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Localized characteristics of lump and interaction solutions to two extended Jimbo–Miwa equations*

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We focus on the localized characteristics of lump and interaction solutions to two extended Jimbo–Miwa equations. Based on the Hirota bilinear method and the test function method, we construct the exact solutions to the extended equations including lump solutions, lump–kink solutions, and two other types of interaction solutions, by solving the underdetermined nonlinear system of algebraic equations for associated parameters. Finally, analysis and graphical simulation are presented to show the dynamical characteristics of our solutions and the interaction behaviors are revealed.

Keywords: Hirota bilinear method, test function method, lump solution, interaction solution, symbolic computation

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1. Introduction

Soliton has the characteristics of maintaining its speed and waveform during propagation, which makes it a hot topic of current researches.[1–5] Soliton is widely studied and applied in many areas, such as fluid mechanic, nonlinear optics, and traffic flow theory.[6–13] Many nonlinear evolution equations (NLEEs) are presented and studied associated with solitons and integrable systems, which are closely linked with mathematical physics method and variety of mechanical phenomena.[14–20] It is meaningful to obtain the exact solutions to nonlinear evolution equations. Lump solution defined differently from soliton solutions is a type of rational solution,[21,22] which are localized in all directions in space.[23,24] In 2015, lump solutions have been constructed to the Kadomtsev–Petviashvili (KP) equation via substituting the positive quadratic function to bilinear equation.[25] In 2006, lump solutions were studied with the variable separation method.[26] Moreover, lump or multi-lump solutions to the Boussinesq equation,[27] the KPI equation,[25] the BKP equation,[28] and the potential-YTSF equation[29] have been deduced.[30]

The well-known (3+1)-dimensional Jimbo–Miwa (JM) equation

$$u_{xxxy}+3\left(u_x u_y\right)_x+2u_{yt}-3u_{xz}=0 \tag{1}$$

is firstly introduced by Jimbo and Miwa as the second member of the entire KP hierarchy in 1983.[31] Nevertheless, the (3+1)-dimensional JM equation does not pass the Painlevé test of integrability, which makes it different from the KP equation. Based on the (3+1)-dimensional JM equation, various methods have been used to deduce the exact solutions such as multi-soliton solutions, rational solutions, and determinant solutions.[32–34] Analysis on these exact solutions plays an important role in the study on the (3+1)-dimensional JM equation.[35,36]

In the present paper, we will study two extended (3+1)-dimensional JM equations as given below:

$$u_{xxxy}+3u_y u_{xx}+3u_x u_{xy}+2u_{yt}-3(u_{xz}+u_{yz}+u_{zz})=0, \tag{2}$$

$$u_{xxxy}+3u_y u_{xx}+3u_x u_{xy}+2(u_{xt}+u_{yt}+u_{zt})-3u_{xz}=0, \tag{3}$$

which are introduced by Wazwaz, and the last two linear terms u_{xz} and u_{yt} of Eq. (1) are extended to $u_{xz}+u_{yz}+u_{zz}$ and $u_{xt}+u_{yt}+u_{zt}$, respectively.[2] Through the following transformation:

$$u=2[\ln f(x,y,z,t)]_x, \tag{4}$$

equations (2) and (3) can be cast into the bilinear form, respectively, as follows:

$$(D_x^3D_y+2D_yD_t-3D_yD_z-3D_xD_z-3D_z^2)f\cdot f=0, \tag{5}$$

$$(D_x^3D_y+2D_xD_t+2D_yD_t+2D_zD_t-3D_xD_z)f\cdot f=0, \tag{6}$$

where D_x, D_y, D_z, D_t, D_x^3, and D_z^2 are bilinear operators[11] defined by the following equation:

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$$D_x^{\alpha}D_y^{\beta}D_z^{\gamma}D_t^{\delta}(f\cdot g)=\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^{\alpha}\left(\frac{\partial}{\partial y}-\frac{\partial}{\partial y'}\right)^{\beta}\left(\frac{\partial}{\partial z}-\frac{\partial}{\partial z'}\right)^{\gamma}\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^{\delta}f(x,y,z,t)g(x',y',z',t')\Big|_{\substack{x'=x,y'=y,\\ z'=z,t'=t}}. \tag{7}$$

The structure of this paper is as follows: In Section 2, we will solve Eq. (5) and obtain the lump and lump–kink solutions to the first extended (3+1)-dimensional JM equation. In Section 3, equation (6) will be solved to deduce interaction solutions to the second extended (3+1)-dimensional JM equation. In Section 4, we will give analysis and discussion on our solutions. Figures describing the characteristics of solutions and the interaction behaviors will also be presented. In Section 5, we will conclude our results.

2. Lump and lump–kink solutions to the first extended equation

2.1. Lump solutions

According to the structure of lump solutions, we substitute the following form of the function f into Eq. (5),

$$f=g^2+h^2+a_{11}, \tag{8}$$

where $g=a_1x+a_2y+a_3z+a_4t+a_5$, $h=a_6x+a_7y+a_8z+a_9t+a_{10}$, $a_{11}>0$, and a_i $(1\le i\le 11)$ are some constants to be determined.

With symbolic computation, we obtain three sets of the parameters a_i $(1\le i\le 11)$.

Case 1

$$\left\{a_1=-\frac{a_6a_7}{a_2},a_4=\frac{(3a_2^2+3a_2a_3-3a_6a_7)a_3}{2a_2^2},a_8=\frac{a_3a_7}{a_2},a_9=\frac{3a_3(a_2a_6+a_2a_7+a_3a_7)}{2a_2^2}\right\}, \tag{9}$$

where a_2, a_3, a_5, a_6, a_7, a_{10}, a_{11} are real constants, and $a_2a_6\neq 0$.

Parameters in Eq. (9) give us the exact form of

$$\begin{aligned}f=&\left[-\frac{a_6a_7}{a_2}x+a_2y+a_3z+\frac{(3a_2^2+3a_2a_3-3a_6a_7)a_3}{2a_2^2}t+a_5\right]^2\\&+\left[a_6x+a_7y+\frac{a_3a_7}{a_2}z+\frac{3a_3(a_2a_6+a_2a_7+a_3a_7)}{2a_2^2}t+a_{10}\right]^2+a_{11},\end{aligned}$$

and the lump solution to the first extended equation as

$$u=\frac{4[(a_6a_7/a_2)A_1+a_6B_1]}{A_1^2+B_1^2+a_{11}}, \tag{10}$$

where

$$A_1=\frac{a_6a_7}{a_2}x-a_2y-a_3z-\frac{(3a_2^2+3a_2a_3-3a_6a_7)a_3}{2a_2^2}t-a_5,\quad B_1=a_6x+a_7y+\frac{a_3a_7}{a_2}z+\frac{3a_3(a_2a_6+a_2a_7+a_3a_7)}{2a_2^2}t+a_{10}.$$

Case 2

$$\left\{a_1=-a_3,a_2=0,a_4=\frac{3a_3(a_7+a_8)}{2a_7},a_6=0,a_9=\frac{3a_8(a_7+a_8)}{2a_7}\right\}, \tag{11}$$

where a_3, a_5, a_7, a_8, a_{10}, a_{11} are real constants, and $a_3a_7\neq 0$.

Parameters of Eq. (11) give us the exact form of

$$f=\left[-a_3x+a_3z+\frac{3a_3(a_7+a_8)}{2a_7}t+a_5\right]^2+\left[a_7y+a_8z+\frac{3a_8(a_7+a_8)}{2a_7}t+a_{10}\right]^2+a_{11},$$

and the lump solutions to the first extended equation as

$$u=\frac{4a_3\{a_3x-a_3z-[3a_3(a_7+a_8)/2a_7]t-a_5\}}{\{-a_3x+a_3z+[3a_3(a_7+a_8)/2a_7]t+a_5\}^2+\{a_7y+a_8z+[3a_8(a_7+a_8)/2a_7]t+a_{10}\}^2+a_{11}}. \tag{12}$$

Case 3

$$\left\{a_1=0,a_4=\frac{3(a_2+a_3)a_3}{2a_2},a_6=-a_8,a_7=0,a_9=\frac{3a_8(a_2+a_3)}{2a_2}\right\}, \tag{13}$$

where a_2, a_3, a_5, a_8, a_{10}, a_{11} are real constants, and $a_2a_8\neq 0$.

Parameters of Eq. (13) give us the exact form of

$$f=\left[a_2y+a_3z+\frac{3(a_2+a_3)a_3}{2a_2}t+a_5\right]^2+\left[-a_8x+a_8z+\frac{3a_8(a_2+a_3)}{2a_2}t+a_{10}\right]^2+a_{11},$$

and the lump solution to the first extended equation as

$$u=\frac{4a_8\{a_8x-a_7y-a_8z-[3a_8(a_2+a_3)/2a_2]t-a_{10}\}}{\{a_2y+a_3z+[3(a_2+a_3)a_3/2a_2]t+a_5\}^2+\{-a_8x+a_8z+[3a_8(a_2+a_3)/2a_2]t+a_{10}\}^2+a_{11}}. \tag{14}$$

Based on the expression of u in Eq. (14), conditions $\partial u/\partial x=0$, $\partial u/\partial y=0$, and $\partial u/\partial z=0$ leads to the extremum points. When $y=0$, there are two sets of extremum points

$$\left(x_1=-\frac{a_5}{a_3}+\frac{a_{10}+\sqrt{a_{11}}}{a_8},\ z_1=-\frac{3(a_2+a_3)}{2a_2}t-\frac{a_5}{a_3}\right),$$
$$\left(x_2=-\frac{a_5}{a_3}+\frac{a_{10}-\sqrt{a_{11}}}{a_8},\ z_2=-\frac{3(a_2+a_3)}{2a_2}t-\frac{a_5}{a_3}\right).$$

2.2. Lump–kink solutions

According to the structure of lump–kink solutions, we substitute the following form of the function f into Eq. (5),

$$f=g^2+h^2+\cosh(\eta)+b_{16}, \tag{15}$$

where $g=b_1x+b_2y+b_3z+b_4t+b_5$, $h=b_6x+b_7y+b_8z+b_9t+b_{10}$, $\eta=b_{11}x+b_{12}y+b_{13}z+b_{14}t+b_{15}$, $b_{16}>0$, and b_i $(1\leq i\leq 16)$ are some constants to be determined.

With symbolic computation, we get three sets of the parameters b_i $(1\leq i\leq 16)$.

Case 1

$$\left\{b_1=-b_3,b_2=0,b_4=\frac{3b_3(b_7+b_8)}{2b_7},b_6=0,b_9=\frac{3b_8(b_7+b_8)}{2b_7},b_{11}=-b_{13},\ b_{12}=0,b_{14}=\frac{b_{13}(b_{13}^2b_7+3b_7+3b_8)}{2b_7}\right\}, \tag{16}$$

where b_3, b_5, b_7, b_8, b_{10}, b_{13}, b_{15}, b_{16} are real constants, and $b_3b_7b_{13}\neq 0$.

Parameters in Eq. (16) give us the exact form of

$$f=\left[-b_3x+b_3z+\frac{3b_3(b_7+b_8)}{2b_7}t+b_5\right]^2+\left[b_7y+b_8z+\frac{3b_8(b_7+b_8)}{2b_7}t+b_{10}\right]^2$$
$$+\cosh\left(-b_{13}x+b_{13}z+\frac{b_{13}(b_{13}^2b_7+3b_7+3b_8)}{2b_7}t+b_{15}\right)+b_{16},$$

and the lump–kink solution to the first extended equation as

$$u=\frac{2(2b_3A_2-b_{13}\sinh(C_2))}{A_2^2+B_2^2+\cosh(C_2)+b_{16}}, \tag{17}$$

where

$$A_2=b_3x-b_3z-\frac{3b_3(b_7+b_8)}{2b_7}t-b_5,\ B_2=b_7y+b_8z+\frac{3b_8(b_7+b_8)}{2b_7}t+b_{10},$$
$$C_2=-b_{13}x+b_{13}z+\frac{b_{13}(b_{13}^2b_7+3b_7+3b_8)}{2b_7}t+b_{15}.$$

When $t\to\pm\infty$, we have $\lim_{x\to\pm\infty}u=-2b_{13}$.

Case 2

$$\left\{b_1=-\frac{b_6b_7}{b_2},\ b_4=\frac{3b_3(b_2^2+b_2b_3-b_6b_7)}{2b_2^2},\ b_8=\frac{b_3b_7}{b_2},\ b_9=\frac{3b_3(b_2b_6+b_2b_7+b_3b_7)}{2b_2^2},\ b_{11}=0,\right.$$

$$b_{13}=\frac{b_3b_{12}}{b_2},\ \ b_{14}=\frac{3b_3b_{12}(b_2+b_3)}{2b_2^2}\Bigg\}, \tag{18}$$

where b_2, b_3, b_5, b_6, b_7, b_{10}, b_{12}, b_{15}, b_{16} are real constants, and $b_2b_6b_{12}\neq 0$.

Parameters in Eq. (18) give us the exact form of

$$f=\left[-\frac{b_6b_7}{b_2}x+b_2y+b_3z+\frac{3b_3(b_2^2+b_2b_3-b_6b_7)}{2b_2^2}t+b_5\right]^2+\left[b_6x+b_7y+\frac{b_3b_7}{b_2}z+\frac{3b_3(b_2b_6+b_2b_7+b_3b_7)}{2b_2^2}t+b_{10}\right]^2$$
$$+\cosh\left(b_{12}y+\frac{b_3b_{12}}{b_2}z+\frac{3b_3b_{12}(b_2+b_3)}{2b_2^2}t+b_{15}\right)+b_{16},$$

and the lump–kink solution to the first extended equation as

$$u=\frac{4[(b_6b_7/b_2)A_3+b_6B_3]}{A_3^2+B_3^2+\cosh(C_3)+b_{16}}, \tag{19}$$

where

$$A_3=\frac{b_6b_7}{b_2}x-b_2y-b_3z-\frac{3b_3(b_2^2+b_2b_3-b_6b_7)}{2b_2^2}t-b_5,\ \ B_3=b_6x+b_7y+\frac{b_3b_7}{b_2}z+\frac{3b_3(b_2b_6+b_2b_7+b_3b_7)}{2b_2^2}t+b_{10},$$
$$C_3=b_{12}y+\frac{b_3b_{12}}{b_2}z+\frac{3b_3b_{12}(b_2+b_3)}{2b_2^2}t+b_{15}.$$

When $t\to\pm\infty$, we have $\lim_{x\to\pm\infty}u=0$.

Case 3

$$\left\{b_1=0,\ b_4=\frac{3b_3(b_2+b_3)}{2b_2},\ b_6=-b_8,\ b_7=0,\ b_9=\frac{3b_8(b_2+b_3)}{2b_2},\ b_{11}=0,\ b_{13}=\frac{b_3b_{12}}{b_2},\ b_{14}=\frac{3b_3b_{12}(b_2+b_3)}{2b_2^2}\right\}, \tag{20}$$

where b_2, b_3, b_5, b_8, b_{10}, b_{12}, b_{15}, b_{16} are real constants, and $b_2b_8b_{12}\neq 0$.

Parameters in Eq. (20) give us the exact form of

$$f=\left[b_2y+b_3z+\frac{3b_3(b_2+b_3)}{2b_2}t+b_5\right]^2+\left[-b_8x+b_8z+\frac{3b_8(b_2+b_3)}{2b_2}t+b_{10}\right]^2$$
$$+\cosh\left(b_{12}y+\frac{b_3b_{12}}{b_2}z+\frac{3b_3b_{12}(b_2+b_3)}{2b_2^2}t+b_{15}\right)+b_{16},$$

and the lump–kink solution to the first extended equation as

$$u=\frac{4b_8A_4}{A_4^2+B_4^2+\cosh(C_4)+b_{16}}, \tag{21}$$

where

$$A_4=b_2y+b_3z+\frac{3b_3(b_2+b_3)}{2b_2}t+b_5,\ \ B_4=b_8x-b_8z-\frac{3b_8(b_2+b_3)}{2b_2}t-b_{10},\ \ C_4=b_{12}y+\frac{b_3b_{12}}{b_2}z+\frac{3b_3b_{12}(b_2+b_3)}{2b_2^2}t+b_{15}.$$

When $t\to\pm\infty$, we have $\lim_{x\to\pm\infty}u=0$.

3. Interaction solutions to the second extended equation

In this section, we will derive two types of interaction solutions to the second extended equation, *i.e.*, Eq. (3), and employ two test functions in the form of

$$f=\cos(c_1x+c_2y+c_3z+c_4t+c_5)+\cosh(c_6x+c_7y+c_8z+c_9t+c_{10})+c_{11}, \tag{22}$$
$$f=\sin(d_1x+d_2y+d_3z+d_4t+d_5)+\cosh(d_6x+d_7y+d_8z+d_9t+d_{10})+d_{11}, \tag{23}$$

respectively, where c_i and d_i $(1\leq i\leq 11)$ are some constants to be determined, $c_{11}\geq 1$ and $d_{11}\geq 1$.

With symbolic computation, we can get parameters in Eq. (22) as

$$\left\{c_2=0,c_4=\frac{3c_1c_3}{2(c_1+c_3)},c_6=0,c_7=-\frac{3c_1c_8}{c_1^3+c_1^2c_3-3c_3},c_9=0\right\}, \tag{24}$$

where c_1, c_3, c_5, c_8, c_{10}, c_{11} are real constants, $c_1c_8(c_1+c_3)(c_1^3+c_1^2c_3-3c_3)\neq 0$, and the exact form of f is as follows:

$$f=\cos\left(c_1x+c_3z+\frac{3c_1c_3}{2(c_1+c_3)}t+c_5\right)+\cosh\left(-\frac{3c_1c_8}{c_1^3+c_1^2c_3-3c_3}y+c_8z+c_{10}\right)+c_{11},$$

which leads to the interaction solution to Eq. (3) as

$$u=\frac{-2c_1\sin(c_1x+c_3z+(3c_1c_3/2(c_1+c_3))t+c_5)}{\cos(c_1x+c_3z+(3c_1c_3/2(c_1+c_3))t+c_5)+\cosh(-(3c_1c_8/c_1^3+c_1^2c_3-3c_3)y+c_8z+c_{10})+c_{11}}. \tag{25}$$

We can find that u is a periodic function $u(x,y,z,t=t_0)=u(x,y,z,t=t_0+T)$ and the value of the period is determined by the expression $T=(4\pi/3)[(c_1+c_3)/(c_1c_3)]$.

In the same manner, we can derive parameters in Eq. (23) as

$$\left\{d_2=0,d_4=\frac{3d_1d_3}{2(d_1+d_3)},d_6=0,d_7=-\frac{3d_1d_8}{d_1^3+d_1^2d_3-3d_3},d_9=0\right\}, \tag{26}$$

where d_1, d_3, d_5, d_8, d_{10}, d_{11} are real constants, $d_1d_8(d_1+d_3)(d_1^3+d_1^2d_3-3d_3)\neq 0$, and hereby the interaction solution to Eq. (3) as

$$u=\frac{2d_1\cos(d_1x+d_3z+(3d_1d_3/2(d_1+d_3))t+d_5)}{\sin(d_1x+d_3z+(3d_1d_3/2(d_1+d_3))t+d_5)+\cosh(-(3d_1d_8/d_1^3+d_1^2d_3-3d_3)y+d_8z+d_{10})+d_{11}}. \tag{27}$$

We can find that u is a periodic function $u(x,y,z,t=t_0)=u(x,y,z,t=t_0+T)$ and the value of the period is determined by the expression $T=(4\pi/3)[(d_1+d_3)/(d_1d_3)]$.

4. Analysis and discussion

In this section, we will study the dynamic characteristics and interaction behaviors of the exact solutions with graphical simulation. Based on the test function method and symbolic computation, we have solved the under-determined nonlinear systems of algebraic equations and obtained exact solutions to the two extended (3+1)-dimensional JM equation. We have derived three sets of lump solutions, that is, Eqs. (10), (12), and (14) to Eq. (2), three sets of lump–kink solutions to Eq. (2) as seen in Eqs. (17), (19), and (21), and interaction solutions Eqs. (25) and (27) to Eq. (3). The 3-dimensional plots and contour plots of these solutions can be found in Figs. 1–5.

4.1. Features of lump solutions

In Figs. 1–3, the features of the lump solutions are presented. We can see that the lump solutions have two peaks corresponding to the two extremum points of the solution, the waves are localized in all directions in the space and decay in a rational way.

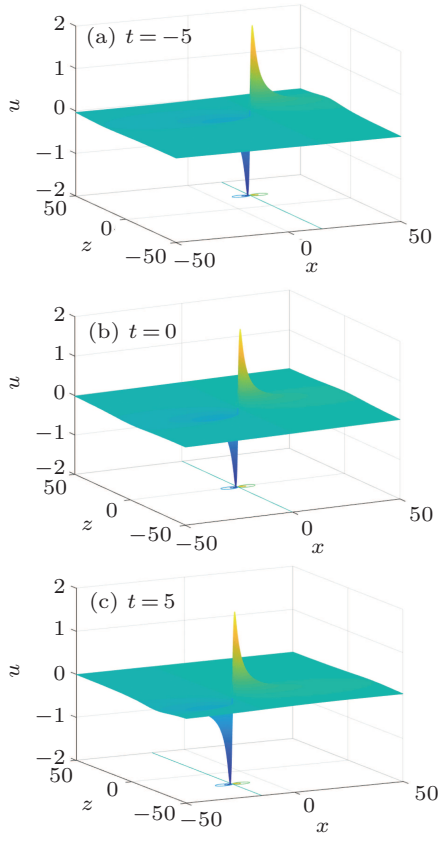


Fig. 1. The three-dimensional plots and contour plots of u via Eq. (10) at the time $t=-5$, $t=0$, and $t=5$ with $y=0$, a_2=1, $a_3=2$, $a_5=1$, $a_6=1$, $a_7=1$, $a_{10}=1$, and $a_{11}=2$.

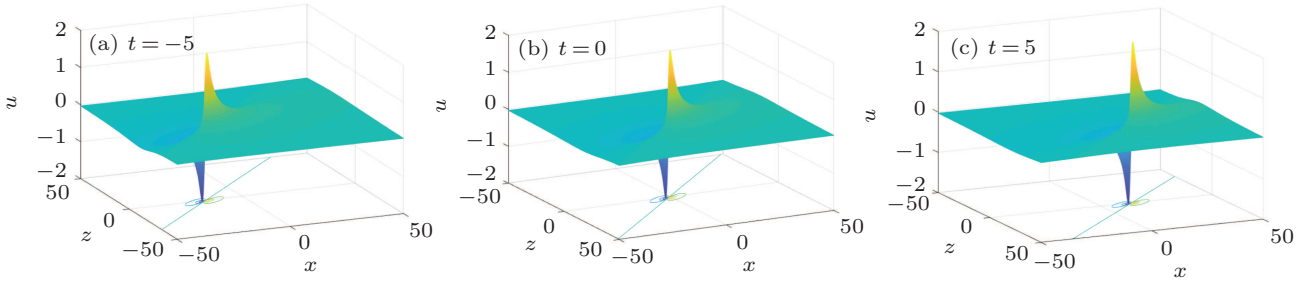


Fig. 2. The three-dimensional plots and contour plots of u via Eq. (12) at the time $t=-5$, $t=0$, and $t=5$ with $y=0$, a_3=-1, $a_5=1$, $a_7=1$, $a_8=1$, $a_{10}=1$, and $a_{11}=1$.

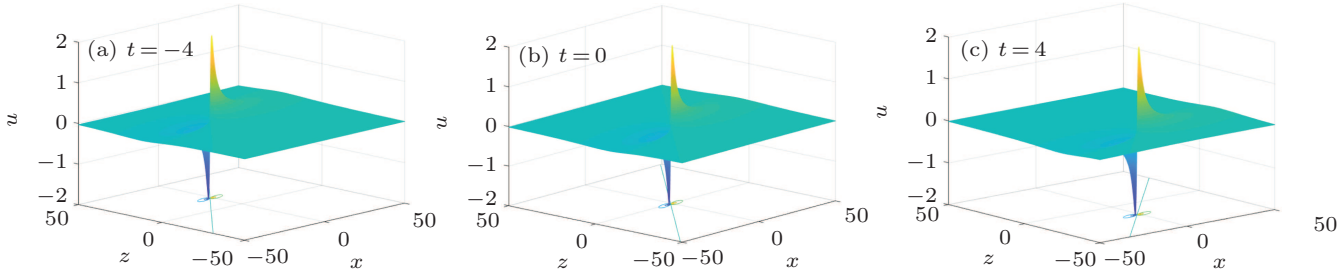


Fig. 3. The three-dimensional plots and contour plots of u via Eq. (14) at the time $t=-4$, $t=0$, and $t=4$ with $y=0$, a_2=1, $a_3=2$, $a_5=1$, $a_8=-1$, $a_{10}=1$, and $a_{11}=1$.

4.2. Interaction behaviors

4.2.1. Lump–kink solutions

Based on the expressions of u given by Eqs. (17), (19), and (21), figures of the lump–kink solutions are obtained. Figure 4 illustrate the interaction phenomena between lump and kink. As can be seen, the wave is consist of two parts including the lump wave and the kink wave. With the increase of t, the lump first appears from one kink, then it begins to move to the other kink and finally the lump disappears again.

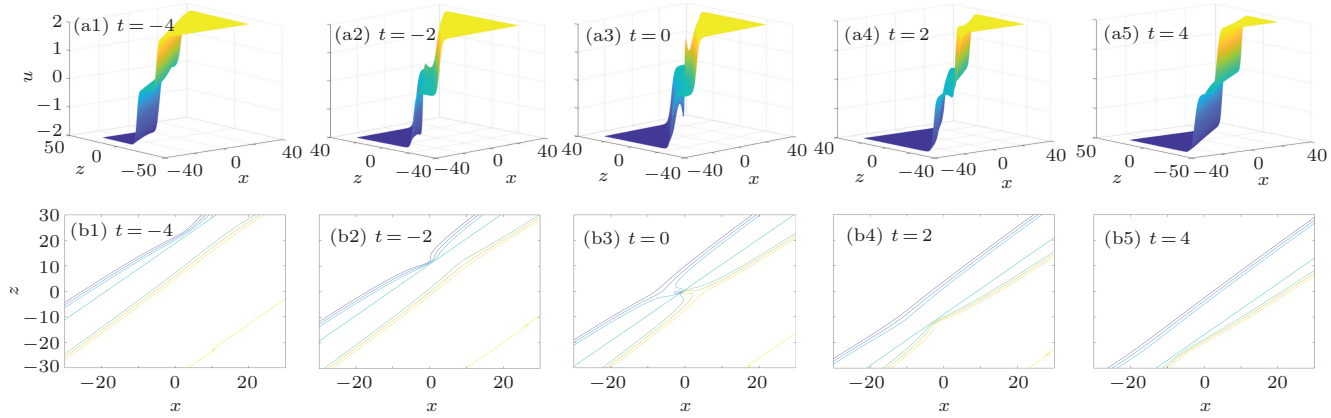


Fig. 4. (a1)–(a5) The three-dimensional plots of u via Eq. (17) at the time $t=-4$, $t=-2$, $t=0$, $t=2$, and $t=4$ with $y=0$, $b_3=-2$, $b_5=1$, $b_7=1$, $b_8=3$, $b_{10}=1$, $b_{13}=-1$, $b_{15}=1$, and $b_{16}=2$; (b1)–(b5) the corresponding contour plots.

4.2.2. Cos–cosh and sin–cosh type solutions

From the three-dimensional plots and contour plots in Figs. 5 and 6, we can clearly find that the waves exist in both sides of the plane $u=0$, they move periodically associated with the valuable t, and the value of the period is 2π. The behaviors of this type waves can be explained as the interaction between the soliton solution and trigonometric function solution. The speed and waveform do not change during the propagation.

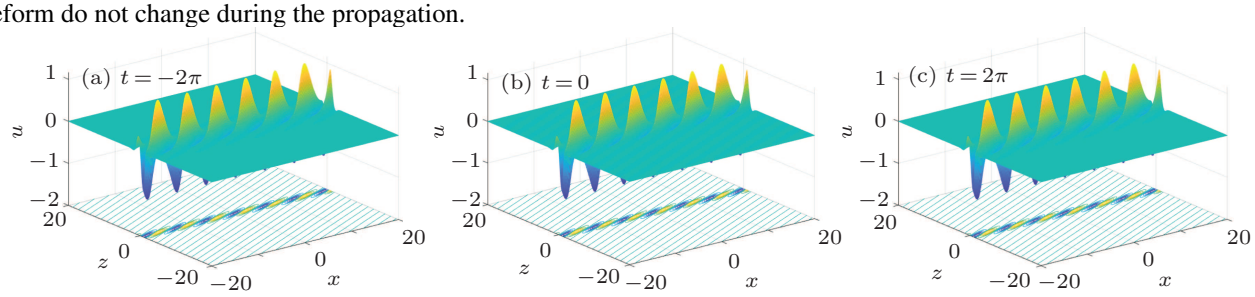


Fig. 5. The three-dimensional plots and contour plots of u via Eq. (25) at the time (a) $t=-2\pi$, (b) $t=0$, and (c) $t=2\pi$ with $y=0$, $c_1=1$, $c_3=2$, $c_5=1$, $c_8=1$, $c_{10}=1$, and $c_{11}=1$.

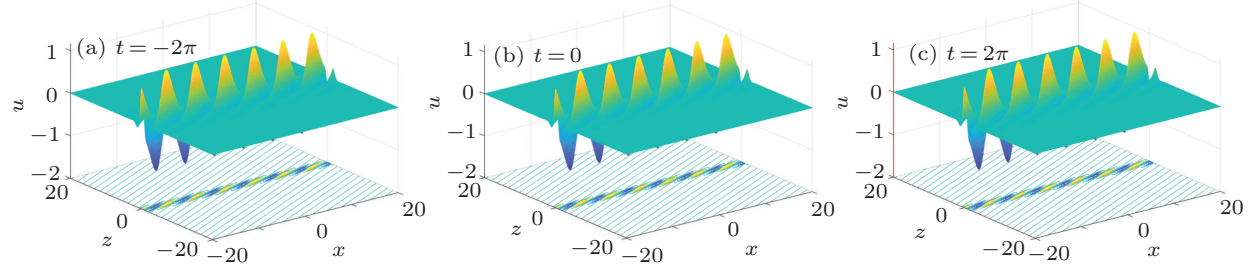


Fig. 6. The three-dimensional plots and contour plots of u via Eq. (27) at the time (a) $t=-2\pi$, (b) $t=0$, and (c) $t=2\pi$ with $y=0$, $d_1=1$, $d_3=2$, $d_5=1$, $d_8=1$, $d_{10}=1$, and $d_{11}=1$.

5. Concluding remarks

Based on the Hirota bilinear representation, the test function method provides a feasible and efficient way to obtain exact solutions to high-dimensional NLEEs. In this paper, we have mainly studied two extended (3+1)-dimensional JM equation. Firstly, we cast the extended equations including Eqs. (2) and (3) into their Hirota bilinear forms by a dependent transformation. With symbolic computation, we have substituted test functions into the bilinear equations to solve the lump solutions, lump–kink solutions, and interaction solutions. Finally, graphical simulation has been presented to reveal dynamical characteristics of lump solutions, and analysis has also been given to show the interaction behavior of lump–kink solutions and two other types of interaction solutions.

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