discussed. Our findings reveal that increasing the magnetic field leads to the faster quantum evolution. In addition, the temperature increment causes the longer quantum speed limit time as well as the entanglement degradation.

Keywords: quantum speed limit time, quantum entanglement, open quantum systems, quantum dots

PACS: 42.50.Dv, 11.10.Nx, 42.50.Pq **DOI:** 10.1088/1674-1056/abab75

1. Introduction

Quantum mechanics acting as a fundamental theory of nature imposes limits on the evolution of physical systems. Heisenberg uncertainty principle is one of the fundamental results in quantum mechanics limiting the simultaneous knowledge of non-commuting variables. In particular, the time-energy uncertainty relation imposes some limitations to obtain the time needed for a quantum process which was investigated by Mandelstam and Tamm.[1] This provides a limit for the evolution of observables, which is known as quantum speed limit (QSL).[2] The minimum time required to transfer a quantum state into a distinguished and distanced desired final state is known as the QSL time (QSLT). Therefore, this quantity determines how fast a quantum system will evolve. In quantum information processing, the transfer of a state to an orthogonal one is regarded as an elementary step of a computation process.[3] Thus, the concept of QSLT possesses a remarkable role in quantum technologies such as quantum computation[4] and optimal control theory.[5]

The initial studies of the QSLT have been mainly focused on the closed quantum systems which possess a unitary evolution. The lower bound of the QSLT in this kind of systems was obtained as $\tau_{\rm QSL} = \max\{\frac{\pi\hbar}{2\Delta E}, \frac{\pi\hbar}{2E}\}$.[6,7] The first quantity is known as Mandelstam–Tamm bound with $(\Delta E)^2 = \langle H^2\rangle - \langle H\rangle^2$, whereas the second term is referred to Margolus–Levitin bound with $E = \langle H\rangle$,[8] H is the Hamiltonian of the system. On the other hand, in realistic physical systems, the interaction between the quantum systems and their surrounding environment is inevitable, and this leads to the notion of open quantum systems.[9] Because of this fact, the concept of the QSLT has been extended to open quantum systems.[10–12] In these studies, the QSLT is investigated for several typical noisy channels and quantum metrology under noise. As is known, according to the nature of the environment, a quantum system would experience the Markovian or non-Markovian processes. Therefore, in generalization of the QSLT to open quantum systems, the nature of the system dynamics should be taken into account. In the introduced approaches in Refs. [10,11], the QSLT is obtained in terms of the variance of the evolution generator which reduces to the Mandelstam–Tamm bound in the case of closed quantum systems. Moreover, using a geometric approach, the QSLT of an open quantum system was obtained depending on the mean of the evolution generator,[12] reducing to the Margolus–Levitin bound for unitary processes. Additionally, in the last reference, by making use of the von Neumann trace inequality, the Mandelstam–Tamm bound was derived, too. Therefore, by employing the introduced approach in Ref. [12], a unified QSLT for a generic positive open quantum system can be obtained. We apply these results to investigate the QSLT for a two-qubit system under non-Markovian dynamics. Moreover, the QSLT in the presence of leakage and decoherence is studied.[13] Additionally, the QSL was introduced recently as a measure of asymmetry in quantum systems.[14] QSLT is an important concept in quantum systems with which the speedup and slow down of a quantum system can be investigated. Therefore, it is also an important concept in the quantum control field. In addition to the above-mentioned studies, the generalized geometric QSLT was introduced.[15] For a rigorous study about the quantum speed limits and its future direction, one would refer to Ref. [16]. There are theoretical investigations which generalize the QSLT to mixed states[17] and sys-

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tems without rotating-wave approximation.[18] Also, by introducing an alternative fidelity for quantum states, an improved bound for QSLT in open quantum system was obtained.[19] In addition to theoretical investigations, to obtain a bound for the QSLT, the notion of QSLT for specific physical systems has attracted a great deal of attention. As an example, the QSLT for a multiqubit open system was studied in Ref. [20]. The QSLT of a central spin coupled to a spin star configuration as the environment, a typical non-Markovian system, was investigated in Ref. [21]. For a complex interacting many-body system where the effects of interactions have to be taken into account, the concept of QSLT was studied.[22] Moreover, the QSLT for a coupled system consisting of a central spin and its surrounding environment which is described by a general XY spin-chain model is explored.[23] Furthermore, the QSLT for qubits in a finite XY spin chain which are imposed by an external transverse magnetic field was investigated recently.[24] Moreover, there are a great number of theoretical studies which are devoted to the notion of QSLT.[25–30] The notion of speedup evolution of open quantum systems with non-Markovian dynamics and structured reservoir is studied.[31–33] In addition, also the QSLT in qubit systems is studied.[34,35]

One of the most remarkable characteristics of composite quantum systems is quantum entanglement, which is the essence of quantum theory.[36] Quantum entanglement is an essential ingredient of many quantum information protocols, such as quantum teleportation[37] and quantum cryptography.[38] Moreover, the quantum entanglement can be employed for the high-precision spectroscopy[39] and quantum simulation.[40] According to the vast application of the quantum entanglement in quantum technologies, the generation, control and preservation of entanglement are important issues. Therefore, the introduction and investigation of systems which provide long lasting and controllable quantum entanglement become the backbone of quantum technologies. There is a close relation between entanglement and quantum evolution time. This issue concerns quantum systems composed of two or more subsystems. It is shown that in closed systems with unitary evolution, entanglement is a resource that allows for a speed-up quantum evolution.[41–44]

Quantum mechanics imposes a fundamental limit on the processing speed of each quantum device. Therefore, investigating the QSLT and the parameters which affect its behavior will determine the functionality of the quantum systems. The QSLT has been studied in some physical systems.[45,46] The simplest quantum system for computation proceeds by a succession of two-qubit quantum gates.[47] One of the famous two-qubit quantum gates is a system composed of two coupled quantum dots (QDs). In this system the spin of the excess electron in each QD is realized as the qubit.[48–52] There are several experimental realization of quantum information applications of QDs.[53–55] Because of the relevance of this system, the study of the QSLT in this system provides us valuable information about the operation of this system. This motivates us to investigate the QSLT and the physical parameters, which possess crucial role on the behavior of the QSLT and entanglement in this system. Therefore, in the present contribution, we investigate a coupled QDs system in an external magnetic field. We also study the influences of an external magnetic field and temperature on the behavior of QSLT, based on the introduced approach in Ref. [12]. The obtained results provide some information about the maximal speed of the evolution of this quantum system. In this study, the spin relaxation is taken into account. Since the spin relaxation in QDs is a non-Markovian process, we study the QSLT in the context of the non-Markovian open quantum systems. Moreover, in the introduced two-qubit system, we study the quantum entanglement and its dependence on the physical parameters of the system. This study provides us to draw a comparison between the evolution of the QSLT and entanglement. In the present contribution, we select the initial state of qubits as a maximally entangled state.

The remained part of the present contribution is organized as follows: In section 2, a brief introduction about the QSLT in non-Markovian processes is presented. Section 3 describes the physical system and its dynamics. In that section, the QSLT is obtained and the influence of the effective parameters on its behavior are studied. Moreover, the time evolution of entanglement and its dependence on the physical parameters in the system are studied in Section 3. Section 4 is devoted to summary and conclusion.

2. Physical system

In the following we develop a scenario to calculate the QSLT and quantum information processing in the coupled-QDs system.[48,49] In this model, the qubit is realized as the spin of the excess electron in a single-electron QD, Fig. 1. In a chain consisting of such QDs, the tunneling barrier between neighboring single-electron QDs provides our physical system. If the barrier potential is high, the tunneling between the QDs is forbidden. If this barrier is selected to a low voltage, the electron would tunnel to the adjacent QD. According to the usual physical content of Hubbard model,[56] the spins would be subjected to a transient Heisenberg coupling,

$$H_{\rm s} = J\boldsymbol{S}_1 \cdot \boldsymbol{S}_2, \tag{1}$$

where J is the exchange constant, which can be assumed as a time-dependent quantity as a consequence of the turning off and on of the tunneling matrix element. The $\boldsymbol{S}_i$ $(i = 1,2)$ are the spin-$1/2$ operators for the i-th dot. In addition to the above interaction, we suppose that an external constant

magnetic field, $\boldsymbol{B}$, is imposed to control the electron spin of each dot. This interaction has the form $H_{\rm B} = \hbar\omega \boldsymbol{B}\cdot\boldsymbol{S}$, where $\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$ is the total spin operator of the coupled-QDs and ω refers to the electron Bohr magneton. We assume that the constant magnetic field is aligned in the z-direction, therefore $H_{\rm B} = \hbar\omega B(S_{1z} + S_{2z})$. Accordingly, the Hamiltonian which governs the unitary operation of the system is given as $H = H_{\rm s} + H_{\rm B}$.

The concept of the isolated quantum system is an ideal concept. Every quantum system is immersed in an environment (its surrounding medium). Since the quantum systems are fragile and weak, the surrounding environment has crucial impacts on their dynamics. For the charged QDs, the dissipation and dephasing caused by coupling to the local nuclear spins through the hyperfine interaction. In order to study this magnetic environment influences through the master equation, one would use a Caldeira–Leggett type model. In this model, the magnetic environment is modeled as a set of harmonic oscillators, $H_{\rm en} = \sum_\xi \hbar\omega_\xi a_\xi^\dagger a_\xi$.[50] The interaction between the environment and the spin system is assumed as a linear interaction, $H_{\rm int} = \hbar\lambda\sum_{i=1,2}\boldsymbol{S}_i\cdot\boldsymbol{b}_i$. In this equation $b_i^j = \sum_\xi g_\xi^{ij}(a_{\xi i}^j + a_{\xi i}^{j\dagger})$ is a fluctuating quantum field. The quantity g_ξ^{ij} is the coupling constant between the magnetic environment and the spin system which is characterized through the spectral density. The operator $a_{\xi i}^j$ is the bosonic annihilation operator of mode ξ which interacts with the jth spin direction in the ith QD. We suppose that as a consequence of the system-environment interaction, the system experiences a non-Markovian dynamics.[57]

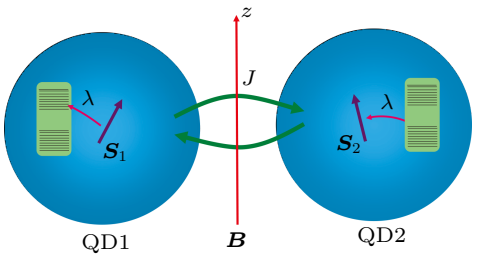


Fig. 1. A schematic representation of the model system. Two coupled QDs in an external magnetic field in which their electron spins experience relaxation dynamics as a consequence of interaction with a magnetic environment.

In the present contribution, our purpose is to investigate the QSLT for a two-qubit system. The QSL time relates how distant the initial and final quantum states can evolve to each other with the time necessary for such a distance to be achieved.[1,2] This means that the QSLT determines a bound on the time needed for a state to reach a fidelity relative to the initial state.

To calculate the QSLT, we have to obtain the density matrix of the system governed by the following time convolutionless master equation[9]

$$\frac{{\rm d}\rho(t)}{{\rm d}t} = -\frac{\rm i}{\hbar}[H,\rho(t)] - \frac{1}{\hbar^2}\int_0^t {\rm d}s\,{\rm Tr}_{\rm B}[H_{\rm I}(t),[H_{\rm I}(s),\rho(t)\otimes\rho_{\rm B}]]. \quad (2)$$

In this equation, ${\rm Tr}_{\rm B}$ refers to the partial trace over the environment degrees of freedom, and $\rho_{\rm B}$ is the environment state. $H_{\rm I}(t)$ is the interaction Hamiltonian between the system and environment, $H_{\rm int}$, in the interaction picture with respect to the environment free Hamiltonian. It is worth noting that the time convolutionless master equation has been used to investigate the electron-phonon interaction[58] and spin-boson model under non-Markovian dynamics.[59] Inserting the $H_{\rm I}(t)$ in Eq. (2), the following integro-differential master equation under the second order Born approximation governs the dynamics of the system,

$$\frac{{\rm d}\rho}{{\rm d}t} = \frac{\rm i}{\hbar}\left[\rho, J\boldsymbol{S}_1\cdot\boldsymbol{S}_2 + \frac{\hbar B}{2}(S_{1z}+S_{2z})\right] + \mathfrak{L}\rho. \quad (3)$$

In this equation, the first term originates from the unitary evolution of the system whereas the second term corresponds to the relaxation dynamics. The relaxation term originates from the second term on the right-hand side of Eq. (2). By inserting the interaction Hamiltonian between the system and environment in the interaction picture into Eq. (2), the second term on the right-hand side, the following result is obtained:

$$\begin{aligned}\mathfrak{L}\rho \;=\; & -\lambda^2\sum_{i,j}[S_{ij}S_{ij}\rho\,K_+(t) - 2S_{ij}\rho S_{ij}\,K(t) \\ & + \rho S_{ij}S_{ij}\,K_+^*(t)]. \end{aligned} \quad (4)$$

In this equation, $K_+(t)$ and $K(t)$ are environment correlation functions defined as

$$\begin{aligned} K_+(t) \;&=\; \sum_\xi {g_\xi}^2\left[\coth\left(\frac{\hbar\omega_\xi}{2k_{\rm B}T}\right)\frac{\sin(\omega_\xi t)}{\omega_\xi} + {\rm i}\frac{\cos(\omega_\xi t)}{\omega_\xi}\right], \\ K(t) \;&=\; \sum_\xi {g_\xi}^2\coth\left(\frac{\hbar\omega_\xi}{2k_{\rm B}T}\right)\frac{\sin(\omega_\xi t)}{\omega_\xi}. \end{aligned} \quad (5)$$

It should be noted that we have assumed a homogeneous environment which has similar features in each quantum dot. By defining the spectral density of environment as $j(\omega) = \pi\lambda^2\sum_\xi {g_\xi^{ij}}^2\delta(\omega-\omega_\xi^{ij})$, the environment correlation functions can be written as

$$\begin{aligned} K_+(t) &= \int_0^\infty {\rm d}\omega\frac{j(\omega)}{\omega^2}\left[\coth\left(\frac{\hbar\omega}{2k_{\rm B}T}\right)\sin(\omega t) + {\rm i}\cos(\omega t)\right], \\ K(t) &= \int_0^\infty {\rm d}\omega\frac{j(\omega)}{\omega^2}\coth\left(\frac{\hbar\omega}{2k_{\rm B}T}\right)\sin(\omega t). \end{aligned} \quad (6)$$

These correlation functions describe all associated effects of the environment. In the above equation, $k_{\rm B}$ is the Boltzmann constant and T refers to the temperature. In addition, $j(\omega)$ is the spectral density of environment which completely describes the interaction between the spins and environment.[60] We suppose a spherical QD, therefore this spectral function may be written as $j(\omega) = \alpha\omega^3{\rm e}^{-\beta\omega^2}$.[66] In our system we have selected them as $\alpha = 0.160830$ ps^2 and $\beta = 1.1452$ ps^2.

3. Quantum speed limit time of the coupled QDs

Our purpose is to calculate the QSLT for an open two-qubit system. The initial and target states are governed by the master equation. The system under investigation is an open quantum system which experiences a non-Markovian evolution. There are different approaches to obtain the QSLT for the open systems.[10–12,67] Each of these approaches leads to the different bounds for the QSL time. Since a unified approach to obtain both the Mandelstam–Tamm and Margolus–Levitin bounds is introduced in Ref. [12], we employ it to obtain the QSLT. The start point of this approach is based on the Bures angle between the initial and final states of the quantum system which is defined as[61]

$$L(\rho_0,\rho_\tau)=\arccos\left(\sqrt{\langle\psi_0|\rho_\tau|\psi_0\rangle}\right). \tag{7}$$

Without loss of generality, we assume that the initial state is pure, $\rho_0=|\psi_0\rangle\langle\psi_0|$. The dynamical velocity which governs the evolution of the density matrix determines the QSL time. This velocity is given by the time derivative of the Bures angle

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}L(\rho_0,\rho_t)&\leq\left|\frac{\mathrm{d}}{\mathrm{d}t}L(\rho_0,\rho_t)\right|\\&=\frac{1}{\sqrt{1-\langle\psi_0|\rho_\tau|\psi_0\rangle}}\frac{|\langle\psi_0|\dot{\rho}_t|\psi_0\rangle|}{2\sqrt{\langle\psi_0|\rho_\tau|\psi_0\rangle}}.\end{aligned} \tag{8}$$

Applying the definition (7), this relation can be written as

$$2\dot{L}\cos L\sin L\leq|\langle\psi_0|\dot{\rho}_t|\psi_0\rangle|. \tag{9}$$

The time evolution of the open quantum systems is determined through the following master equation, in the present system Eq. (3),

$$\dot{\rho}_t=\mathfrak{L}_t\rho_t, \tag{10}$$

in which $\mathfrak{L}$'s are positive generators of the dynamical semigroup $\Lambda_t=\exp(\mathfrak{L}_t t)$. Applying the introduced approach of Ref. [12] the QSLT is obtained as

$$\tau_{\mathrm{QSL}}=\max\{\tau_1,\tau_2,\tau_\infty\}. \tag{11}$$

Here, the following notation is used:[45]

$$\tau_p=\frac{1}{\Gamma_\tau^p}\sin^2[L(\rho_0,\rho_\tau)],\quad(p=1,2,\ldots,\infty) \tag{12}$$

where

$$\Gamma_\tau^p=\frac{1}{\tau}\int_0^\tau \mathrm{d}t||\mathfrak{L}_t\rho_t||_p,\quad||\mathfrak{L}_t\rho_t||_p=\left(\sum_i\sigma_i^p\right)^{1/p}.$$

The last equation denotes the p-norm of the operator $\dot{\rho}_t$ with singular values σ_i^p. It is evident that in the case of $p=1$ this definition leads to the trace norm and if $p=2$ the definition of Hilbert-Schmidt norm will be recovered.[62,63] It is worthwhile to note that the above results correspond to the initial pure state. Moreover, the QSLT for arbitrary initial states is studied.[67]

The studied system in the present contribution, the two coupled qubits, is the basic system to prepare the entangled states. Therefore, the time evolution of entanglement in these systems is an important feature of interest. Thus we study the QSLT and entanglement dynamics. This investigation provides a comparison between the entanglement evolution and the time behavior of QSLT. In order to quantify the entanglement we should utilize an appropriate measure.[64] For the two-qubit systems concurrence is a suitable measure to quantify the entanglement.[65] To proceed further to define the concurrence, suppose that the state of the whole system (two-qubit system) is given as ρ. The concurrence is defined as $C(\rho)=\max\{0,\lambda_1-\lambda_2-\lambda_3-\lambda_4\}$, where λ_i's are eigenvalues of a matrix R which is defined as $R=\sqrt{\sqrt{\rho}\widetilde{\rho}\sqrt{\rho}}$, where $\widetilde{\rho}=(\sigma_y\otimes\sigma_y)\rho^*(\sigma_y\otimes\sigma_y)$. In this relation, σ_y is the y component of the Pauli matrices and ρ^* refers to the complex conjugate of the system density matrix, ρ. In the following we are going to investigate the influences of physical parameters on the QSLT and entanglement in the system under consideration.

Following the system dynamics, we can obtain the interested physical quantities. Comparing the $\mathfrak{L}_t$ given in Eq. (10) with Eq. (3) we can calculate the QSLT in the present system. In this model we can study the associated effects of the initial state, the external magnetic field, the temperature and the coupling strength between the QDs. It is worthwhile to note that the above-mentioned approach to study the QSLT is based on the initial pure state. Therefore, we choose the initial state of the system as $|\psi_0\rangle=\frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle+|\downarrow\downarrow\rangle)$. According to the concept of QSLT, at times shorter than the τ_{QSL}, the target state cannot be fully reached and the high speed operation will lead to an incomplete fidelity. Since the initial state is selected as a Bell state, maximally entangled state, our aim is controlling this entanglement via the introduced decoherence process. That is, our purpose is preserving the initial state. In other words, we are going to study the robustness of initial entangled state against the noisy dynamics.

The corresponding QSLT and entanglement evolutions are plotted in Fig. 2. Figure 2(a) illustrates the QSLT evolution for different magnetic fields. It shows that the smaller the magnetic field, the larger the QSLT. That is, decrement in magnetic field leads to larger minimum time which is needed to transfer to a desired state. Figure 2(b) corresponds to the entanglement time evolution. It illustrates the disentanglement in this system. Moreover, the disentanglement does not affected by magnetic field. The time evolution of concurrence exhibits the entanglement degradation to zero. That is, via the noisy dynamics, the initial entangled state is changed to an unentangled state.

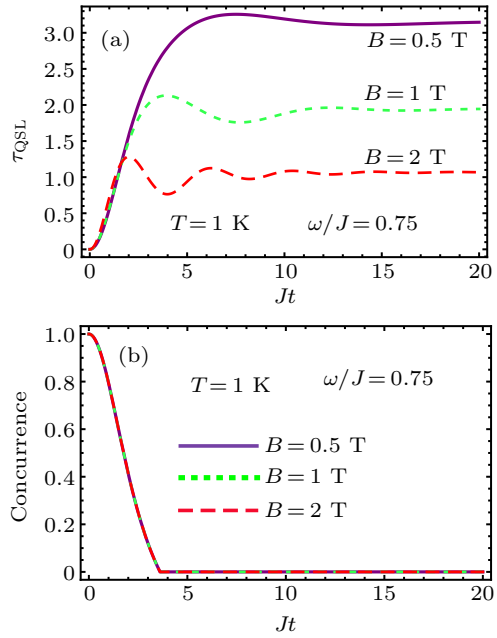


Fig. 2. The time evolution of the QSLT (a) and entanglement (b). Different plots in each panel correspond to the given magnetic field. The temperature is selected as $T=1$ K, and $\omega/J=0.75$.

The magnetic field dependence of the QSLT and entanglement is shown in Fig. 3. Figure 3(a) illustrates the influence of the magnetic field on the QSLT. The magnetic field increment leads to the QSLT decrement. That is, by increasing the magnetic field, one can speedup the evolution of present quantum system. Also, this panel exhibits the temperature increasing leads to the QSLT decrement. The influence of temperature is investigated in the next figure. An important feature of Fig. 3 is insensitivity of entanglement with respect to the magnetic field. Figure 3(b) demonstrates that in the studied system the magnetic field does not affect the entanglement. It is worth noting that this result corresponds to the introduced system with the specific selected initial state.

In order to obtain more insights about the system, the influences of temperature on the desired physical quantities in the present system are studied through Fig. 4. Figure 4(a) shows the temperature dependence of the QSLT. For small values of the magnetic field, the temperature increment leads to the increment of the QSLT. That is, by increasing the temperature, the system will never reach the desired state. The contribution of the temperature originates from the magnetic environment. In the present model, the quantum system experiences a decoherence phenomenon, as a consequence of environment interaction. If the decoherence time is smaller than the QSLT then the system will never reach the desired state. Therefore, the quantumness nature of the system, usually used in the quantum information and computation, will wash out. Thus, in order to provide an appropriate state transformation, the QSLT should be smaller than the decoherence time. At elevated temperatures the decoherence impacts on the quantum systems will increase. Accordingly, in high temperature regime the decoherence will affect in shorter time scales. As a consequence of this fact, if the main purpose is to obtain a full quantum computation, the quantum system should be manipulated in shorter time scales than the decoherence time. Moreover, by increasing the magnetic field the effects of the temperature can be compensated. Drawing an attention to Fig. 4(a) shows the increasing rate of the QSLT, as a consequence of temperature increasing, can be rectified by the magnetic field increment. Therefore, in the present system, the influence of the environment will be compensated with the magnetic field.

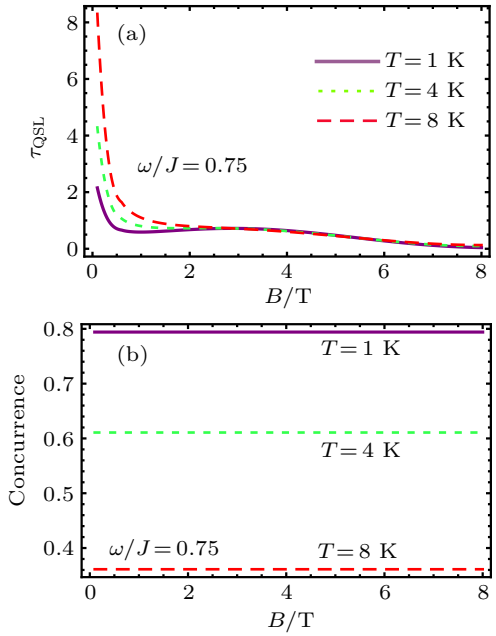


Fig. 3. The magnetic field dependence of the QSLT (a) and entanglement (b). In each panel different plots correspond to different temperatures. In all panels we have chosen $\omega/J=0.75$ and also $Jt=1$.

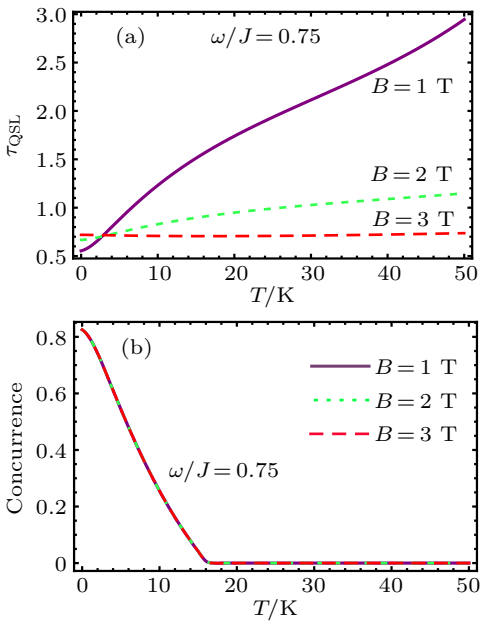


Fig. 4. The temperature dependence of the QSLT and entanglement. In each panel different plots correspond to different magnetic fields. In all panels the quantity ω/J is selected equal to 0.75 and also $Jt=1$.

As an important feature of the present contribution, the

temperature dependence of the entanglement is illustrated in Fig. 4(b). Quantum entanglement as a kind of quantum correlation is sensitive to the decoherence phenomena. Quantum entanglement is a shallow correlation which will wash out in interaction with the environment. The influences of the environment on the quantum entanglement would be followed through the temperature dependence. Figure 4(b) shows that the temperature increasing leads to the disentanglement. It is worth noting that the magnetic field increasing does not contribute any role in the entanglement degradation. This result is in agreement with the obtained results from Figure 3(a).

4. Summary and conclusion

A QD-based system which is appropriate for quantum state transfer and computation is studied and the QSLT and entanglement are investigated in the present contribution. The physical system is the electron spin in the separated coupled QDs which are imposed by an external magnetic field. The relaxation dynamics as a consequence of the hyperfine interaction between the electron spin and local nuclear spins is modeled through the magnetic environment. We have supposed that the system experiences a non-Markovian dynamics and we have derived the QSLT for non-Markovian dynamics. Applying the obtained results to the coupled QDs system provides some information about the quantum speed evolution of this system.

QSLT bounds the speed of the quantum evolution. The influences of some physical parameters such as the external magnetic field and temperature are investigated for selected initial conditions. It is shown that the stronger the magnetic field, the smaller the QSLT. Therefore, the increment of the magnetic field will lead to the faster quantum state transfer. Moreover, it is shown that the temperature increasing will cause the increment of QSLT. At elevated temperatures the quantum features of the system will disappear. Thus, as a consequence of decoherence, in the high temperature limit the QSLT is increased monotonically. In order to operate a quantum system in high temperature limit, it should be manipulated in shorter time scales than the decoherence time. On the other hand, the obtained results reveal that the influences of the high temperature can be compensated for by the magnetic field increment.

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